

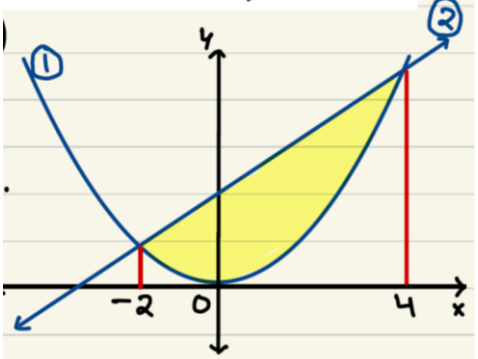
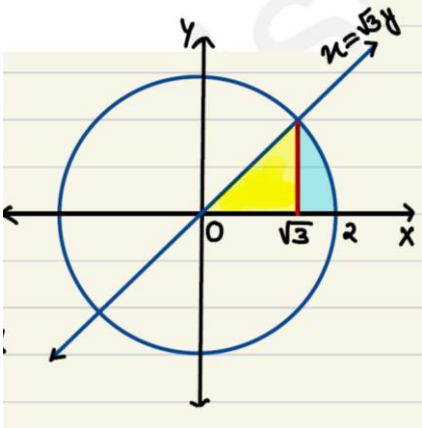


**EEE CONSORTIUM**  
**PREBOARD EXAMINATION 2021-2022**  
**Marking scheme (TERM -II)**  
**Set - I**

<b>Section- A</b>		
<b>1.</b>	Let $I = \int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] dx = \int \log(\log x) dx + \int \frac{dx}{(\log x)^2}$	<b>1</b>
	$= I_1 + I_2 + C$ (say) {C is arbitrary constant}	
	Where $I_1 = \int \log(\log x) dx$ and $I_2 = \int \frac{dx}{(\log x)^2}$	<b>1</b>
	$I_1 = \int \log(\log x) \cdot 1 dx$ $= \log(\log x) \int 1 dx - \int \left[ \frac{d[\log [\log x]]}{dx} \int 1 dx \right] dx$ $= \log(\log x)(x) - \int \left[ \frac{1}{\log x} \cdot \frac{d(\log x)}{dx} \int 1 dx \right] dx$ $= \log(\log x)(x) - \int \left[ \frac{1}{\log x} \right] dx$ $I_1 = x \log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2} dx$ <p>Now <math>I = I_1 + I_2 + C</math></p> $I = I_1 + \int \frac{dx}{(\log x)^2}$ $= x \log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2} dx + \int \frac{dx}{(\log x)^2} + C$ $= x \log(\log x) - \frac{x}{\log x} + C$ <p>Hence <math>I = x \log(\log x) - \frac{x}{\log x} + C</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Let <math>\tan x = t</math>  <math>\Rightarrow \sec^2 x dx = dt</math>  <math>\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 4}}</math>  <math>= \log t + \sqrt{t^2 + 4}  + c</math>  <math>= \log \tan x + \sqrt{\tan^2 x + 4}  + c</math></p>	<b>1</b>
<b>2.</b>	Degree of the given differential equation = 3	<b>1</b>
	Order of the given differential equation = 2 Hence, the sum of order and degree = 2 + 3 = 5	<b>1</b>
<b>3.</b>	Given $ \hat{a} + \hat{b}  = 1$ As we know that $ \hat{a} + \hat{b} ^2 +  \hat{a} - \hat{b} ^2 = 2( \hat{a} ^2 +  \hat{b} ^2)$ $\Rightarrow 1 +  \hat{a} - \hat{b} ^2 = 2(1 + 1)$ $\Rightarrow  \hat{a} - \hat{b} ^2 = 3$	<b>1</b>
		<b>1</b>

	$\Rightarrow  \hat{a} - \hat{b}  = \sqrt{3}$	
4.	<p>We know the direction cosines of the line passing through two points P(x<sub>1</sub> , y<sub>1</sub> , z<sub>1</sub> ) and Q(x<sub>2</sub>, y<sub>2</sub> , z<sub>2</sub> ) are given by</p> $\frac{x_2 - x_1}{PQ} = \frac{y_2 - y_1}{PQ} = \frac{z_2 - z_1}{PQ}$ <p>Where <math>PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}</math> Here P is (-2, 4, -5) and Q is (1, 2, 3). So <math>PQ = \sqrt{(1 - (-2))^2 + (2 - 4)^2 + (3 - (-5))^2}</math> <math>= \sqrt{77}</math> Thus, the direction cosines of the line joining two points is <math>\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}</math></p>	1  

