微積分期末考 F74094083 資訊系113 林欣誠

$A=\frac{1}{2}$, $B=-2$

1.

let $u=2x$, $v=\frac{y}{2}$

$\Rightarrow u^2+v^2=1$

$J=\begin{vmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{vmatrix}=1$

$\Rightarrow \iint_{S} \frac{u^2}{4}\cdot 1\cdot du\,dv$, $0\le r\le 1$, $0\le\theta\le 2\pi$

$=\int_0^{2\pi}\int_0^1 \frac{1}{4}(r^2\cos^2\theta)\cdot r\,dr\,d\theta$

$=\frac{1}{4}\int_0^{2\pi} \frac{1}{4}r^4\Big|_0^1 \cos^2\theta\,d\theta$

$=\frac{1}{16}\int_0^{2\pi}\frac{1+\cos2\theta}{2}d\theta$

$=\frac{1}{32}(\theta+\sin2\theta)\Big|_0^{2\pi}$

$=\frac{1}{32}(2\pi+0-0-0)$

$=\frac{\pi}{16}$

2.

$P(x,y)=9+6xy$, $Q(x,y)=3x^2-9y^3$

$\frac{\partial P}{\partial y}=6x$, $\frac{\partial Q}{\partial x}=6x$

$\because \frac{\partial P}{\partial y}=6x=\frac{\partial Q}{\partial x}$

$\therefore$ F is conservative

$f=\int f_y\,dy=3x^2y-\frac{9}{4}y^4+g(x)$
$+$ constant of y

$9+6xy=f_x=\frac{\partial}{\partial x}(3x^2y-\frac{9}{4}y^4+g(x))$

$=6xy+g'(x)$

$g'(x)=9 \Rightarrow g(x)=9x+C$

$\therefore f(x,y)=3x^3y-\frac{9}{4}y^4+9x+C$

are potential functions

7.

$1\le r\le\sqrt{17}$, $\frac{\pi}{2}\le\theta\le\frac{3}{2}\pi$

$\Rightarrow \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\int_1^{\sqrt{17}}(\frac{1}{2}r\cos\theta-4r^2\sin^2\theta)r\,dr\,d\theta$

$=\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}(\frac{1}{6}r^3\cos\theta-r^4\sin^2\theta)\Big|_1^{\sqrt{17}}d\theta$

$=\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}(\frac{17}{6}\sqrt{17}\cos\theta-289\sin^2\theta-\frac{1}{6}\cos\theta+\sin^2\theta)d\theta$

$=\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\frac{1}{6}(17\sqrt{17}-1)\cos\theta\,d\theta-\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}(289-1)\sin^2\theta\,d\theta$

$=\frac{17\sqrt{17}-1}{6}\cdot\sin\theta\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi}-288\cdot\frac{1-\cos2\theta}{2}\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi}$

$=\frac{17\sqrt{17}-1}{6}\cdot(-2)-144(1-(-1)-1+(-1))$

$=-\frac{17\sqrt{17}-1}{3}$

6.

$\int_C(2y+\cos x^2)dx+(x+e^{\sqrt{y}})dy$

$=\iint_D(1-2)dA=-\iint_D 1\,dA$

$y=x^2\Rightarrow x=x^4\Rightarrow x=0,1$

$\Rightarrow -\int_0^1\int_{x^2}^{\sqrt{x}}1\,dy\,dx$

$=-\int_0^1(\sqrt{x}-x^2)dx$

$=-(\frac{2}{3}x^{\frac{3}{2}}-\frac{x^3}{3})\Big|_0^1 dx$

$=-\frac{1}{3}$

5.

$$\begin{cases} x = r\cos\theta\sin\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\phi \end{cases}, \quad 0 \le r \le 2,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi$$

$$\iiint_\Omega (3x^2+y^2)\sqrt{x^2+y^2+z^2}\,dx\,dy\,dz$$

$$= \int_0^{\pi}\int_0^{2\pi}\int_0^{2} [3(r\cos\theta\sin\phi)^2 + (r\sin\theta\sin\phi)^2]\overbrace{\sqrt{(r\cos\theta\sin\phi)^2 + (r\sin\theta\sin\phi)^2 + (r\cos\phi)^2}}^{r} \cdot r^2\sin\phi\,dr\,d\theta\,d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi}\int_0^{2} (2r^2\cos^2\theta\sin^2\phi + r^2\sin^2\phi)\,r^3\sin\phi\,dr\,d\theta\,d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \left(\frac{1}{3}r^6\cos^2\theta\sin^3\phi + \frac{1}{6}r^6\sin^3\phi\right)\Big|_0^2\,d\theta\,d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \left(\frac{64}{3}\cos^2\theta\sin^3\phi + \frac{32}{3}\sin^3\phi\right)d\theta\,d\phi$$

$$= \int_0^{\pi} \frac{32}{3}\sin^3\phi\,(2\theta + \sin 2\theta)\Big|_0^{2\pi}\,d\phi$$

$$= \int_0^{\pi} \frac{128}{3}\pi\sin^3\phi\,d\phi$$

$$= \frac{128}{3}\pi\int_0^{\pi} (1-\cos^2\phi)\sin\phi\,d\phi, \quad u = \cos\phi,\ du = -\sin\phi\,d\phi$$

$$= \frac{128}{3}\pi\left(-\int_1^{-1}(1-u^2)\,du\right)$$

$$= \frac{128}{3}\pi \cdot \left(u - \frac{u^3}{3}\right)\Big|_{-1}^{1}$$

$$= \frac{512}{9}\pi$$