

1. Consider the function  $f_3$  where  $f_3(0)=1$  and  $f_3(1)=0$ .

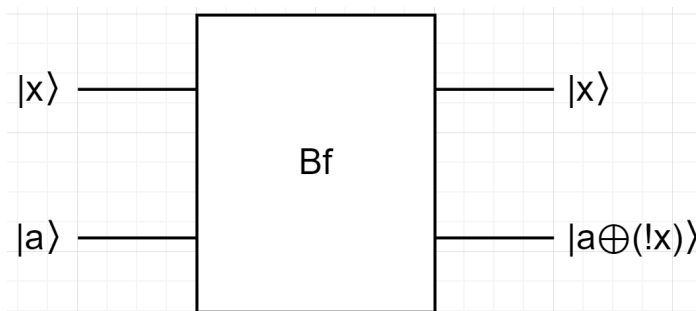
- What would be an oracle associated with this function?
- Build the internal circuit of the oracle. (There is no single way to do this.)
- What is the matrix associated with this quantum operation? (Textbook mentions it, but it is incorrect!)

Answer:

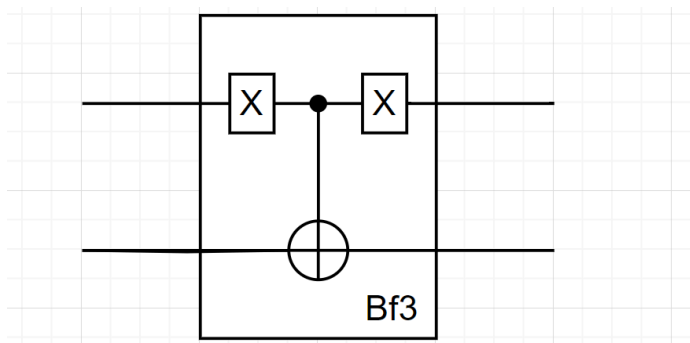
$$f_3(0)=1; \quad f_3(1)=0; \quad f_3(x)=!x$$

$$a \oplus f_3(x) = a \oplus (!x)$$

a)



b)



c)

ax	00	01	10	11
00	0	0	1	0
01	0	1	0	0
10	1	0	0	0
11	0	0	0	1

3. Let's now consider a function that receives two bits as input and outputs a single bit. It works as follows:

$$f(00) = f(01) = 1 \text{ and } f(10) = f(11) = 0.$$

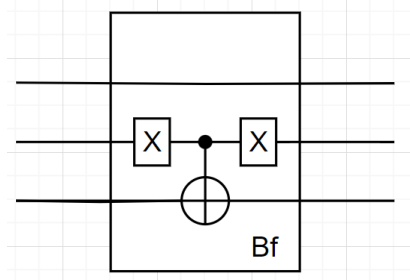
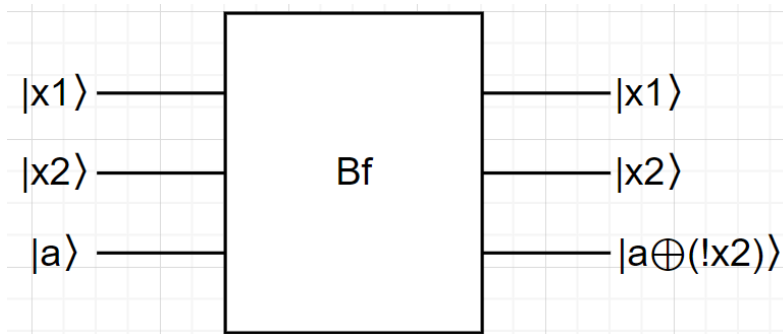
- Is this function balanced, constant, or none?
- What would be an oracle associated with this function?
- What is the matrix associated with this quantum operation? Hint: It should be a  $8 \times 8$  matrix.

Answer:

$$f(00) = f(01) = 1 \quad f(10) = f(11) = 0 \quad f(x_2, x_1) = !x_2$$

a) Balanced.

b)



c)

a x2 x1	000	001	010	011	100	101	110	111
000	0	0	0	0	1	0	0	0
001	0	0	0	0	0	1	0	0
010	0	0	1	0	0	0	0	0
011	0	0	0	1	0	0	0	0
100	1	0	0	0	0	0	0	0
101	0	1	0	0	0		0	0
110	0	0	0	0	0	0	1	0
111	0	0	0	0	0	0	0	1