6 PROBABILITY

SYNOPSIS

- The conditional probability of an event E, given the occurrence of the event F is given by P(E/F)= $\frac{P(E \cap F)}{P(F)}, P(F) \neq 0$
- $0 \leq P(E/F) \leq 1$ P(E'/F) = 1 – P(E/F)
- $P((E \cup F)/G) = P(E/G) + P(F/G) - P((E \cap F)/G)$
- $P(E \cap F) = P(E)P(F/E), P(E) \neq 0$
- If E and F are independent, then $P(E \cap F) = P(E)P(F)$

THEOREM OF TOTAL PROBABILITY

- Let $\{E_1, E_2, E_3, ...E_n\}$ be the partition of a sample space and suppose that each of $E_1, E_2, E_3......E_n$ has nonzero probability. Let A be any event associated with S, then $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + ... + P(E_n)P(A/E_n)$

BAYES' THEOREM

- Let $\{E_1, E_2, E_3, ...E_n\}$ be the partition of a sample space and suppose that each of $E_1, E_2, E_3......E_n$ has nonzero probability. Let A be any event associated with S,and $E_1 \cup E_2 \cup ....E_n = S$, then $P(E_i / A) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$

PROBABILITY

1 mark

1. $P(A)= \frac{1}{2}$ $P(B)= \frac{1}{3}$ and $P(A\cup B)= \frac{2}{3}$. Are events A and B independent?

2. If P(A)=0.4, P(B)=0.8 and P(B|A)=0.6 Find $P(A\cup B)$.

3. In a binomial distribution, the sum of mean and variance of 5 trials is 3.75 Find P.

4. Two cards are drawn at random with out replacement from a pack of cards, find the probability both cards are red.

5. A coin is tossed 3 times. What values x can take if it represents number of heads.

6. If P(A)=0.4, P(B)=p and $P(A\cup B)$=0.6, Find p, if A and B are independent events.

4 marks

7. A purse contains 2silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin pulled out random from one of the two purses. What is the probability that coin is silver?

8. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its color is put in the second bag. A ball is then dawn from the second bag. Find the probability that the ball drawn is blue in color.

9. An urn contains 4 red and 3 blue balls. Find the probability distribution of the number of blue balls in a random draw of 3 balls, without replacement.

10. Two cards are drawn simultaneously from a well shuffled deck of cards. Find the mean of the number of success, when getting an ace is considered as a success.

11. Two numbers are selected at random from the integers 1 through 9. if the sum is even, find the probability that both the numbers are odd

12. Probability of winning when batting coach A and bowling coach B working independently are ½ and 1/3 respectively. If both try for the win independently find the probability that there is a win. **Will the independently working may be effective? And why?**

13. A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point. **Do you think drinking habit can ruin one’s family life?**

14. Two third of the students in a class are sincere about their study and rest are careless. Probability of passing in examination are 0.7 and 0.2 for sincere and careless students respectively. A Student is chosen and is found to be passed. What is the probability that he/ she was sincere. **Explain the importance of sincerity for a student.**

15. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

16. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X = 4) = P(X = 2)$. Find the probability of success.

17. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

(i) all the four cards are spades ?

(ii) only 2 cards are spades ?

18. From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs.

19. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

6 marks

20. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Prove that the probability that it is actually six is 3/8

21. In a test, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The Probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6.The probability that his answer is correct, given that he copied it, is 1/8.Prove that the probability he knew the answer to the question, given that he answered correctly is 24/29

22.A letter is known to come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letters have come from (i) Calcutta (ii) Tatanagar.

23.Three persons A, B, C throw a die in succession till one gets a six and wins the game. Find the respective probability of winning.

24. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

25. Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and variance of this distribution.

26. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly ?

Scoring Key

1.	yes
2.	0.96
3.	$\frac{1}{2}$
4.	$\frac{25}{102}$
5.	X =0,1,2,3
6.	$p = \frac{1}{3}$
7.	$\frac{19}{42}$
8.	$\frac{93}{154}$
9.	P(X=0) = 4/35 P(X =1) = 18/35 P(X =2) = 12/35 P(X=3) = 1/35
10.	Mean =2/13
11.	5/8
12.	2/3 1chances of success increase when ideas flows independently 2 Hard work pays the fruits
13.	126 X $(0.6)^4$ $(0.4)^4$ Yes, addiction of wine or smoking is definitely harmful for a person and its family
14.	7/8 A student is future of a country. If a student is sincere then he/she can serve the country in a better way.
17.	4/11 & 7/11
18.	$\left[\frac{36}{91}, \frac{30}{91}, \frac{25}{91}\right]$
19	(i) 2/3 (ii) 1/2

2 Marks

1) If A and B are two events such that $P(A) = {}^{1}/_{4}$, $P(B) = {}^{1}/_{3}$ and $P(AUB) = {}^{1}/_{2}$, show that A and B are independent events.
2) Let A and B be events such that
   $P(A) = {}^{1}/_{2}$, $P(B) = {}^{7}/_{12}$ and P(not A or not B) = ${}^{1}/_{4}$
   State whether A and B are
   (i) mutually exclusive (ii) independent
3) A coin is tossed and then a die is thrown. Find the probability of obtaining a 4, it being given that a head came up.
4) Let A and B be events such that
   $2P(A) = P(B) = \frac{5}{13}$, $P(A/B) = \frac{2}{5}$
   Find P(AUB).
5) A die is tossed twice. 'Getting an odd number on a toss' is considered a success. Find the probability distribution of number of successes.
6) A die is tossed twice. 'Getting a number greater than 4' is considered a success. Find the probability distribution of number of successes.
7) A coin is tossed 6 times. Find the probability of getting at least 3 heads.
8) A coin is tossed 5 times. What is the probability that a head appears an even number of times?
9) A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
10) A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

<u>**Scoring Key(2 Marks)**</u>

<table>
<tr><td>2.</td><td>(i) No (ii) No</td></tr>
<tr><td>3.</td><td>$\frac{11}{26}$</td></tr>
<tr><td>4.</td><td>$\frac{1}{6}$</td></tr>
<tr><td>5.</td><td><table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p_i</td><td>1/4</td><td>1/2</td><td>1/4</td></tr></table></td></tr>
<tr><td>6.</td><td><table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p_i</td><td>4/9</td><td>4/9</td><td>1/9</td></tr></table></td></tr>
<tr><td>7.</td><td>$\frac{21}{32}$</td></tr>
<tr><td>8.</td><td>½</td></tr>
<tr><td>9.</td><td>3/8</td></tr>
<tr><td>10.</td><td>1/6</td></tr>
</table>

7. Indefinite integral

$$\int \cos x \; dx = \sin x + c$$

$$\int \sin x \; dx = -\cos x + c$$

$$\int \sec^2 x \; dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \; dx = -\cot x + c$$

$$\int a^x \; dx = \frac{a^x}{\log_e a} + c$$

$$\int \sec x.\tan x \; dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \; dx = \operatorname{cosec} x + c$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + c$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$$

$$\int x^n \; dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

$$\int \frac{dx}{x} = \log|x| + c$$

$$\int e^x \; dx = e^x + c$$

II. Integration using Trigonometric Identities

(Even Powers of sin$x$) $\int \sin^2 x \, dx$, $\int \sin^4 x \, dx$, $\int \sin^6 x \, dx$ put $\sin^2 x = \dfrac{1-\cos 2x}{2}$