	<p style="text-align: center;">OR</p> $y=vx$ $x \frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v}$ $\int \frac{v - \cos v}{2\sin v - v^2} dv = \int \frac{dx}{x}$ $-\frac{1}{2} \log u + \log c = \log x \Rightarrow x\sqrt{u} = c$ $2x^2 \sin\left(\frac{y}{x}\right) - y^2 = c^1$	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
9.	<p>Let <math>\vec{c} = ai + bj + ck</math></p> $\vec{a} \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = i(c - b) - j(c - a) + k(b - a)$ $i(c - b) - j(c - a) + k(b - a) = j - k$ $c - b = 0 \Rightarrow c = b; -1 = c - a \Rightarrow a = c + 1; b - a = -1 \Rightarrow a = b + 1$ $a + b + c = 3$ $1 + b + b + b = 3 \Rightarrow b = \frac{2}{3}; c = \frac{2}{3}; a = \frac{5}{3}$ $\vec{c} = \frac{5}{3}i + \frac{2}{3}j + \frac{2}{3}k$	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">+</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">+</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>
10.	<p>Let <math>(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)</math> be the point on the line</p> <p>d.r's of perpendicular line <math>10\lambda + 9, -4\lambda - 1, -11\lambda - 13</math></p> <p>d.r's of the vector on the line <math>10, -4, -11</math></p> $237\lambda = -237 \Rightarrow \lambda = -1$ <p>Coordinate of the foot of the perpendicular is <math>(1, 2, 3)</math></p> <p>Image of A is <math>(0, 3, 5)</math></p> <p style="text-align: center;">OR</p> <p>d.r's <math>\vec{b}_1</math> is <math>1, 2, 3</math>                      d.r's of <math>\vec{b}_2</math> is <math>-3, 2, 5</math></p> $\vec{b} \perp \vec{b}_1, \vec{b} \perp \vec{b}_2 \Rightarrow \vec{b} \parallel \vec{b}_1 \times \vec{b}_2$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix} = 4\hat{i} - 14\hat{j} + 8\hat{k}$ $\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(4\hat{i} - 14\hat{j} + 8\hat{k})$	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
<b>Section- C</b>		
11.	$I = \int_{-1}^2  x^3 - x  dx.$ $I = \int_{-1}^0 (x^3 - x) dx - \int_0^1 (x^3 - x) dx + \int_1^2 (x^3 - x) dx$	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p>

	$I = \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 - \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_0^1 + \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$ $I = -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right) = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4}$	1 1
12.	<p><math>4y = 3x^2 \dots (1)</math> and <math>3x - 2y + 12 = 0 \dots (2)</math></p>  <p>Point of intersection: <math>x = -2, 4</math></p> $Area = \int_{-2}^4 \left( \frac{3x + 12}{2} - \frac{3x^2}{4} \right) dx = \left[ \frac{3x^2}{4} + 6x - \frac{x^3}{4} \right]_{-2}^4$ $Area = \left( 3 \times \frac{16}{4} + 24 - \frac{64}{4} \right) - (3 - 12 + 2) = 20 - (-7) = 27 \text{ sq units}$ <p style="text-align: center;"><b>OR</b></p> <p>line <math>x = \sqrt{3}y \dots (1)</math> and the circle <math>x^2 + y^2 = 4 \dots (2)</math></p>  <p>Point of intersection : <math>x = \pm\sqrt{3}</math></p> $Area = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$ $Area = \frac{1}{2\sqrt{3}} [x^2]_0^{\sqrt{3}} + \left[ \frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2$ $Area = \frac{1}{2\sqrt{3}} [3 - 0] + \left[ \left( 0 + 2 \times \frac{\pi}{2} \right) - \left( \frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{3} \right) \right]$ $Area = \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq units}$	1  1 1 1  1  1
13.	Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ .	

	<p>Equation of line BC :</p> $\frac{x-0}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} \Rightarrow \frac{x-0}{1} = \frac{y+1}{-1} = \frac{z-3}{-2} = \lambda$ <p>Let the foot of perpendicular from A be given by :</p> <p><math>F(\lambda, -\lambda-1, -2\lambda+3)</math></p> <p>Drs of AF : <math>(\lambda+1), (-\lambda-9), (-2\lambda-1)</math></p> <p>Drs of BC : <math>1, -1, -2</math></p> <p><math>AF \perp BC \Rightarrow \lambda+1+\lambda+9+4\lambda+2=0 \Rightarrow 6\lambda+12=0 \Rightarrow \lambda=-2</math></p> <p><math>\therefore</math> Foot of perpendicular : <math>F(-2, 1, -1)</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
14.	<p style="text-align: center;"><b><u>CASE -BASED/ DATA- BASED</u></b></p> <p>Let B1, B2, B3 be the events that the bolts are manufactured by machines M1, M2 and M3 respectively. And E be the event that defective bolts are produced .</p> <p><math>P(B1) = \frac{25}{100}</math> , <math>P(B2) = \frac{35}{100}</math> , <math>P(B3) = \frac{40}{100}</math></p> <p><math>P(E B1) = \frac{5}{100}</math> , <math>P(E B2) = \frac{4}{100}</math> , <math>P(E B3) = \frac{2}{100}</math></p>	
	<p>(i) <math>P(E) = P(B1) \times P(E B1) + P(B2) \times P(E B2) + P(B3) \times P(E B3)</math></p> $P(E) = \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100} = \frac{345}{10000} = \frac{69}{2000}$	<p>1</p> <p>1</p>
	<p>(ii) <math>P(B2 E) = \frac{P(B2) \times P(E B2)}{P(B1) \times P(E B1) + P(B2) \times P(E B2) + P(B3) \times P(E B3)}</math></p> $P(B2 E) = \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}} = \frac{140}{345} = \frac{28}{69}$	<p>1</p> <p>1</p>