Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
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Summary

In the background of climate change and increasing number of extreme weather events and natural disasters, this paper proposes a system of models, aiming at improving the affordability of property insurance and the sustainability of insurance industry.

According to specific characteristics and patterns of a disaster, We establish two specially-designed model for wildfire in California, the U.S., and flood in Zhejiang, China. Each model consists of disaster modelling, loss forecasting, insurance pricing and risk analysis.

In the Flood Model, we apply **Maximal Likelihood Estimate** and **Nonlinear Regression** to historical data and obtain the probability distribution function of future precipitation, with increasing variance. Considering the occurrence of flood as an **Poisson process**, we establish **Surplus Stochastic Process** to estimate the insurance companys' cash flow. Then a **Company Optimization Model** is established to maximize the profits of insurance companies under controllable bankruptcy risk and determine the optimal insurance price and premium reserves. At last, we analyze the influence of climate change on the choice and profit of insurance companies as sensitivity analysis.

In the Wildfire Model, we first **grip** the map of California. Applying **Multiple Linear Regression** and **Random Forest Method**, fire risk level is specified for each unit. We also consider the seasonal factors and heterogeneity of property loss. After two spatial layers and one temporal layer are developed, we can establish Simulation Model based on **Cellular Automata**. According to simulation of 200 paths with $55 \times 29 \times 2500$ units, analysis of **bankruptcy risk** and **minimum premium reserves** is conducted in a comprehensive and quantitative way. We also introduce **cost-effectiveness**, through which we can offer advice to insurance companies, real-estate practitioners and community leaders.

Next, we develop the pricing model of churches. The model consists of a **Church Database** and a **"Similarity" Algorithm**. By introducing 6 key indices and define a "Distance", we can find "the most similar churches" as the basis of pricing. Moreover, our method successfully tackles the problems of incomplete market and implicit value.

Finally, we choose Fort Dick Bible Church (FDBC) in California as an example. Our Insurance Model gives that the annual insurance fee is $135, while our Pricing Model determines that the annual rent is $816.4. This means the **insurance-rent ratio** is 17% and this number can be up be 24% in the real market. In light of this high ratio, it is recommended that the community should buy insurance only in the first several years and utilize this buffer period to take other wild-prevention measures.

Keywords: Disaster Modeling, Surplus Stochastic Process, Cellular Automata, Cost-Effectiveness, "Similarity" Algorithm.

Contents

1 Introduction

1.1 Background and Problem Restatement

Climate change increases the likelihood of extreme weather events and natural disasters, which poses significant challenges for both property owners and insurance companies. For property owners, insurance is not only getting more expensive, but also harder to find. For insurance companies, crisis in profitability due to unpredictable extreme weather events threatens the sustainability of insurance industry. Hence, it is significant for insurance companies to develop reliable and effective disaster models as well as to take sustainable and resilient underwriting strategies. They should properly underwrite policies to cover the expense of future claims while also safeguarding their long-term viability.

Moreover, climate change also poses considerable threat to facilities of important economic, historical, cultural or community significance. For community leaders, they should properly evaluate the value of these facilities and establish insurance coverage or take other effective measures to minimize potential losses due to climate disasters.

1.2 Our Work

In this paper, we consider two typical natural disasters which are closely related to climate change, flood in Zhejiang, China and wildfire in California, the U.S..

We believe that the key to increasing profitability for the insurance companies as well as affordability for property owners is to establish reliable disaster model and to take resilient strategies, both of which rely on insightful and precise description of the particular disaster. Therefore, we establish specially-designed models for wildfire and flood respectively, aiming at capturing the key characteristics and patterns of the disasters. Specifically, we use macro-mechanism in the modelling of flood and micro-mechanism for wildfire. We also evaluate the risk of insurance companies and design operation and pricing strategies for insurance companies.

In the Flood Model, we first apply MLE to historical data to obtain the possibility distribution function of future precipitation and then fit a loss function curve via regression analysis. Once we get the distribution of future risk, we establish a stochastic process, the surplus process to evaluate the bankrupt probability and expected profits. Next, we integrate the factors of insurance market and climate change into our model, establishing an optimization model for company. Via simulation and sensitivity analysis on the severity of climate change, we obtain the optimal premium price, premium reserve and maximal profit under different climate conditions.

In the Wildfire Model, we first divide the map of California into 55×29 geographic units, and each geographic unit is further divided into 2500 basic units. Then we use Multiple Linear Regression and Random Forest method from the historical data of precipitation, temperature, forest coverage and property loss and obtain the risk level of each unit. We also introduce variables that characterize seasonal changes and intensity of property loss in different regions. After constructing two spatial layers and one temporal layer on the grid, we apply Cellular Automata algorithm for wildfire simulation. Following that, surplus process is used again to determine the minimum amount of premium reserves under the constraint of at most 5% probability of bankruptcy. At last, we introduce the notion of cost-effectiveness and make comprehensive and quantitative assessment for each geographic unit, which offers important implications for both the insurance companies and community leaders.