(Even Powers of cos$x$) $\int \cos^2 x \, dx$, $\int \cos^4 x \, dx$, $\int \cos^6 x \, dx$ put $\cos^2 x = \dfrac{1+\cos 2x}{2}$

(Odd powers of sin x) $\int \sin^3 x \, dx$, $\int \sin^5 x \, dx$, $\int \sin^7 x \, dx$ write $\int (\sin^2 x)^n \sin x \, dx$ and put $\sin^2 x = 1 - \cos^2$ x, Let $t = \cos x$

$\int$sin ax sinbxdx , $\int$sin ax cosbx dx , $\int$cosax sinbxdx , $\int$cos ax cosbx dx use 2 sin A sin B, 2sin A cos B, 2 cos A sin B, 2 cos A cos B

III. Special Integrals

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a-x}{a+x}\right| + c$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{x^2-a^2} = \log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \frac{dx}{x^2-a^2} = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

IV. Integrals of the type $\int \dfrac{dx}{ax^2+bx+c}$, $\int \dfrac{dx}{\sqrt{ax^2+bx+c}}$

Working Rule: Express denominator as a sum or difference of perfect squares and use special integrals.

V Integrals of the type $\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$

Working Rule: $px + q = A(2ax + b) + B$, Find A and B.

$$\int \frac{(px+q)dx}{ax^2+bx+c} = A\int \frac{(2ax+b)dx}{ax^2+bx+c} + B\int \frac{dx}{ax^2+bx+c}$$

$$\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}} = A\int \frac{(2ax+b)dx}{\sqrt{ax^2+bx+c}} + B\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad > (\text{typeIII})$$

VI. Integrals of the type $\int \frac{p(x)dx}{ax^2+bx+c}$ where degree $p(x) \geq 2$

Working Rule: Carry out long division and write

$$\int \frac{p(x)dx}{ax^2+bx+c} = \text{a polynomial} + \int \frac{(px+q)dx}{ax^2+bx+c} \quad (\text{type IV})$$

VII. Integrals of the type $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a\sin^2 + b\cos^2 x}$, $\int \frac{dx}{a+b\sin^2 x + c\cos^2 x}$

Working Rule: Divide N and D by $\cos^2 x$. Put $t = \tan x$, $dt = \sec^2 x d x$

VIII. Integrals of the type $\int \frac{dx}{a+b\cos x}$, $\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a\cos x + b\sin x}$

Working Rule: Put $\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$, $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

Substitute $t = \tan\frac{x}{2}$

IX. Integrals of the type $\int \left(\frac{a\sin x + b\cos x}{c\sin x + d\cos x}\right)dx$

Working Rule: $\Rightarrow$ N = A (Denominator) + B (Derivative of denominator) $\Rightarrow$ Numerator = A ($c\sin x + d\cos x$) + B ($c\cos x - d\sin x$)

X. More Special Integrals

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

XI. Integrals of the type $\int (px+q)\sqrt{ax^2+bx+c} + dx$

Working Rule: $px + q = A(2ax + b) + B$. (This reduces to type X)

XII. Integrals of the type $\int \frac{(px^2+qx+r)dx}{\sqrt{ax^2+bx+c}}$

Working Rule: $px + qx + r = A(ax^2 + bx + c) + B(2ax + b) + c$

XIII. Integrals of the parts

$$\int \underset{\text{I}}{f(x)}\ \underset{\text{II}}{g(x)} = f(x)\int g(x)dx - \int\left\{f(x)\int g(x)\,dx\right\}dx$$

Rule for choosing functions I and II.

I LATE
- I = Inverse Trigonometric Functions
- L = Logarithmic Functions
- A = Algebraic Functions
- T = Trigonometric Functions
- E - Exponential Functions

Special Form $\int e^{ax}\ \sin\ bx\,dx \quad \int e^{ax}\ \cos\ bxdx$

ILATE need not be used. e^{ax} may be the 1st function.

Integrals of the form $\int e^{x}\ (f(x)+f'(x))\,dx\ =\ e^{x} f(x)+c$

$$\int e^{kx}\ (kf(x)\ +\ f'(x))\,dx\ =\ e^{kx} f(x)\ +\ c$$

XIV. Integration by Partial Fractions

Factors in Denominator	Corresponding Partial Fraction]
$(x-a)^2$	$\frac{\text{A}}{x-a}+\frac{\text{B}}{(x-a)^2}$
$(x-a)^3$	$\frac{\text{A}}{x-a}+\frac{\text{B}}{(x-a)^2}+\frac{\text{C}}{(x-a)^3}$
$(x-a)(x-b)$	$\frac{\text{A}}{x-a}+\frac{\text{B}}{x-b}$
$(x-a)(ax^2+bx+c)$ || irreducible	$\frac{\text{A}}{x-a}+\frac{\text{B}x+\text{C}}{ax^2+bx+c}$
$(ax^2+bx+c)^2$	$\frac{\text{A}x+\text{B}}{ax^2+bx+c}+\frac{\text{C}x+\text{D}}{(ax^2+bx+c)^2}$
$(ax^2+b)(cx^2+d)$	Put $x^2 = \text{Z}$

XV. Integrals of the form $\int\frac{(x^2+1)dx}{x^4+kx^2+1},\ \int\frac{(x^2-1)dx}{x^2+kx^2+1}$

Working Rule

Divide N and D by x^2

Put $t = x + \frac{1}{x} \Rightarrow dt = \left(1-\frac{1}{x^2}\right)dx$

Put $t = x - \frac{1}{x} \Rightarrow dt = \left(1 + \frac{1}{x^2}\right)dx$

Assignments

1 Mark

1. Evaluate $\int e^x \left(1 - \cot x + \cot^2 x\right)\, dx$.
2. Evaluate $\int \frac{\sin x}{3 + 4\cos^2 x}\, dx$.
3. Evaluate $\int \frac{1}{x + x\log x}\, dx$.
4. Evaluate $\int \frac{\sin x + \cos x}{\sqrt{1 - \sin 2x}}\, dx$.
5. Evaluate $\int \frac{\cos 2x - 1}{\cos 2x + 1}\, dx$.
6. Evaluate $\int \sin(\log x) + \cos(\log x)\, dx$.
7. Evaluate $\int \frac{1}{2x^2 - x - 1} dx$.
8. Evaluate $\int \cos 2x . \cos 3x\, dx$.
9. Evaluate $\int e^x \sqrt{e^{2x} + 1}\, dx$.
10. Evaluate $\int \sqrt{x - x^2}\, dx$.
11. Evaluate $\int \frac{1}{x\left(x^5 - 1\right)}\, dx$.

4 Marks

12. Evaluate $\int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-x/2}\, dx$.
13. Evaluate $\int e^x \frac{1 + x}{(2 + x)2}\, dx$.
14. Evaluate $\int \sin^{-1} \sqrt{\frac{x}{a + x}}\, dx$.
15. Evaluate $\int e^x \cos^2 x\, dx$.
16. Evaluate $\int \frac{1}{\sin^4 x + \cos^4 x}\, dx$.
17. Evaluate $\int \tan^{-1} \sqrt{x}\, dx$.
18. Evaluate $\int \frac{1}{3 + 2\sin x + \cos x}\, dx$.

