Lab06

1. Apply Hadamard gate to $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ and report the result as both Dirac notation and Vector notation.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$a)\ H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha+\beta \\ \alpha-\beta \end{bmatrix} = \frac{1}{\sqrt{2}}((\alpha + \beta)|0\rangle + (\alpha - \beta)|1\rangle)$$

$$b)\ H|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)) + \beta(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)) =$$

$$= \frac{\alpha}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{\beta}{\sqrt{2}}(|0\rangle - |1\rangle) =$$

$$= \frac{\alpha}{\sqrt{2}}|0\rangle + \frac{\alpha}{\sqrt{2}}|1\rangle + \frac{\beta}{\sqrt{2}}|0\rangle - \frac{\beta}{\sqrt{2}}|1\rangle =$$

$$= (\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}})|0\rangle + (\frac{\alpha}{\sqrt{2}} - \frac{\beta}{\sqrt{2}})|1\rangle =$$

$$= \frac{1}{\sqrt{2}}((\alpha + \beta)|0\rangle + (\alpha - \beta)|1\rangle)$$

2. Mathematically show that the resulted qubit from the previous question follows the probability requirement (i.e., the summation of all probabilities of all measurement results should be 1.),

$$H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha+\beta \\ \alpha-\beta \end{bmatrix} = \frac{1}{\sqrt{2}}((\alpha + \beta)|0\rangle + (\alpha - \beta)|1\rangle)$$

$$(\frac{1}{\sqrt{2}}(\alpha + \beta))^2 + (\frac{1}{\sqrt{2}}(\alpha - \beta))^2 = \frac{1}{2}(\alpha + \beta)^2 + \frac{1}{2}(\alpha - \beta)^2 =$$

$$= \frac{1}{2}(\alpha + \beta)^2 + \frac{1}{2}(\alpha - \beta)^2 = \frac{1}{2}(\alpha^2 + 2\alpha\beta + \beta^2) + \frac{1}{2}(\alpha^2 - 2\alpha\beta + \beta^2) =$$

$$= \frac{\alpha^2}{2} + \alpha\beta + \frac{\beta^2}{2} + \frac{\alpha^2}{2} - \alpha\beta + \frac{b^2}{2} = \alpha^2 + \beta^2 = 1$$

3. Mathematically show that the reverse of Hadamard gate is itself. Note that a matrix A is its own reverse if AA = I, where I is the identity matrix.

a) $HH = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+1 & 1-1 \\ 1-1 & 1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$