UNIVERSITÀ DI PISA

Facility Location Problem

AMBULANCE PLACEMENT

Logistics Project work

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1. Problem description

The aim of this projects is to solve a location problem for ambulances, so that the emergency services coordinator can maximize the number of residents covered within four minutes in emergency situations. Due to budget restrictions, the coordinator can only set 2 ambulances in all the five regions. The possible area of location is divided in five regions and the average time required to travel from one region to another is given in the following Table 1.

		To region				
		1	2	3	4	5
From region	1	0	4	6	3	2
	2	4	0	2	3	6
	3	6	2	0	5	3
	4	3	3	5	0	7
	5	2	6	3	7	0

Table 1. Average travel time between regions

Also, the number of residents in each region is specified in the following Table 2.

Population					
1	2	3	4	5	Total
4.500	6.500	2.800	5.200	4.300	**23.300**

Table 2. Population of each region

2. Mathematical formulation

In order to solve this problem, first it was necessary to identify to which kind of problem we are dealing. By reading the problem is easy to identify two key comments. The first, is to locate the ambulances in a possible region and, second, there is an upper limit on the number of facilities that can be opened (budget constraints). With this, we can point that the problem corresponds to a Maximal Covering Location Model (MCLP).

2.1 Integer linear programming model

Define the input data as follows:

- $I = set\ of\ demands\ regions, indexed\ by\ i$
- $J = set\ of\ candidate\ locations, indexed\ by\ j$
- $h_i = population\ in\ region\ i,\ \forall i \in I$
- $p = number\ of\ ambulances\ to\ place$
- $N_i = set\ of\ location\ condidates\ that\ can\ cover\ region\ i$

Next, the following are the decision variables:

$$x_j = \begin{cases} 1\,, & if\ ambulance\ is\ located\ at\ j, \forall j \in J \\ 0\,, & otherwise \end{cases}$$

$$z_i = \begin{cases} 1\,, & if\ ambulance\ covers\ region\ i, \forall i \in I \\ 0\,, & otherwise \end{cases}$$

The formal ILP model is as follows:

$$max \sum_{i \in I} h_i x_i \qquad ①$$

$$\sum_{j \in J} x_j = p \qquad ②$$

$$\sum_{j \in N_i} x_j \geq z_i, \forall i \in I \qquad ③$$

$$\begin{aligned} &x_j \in \{0,1\}, \forall j \in J \qquad ④ \\ &z_i \in \{0,1\}, \forall i \in I \end{aligned}$$

Where each of the formulas has the following meaning:

① The objective function denotes that we are looking to maximize the population covered

② Constraint that limits the quantity of ambulances to place

③ Covering constraint

④ Binary variables

With this, the ILP model for this problem is the following:

$$4.500z_1 + 6.500z_2 + 2.800z_3 + 5.200z_4 + 4.300z_5$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 2$$

$$\begin{aligned} x_1 + x_2 + x_4 + x_5 &\geq z_1 \\ x_1 + x_2 + x_3 + x_4 &\geq z_2 \\ x_2 + x_3 + x_5 &\geq z_3 \\ x_1 + x_2 + x_4 &\geq z_4 \\ x_1 + x_3 + x_5 &\geq z_5 \end{aligned}$$

$$\begin{aligned} &x_j \in \{0,1\}, \forall j \in J \quad J = 1,2,3,4,5 \\ &z_i \in \{0,1\}, \forall i \in I \quad I = 1,2,3,4,5 \end{aligned}$$

2.2 CPLEX results

After the AMPL implementation (see annex) the values obtained using CPLEX Solver for each function is the following:

- Regions where the 2 ambulances will be set (x values):

Ambulances to set				
1	**2**	**3**	**4**	**5**
1	1	0	0	0

Table 3. Ambulance location

- Regions covered by the ambulances (z values):

Regions covered				
1	**2**	**3**	**4**	**5**
1	1	1	1	1

Table 4. Regions covered

- $Total\ population\ covered\ = 23.300$

This solution implies that by placing 2 ambulances, all regions are covered and therefore the population covered is equal to the total population of all the regions.