19. Evaluate $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\ dx$.

20. Evaluate $\int \frac{3x+1}{\sqrt{5-2x-x^2}}\ dx$.

21. Evaluate $\int \frac{x^2}{(x^2+1)(3x^2+4)}\ dx$.

22. Evaluate $\int \frac{\sin x}{\sin 4x}\ dx$.

23. Evaluate $\int \frac{1}{1+x+x^2+x^3}\ dx$.

24. Evaluate $\int \frac{1}{x^2(x^4+1)^{3/4}}\ dx$. **Discuss the importance of integration (unity) in life.**

2 Marks

25. $\int(9\sin x - 7\cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x\ )\ dx$

26. $\int(\frac{\cot x}{\sin x} - \tan^2 x - \frac{\tan x}{\cot x} + \frac{2}{\cos^2 x}\ )\ dx$

27. $\int \sec x(\sec x + \tan x)\ dx$

28. $\int \operatorname{cosec} x(\operatorname{cosec} x - \cot x)\ dx$

29. $\int \frac{1}{(1-\cos 2x)}\ dx$

30. $\int \left(\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\right)\ dx$

31. $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)\ dx$

32. $\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) dx$

Scoring Key

1.	$-e^{x}\cot x\ \ ec$
2.	$--\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)\ +c$
3.	$\log\lvert 1+\log x\rvert\ +c$
4.	$\log\lvert\sin x-\cos x\rvert\ +c$
5.	$x-\tan x\ \ +c$
6.	$x\sin(\log x)\ +c$
7.	$\frac{1}{3}\log\left\lvert\frac{x-1}{2x+1}\right\rvert\ +c$
8.	$\frac{-1}{8}[\sin 4x+2\sin 2x]$
9.	$\frac{1}{2}e^{x}\sqrt{e^{2x}+1}+\frac{1}{2}\log\left\lvert e^{x}+\sqrt{e^{x}+1}\right\rvert$
10.	$\frac{1}{4}(2x-1)\sqrt{x-x^{2}}+\frac{1}{8}\sin^{-1}(2x-1)\ +c$
11.	$\frac{1}{5}\log\left\lvert\frac{x^{5}-1}{x^{5}}\right\rvert$
12.	$-e^{-x/2}\sec(x/2)\ +c$
13.	$\frac{e^{x}}{x+2}\ \ +c$
14.	$x\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+a\tan^{-1}\sqrt{\frac{x}{a}}$
15.	$\frac{e^{x}}{2}+\frac{e^{x}}{10}(\cos 2x+2\sin 2x)$
16.	$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^{2}x-1}{\sqrt{2}\tan x}\right)\ +c$
17.	$(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}\ \ +c$
18.	$\tan^{-1}(1+\tan x/2)$
19.	$\tan^{-1}(\tan^{2}x)\ +c$
20.	$-3\sqrt{5-2x-x^{2}}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)$

21.	$\frac{2}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2}x - \tan^{-1}x$
22.	$\frac{-1}{8}\log\frac{1+\sin x}{1-\sin x} + \frac{1}{4\sqrt{2}}\log\left\|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right\|$
23.	$\frac{1}{2}\log\|x+1\| - \frac{1}{4}\log\|1+x^2\| + \frac{1}{2}\tan^{-1}x \quad + c$
24.	$-\left(1+\frac{1}{x^4}\right)^{1/4} \quad + c$ a)United we stand, divided we fall b)Union is strength
25.	– 9cosx – 7sinx – 6tanx – 3cotx – x + C
26.	– cosecx + tanx + x – secx + C
27.	tanx + secx + C
28.	– cotx + cosecx + C
29.	$-\frac{1}{2}$ cotx + C
30.	secx – cosecx + C
31.	$\frac{x^2}{2}$ + C
32.	x^2 + C

Definite Integral

$\int f(x)dx = (\mathrm{F}(x))_a^b = \mathrm{F}(a)$ where $\int f(x)dx = \mathrm{F}(x) + c$

Properties of Definite Integrals

1. $\int_a^b f(x)dx = \int_a^b f(y)dy$
2. $\int_a^b \mathrm{f(x)dx} = -\int_b^a f(x)\, dx$
3. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
5. $\int_o^a f(x)dx = \int_o^a f(a-x)dx$"
6. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if $f(x)$ is even function
   $= 0$ if $f(x)$ is odd function
7. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx,$ if $(x) = f(2a-x)$
   $= 0$, if $f(2a-x) = -f(x)$

Definite Integral as a Limit of Sums:

$$\int_a^b f(x)dx = it_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) \ldots\ldots f(a+\overline{n-1}h)\right]$$

Where $nh=b-a$ and $n \to \infty$ when $h \to 0$

$$e^a + e^{a+h} \ldots\ldots\ldots + e^{a+\overline{n-1}h} = \frac{e^a(e^{nh}-1)}{e^h-1}$$

$$1+2 \ldots\ldots\ldots (n-1) = \sum_{x=0}^{n-1} x = \frac{n(n-1)}{2}$$

$$1^2+2^2 \ldots\ldots + (n-1)^2 = \sum_{x=0}^{n-1} x^2 = \frac{n(n-1)(2n-1)}{6}$$

$$1^3+2^3 \ldots\ldots (n-1)^3 = \sum_{x=0}^{n-1} x^3 = \left[\frac{n(n-1)}{2}\right]^2$$

ASSIGNMENTS

1Mark

1. If $\int_0^a \sqrt{x}\, dx = 2a \int_0^{\pi/2} \sin^3 x\, dx$.

2. Evaluate $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2}+\pi/4\right)\, dx$.

3. If $\int_0^k \frac{1}{2+8x^2}\, dx = \pi/16$ find k.

4. Evaluate $\int_{-1}^{1} e^{|x|}\, dx$.

5. Evaluate $\int_0^2 \left[x^2\right]\, dx$.

6. Evaluate $\int_{-1}^{1} \log\left[\frac{2+x}{2-x}\right]\, dx$.

4 Marks

7. Evaluate $\int_0^2 \left|x^2+2x-3\right|\, dx$.

8. $\int_2^5 f(x)\, dx$ Where $f(x)=|x-2|+|x-3|+|x-4|$.

9. Evaluate $\int_0^{\pi/2} \frac{dx}{6-\cos x}$.

10. Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\, dx$.

11. Evaluate $\int_{-1}^{\frac{3}{2}} |x\cos \pi x| dx$

12. $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$.

13. $\int_0^1 \cot^{-1}\left(1-x+x^2\right)\, dx$.

14. $\int_0^{5/2} \frac{x\tan x}{\sec x \cos ec x}\, dx$.