In order to take appropriate preservation measures for buildings of special significance, a pricing model of churches is established. We first build the database of church transactions, summarizing price information and other 6 important features of a church. Then we develop an algorithm to determine "the most similar" churches and take their average price as the value of the targeted church. This method successfully tackles the problems of incomplete market and implicit value.

Finally, we choose the Fort Dick Bible Church (FDBC) in California, the U.S., which is important for local community, as an example and apply our models. We use the Insurance Model to obtain the estimated insurance fee for FDBC and use the Pricing Model to determine the value of this church. By analyzing Insurance-Rent ratio, we give recommendations to the local community.

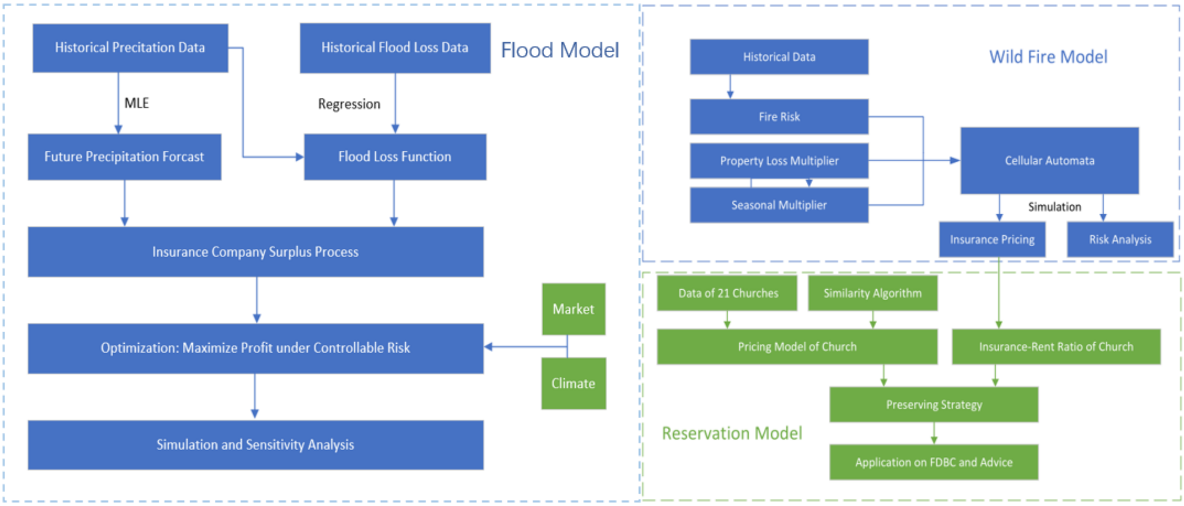

Figure 1: Flow Chart of Our Models

2 Assumptions and Justifications

To simplify the problem, we make the following basic assumptions.

- Insurance companies and insurance applicants are rational and they are in an efficient market.

  Justification: The hypothesis of Rational Economic Agent and the hypothesis of Efficient Market are commonly used in economic and financial analysis, laying the foundation of the pricing of financial instruments.

- The insurance company will compensate for all property losses that are insured.

  Justification: Insurance that compensates for all property losses is prevailing. Despite the existence of other kinds of products like partial compensation or fixed compensation, the kind of full-compensation is fundamental in insurance industry.

- The discount rate is zero.

  Justification: In times of moderate inflation, discount rate has a relatively small influence on our insurance model. To avoid complicated discussions about what level of discount rate should be used in our model and how its change influences our results, we ignore its influence.

3 Notations

The key mathematical notations used in this paper are listed below.

Table 1: Notations

Symbols	Descriptions
x_{ijk}	The precipitation of $i-$th area in month j and year k,$1 \leq k \leq 9$
r_{ijk}	The random variable of predicted precipitation of $i-$th area in month j and year k
S_t	The random variable of the total loss at time t
α	The increasing rate of the variance of precipitation
u	The premium reserves
U_t	Cash flow of insurance company at time t.
c	The premium revenue of insurance company
n_s	The annual sale product of insurance company
p	The price of insurance product
τ	The stopping time when the insurance company goes bankrupt
P	The total profit of insurance company in the k-th year
F_{pq}	Fire Risk Indicator of geographic unit (p, q)
PLM_{pq}	The property loss multiplier of the geographic unit (p, q)
$R_{pq}(t)$	The wildfire risk level of geographic unit (p, q) at time t
$SM(t)$	Seasonal multiplier at time t
L_{pqt}	The loss incurred at day t in geographic unit (p, q)
IE	The indicator of fund inefficiency of an insurance company
I^a, I^u	Insurance fee for one acre, insurance fee for one geographic unit
I	Annual insurance fee
V^a, V^u	The average property value of one acre, the value of a housing unit
V	The value of a certain property
Rt	The rent of a certain property

