Lab06

1. Apply Hadamard gate to $|\psi\rangle=\alpha|\mathbf{0}\rangle+\beta|\mathbf{1}\rangle$ and report the result as both Dirac notation and Vector notation.

a) $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

b) $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

2. Mathematically show that the resulted qubit from the previous question follows the probability requirement (i.e., the summation of all probabilities of all measurement results should be 1.),

a) $\alpha^2 + \beta^2 = \frac{1}{\sqrt{2}}^2 + \frac{1}{\sqrt{2}}^2 = \frac{1}{2} + \frac{1}{2} = 1$

b) $\alpha^2 + \beta^2 = \frac{1}{\sqrt{2}}^2 + (-\frac{1}{\sqrt{2}})^2 = \frac{1}{2} + \frac{1}{2} = 1$

3. Mathematically show that the reverse of Hadamard gate is itself. Note that a matrix A is its own reverse if AA = I, where I is the identity matrix.

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{2}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \frac{2}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$