15. Evaluate using limit as a sum

2Marks

Prove the following

16. $\int_0^{\frac{\pi}{2}} \frac{cosx}{(sinx+cosx)} dx = \frac{\pi}{4}$
17. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{(\sqrt{sinx}+\sqrt{cosx})} dx = \frac{\pi}{4}$
18. $\int_0^{\frac{\pi}{2}} \frac{dx}{(1+tanx)} = \frac{\pi}{4}$
19. $\int_0^{\frac{\pi}{2}} \frac{dx}{(1+cotx)} = \frac{\pi}{4}$
20. $\int_0^1 x(1-x)^5 dx = \frac{1}{42}$
21. $\int_0^{\pi} \frac{xtanx}{(secx+cosx)} dx = \frac{\pi^2}{4}$

Scoring Key

1.	a=0, 4	
2.	0	
3.	$k = \frac{1}{2}$	
4.	$2e$-2	
5.	$5-\sqrt{2}-\sqrt{3}$	
6.	0	
7.	4	
8.	9.5	
9.	$\frac{2}{\sqrt{35}}\tan^{-1}\sqrt{\frac{7}{5}}$	
10.	$\pi/12$	
11.	$\frac{3\pi+1}{\pi^2}$	
12.	$\frac{\pi}{8}\log 2$	
13.	$\frac{\pi}{2}-\log 2$	
14.	$\frac{\pi^2}{4}$	
15.	a.	$112/3$
	b.	$118/3$
	c.	$\frac{15+e^8}{2}$

APPLICATION OF INTEGRALS
SYNOPSIS

- The area of the region bounded by the curve y=f(x), x axis and the lines x = a and x= b (b>a) is given by the formula : Area $\int_a^b y dx = \int_a^b f(x) dx$
- The area of the region bounded by the curve x = h(x) , y axis and the lines y = c, y = d is given by the formula : Area $\int_c^d x dy = \int_c^d h(x) dy$
- The area of the region enclosed between two curves y = f(x) , y = g(x) and the x =a , x=b is given by the formula : Area = $\int_a^b [f(x) - g(x)] dx$, where f(x) ≥g(x) in [a,b]
- If f(x) ≥ g(x) in [a,c] and f(x) ≤ g(x) in [c,b] , a < c < b then Area = $\int_a^c [f(x) - g(x)] dx + \int_c^b [g(x) - f(x)] dx$

Application of integration

1. Using integration, find the area of the region bounded by the following curves, draw a rough sketch also. $y = |x+1| + 1$; $x = -3$; $x = 3$ and $y = 0$.
2. Find the area bounded by the lines $x + 2y = 2$; $y - x = 1$ and $2x + y = 7$.
3. Find the area enclosed between the parabola $y^2 = x$ and the line $x + y = 2$.
4. Find the area of the region $\{(x, y): x^2 + y^2 \le 2ax;\ y^2 \ge ax;\ x, y \ge 0\}$.
5. Make a sketch of the region given below and find its area using integration $\{(x, y): 0 \le y \le x^2 + 3;\ 0 \le y \le 2x + 3;\ 0 \le x \le 3\}$.
6. Using integration, find the area of the region bounded by the ΔABC whose vertices A, B, C are (-1, 1), (0, 5) & (3, 2).
7. Find the area bounded by the curves $x = y^2$ and $x = 3 - 2y^2$
8. Find the area bounded by the curves $y = \sqrt{1 - x^2}$, the line $y = x$ and the positive axis.
9. Find the area of the region enclosed between the circles $x^2 + y^2 = 1$ and $(x - \frac{1}{2})^2 + y^2 = 1$.

10. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

11. A circular Olympic gold medal has a radius 2cm and taking the centre at the origin, Find its area by method of integration. **What is the importance of Olympic Games for a sportsman? and why?**

12.A poor deceased farmer has agriculture land bounded by the curve y = cosx, between x=0 and x= 2π He has two sons. Now they want to distribute this land in three parts .Find the area of each part. **Which parts should be given to the farmer and Why? Justify your answer?**

6 Marks

1. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4, and y = 0 into three equal parts.

2. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

3. Find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$, using integration.

4. Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

5. Find the area of the region in the first quadrant enclosed by the y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration.

Scoring Key

1.	16
2.	6 sq. ml
3.	$\frac{9}{2}$
4.	$\left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$ sq. units
5.	$\frac{50}{3}$ sq. units
6.	$\frac{15}{2}$ sq. units
7.	4 sq. unit
8.	$\frac{\pi+1}{8}$ sq. ml
9.	$-\frac{2\sqrt{3}+\sqrt{15}}{16}-2\sin^{-1}\frac{1}{4}+\pi$
10.	$\frac{4}{3}\left(8\pi-\sqrt{3}\right)$
11.	4π cm^2 Olympic game is a supreme platform for a sportsman. In Olympic Games all countries of the world participate and try their best and make their country proud.
12.	1. Respect the parents 2. Help the elders (parents)

DIFFERENTIAL EQUATIONS
SYNOPSIS

- **Order of differential equation** : is defined as the order of the highest order derivative of dependent variable w.r.t independent variable, involved in the given differential equation.

- **Degree of differential equation** : Is defined as the highest power of the highest order derivative involved in the given differential equation.

- **Solution of a differential equation** :
    1. General Solution – solutions which contains as many arbitrary constants as the order of the differential equation .
    2. Particular Solution – solutions free from arbitrary constants.

- **Formation of a differential equation :** To form a differential equation from a given function differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

- **Methods of solving first order ,first degree differential equations :**

    1. Variable Seperable Method: is used to solve such an equation in which variables can be separated completely i.e. terms containing y should remain with dy and terms containing x with dx.

    2. Homogeneous Differential Equations : A differential equation that can be expressed in the form $\frac{dy}{dx} = f(x, y)$ or $\frac{dx}{dy} = g(x, y)$ where f(x,y) & g(x,y) are homogeneous functions of degree zero is called homogeneous differential equation.

    3. **Linear Differential Equation :** A differential equation of the form $\frac{dy}{dx} + Py = Q$ or $\frac{dx}{dy} + Px = Q$ where P & Q are functions of x or functions of y or constants is called a first order linear differential equation.

DIFFERENTIAL EQUATION

1 Mark

I. Write the order and degree of the following differential equation

a) $x^2\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2+3=0$

b) $y^2\left(\frac{d^3y}{dx^3}\right)^2-8\left(\frac{dy}{dx}\right)^4+7x=0$

c) $y-x\frac{dy}{dx}+\sqrt{1+\frac{dy}{dx}}=0$

d) $\frac{d^3y}{dx^3}-\sin\left(\frac{dy}{dx}\right)+8\frac{dy}{dx}=3$

e) $\left[2-\left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{2}}=3x\frac{d^3y}{dx^3}$

2.Form the differential equation of the family of parabolas $y^2=4ax$,where 'a' is an arbitrary constants.

3. Form the differential equation representing the family of curves y= A cos(x+b) where A & B are parameters.

4.Form the differential equation of all the circles which pass through the origin and whose centers lie on y-axis.

5.Write the integrating factor of the following differential equation

a) $\cos^2x\, dy = (x-y\sin x)dx$ (b) $x\, dy+ y\, dx= x\cot x\, dx$

6. Write the integrating factor of the following differential equation :

$$(1+y^2) + (2xy-\cot y)\frac{dy}{dx} = 0$$

7. Write the sum of the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0.$$

8. Write the solution of the differential equation
$$\frac{dy}{dx} = 2^{-y}.$$

9. Write the sum of the order and degree of the following differential equation :
$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$$

10. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves $xy = C \cos x$.

