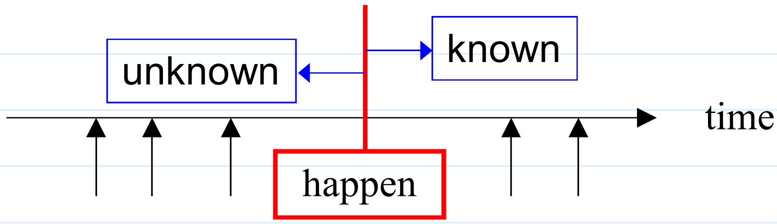
Introduction to Probability

- Uncertainty/Randomness (不確定性/隨機性) in our life
  - ➢ Many events are random in that their result is unknowable *before* the event happens.



  - ▪ Will it rain tomorrow?
  - ▪ How many wins will a player/team achieve this season?
  - ▪ What numbers will I roll on two dice?
  - ▪ **Q**: Is your height/weight measure random?
  - ➢ We often want to assess *how likely* it is the outcomes of interest occur. Probability is that measurement.

Random v.s. Deterministic Patterns

random	隨機	noise (雜訊)	uncertain result
deterministic	規律	signal (信號)	predictable result

- Consider the two cases:
  - ➢ Case I (← random pattern?)

1	2	3	4	5	6	7	8	9
●	●	●	●	●	●	●	●	●
R	R	G	R	G	R	R	R	G

  - ➢ Case II (← deterministic pattern?)

1	2	3	4	5	6	7	8	9
●	●	●	●	●	●	●	●	●
R	R	G	R	R	G	R	R	G

  - ➢ Note. #R : #G = 2 : 1

- (Possible) modeling:
  - Case I. $X_1, X_2, \ldots, X_n, \ldots$ are independent, for i=1, 2, …,

$$X_i = \begin{cases} R, & \text{with prob. } 2/3, \\ G, & \text{with prob. } 1/3. \end{cases}$$

  - Case II. For i=3, 4, …,

$$X_i = \begin{cases} R, & \text{if } (X_{i-2}, X_{i-1}) \in \{(R,G),(G,R)\}, \\ G, & \text{if } (X_{i-2}, X_{i-1}) = (R,R). \end{cases} \qquad (*)$$

- Prediction strategy:
  - Case I: always guess $X_i=R$ (why? next slide)
  - Case II: decide X_i by X_{i-1}, X_{i-2} using $(*)$

- **Q**: why always guess $X_i=R$ for Case 1?
  - Let

$$X_i = \begin{cases} R, & \text{with prob. } p, \\ G, & \text{with prob. } 1-p. \end{cases}$$

$$Y_i = \begin{cases} R, & \text{with prob. } q, \\ G, & \text{with prob. } 1-q. \end{cases}$$

  - Then,

$$\begin{aligned} P(X_i = Y_i) &= P((X_i, Y_i) \in \{(G,G),(R,R)\}) \\ &= pq + (1-p)(1-q) \\ &= 1 - p + (2p-1)\,q \end{aligned}$$

		Y_i	
		R	G
X_i	R	✓	×
	G	×	✓

  - The $P(X_i=Y_i)$ is maximized at

$$q = \begin{cases} 1, & \text{if } p > 0.5, \\ 0, & \text{if } p < 0.5. \end{cases}$$

and

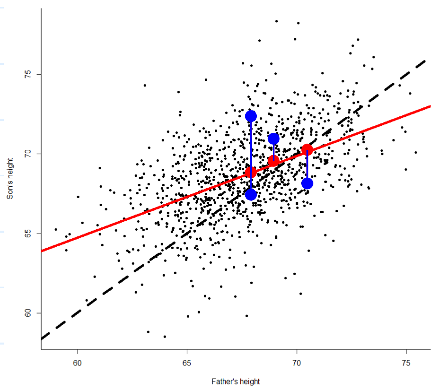
$$\max_q P(X_i = Y_i) = \begin{cases} p, & \text{if } p > 0.5, \\ 1-p, & \text{if } p < 0.5. \end{cases}$$

