1.

1.

$$\log(n!) = \log 1 + \log 2 + \cdots + \log n$$

$$\log 4^{\log n} = \log n \, (\log 4) = 2 \log n$$
$$= \log n^2$$

$n^2 > \log n!$

$\therefore a > b$

又. $d > b$

$n! > n^2$

$\Rightarrow d > a$

$\Rightarrow d > a > b$

$2^n > n^2$ $\because \log n > 4$

$\therefore c > a$ $\Rightarrow f > a$

令 $\log n = k$

$\log f : k \log k$

$\log c : 2^k$

$\because 2^k > k \log k$

$\Rightarrow c > f$

$n! > 2^n$

$\therefore d > c$

$\Rightarrow d > c > f > a > b > e$ $\therefore$ Ans $= 2$.

2.

2.

(a) Let $n = 2^k$, $S(k) = T(n)$

$\Rightarrow S(k) = T(2^k)$

$= 2T(\frac{k}{2}) + k$

By master theorem, $a=2$, $b=2$

$\Rightarrow \log_b a = 1 \Rightarrow k^{\log_b a} = f(k) = k$

$\Rightarrow S(k) = k \cdot \log k$

$\Rightarrow T(n) = S(\log n)$

$= \log n \cdot \log(\log n)$

(b)

By master theorem, $a = b = 2$,

$\log_b a = 1 \Rightarrow n^{\log_b a} = f(n) = n$

$\Rightarrow T(n) = n \cdot \log n$

(c)

Guess: $T(n) \leq dn^3$ for positive constant d

$$\Rightarrow T(n) = 4T(\tfrac{n}{3}) + n \leq 4(d(\tfrac{n}{3})^3) + n$$
$$= \frac{4}{27}dn^3 + n$$
$$\leq dn^3$$

$\Rightarrow T(n) = O(n^3)$

3.

3.

$$\log n! = \log 1 + \log 2 + \cdots + \log n \le n \cdot \log n$$

$$\Rightarrow \log n! = O(\log n)$$

$$\log 1 + \log 2 + \cdots + \log(\tfrac{n}{2}) + \cdots + \log n \ge \frac{n}{2} \log(\tfrac{n}{2})$$

$$\therefore \log(n!) = \Omega(n \log n)$$

4.

4.

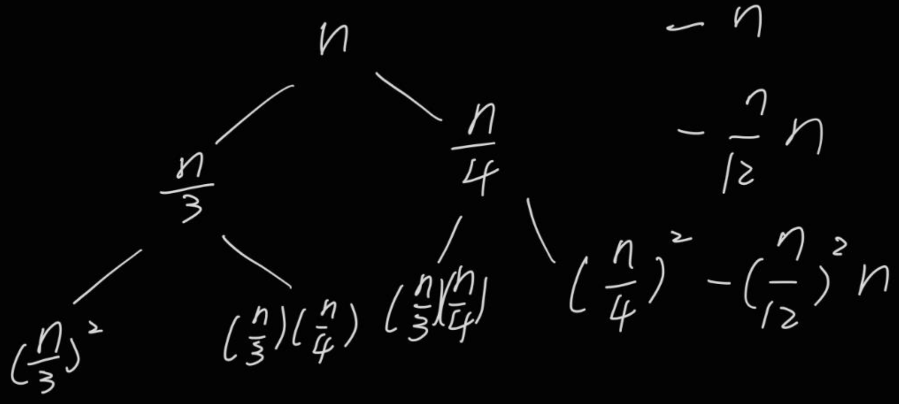
By master theorem, $a = b = 3$, $\log_b a = 1$

$$f(n) = \frac{n}{\log^2 n} < n$$

$$\Rightarrow T(n) = \Theta(n)$$

5.

5.



$$\Rightarrow T(n) = n\left(1 + \frac{7}{12} + \left(\frac{7}{12}\right)^2 + \cdots\right)$$

$$T(n) \leq n\left(1 + \frac{7}{12} + \cdots + \left(\frac{7}{12}\right)^{\log_3 n - 1}\right) \leq cn$$

$$\text{for } c \geq \left(1 + \frac{7}{12} + \cdots + \left(\frac{7}{12}\right)^{\log_3 n - 1}\right)$$

$$\Rightarrow T(n) = O(n)$$

$$T(n) \geq n\left(1 + \frac{7}{12} + \cdots + \left(\frac{7}{12}\right)^{\log_4 n - 1}\right) \geq dn$$

$$\text{for } d \leq \left(1 + \frac{7}{12} + \cdots + \left(\frac{7}{12}\right)^{\log_4 n - 1}\right)$$

$$\Rightarrow T(n) = \Omega(n)$$

$$\therefore T(n) = \Theta(n)$$