11. Write the sum of the order and degree of the differential equation
$$1 + \left(\frac{dy}{dx}\right)^4 = 7\left(\frac{d^2y}{dx^2}\right)^3$$

<u>4 Marks</u>

12.Solve $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

13. **Solve $(1+e^{2x})dy + (1+y^2)\,e^x dx = 0$; given that x=0 , y=1**

14. **Solve** $\frac{dy}{dx} = \frac{1}{(x+y)^2}$

15. **Solve y dx +xlog** $(\frac{y}{x})dy - 2xdy = 0$

16. **Solve** $(x\sqrt{x^2+y^2} - y^2)dx + xydy = 0.$

17. **Solve** $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

18. **Solve** $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$

19. **Solve** $\left(2x-10y^3\right)\frac{dy}{dx} + y = 0$

20. **Solve** $\sqrt{1-y^2}\,dx = (\sin^{-1}y - x)dy$ **; given that y(0)=0**

21. **Solve** $x\frac{dy}{dx} - y = (x+1)e^{-x}$ **; given that y(1)=0**

22. **Solve the differential equation** (x+ $2y^2$)dy/dx =y,given that x=2 and y=1.If x denotes the % of people and y denotes the % of people who are polite and y denotes the % of people who are intelligent .Find x when y = 2%.**A polite child is always liked by all in society.Do you agree ?Justify your answer?**

23. dy/dx + y/x =0 , where x denotes the percentage population living in a city & y denotes the area for healthy life of population. Find the particular solution when x=100 and y =1.**Is higher density of population harmful? Justify your answers.**

<u>6 Marks</u>

24. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

25. Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that x = 1 when y = 0.

25. Solve the following differential equation :

$$\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$$

26. Solve the following differential equation :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\, dy = 0$$

27. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

28. Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$, given that x = 1 when y = 0.

29. Find the particular solution of the differential equation

$$x\frac{dy}{dx} + y - x + xy\cot x = 0;\ x \neq 0, \text{ given that when } x = \frac{\pi}{2},\ y = 0$$

30. Solve the differential equation $x^2\, dy + (xy + y^2)\, dx = 0$ given y = 1, when $x = 1$

Answer Key

Qu.nos	Answers
1a	Order=2 ;degree=1
b	Order=3; degree=2
c	Order=1 ; degree =2
d	Order=3;degree=not defined
e	Order=3; degree=2
2	$2xy_1=y$
3	$y_2+y=0$
4	$(x^2-y^2)y_1=2xy$
5	$e^{\sec x}$; e^x
6	$xe^x-e^x=\sqrt{1-y^2}+c$
7	$e^xy=1$
8	$y-\tan^{-1}(x+y)=c$
9	$cy=\log\left\|\frac{y}{x}\right\|-1$
10	$\sqrt{x^2+y^2}=x\log\left\|\frac{c}{x}\right\|$
11	$x+ye^{\frac{x}{y}}=c$
12	$2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$
13	$x=2y^3+cy^{-2}$
14	$x-\sin^{-1}y+1=e^{\sin^{-1}y}$
15	Y=x-1-logx
16	X= 2y (8) Yes polite child has a peaceful mind and peaceful mind grasps the ideas easily and understand the complicated concept2
17	xy =100 Yes, as the population increases area for living decreases, that is very it is harmful for us.

(2 Marks)

1. Verify that y = Acosx –Bsinx is a solution of the differential equation $\frac{d^2y}{dx^2}$ +y = 0
2. Form the differential equation satisfied by the equation y = a sin (a+b), where a and b are arbitrary constants.
3. Form the differential equation of the family of straight line y = mx +c, where m and c are arbitrary constants,.
4. Find the differential equation of all the circles which touch the y-axis at the origin.
5. Find the differential equation of the family of all circles in second quadrant and touching the coordinate axis.

Scoring Key

2.	$\frac{d^2y}{dx^2}$ +y = 0
3.	x$\frac{dx}{dy}$ = y
4.	y^2-x^2= 2xyy_1
5.	$(x+y)^2$.[y'^2 +1] = $(x+ yy')^2$

10.VECTORS

1. A directed line segment represents a vector with its direction representing direction of vector and its length being proportional to the magnitude of the vector.

2. A vector with magnitude 1 is called a unit vector.

3. Vectors along same line are called *collinear* vectors. Vectors with same magnitude and direction are called equal vectors.

4. Vector having same magnitude as $\vec{a}$ but opposite in direction is called negative of vector $\vec{a}$.

5. Vectors having same direction are called like vectors and having opposite direction are called unlike vectors.

6. If O is taken as origin of reference, then position vector of any point p is vector $\overrightarrow{OP}$.

7. If the position vectors of end points of a line segment are $\vec{a}$ and $\vec{b}$ then the position vector of a point dividing it in the ratio m: n is $\frac{a\vec{b} + n\vec{a}}{m+n}$.

8. Cosine of angle, made by a vector with x - axis y-axis and z-axis are called direction cosines of the vector.

9. Scalar or dot product of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

10. If $\vec{a}\,.\,\vec{b} = \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$ and conversely if $\vec{a} \perp \vec{b}$ then $\vec{a}.\vec{b}$=0.

11. $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (commutative property)

12. $\vec{a}.\left(\vec{a} + \vec{c}\right) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ (Distributive property)

13. $\vec{a}.\vec{a} = \left|\vec{a}\right|^2$

14. Vector or cross product of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta.\hat{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

15. $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|.|\vec{b}|}$ and $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

16. $a\times b \neq b\times a$

17. $\vec{a}\times\vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \parallel \vec{b}$ than $\vec{a}\times\vec{b} = 0$.

18. If $\vec{a}$ and $\vec{b}$ represent adjacent sides of $\|^{gm}$ then area of $\|^{gm}$ is given by $|\vec{a}\times\vec{b}|$.

19. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

20. A unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is represented by $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\hat{b}|}$

Assignments

1 Mark

1. Find the value of λ so that the vectors a=2i+λ j +k and b=i+-2j +3k are perpendicular to each other.
2. Find the area of a parallelogram having diagonals 3i+j-2k and i-3j+4k.
3. Find $|a-b|$, two vectors a and b are such that $|a|$=2, $|b|$=3 and a.b=4
4. Find $|axb|$,if a=2i+j+3k and b=3i+5j-2k.
5. Find a vector in the direction of the vector a=(3i+j) that has magnitude 5 units
6. Find the projection of the vector i-j on the vector i+j.
7. Find the angle between the vectors a and b when a.b = 6,1a1=3 1b1=4
8. If $\vec{a}$ and $\vec{b}$ are unit Vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\sqrt{2}\vec{a} - \vec{b}$ is a unit vector ?
9. Find a Vector in the direction of $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.
10. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

2 Marks

11. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c}$ = o and $|\vec{a}|$ = 3, $|\vec{b}|$ = 5 and $|\vec{c}|$ = 7, then find the angle between $\vec{a}$ and $\vec{b}$.

12. If a=5i-j+7k and b=i-j- λ k, Find the value of λ for which a + b and a-b are orthogonal.

13. Find the angle between the vectors i-2j+3k and 3i-2j+k
14. Show that the area of a parallelogram having diagonals 3i+j-2k and i-3j+4k is $5\sqrt{3}$
15. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} + \hat{b} + \hat{c} = 0$, then find the value of $(\hat{a}.\hat{b} + \hat{b}.\hat{c} + \hat{c}.\hat{a})$.
16. Find $|\vec{x}|$ if $(\vec{x} - \hat{a}).(\vec{x} + \hat{a}) = 12$
17. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors and θ is the angle between them, then prove that $|\vec{a} \times \vec{b}| = (\vec{a}.\vec{b})\tan\theta$.
18. Find x such that the four points A(4,1,2) , B(5,x,6),C(5,1,-1) and D(7,4,0) are coplanar.
19. If $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then find $|\vec{b}|$.

