HY/APMQP/1222/C **22-SEP-2022**

HALF YEARLY EXAMINATION (2022-2023)

SUBJECT: Applied Mathematics **Maximum Marks: 80**

GRADE: XII

Time Allowed: 3 Hours

General Instructions:

1. This question paper contains It consists of four sections- **I, II, III and IV.**
2. Section **I** comprises of 10 questions of **1 mark** each**.**
3. Section **II** comprises of 10 questions of **2 marks** each**.**
4. Section **III** comprises of 10 questions of **3marks** each**.**
5. Section **IV** Comprises of 4 questions of **5 marks** each out of which two are case study questions

SECTION I

Each question carries 1 mark

1. If $x=3at^2$ and $y=4a^2t^5$ find $\frac{dy}{dx}$

2. If $\Delta= \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the cofactor of the element a_{23}

3. Evaluate $\int \frac{(1+\log x)^2}{x} dx$

4. If $y=\log\left(\frac{1-x^2}{1+x^2}\right)$ Find $\frac{dy}{dx}$

5. If A = $\begin{bmatrix} 4 & 6 \\ 7 & 5 \end{bmatrix}$, then what is the value of A (Adj A)

6. $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$

7. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

8. $For\ the\ curve\ \sqrt{x} + \sqrt{y} = 1\ find\ \ ,\frac{dy}{dx}\ at\ \left(\frac{1}{4}, \frac{1}{4}\right)$

9. The corner points of the feasible region of a LPP are (0,0), (0, 8) ,(2, 7) , (5,4) and (6, 0)
The maximum profit Z = 3x + 2y occurs at the point

10. Find the slope of the tangent to the curve y = $x^3 - x + 1$at the point whose x coordinate is 2

SECTION II

All questions are compulsory. In case of internal choice attempt any one

Each question carries 2 marks

11. Find a matrix A such that 2 A – 3 B + 5 C = O , where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

OR

If x and y are 2 x 2 matrices, then solve the following matrix equations for X and Y :

2 X + 3 Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$, 3 X + 2 Y= $\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$

12 The sides of an equilateral triangle is increasing at the rate 2cm/sec. The rate at which the area increases when the side is 10cm is

13 Evaluate $\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}dx$

14 Find the value of x if the matrix A = $\begin{bmatrix} x & 1 & 2 \\ 1 & 0 & 3 \\ 5 & -1 & 4 \end{bmatrix}$ is singular

15 $x^2 + y^2 = 3, Find\ \frac{d^2y}{dx^2}$

16 $If\ y = \sqrt{logx + \sqrt{logx + \sqrt{logx}}}$, then find $\frac{dy}{dx}$

17 Find the maximum and minimum values of the function f(x)= $-(x-1)^2+10$

18 If $xy = x^3 + y^3$, find $\frac{dy}{dx}$

19 $If\ x = e^{\frac{x}{y}}$ Find $\frac{dy}{dx}$

20 Evaluate $\int \frac{6x^2+1}{(4x^3+2x-5)}dx$

SECTION III

All questions are compulsory. In case of internal choice attempt any one

Each question carries 3 marks

21 $x^2 + 3xy + y^2 = -xy^2+3$, Find $find$, $\frac{dy}{dx}$

22 A company produces and sells a product and fixed costs of the company are Rs. 24000 and variable cost is 25% of the total revenue on selling the product at Rs. 8 per unit.

i) Find the total cost function. ii) Find the total revenue function. iii) Break-even point

23 Express the following matrix as the sum of a symmetric matrix and a sew symmetric matrix and verify your result

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

24 For the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Show that $A^2-5A + 4I = O$. Hence, find A^{-1}

25 Find the interval in function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is increasing or decreasing

OR

Prove that the function f(x) = log (1+x) - $\frac{2x}{x+2}$ is strictly increasing throughout its domain

26 $\int \frac{dx}{x^3(x^5+1)^{\frac{3}{5}}}$

27 If $y = \frac{log\,x}{x}$, Show that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$

28 Find the equation of the tangent line to the curve y = $x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$

29 $\int \frac{e^{3logx}}{(x^4+1)^3} dx$

30 A 10-ft ladder is leaning against a wall on flat ground. The base of the ladder starts to slide away from the wall at the rate of 2 ft/sec. How fast is the ladder's top sliding down the wall when the base is 8 ft from the wall?

SECTION IV

Qns. 30 to 31 carries 5 marks. Both the Case study-based questions are compulsory

31. Two instituions decided to award their employees for three values resourcefulnessm competence and determinationat the rate Rs,x,y,z respectively per person. The first instituion decided to award respectively 4,3,2 employees a sum of Rs.37000and second instituion decided to award respectively 5,3,2 employess a sum of Rs.47000.All the three prizes per person amounts to Rs.12000.
Based on the above information answer the following questions
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(i) Based on the above information,

(*i*) The model linear equations representing the above information are

(*a*) $4x + 3y + 2z = 37000$
$5x + 3y + 4z = 47000$
$x + y + z = 12000$

(*b*) $5x + 3y + 4z = 37000$
$4x + 3y + 2z = 27000$
$x + y + z = 12000$

(*c*) $4x + 5y + z = 47000$
$3x + 3y + z = 37000$
$2x + 4y + z = 12000$

(*d*) none of these

(ii) The value of the determinant as per the above matrix equation is
a) -1 b) 3
c) -3 d) -5

(iii) The linear equation representing the above information has

a) No solutions b) Infinitely many solution
c) Unique solution d) Three solutions

(iv) Using Cramer's rule the value of x
a) 5000 b) 3000
c) 4500 d)4000

(v) Using Cramer's rule the value of y and z are

a) 3000,5000 b) 5000,3000
c) 4000 ,5000 d)5000,4000

32. A firm has the cost function C(x)= $\frac{x^3}{3} - 7x^2$-111x+50 and the demand function x=100-p
Based on the above information answer the following questions

(i) The total revenue function is

a) R(x)= $x^2 - 100x$ b) R(x)= $100x - x^2$
c) R(x)= $100 - x$ d) none of these

(ii) The total profit function is

a) C(x)=- $\frac{x^3}{3} + 6x^2$-11x+50 b) C(x)= $\frac{x^3}{3} - 6x^2$-11x+50

c) C(x)= $\frac{x^3}{3} + 6x^2$-11x+50 d) $C(x) = -\frac{x^3}{3} - 6x^2 + 11x + 50$

(iii) The value of x for which the profit function is maximum

a)8 b)9 c)10 d) 11

(iv) The maximum profit is

a) Rs.131.11 b)Rs.113.11 c)Rs.111.33 d) Rs.133.11

(v) The marginal revenue when x=10 is

a)900 b)80 c)90 d) 800

33. If $y\sqrt{x^2+1} = \log\left[\sqrt{x^2+1} - x\right]$, show that $\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

OR

If $y = \left\{x + \sqrt{x^2+1}\right\}^m, show \left(x^2+1\right)y_2 + xy_1 - m^2 y = 0$

34. A manufacturer produces nuts and bolts for industrial machinery, It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 2.50 per package of nuts and Re1.00 per package of bolts. How many packages of each should he produce each day so as to maximize his profit, if he operates his machines for at most 12 hours a day? Formulate this as an LPP sum and then solve it graphically.

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