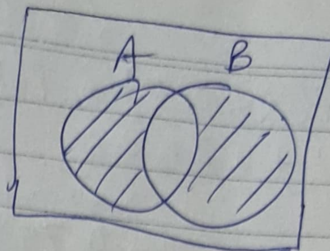
Assign 1

Q1)

$P(A) = 0.3$
$P(B) = 0.4$
$P(A \cap B) = 0.2$



(a) $P(E) = (0.3 - 0.2) + (0.4 - 0.2)$
$= 0.1 + 0.2 = 0.3$

(b) $P(E) = 0.3 + 0.4 - 0.2 = 0.5$

(c) $P(E) = 1 - 0.5 = 0.5$

Q2)

1 2 3
car

$D_i \rightarrow$ event that car is behind door i.
$P(D_i) = 1/3$
$B \rightarrow$ Monty open door 2.
also $P(B|D_2) = 0$
$P(D_1) = P(D_2) = P(D_3) = 1/3$

$$P(D_1|B) = \frac{P(B|D_1)}{P(B|D_1) + P(B|D_3)}$$

$P(B|D_1) = 1/2$ Monty would have opened 2 or 3
$P(B|D_3) = 1$ Monty's only choice was to open door 2.

$\left\{ \begin{array}{l} P(D_1|B) = 1/3 \\ P(D_3|B) = 2/3 \end{array} \right\}$ 2/3 probability to ~~st~~ switch.

Q3)

$P(Red) = \frac{1}{2}\left(\frac{3}{4} \times \frac{2}{5} \times \frac{1}{4}\right)$ or $\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4}$ or $\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4}$ or $\frac{6}{6} \times \frac{5}{5} \times \frac{4}{4}$

$\frac{6}{6} \times \frac{5}{5} \times \frac{2}{4}$

$$= \frac{1}{\frac{1}{20} + \frac{1}{5} + \frac{1}{2} + 1} = \frac{1}{\frac{1}{20} + \frac{4}{20} + \frac{10}{20} + \frac{20}{20}} = \frac{20}{35} = \frac{4}{7}$$

Q4) (a) $P(X<0.5) = P(X=0.2) + P(x=0.4)$

$= 0.1 + 0.2 = 0.3$

(b) $P(0.25 < x < 0.75)$

$= P(X=0.4) + P(X=0.5)$

$= 0.2 + 0.2 = 0.4$

(c) $P(X=0.2 \mid X<0.6)$

$= \dfrac{0.1}{0.1+0.2+0.2} = \dfrac{0.1}{0.5} = 0.2$

Q5)

1) $\dfrac{2}{3} = \dfrac{7-6c}{6} \Rightarrow 7-6c = 4 \Rightarrow 6c = 3$

$\boxed{c = 1/2}$

2) $P(1<x<2)$

Q5) $\dfrac{2}{3} + \dfrac{7-6c}{6} + \dfrac{4c^2-9c+6}{4} = 1$

$\Rightarrow \dfrac{2}{3} + \dfrac{7}{6} - c + c^2 - \dfrac{9}{4}c + \dfrac{4}{6} = 1$

$\Rightarrow \dfrac{4}{3} + \dfrac{7}{6} + c^2 - \dfrac{13}{4}c = 1$

$\Rightarrow c^2 - \dfrac{13}{4}c + \dfrac{9}{6} = 0$

$\Rightarrow 12c^2 - 39c + 18 = 0$

0.096 ~~0.574~~

≈ 0.57

$\leftarrow 0.24$

Q5) $\frac{2}{3} + \frac{7-6c}{6} + \frac{4c^2 - 9c + 6}{4} = 1$

1.071
2.179

$\Rightarrow \frac{2}{3} + \frac{7}{6} - c + \left(c^2 - \frac{9}{4}c + \frac{6}{4}\right) = 1$

$\Rightarrow \frac{1}{2} + \frac{2}{3} + \frac{7}{6} + c^2 - \frac{13}{4}c = 0$

$\Rightarrow \frac{14}{6} + c^2 - \frac{13}{4}c = 0 \quad \Rightarrow 28 + 12c^2 - 39c = 0$

$\frac{39 \pm \sqrt{39^2 - 4 \times 12 \times 28}}{24}$

$0 < 39^2$

$0-1$

0.666

0.096 $\leftarrow$ $1-2$

0.238 $\leftarrow$ $2-3$

1.002

$P(1 < X < 2) = 0.096$
$P(2 \le X < 3) = 0.238$
$P(0 < X \le 1) = 0.666$
$P(1 \le X \le 2) = 0.096$
$P(X \ge 3) = 0$.

Q6) $p(x) = 1 \qquad x \in [0, 1]$
$= 0 \qquad x < 0$ or $x > 1$

$E[X^2 + Y^2] = 1$
$Var[Y] = \frac{5}{9}$

$$E[X] = \int_0^1 f(x)\cdot x\,dx = \int_0^1 1\cdot x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$E[X^2] = \int_0^1 f(x)\cdot x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$Var[X] = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

$$E[Y] = ?$$

$$E[X^2+Y^2] = 1 \Rightarrow E[X^2] + E[Y^2] = 1$$

$$\frac{1}{3} + E[Y^2] = 1$$

$$E[Y^2] = \frac{2}{3}$$

$$Var(Y) = \frac{5}{9} = E[Y^2] - (E[Y])^2$$

$$\frac{5}{9} = \frac{2}{3} - (E[Y])^2 \Rightarrow \frac{6}{9} - \frac{5}{9} = (E[Y])^2$$

$$\Rightarrow E[Y] = \frac{1}{3}$$

$$E[X+Y] = E[X] + E[Y] = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$E[X] = \frac{1}{2}$, $Var[X] = \frac{1}{12}$, $E[Y] = \frac{1}{3}$, $E[X+Y] = \frac{5}{6}$