20. Find the volume of the paralleopiped whose edges are represented by $\vec{a} = 2\hat{\imath}-3\hat{\jmath}+4\hat{k}$, $\vec{b} = \hat{\imath}+2\hat{\jmath} - \hat{k}$ and $\vec{c} = 3\hat{\imath}-\hat{\jmath}+2\hat{k}$,

4 Marks

21 Show that the points whose position vectors are 5i+6j+7k, 7i-8j+9k and 3i+20j+5k are collinear.
22.If a= i+j+k and b= j-k. find a vector c such that axc=b and a.c=3
23 Find a unit vector perpendicular to each of the vectors (a+b) and (a-b), where a=i +j+ k and b =i+2j+3k..
24 .Find the area of the parallelogram whose adjacent sides are given by the vectors a=3i+j+4k and b=i-j+k .
25. Let a,b be the three vectors such that 1a1=3,1b1=4 1c1=5 and each is perpendicular to the other two, find $|a+b+c|$

26 Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ respectively are coplanar.

27. Prove that $[\vec{a}, \vec{b} + \vec{c}, \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$

6 Marks

28. Let a=i-j , b=3j-k and c=7i-k.Find the vector a such that it is perpendicular to both a and b, and c.d=1.

29.Using vector method, find the area of the triangle whose vertices are A(1,1,1) B(1,2,3) and C(2,3,1).

30. Find a vector whose magnitude is 3 and which is perpendicular to each of the vectors a=3i+j-4k and b=6i+5j-2k
31. If a and b are unit vectors and θ is the angle between them, Prove that $\tan\frac{\theta}{2} = \left|\frac{a-b}{a+b}\right|$
32. If a=3i-j and b=2i+2j-3k, Express b as a sum of b_1 and b_2 where b_1 is parallel to a and b_2 is perpendicular to a.

SCORING KEY:

1.	5/2
2.	$5\sqrt{3}$
3.	$\sqrt{5}$
4.	$\sqrt{507}$
5	$1/\sqrt{10}(15i+5j)$
6.	0
7.	60
8	45
9	7/5(i-2j)
10.	-3/2
11.	60
12	$\sqrt{73}$
13	$Cos^{-1}(5/7)$
15	-3/2
16	$\sqrt{13}$
18	4
19	3
20	7 cubic units
22.	1/3(5i+2j+2k)
23	$-(i-2j+k)/\sqrt{6}$
24	$\sqrt{42}$
25	$5\sqrt{2}$
28	¼(i+j+3k)
29	$\sqrt{21}$
30	2i-2j+k
32.	b_1=3/2i-1/2j b_2=1/2(i+3j-6k)

THREE DIMENSIONAL GEOMETRY
SYNOPSIS

- **SKEW LINES:** are lines in space which are neither parallel nor intersecting. They lie in different planes.

- **ANGLE BETWEEN TWO LINES :**
  1. $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$ where l_1, m_1, n_1 & l_2, m_2, n_2 are direction cosines of the lines.
  2. $\cos\theta = \left|\dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$ where a_1, b_1, c_1 & a_2, b_2, c_2 are direction ratios.

- **EQUATIONS OF LINE :**
  1. Vector Equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$
  2. Equation of a line through a point (x_1, y_1, z_1) and having d.c's l,m,n is $\dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n}$
  3. Vector equation of a line which passes through two points whose position vectors are $\vec{a}$ & $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
  4. Cartesian equation of a line which passes through two points (x_1, y_1, z_1) & (x_2, y_2, z_2) is $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

- If θ is the acute angle between $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ & $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ then $\cos\theta = \left|\dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

- **SHORTEST DISTANCE :** between two skew lines is the line segment perpendicular to both the lines. Shortest distance between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ & $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is
$$\left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

- **DISTANCE BETWEEN TWO PARALLEL LINES :** $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ & $\vec{r} = \vec{a}_2 + \mu \vec{b}$ is

$$\left| \frac{\vec{b} \times . \left(\vec{a}_2 - \vec{a}_1 \right)}{\left| \vec{b} \right|} \right|$$

- **EQUATIONS OF PLANE :**
  1. In the vector form , equation of a plane which is at a distance d from the origin, and n is a unit vector normal to the plane through the origin is $\vec{r} \cdot \hat{n} = d$
  2. The equation of plane through a point whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ *is* $\left(\vec{r} - \vec{a} \right) . \vec{N} = 0$ and Cartesian form is $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$
  3. Equation of a plane through three non collinear points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ & (x_3, y_3, z_3) *is* $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$, which is in Cartesian form. Equation in vector form $\left(\vec{r} - \vec{a} \right) . \left[\left(\vec{b} - \vec{a} \right) \times \left(\vec{c} - \vec{a} \right) \right] = 0$ where $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points.

- Vector equation of a plane that passes through the intersection planes $\vec{r} . \vec{n}_1 = d_1$ & $\vec{r} . \vec{n} = d_2$ is $\vec{r} . \left(\vec{n}_1 + \lambda \vec{n}_2 \right) = d_1 + \lambda d_2$ where λ is any non zero constant.
- Cartesian equation of a plane that passes through the intersection of two given planes $A_1 x + B_1 y + C_1 z + D_1 = 0$ & $A_2 x + B_2 y + C_2 z + D_2 = 0$ is $(A_1 x + B_1 y + C_1 z + D_1) + \lambda (A_2 x + B_2 y + C_2 z + D_2) = 0$
- Two planes are coplanar if $\left(\vec{a}_2 - \vec{a}_1 \right) . \left(\vec{b}_1 \times \vec{b}_2 \right) = 0$ where two planes are $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ & $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$
- **ANGLE BETWEEN TWO PLANES :** If two planes are $\vec{r} . \vec{n}_1 = d_1$ & $\vec{r} . \vec{n} = d_2$ then angle is

$$\cos\theta = \left| \frac{\vec{n}_1 . \vec{n}_2}{\left| \vec{n}_1 \right| \left| \vec{n}_2 \right|} \right|$$

1 MARK

1. The direction ratios of the vector are -18, 12, - 4. Find the direction cosines.
2. Find k if the lines $\frac{x-1}{3}=\frac{y+1}{2k}=\frac{z}{3}\ and\ \frac{x+5}{-2}=\frac{y}{4}=\frac{z-3}{k}$ are perpendicular
3. The equations of the line are $5x-3=15y+7=3-10z$.Write the direction cosines of the line
4. If a line makes angles 90^0 , 60^0 and θ with x, y and z axis respectively, where θ is acute, find θ.
5. If a line makes angles α, β, γ with the axes, prove that$sin^2\alpha+sin^2\beta+sin^2\gamma=2$.
6. If the Cartesian equation of the line is$\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$, write the vector equation of the line.
7. Find the angle between the lines $2x=3y=-z\ and\ 6x=\ -y=\ -4z$.
8. Write the direction cosines of normal to the plane 3x + 4y +12z = 52.
9. Write the intercepts cut off by the plane 2x + y – z = 5 on x-axis.
10. Find the distance of the plane 3x – 4y+12z = 3 from the origin.
11. Write the Cartesian equation of the plane, bisecting the line segment joining the points A (2, 3, 5) and B(4,5,7) at right angles.
12. Write the vector equation of the plane, passing through the point (a, b, c) and parallel to the plane