3. One ambulance constraint

As we have seen above, when constraining the problem to 2 ambulances, all the population is covered. This means that the optimal solution is equal to the maximum value that the objective function can take. In the following lines it is possible to see what happens with the optimal value when changing the constraint to just 1 ambulance.

3.1 ILP model for 1 ambulance

$$4.500z_1 + 6.500z_2 + 2.800z_3 + 5.200z_4 + 4.300z_5$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$\begin{aligned}
x_1 + x_2 \quad\quad + x_4 + x_5 &\geq z_1 \\
x_1 + x_2 + x_3 + x_4 \quad\quad &\geq z_2 \\
x_2 + x_3 \quad\quad + x_5 &\geq z_3 \\
x_1 + x_2 \quad\quad + x_4 \quad\quad &\geq z_4 \\
x_1 \quad\quad + x_3 \quad\quad + x_5 &\geq z_5
\end{aligned}$$

$$x_j \in \{0,1\}, \forall j \in J \quad J = 1,2,3,4,5$$
$$z_i \in \{0,1\}, \forall i \in I \quad I = 1,2,3,4,5$$

3.2 CPLEX results

After the AMPL implementation (see annex) the values obtained using CPLEX Solver for each function is the following:

- Region where 1 ambulance will be set (x values):

Ambulances to set				
1	**2**	**3**	**4**	**5**
1	0	0	0	0

Table 5. Region of location of the ambulance

- Regions covered by the ambulance (z values):

Regions covered				
1	**2**	**3**	**4**	**5**
1	1	0	1	1

Table 6. Regions covered by the ambulance

- $Total\ population\ covered\ = 20.500$

These last results obtained means that if only 1 ambulance is set, almost all regions are covered with a coverage of almost 88% of the total population, leaving only the region number 3 uncovered, with a population 2.800. This is not a very feasible solution for the Emergency Services Coordinator because Region 3, with 2.800 people, would not eventually receive any support.

Annex

Mod file for 2 ambulances

```
# model
param To;
param From;
set to_area:= 1..To;
set from_area:= 1..From;

param tot_pop {from_area};
param coverage {from_area, to_area};

var Open {from_area} integer binary;
var Cov {from_area} integer binary;

maximize pop_cov: sum {i in from_area} tot_pop[i]*Cov[i];

subject to cover_const {j in to_area}: sum {i in from_area} coverage[i,j]*Open[i] >= Cov[j];
subject to tot_facilities: sum {i in from_area} Open[i] = 2;
```

Dat file

```
data;
param To := 5;
param From := 5;

param coverage:
    1   2   3   4   5:=
1   1   1   0   1   1
2   1   1   1   1   0
3   0   1   1   0   1
4   1   1   0   1   0
5   1   0   1   0   1;

param tot_pop :=
1 4500
2 6500
3 2800
4 5200
5 4300;
```

Run file

```
reset;
model 20211214_one_ambulance.mod;
data 20211214_one_ambulance.dat;
option solver cplex;
#option solver cplexamp;
solve;
display pop_cov;
display Open;
display Cov;

reset;
model 20211214_one_ambulance_v2.mod;
option solver cplex;
#option solver cplexamp;
solve;
display pop_cov;
display x1,x2,x3,x4,x5;
display z1,z2,z3,z4,z5;
```

Alternative AMPL formulation for 2 ambulances

```
var x1 binary;
var x2 binary;
var x3 binary;
var x4 binary;
var x5 binary;
var z1 binary;
var z2 binary;
var z3 binary;
var z4 binary;
var z5 binary;

maximize pop_cov: 4500*z1+6500*z2+2800*z3+5200*z4+4300*z5;

subject to a: x1+x2+x4+x5 >= z1;
subject to b: x1+x2+x3+x4 >= z2;
subject to c: x2+x3+x5 >= z3;
subject to d: x1+x2+x4 >= z4;
subject to e: x1+x3+x5 >= z5;
subject to f: x1+x2+x3+x4+x5 = 2;
```