مــدرسة دلهــي الخــاصــة ذ.م.م.

DELHI PRIVATE SCHOOL L.L.C.

Affiliated to C.B.S.E., DELHI

(Approved & Recognized By Ministry of Education - United Arab Emirates)

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PERIODIC TEST I (2023-24)MATHEMATICS	
Section A(1 mark each)	
1.	Option-c 81
2.	Option-d $[5, \infty)$
3.	Option-c- $\frac{5\pi}{6}$
4.	Option-a
Section B(2marks)	
5	{0,2,4,}
6	$\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)= \sin^{-1}(sin(x+45))=x+\frac{\pi}{4}$
7	$\sec^2(\tan^{-1}2)+\mathrm{cosec}^2(\cot^{-1}3)$. A= $\tan^{-1}2$ and B= $\cot^{-1}3$ $=\sec^2(A)+\mathrm{cosec}^2(B)$ =15
Section C (3 marks)	
8	$A = P^{-1}Q^{-1}$. $\therefore P^{-1}=\frac{\text{adj } P}{\lvert P\rvert}=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$ $\therefore Q^{-1}=\frac{\text{adj } Q}{\lvert Q\rvert}=\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ A= $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

9	$$I+A=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}0&-\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2}&0\end{bmatrix}=\begin{bmatrix}1&-\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2}&1\end{bmatrix}$$ and, $$I-A=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}0&\tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2}&0\end{bmatrix}=\begin{bmatrix}1&\tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2}&1\end{bmatrix}.$$ $\therefore$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&\tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2}&1\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}$$ $\Rightarrow$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&\tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2}&1\end{bmatrix}\begin{bmatrix}\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}&-\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\\ \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}&\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{bmatrix}$$ $\Rightarrow$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&t\\ -t&1\end{bmatrix}\begin{bmatrix}\frac{1-t^2}{1+t^2}&-\frac{2t}{1+t^2}\\ \frac{2t}{1+t^2}&\frac{1-t^2}{1+t^2}\end{bmatrix};\text{ where } t=\tan\frac{\alpha}{2}.$$ $\Rightarrow$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}\frac{1-t^2+2t^2}{1+t^2}&\frac{-2t+t-t^3}{1+t^2}\\ \frac{-t+t^3+2t}{1+t^2}&\frac{2t^2+1-t^2}{1+t^2}\end{bmatrix}$$ $\Rightarrow$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}\frac{1+t^2}{1+t^2}&\frac{-t(1+t^2)}{1+t^2}\\ \frac{t(1+t^2)}{1+t^2}&\frac{1+t^2}{1+t^2}\end{bmatrix}=\begin{bmatrix}1&-t\\ t&1\end{bmatrix}$$ $\Rightarrow$ $$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\ \sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&-\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2}&1\end{bmatrix}=I+A$$

10	*R is reflexive.* For every $a \in \mathbf{R}$, $a - a + \sqrt{3} = \sqrt{3} \in S \Rightarrow (a, a) \in R$ $\Rightarrow$ R is reflexive. *R is not symmetric.* Take $a = \sqrt{3}$ and $b = 1$. As $\sqrt{3} - 1 + \sqrt{3} = 2\sqrt{3} - 1 \in S$, $(\sqrt{3}, 1) \in R$. But $1 - \sqrt{3} + \sqrt{3} = 1 \notin S \Rightarrow (1, \sqrt{3}) \notin R$ $\Rightarrow$ R is not symmetric. *R is not transitive.* Take $a = 1$, $b = \sqrt{2}$ and $c = \sqrt{3}$. As $1 - \sqrt{2} + \sqrt{3} \in S$, $(1, \sqrt{2}) \in R$. Also $\sqrt{2} - \sqrt{3} + \sqrt{3} = \sqrt{2} \in S$, $(\sqrt{2}, \sqrt{3}) \in R$. But $1 - \sqrt{3} + \sqrt{3} = 1 \notin S$, $(1, \sqrt{3}) \notin R$. Thus, $(1, \sqrt{2}) \in R$ and $(\sqrt{2}, \sqrt{3}) \in R$ but $(1, \sqrt{3}) \notin R$ $\Rightarrow$ R is not transitive.
11	**Solution.** *The function f is onto.* *Reason :* Consider any $n \in \mathbf{N}$ (codomain of f). Certainly $2n \in \mathbf{N}$ (domain of f). Also $2n$ is even. Thus, for all $n \in \mathbf{N}$ (codomain of f), there exists $2n \in \mathbf{N}$ (domain of f) such that $f(2n) = \frac{2n}{2} = n \Rightarrow f$ is onto. *The function f is not one-one.* *Reason :* As $1, 2 \in \mathbf{N}$ (domain of f) and $f(1) = \frac{1+1}{2} = 1$, $f(2) = \frac{2}{2} = 1$, so that the different elements 1 and 2 of the domain of f have same image 1. Therefore, f is not one-one. Hence, f is not bijective.
Section D (4 Marks)	