- **Q**: Is Case II really a deterministic pattern?
  - Under the model for Case I,

$$P(RRGRRGRRG) = \left[\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)\right]^3 = 0.325\%$$

  - Radom pattern $\longrightarrow$ Deterministic pattern
  - Deterministic pattern $\longrightarrow$ Random pattern
- System containing both random and deterministic patterns
  - Galton (1875): regression effect

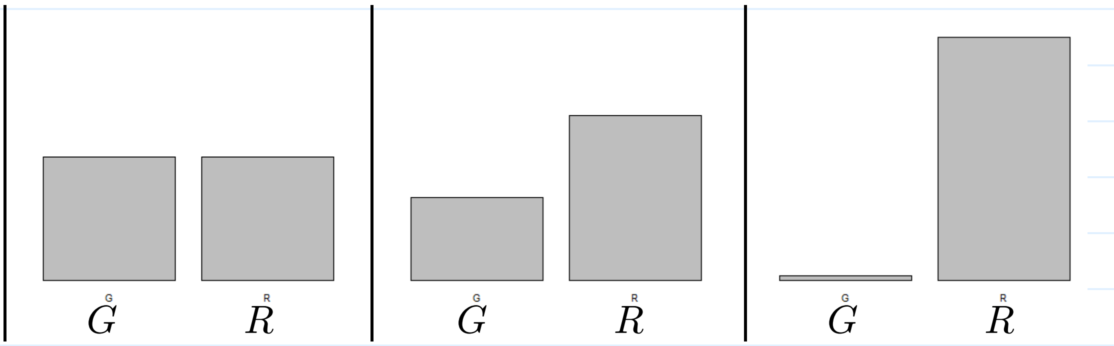
son's height



father's height

Random = totally unpredictable?

- Entropy: $-[p\log(p) + (1-p)\log(1-p)]$



$\max_q P(X_i = Y_i)$

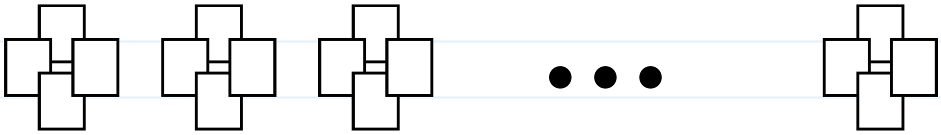
more random	more deterministic
more unpredictable	more predictable
larger entropy	smaller entropy

- Black swan effect

Should everyone have the same probability for an event?

- Example: 52 cards

	1	2	3		13



Player 1	32	24	10	• • •	deterministic
Player 2	X	X	X	• • •	random

- Conditional probability
- Subjective (Bayesian) probability:

紅樓夢的作者是曹雪芹嗎?

信者恆信，不信者恆不信

It's the Chance (Probability, Proportion, Frequency), Stupid

- Bill Clinton, 1992, Campaign slogan

It's the Economy, Stupid.

- Examples
  - 該買某保險嗎?
  - 發生飛機失事事件後，該改成開車嗎?
  - 規畫謬誤：蚊子館、該創業嗎?
  - 敍述謬誤：偉人（成功）的故事
  - 賭徒謬誤：擲筊多次未成, 則擲出聖筊機會變大?
  - 馬路三寶? 汽車保險金額, 男>女
  - 車禍先問酒駕? 酒駕易肇事, yes, 但肇事者多酒駕?
  - 屏東人：你怎麼不黑?

Distinction between Discovery (發現) and Invention (發明)

- Examples
  - 哥倫布 "發現" 新大陸
  - 愛迪生 "發明" 電燈泡
  - **Q**: 相對論是發明還是發現?
- 機率論是人類 "發明" 來處理生活中的不確定性之理論
- 愛因斯坦: "上帝永遠不會擲骰子"

❖ **Further Readings**:

✓ Kahneman (2011), *Thinking, Fast and Slow*. (快思慢想)

✓ Silver (2012), *The Signal and the Noise*. (精準預測)