F74094083 資訊系113 林俊誠

1.

For $n=1$, $S(1)=2-1=1$

$\therefore S(1)$ is true

Assume $n=k$ is true for some $k \in \mathbb{Z}^+$

$\Rightarrow S(k)=1+3+5+\cdots+2k-1=k^2$

$S(k+1)=1+3+5+\cdots+2k-1+2k+1$

$=S(k)+2k+1$

$=k^2+2k+1$

$=(k+1)^2$

By the Principle of Finite Induction,

$S(n)$ is true for all $n \in \mathbb{Z}^+$

2.

$\because 17 \mid (2a+3b) \Rightarrow 17 \mid (-4)(2a+3b)$

and $17 \mid (17a+17b)$

$\Rightarrow 17 \mid (17a+17b-8a-12b)$

$\Rightarrow 17 \mid (9a+5b)$

3.

$7 \underline{| 658}$

$7 \underline{| 94} \cdots 0$

$7 \underline{| 13} \cdots 3$

$1 \cdots 6$

$\therefore 658_{10} = 1630_7$

4.

$3x+5y=1$

$\Rightarrow x=2, y=-1$ is a solution

$\Rightarrow$ let $x=2-5k$, $y=-1+3k$ for some $k \in \mathbb{Z}$

$\Rightarrow 3(2-5k)+5(-1+3k)=6-5=1$

$\Rightarrow x=2-5k$, $y=-1+3k$ is also a solution

$\because k \in \mathbb{Z}$

$\Rightarrow x$ and y have infinite solution

5.

$8n+3=1\cdot(5n+2)+(3n+1)$

$5n+2=1\cdot(3n+1)+(2n+1)$

$3n+1=1\cdot(2n+1)+n$

$2n+1=2\cdot(n)+1$

$n=n(1)+0$

$\because$ The last remainder is 1

$\Rightarrow \gcd(8n+3, 5n+2)=1$

$\Rightarrow 8n+3$ and $5n+2$ are relatively prime.

6.

$\gcd(m, m+1)=1=\gcd(m+1, m+2)$

For any prime number p, If $p \mid m+1$,

$p \nmid m$ and $p \nmid m+2$ and $p \mid n^2$

$\because n^2$ is perfect square

$\therefore m+1$ is perfect square

$\Rightarrow m(m+2)$ is perfect square

$\because m^2 < m(m+2) = m^2+2m < (m+1)^2$

$\therefore \Rightarrow m(m+2)$ cannot be a perfect square

$\therefore m(m+1)(m+2) \neq n^2$ for any $m, n \in \mathbb{N}$

7.

2^{mn}

8.

True

9.

R_1 : 3 doesn't in R_1

R_2 : 2 appears twice.

10.
n^m

11.
$P(n, m)$
$= \frac{n!}{(n-m)!}$

12.

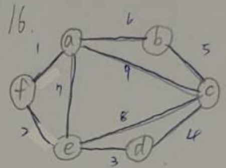
	x	a	b	c	d	e
x	x	a	b	c	d	e
a	a	-	-	-	-	-
b	b	-	-	-	-	-
c	c	-	-	-	-	-
d	d	-	-	-	-	-
e	e	-	-	-	-	-

$\therefore 5^{25}$

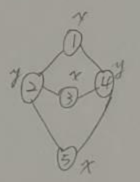
13.
maximum number of $x = m-n+1$

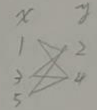
14.
Let $f: V_1 \to V_2$
$\Rightarrow f(u_1) = v_1,\ f(u_2) = v_4,\ f(u_3) = v_2,$
$f(u_4) = v_5,\ f(u_5) = v_3,\ f(u_6) = v_6.$
In V_1, there is a cycle
$u_1 \to u_4 \to u_5 \to u_2 \to u_3 \to u_6 \to u_1$ — 1
In V_2, there is also a cycle
$v_1 \to v_5 \to v_4 \to v_3 \to v_2 \to v_6 \to v_1$ — 2
$\because$ 2 is the corresponding path of 1
$\therefore V_1$ and V_2 is isomorphism

15.
No, it's impossible.
$\because$ 5-regular
$\Rightarrow |V| \geq 6$
$\Rightarrow |E| \geq \frac{6 \times (6-1)}{2} = 15 > 10$
$\therefore$ Impossible

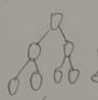
16.


$(a \to f \to e \to d \to c \to b \to a \to e \to c \to a)$

17.
No.




$\because |x| = 3 \neq |y|$
$\therefore$ No.

18.


← A color
← B color
← A color

$\therefore \chi(T) = 2$

19. x-y walk:

$$x \to x_1 \to x_2 \to x_3 \cdots \to x_k \to x_{k+1} \to \cdots \to x_m \to x_{m+1} \to \cdots \to y$$

If none of vertices is repeated, x-y walk is equal to x-y path

If $x_k = x_m$, $x_k \to x_{k+1} \to \cdots \to x_m$ is a closed walk.

If the closed walk is removed, x still can arrive y.

$\therefore$ If remove all closed walk, x-y walk will become x-y path

20. $f: A \to B$ is invertible $\Leftrightarrow$ f is one-to-one and onto

$\therefore$ If a function f' isn't one-to-one or onto, it is not invertible

$\Rightarrow$ Not every function has inverse function. —①

Suppose g is the inverse function of f.

$\Rightarrow$ f is also the inverse function of g. —②

By ① and ②, not every function is an inverse function.