



HALF YEARLY EXAMINATION (2022-23)

Subject: Mathematics

Max. Marks:80

Grade: XII

Time:3 Hrs

Name:

Section:

Roll No:

General Instructions:

- This question paper consists of 3 printed pages.
- All answers to be written in the answer sheet provided.

SECTION A (10 x 1=10)

I. Multiple Choice Questions:

1. Differentiate $2^{\cos^2 x}$
2. Find the value of $\sin \left[\frac{\pi}{3} - \sin^{-1} \left(\frac{-1}{2} \right) \right]$
3. If $B = \begin{bmatrix} -1 & 5 \\ 0 & 3 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A
4. Show that the function f in the set N, the set of natural numbers defined as $f(x) = y = 3x$, is not onto.
5. Find the value of A^2 , where A is a 2x2 matrix whose elements are given by
$$a_{ij} = \begin{cases} 1, & i \neq j \\ 0, & i = j \end{cases}$$
6. Differentiate with respect to x
$$f(x) = \log_7(\log x)$$
7. If A and B are two independent events with $P(A) = 0.3$ and $P(B) = 0.6$, then find $P(\text{neither A nor B})$
8. The length x of a rectangle is decreasing at the rate of 3 cm/min and the width y is increasing at the rate of 2 cm/minute. When $x = 10$ cm and $y = 6$ cm, find the rate of change of the perimeter.
9. Find the value of $\cos^{-1} \left(\cos \frac{7\pi}{6} \right)$
10. If R is an equivalence relation defined in set $A = \{1, 2, 3, \dots, 10\}$ as $R = \{ (a, b) : |a - b| \text{ is a multiple of } 3 \}$. write the equivalence class [1]

SECTION B (10 x 2 = 20)

11. Find the co factors of the elements of the third row of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and evaluate that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

12. Find the value of k so that the function f defined by
- $$F(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \text{ is continuous at } x = \frac{\pi}{2}$$
13. Find the derivative of $f(x) = \sin(\sin x^2)$ at $x = \sqrt{\frac{\pi}{2}}$
14. Let R be the relation defined On N (set of natural numbers) defined by $R = \{ (a, b) : a, b \in N \text{ and } b \text{ is divisible by } a \}$. Check the relation R for reflexivity , symmetry and transitivity
15. Show that the function $f : R \rightarrow R$ defined by $f(x) = x - [x]$ is neither one -one nor onto .
16. Find the value of $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$
17. Evaluate $\cot^{-1} \left[\cot \frac{-9\pi}{4} \right]$
18. The side of an equilateral triangle is increasing at the rate of 2 cm / sec .At what rate its area increasing when the side of the triangle is 20 cm .
19. An urn contains 10 black and 5 white balls .Two balls are drawn from the urn one after the other without replacement .What is the probability that both drawn balls are black.
20. A die is thrown three times. Events A and B are defined as below.
 A : 5 on the first and 6 on the second throw.
 B : 3 or 4 on the third throw .
 Find the probability of B , given that A has already occurred.

Section C (10 x 3 =30)

21. Prove that the function $f(x) = \log(1+x) - \frac{2x}{x+2}$ is strictly increasing throughout its domain
22. Express the following matrix as the sum of a symmetric matrix and a skew symmetric matrix and verify your result
- $$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$
23. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 4I = O$. Hence , find A^{-1}
24. Differentiate $\tan^{-1} \left(\frac{2x}{1-x^2} \right)$ with respect to $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$
25. If $x = \sin t$, $y = \sin pt$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$
26. Consider $f : R_+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$ where R_+ is the set of all non negative real numbers x . Prove that f is one -one and onto

- 27 Simplify $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$
- 28 If $y = (\sin x)^x + (\cos x)^{\tan x}$, find $\frac{dy}{dx}$
- 29 The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when two equal sides are equal to the base
- 30 Suppose a girl throws a die. If she gets a 1 or 2, she tosses a coin 3 times and notes the number of heads. If she gets a 3, 4, 5 or 6, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, then what is the probability that she threw a 3, 4, 5 or 6 with the die

SECTION D (4x 5 = 20)

- 31 A manufacturer makes two types of toys, say A and B. Three machines are needed for their manufacturing, and the time (in minutes) required for each toy on the machines is given below. Each machine is available for a maximum of 6 hours per day. If the profit on each toy A is Rs 7.50 and on each toy B is Rs 5. Formulate this as an LPP and then solve it graphically to maximize the profit.

Toy	Machines		
	I	II	III
A	12	18	6
B	6	0	9

- 32 Differentiate the following function with respect to x
 $f(x) = \sin^{-1} \frac{2^{x+1}}{1+4^x}$
- 33 During the time of need and otherwise also people help the needy. It was found in survey that out of 200 people surveyed in a city 50 help the needy on regular basis, 120 contribute to prime Minister relief fund and the rest help through NGO's. A person is selected who is in need of a help, the probabilities of help through person on regular basis from Prime Minister relief fund and through NGO's are 0.15, 0.06 and 0.10 respectively.

Based on the above information answer the following questions

- (i) The probability of a help through NGO's is
 a) $\frac{1}{4}$ b) $\frac{17}{20}$ c) $\frac{5}{12}$ d) $\frac{3}{20}$
- (ii) The conditional probability of helping the needy through Prime Minister's relief fund is
 a) 0.15 b) 0.16 c) 0.10 d) 0.69
- (iii) The probability that the needy person received the help is
 a) 0.177 b) 1.77 c) 0.0885 d) 0.67
- (iv) The probability that needy person was helped through person on regular basis is
 a) $\frac{75}{177}$ b) $\frac{72}{177}$ c) $\frac{1}{59}$ d) $\frac{6}{59}$
- (v) The city of population 1 Lakh how many are expected to help on regular basis
 a) 25000 b) 60000 c) 20000 d) 5000

- 34 Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs x each, Rs y each and Rs z each. Three respective values to its 3, 2 and 1 students with a total award money of values (by giving same award money for three values as before.) The total amount of awards for one prize on each value is Rs 600

Based on the above information, answer the following questions.

- (i) What is the matrix equation representing the above situation
- (ii) What is the value of x
- (iii) What is the value of z
- (iv) What is the value of y
- (v) What is A^{-1} (adj A) in the above situation?