ANSWERS

1	-9/11,6/11,2/11
2	k=6/11
3	6/7,2/7,3/7
4	30^0
5	proof
6	$\vec{r}=3\hat{\imath}-4\hat{\jmath}+3\hat{k}+\lambda(-5\hat{\imath}+7\hat{\jmath}+2\hat{k})$
7	90^0
8	3/13,4/13,12/13
9	5/2
10	3/13
11	(x + y + z=13
12	$\vec{r}.\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)=2.(\vec{r}.\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)=a+b$ +c

2 MARKS

1. Find the equation of a line which is parallel to the vector $3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ and which passes through the point (1,-2, 3).
2. Find the equation of a plane which bisects perpendicularly the line joining the points A(2,3,4) and B(4,5,8) at right angles.
3. If the line drawn from the point (-2,-1,-3) meets a plane at right angle at the point (1,-3,3) find the equation of the plane.
4. O is the origin and A is (a, b, c). Find the direction cosines of the line OA and the equation of plane through A at right angles to OA.
5. Find the angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$.
6. Find the coordinates of the point where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane $x + y + 4z = 6$.
7. Find the equation of a plane which is at a distance of 3√3 units from origin and the normal to which is equally inclined to the coordinate axis.
8. If a plane meets the coordinate axes in points A, B, C and the centroid of the triangle ABC is(α, β, γ), find the equation of the plane.
9. For what values of p and q will the line joining the points A(3,2,5) and B(p,5,0) be parallel to the line joining the points C(1,3,q) and D(6,4,-1).
10. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the points P (1, 3, 3).

ANSWERS

Q.NO.	ANSWERS
1	$\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z-3}{6}$
2	$x + y + 2z = 19$
3	$3x - 2y + 6z = 27$
4	$ax + by + cz = a^2 + b^2 + c^2$
5	$sin\theta = \frac{\sqrt{2}}{10}$
6	(1, 1, 1)
7	$x + y + z = \pm 9$
8	$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$
9	P=18, q=2/3.
10	(-2,-1,3) OR (4,3,7)

4 MARKS

1. Find k if the lines $\frac{1-x}{3}=\frac{y-2}{2k}=\frac{z-3}{2}\ and\ \frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular.
2. Find the foot of the perpendicular from the point P(7,14,5) to the plane $2x+4y-z=2$.
3. Find the image of the point (1,2,3) in the plane $x+2y+4z-38=0$
4. Show that the lines$\frac{x-1}{3}=\frac{y-1}{-1},z+1=0\ and\ \frac{x-4}{2}=\frac{z+1}{3},y=0$, intersect each other and also find the point of intersection.
5. Determine the Cartesian equation of the line passing through the point (-1, 3,-2) and perpendicular to the lines$\frac{x-2}{1}=\frac{y}{2}=\frac{z+3}{3}\ and\ \frac{x-2}{-3}=\frac{y+7}{2}=\frac{z}{5}$.
6. Find the shortest distance between the lines $x+1=2y=-12z\ and\ x=y+2=6z-6$.
7. Find the equation of the plne through the points (2, 1,-1) and (-1, 3, 4) and perpendicular to the plane x – 2 + 4z = 10.
8. Find the equation of the plane which passes through the point (3, 2, 0) and contains the line$\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$.
9. From the point P (a, b, c) perpendiculars PL and PM are drawn to YZ-plane and ZX-plane respectively. Find the equation of the plane OLM.
10. Show that the points (0,-1,0),(1,1,1,),(3,3,0)and (0,1,3) are coplanar. Also, find the plane containing them.
11. Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0, x+y+z-2=0$ and the point (2,2,1)
12. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3\ and\ 5x-3y+4z+9=0$ and parallel to the ine $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.
13. Show that the lines $\vec{r}=\hat{\imath}+\hat{\jmath}+\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})and\ \vec{r}=4\hat{\jmath}+2\hat{k}+\mu(2\hat{\imath}-\hat{\jmath}+3\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.
14. Considering the earth as a plane having equation 5x+9y-10z+138=0, a monument is standing vertically such that its peak is at the point (1, 2-3). Find the height of the monument. How can we save our monuments?
15. Let the point P (5, 9, 3) lie on the top of Qutab Minar, Delhi. Find the image of the point on the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Do you think that the conservation of monuments is important and why?
16. Find the equation of the plane passing through the pins (-1, 2, 0), (2, 2,-1) and parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{1}$.

17. Find the equation of the plane passing through the line of intersection of the planes having equations x + 2y + 3z – 5 = 0 and 3x - 2y – z + 1=0 and cutting off equal intercepts on the X and Z axes.
18. Find the equation of the plane through the points (0, 4,-3) and (6.-4, 3), if the sum of their intercepts on the three axes is zero.

ANSWERS

1	k=-2
2	(1,2,8)
3	(3,5,11)
4	(4,0,-1)
5	$\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$
6	2units
7	18x + 17y + 4z – 49 = 0
8	x-y+z-1=0
9	$bcx+acy-abz=0$
10	4x-3y+2z=3
11	7x-5y+4z-8=0
12	7x+9y-10z-27=0
13	-2x-y+z+2=0
14	$\frac{191}{(206)^{\frac{1}{2}}}$
15	(3,5,7).Conservation of monuments is very important because it is a part o out history and their contribution,
16	x+2y+3z=3
17	5x + 2y + 5z -9=0
18	x/3 – y/2 - z = 1

6MARKS

1. Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$
2. Find the coordinates of the point where the line through the points A(3,4,1) and B(5,1,6) crosses the plane determined by the points P(2,1,2),Q(3,1,0) and R(4,-2,1).
3. Find the equation of the plane that contains the point (1,-1, 2) and is perpendicular to both the planes 2x+3y-2z=5 and x+2y-3z=8. Hence, find the distance of the point P (-2, 5, 5) from the plane obtained above.
4. Find the equation of the plane passing through the point (-1,3,2) and perpendicular to each of the planes x+2y+3z=5 and 3x+3y+z=0.
5. Find the equation of a plane passing through he point P(6,5,9) and parallel to the plane determined by the points A(3,-1,2), B(5,2,4) and C(-1,-1,6). Also, find the distance of this plane from the point A.
6. The coordinates of the foot of the perpendicular drawn from the origin in a plane are (4,-2,-5).Find the equation of the plane.
7. Find the distance of the point (-1,-5,-10) from the plane x –y+ z =5 measured parallel to the line$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$
8. Find the image of the point having position vector$\hat{\imath}+3\hat{\jmath}+4\hat{k}$ in the plane $\vec{r}.\left(2\hat{\imath}-\hat{\jmath}+\hat{k}\right)=3=0$
9. Show that the lines $\frac{x-1}{2}=\frac{y-3}{4}=-z\ and\ \frac{x-4}{3}=\frac{1-y}{2}=z-1$ are coplanar and also find the equation of the plane containing these lines.