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1. If ₹15000 is invested in bond X, then the amount invested in bond Y = ₹(35000 - 15000) = ₹20000

   Investment A = $\begin{array}{cc} X & Y \\ \end{array}$ $\begin{bmatrix} 15000 & 20000 \end{bmatrix}$; B = $\begin{array}{c} \text{Invest rate} \\ \begin{array}{c} X \\ Y \end{array} \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} \end{array}$

2. The amount of interest received on each bond is given by

   AB = [15000 20000] $\times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$

   = [15000 $\times$ 0.1 + 20000 $\times$ 0.08] = [1500 + 1600] = 3100

3. Let ₹x be invested in bond X and then ₹(35000 - x) will be invested in bond Y.

   Now, total amount of interest is given by

   [x 35000 - x] $\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ = [0.1x + (35000 - x) 0.08]

   But, it is given that total amount of interest = ₹3200

   $\therefore$ 0.1x + 2800 - 0.08x = 3200

   $\Rightarrow$ 0.02 x = 400 $\Rightarrow$ x = 20000

   Thus, ₹20000 invested in bond X and ₹35000 - ₹20000

   = ₹15000 invested in bond Y.

4. Let ₹x invested in bond X, then we have

   x $\times \frac{10}{100}$ = 500 $\Rightarrow$ x = 5000

   Thus, the amount invested in bond X is ₹5000 and so investment in bond Y be ₹(35000 - 5000) = ₹30000

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R is not reflexive.

Take $a = \frac{1}{2}$. As $\frac{1}{2} \le \left(\frac{1}{2}\right)^2$ *i.e.* $\frac{1}{2} \le \frac{1}{4}$ is false, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin$ R. Therefore, R is not reflexive.

R is not symmetric.

Take $a = 1$, $b = 2$. As $1 \le 2^2$ *i.e.* $1 \le 4$ is true, $(1, 2) \in$ R.

But $2 \le 1^2$ *i.e.* $2 \le 1$ is false, $(2, 1) \notin$ R. Therefore, R is not symmetric.

R is not transitive.

Take $a = \frac{1}{3}$, $b = -2$ and $c = \frac{1}{2}$.

As $\frac{1}{3} \le (-2)^2$ *i.e.* $\frac{1}{3} \le 4$ is true, $\left(\frac{1}{3}, -2\right) \in$ R.

Also $-2 \le \left(\frac{1}{2}\right)^2$ *i.e.* $-2 \le \frac{1}{4}$ is true, $\left(-2, \frac{1}{2}\right) \in$ R.

But $\frac{1}{3} \le \left(\frac{1}{2}\right)^2$ *i.e.* $\frac{1}{3} \le \frac{1}{4}$ is false, $\left(\frac{1}{3}, \frac{1}{2}\right) \notin$ R.

Therefore, R is not transitive.

b)

The function f is not one-one.

: Since $f\left(\frac{1}{2}\right) = \left[\frac{1}{2}\right] = 0$ and $f\left(\frac{1}{3}\right) = \left[\frac{1}{3}\right] = 0$, so the two different elements $\frac{1}{2}, \frac{1}{3}$ of **R**

The function f is not onto.

Reason : Range of $f = \mathbf{I}$ (set of integers), which is a proper subset of **R** (codomain of f)

$\Rightarrow$ f is not onto.

SectionE(5 marks)

14	$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ $= 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$ The given system has the unique solution $X = B^{-1} C = \frac{1}{8}AC$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\Rightarrow x = 3, y = -2, z = -1.$