ANSWERS

Q.No.	Answers
1	17/2units
2	(5/3,6,-7/3)
3	5x+4y+z+7=0,√42 units
4	7x-8y+3z+25=0
5	3x-4y+3z=25,6/√34 units
6	4x – 2y – 5z = 45
7	13 units
8	(-3,5,2)
9	2x-5y-16z+13=0

LINEAR PROGRAMMING
Synopsis

- A linear programming problem is concerned with finding the optimal value(maximum or minimum)of a linear function of several variables(called objective function) subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities(called linear **constraints**)Variables are sometimes called **decision variables** and are **non negative**.
- A few important linear programming problems are:
  - (i) Diet problem
  - (ii) Manufacturing problems
  - (iii) Transportation problems
- The common region determined by all the constraints including the non negative constraints of a linear programming problem is called the feasible region of the problem.
- Points within and on the boundary of the **feasible** region represent feasible solutions of the constraints.
- Any point outside the feasible region is an **infeasible solution**.
- Any point within the feasible region that gives the optimal value is called an **optimal solution.**
- **Theorem 1:** Let **R** be a feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. The optimal value of Z under the constraints given will occur at a corner point of the feasible region**.**
- **Theorem 2:** Let **R** be a feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. If **R** is bounded then the objective function has both a maximum and a minimum value on **R** and each of these occurs at the corner point of **R**.
- If the feasible region is unbounded then the maximum or minimum may not exis**t.** If it occurs it must occur at a corner point of **R**.
- Corner Point Method for solving a linear programming problem. The method comprises of the following steps:
  - (i) Find the feasible region of the linear programming problem and determine its corner points.
  - (ii) Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m respectively be the largest and the smallest values at these points.
  - (iii) If the region is bounded M and m respectively are the maximum and minimum values of the objective function.
- If two points of the feasible region are both optimal solutions of the same type then any point on the line segment joining these points is also an optimal solution of the same type.

LINEAR PROGRAMMING

6 Marks

1. A company produces two types of products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit on each unit of A is Rs.20 And on B is Rs.15. How many units of A and B should be produced to maximize the profit? Form an L.P.P and solve it graphically.

2. A man owns a field of areas 1000 sq.m . He wants to plant fruit trees in it. He has a sum of Rs.1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq.m of ground per tree and costs Rs.20 sq.m of ground per tree and costs Rs.25 per tree. When full grown, a type a tree produces an average of 20kg, of fruit which can be sold at a profit of Ts.2 per kg and a type B tree produces an average of 40kg of fruit which can be sold at a profit of Rs.1.50 per kg. How many of each type should be planted to achieve maximum profit when trees are fully grown?. What is the maximum profit?.

3. A gardener has a supply of fertilizers of the type I which consists of 10% nitrogen and 6% phosphoric acid, and of the type II which consists of 5% nitrogen and 10%phosphoric acid. After testing the soil conditions, he finds that he needs at least 14kg of nitrogen and 14kg of phosphoric acid for his crop. If the type-I fertilizer costs Rs . 2 per kg and the type-II fertilizer costs Rs.3 per kg, determine how many kgs of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost. What is the minimum cost? **What are the side effects of using excessive fertilizers?**

4. A dietician wishes to mix two types of food, X and Y in such a way that the vitamin contents of the mixture contains atleast 8 units/kg of vitamin A and 1 unit /kg of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 5 per kg to purchase the food X and Rs 7 per kg to purchase the food Y. Determine the minimum cost of such a mixture. **Why a person should take balanced food?**

5. A diet for a sick person must contain at east 4000 units of vitamin, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs.4 and Rs.3 respectively . One unit of food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost satisfying the requirements.

6. If a young man drives his vehicle at 25 km/hr, he has to spend Rs.2/km on petrol. If he drives it at a faster speed of 40 km/hr, the petrol cost increases to Rs.5/km.He has Rs.100 to spend on petrol and travel within one hour. Express this as an L.P.P and solve.

7. An aero plane can carry a maximum of 200 passengers. A profit of Rs.400 is made on each executive class ticket and a profit of Rs.300 is made on each economy class ticket. The airline

reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many buy tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

8. Anil wants to invest atmost Rs. 12,000 in bond A and B . According to the rules , he has to invest atleast Rs.2000 in bond A and atleast Rs. 4000 in bond B If the rate of interest of bond A is 8% per annum and on bond B is 10% per annum, how should he invest his money for maximum interest ?

9. An oil company requires 13,000 20,000 and 15,000 barrels of high grade medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs.400 per day and B costs Rs.300 per day to operate, how many days should each be run to minimize the cost of requirement?

10. A medicine company has factories at two places A and B. From these places, supply is made to each of its three agencies situate at P, Q and R. The monthly requirements of the agencies are respectively 40,40and 50 packets of the medicines, while the production capacity of the factories at A and B are 60 and 70 packets respectively. The transportation costly per packet from the factories to the agencies is given.

Transportation cost per packet in Rs		
To/From	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency so that the cost transportation is minimum? Also find the minimum cost.

11.If a class XII student aged 17 years ,rides his motor cycle at 40km/hr, the petrol cost is Rs 2per km.If he rides at a speed of 70km/hr, the petrol cost increases Rs. 7 km/hr.He has Rs 100 to spend on petrol and wishes to cover maximum distance within one hour. Express it as an L.P.P and solve it graphically. **What is benefit of driving at an economical speed ?Should a child below 18 years allowed to drive a motor cycle ? Give reasons?**

12. An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but there are 4500 bottles into which either of the medicine can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B and there are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan his production to cover maximum patients? **How can you help others in case of natural disaster.**

13. A company manufactures three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is ₹ 12,000 and of factory II is ₹ 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost ? Formulate this problem as an LPP and solve it graphically.

14. A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of ₹ 24 per package on nuts and ₹ 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 10 hours a day. Make an L.P.P. from above and solve it graphically ?

Scoring Key

1.	Product A - 25 units , Product B - 125 units , Maximum profit - Rs.2375
2.	Type A - 20 trees , Type B - 40 trees , Maximum profit –Rs.2200
3.	Type I – 100 kgs , Type II – 80 kgs , Minimum Cost – Rs.92 Excessive use of fertilizers can spoil the quality of crop also it may cause infertility of land.
4.	Food X – 2kg , Food Y – 4kg , Minimum Cost – Rs.38 Balanced diet gives fit, healthy and disease free life to a person.
5.	Food X – 5units , Food Y – 30 units , Minimum Cost – Rs. 110
6.	At 25 km/hr – 50/3 km , At 40 km/hr – 40/3 km , Maximum distance – 30 km
7.	Executive class – 40 , Economy class – 160 , Maximum Profit – Rs. 64000
8.	Bond A – Rs. 2000 , Bond B – Rs.10000
9.	Refinery A – 57 days , Refinery B – 37 days
10.	From A – 10 packets, 0 packets , 50 packets to P ,Q, R respectively From B - 30 pac kets , 40 packets , 0 packets to P , Q , R respectively. Minimum Cost – Rs. 400
11.	$Z=X+Y, x/40 + y/70 \leq 1$ $2x+7y\leq 100$ $x\geq 0, y\geq 0$ 1.X=1560/41 km and y= 140/41 km 2. It Saves petrol. It saves money.

	3. No because according to the law driving license is issued when a person is above the 18 years of age
12.	10500 bottles of medicine A and 34500 bottles of medicine B and they can cover 325500 patients. We should not get panic and should not create panic in case of natural disaster. Must have the helpline numbers of government agencies and NGO working in case of Natural Disaster.