4 Flood Model

4.1 Future Precipitation Prediction

Since the most influential factor that increases the risk of flood is precipitation, a precipitation prediction model is established based on the historical data. Assume that the monthly precipitation of a certain area follows a normal distribution $N(\mu, \sigma)$. Via utilizing the historical data of precipitation of the $i-$th area in month j $(1 \leq j \leq 12)$ and year k $(1 \leq k \leq n)$, which is denoted by x_{ijk}, we can apply the Maximum Likelihood

Estimation for normal distribution to determine the optimal parameter pair (μ_{ij}, σ_{ij})

$$(\mu_{ij}, \sigma_{ij}) = \arg\max_{\mu,\sigma} \prod_{k=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_{ijk}-\mu)^2}{2\sigma^2}} \tag{1}$$

The parameters (σ_{ij}, μ_{ij}) vary across different areas and years. Additionally, since climate change will increase the frequency of extreme weather events, the precipitation variance in year k, which is denoted by σ_k, will increase linearly to reflect the influence of climate change.

$$\sigma_k = \sigma_0 + \alpha(k - n) \tag{2}$$

where α manifests the severity of climate change. Larger α represents larger speed of climate change.

After that, we can estimate the precipitation of year k $(k > n)$, which is denoted by r_{ijk}, via normal distribution $N\big(\mu_{ij}, \sigma_{ij} + \alpha(k-n)\big)$.

4.2 Economic Loss Estimation

From government database we can obtain data on flood losses in different regions. We employ a nonlinear regression model to attain the relationship between monthly precipitation x and economic loss $f(x)$

$$f(x) = \begin{cases} ae^{\frac{x-b}{c}} & \text{if } x > b \\ 0 & \text{if } x < b. \end{cases} \tag{3}$$

where b is the threshold of flood damage. If the precipitation is less than b, flood will not occur and the loss is 0. However, once precipitation exceeds b, the flood occurs. Additionally, we choose exponential loss function because we believe that an increase of precipitation above the threshold b leads to a rapidly escalating level of damage.

4.3 Future Loss Prediction

To simulate the uncertainty of extreme weather event, we regard the occurrence of flood as random event and the evolution of the flood over time as a stochastic process. Specifically, in year k, we assume that the number of floods in a certain area is N_k, which follows a Poisson distribution with parameter λ. Here we assume that λ also increases linearly over time at rate α to manifest the influence of climate change.

$$N_k \sim Poisson(\lambda_0 + \alpha(k - n)) \tag{4}$$

Consequently, the total loss in year k is also a random variable

$$S_k = \sum_{j=1}^{12} \sum_{p=1}^{N_k} f(r_{ijkp}) \tag{5}$$

where r_{ijkp} is an i.i.d random variable with the same distribution function as r_{ijk}. Denote $X_k = \sum_{j=1}^{12} f(r_{ijkp})$, then we can calculate the expectation and variance of S_k as

$$\mathbf{E}(S_k) = (\lambda_0 + \alpha(k-n))\mathbf{E}(X_k) \tag{6}$$

$$\mathbf{D}(S_k) = (\lambda_0 + \alpha(k-n))\mathbf{E}(X_k^2) \tag{7}$$

4.4 Surplus Process and Bankruptcy Risk

In part $4.1 \sim 4.3$ we have established the model to predict the risk of losses caused by flood in the future. Now we consider its influence on the strategies of a insurance company. We first employ a stochastic process to simulate the cash flow of the company. We denote the cash flow at year k by U_k with premium reserves $U_0 = u$. The surplus profit of the company can be then regarded as a stochastic process

$$U_k = u + ck - n_s \sum_{t=1}^{k} S_t \tag{8}$$

where n_s is the sales volume of insurance, c is the total premium revenue in the region and p is the price of insurance. According to classical economic theory of demand curve, we assume that sales volume n_s and price p has negative linear correlation. By multiplying the sales volume by price, we obtain premium revenue c.

$$n_s = a - bp, c = pn_s = p(a - bp) \tag{9}$$

where a and b are constant coefficients.

In this model, $U_k < 0$ represents that the insurance company goes bankrupt. Denote

$$\tau = \min_{k>0}\{U_k < 0\} \tag{10}$$

then τ is a time-of-ruin random variable, indicating the stopping time of bankruptcy. Also denote

$$\Phi_u = \mathbf{P}(\tau < \infty)$$

as the probability of ultimate ruin. According to Lundberge inequality[6],

$$\Phi_u \leq e^{-Tu} \tag{11}$$

where T is the accommodation coefficient with upper bound

$$T < \inf_{k>0} \frac{2(c - \lambda \mathbf{E}(n_s S_k))}{\mathbf{E}(n_s^2 S_k^2)}. \tag{12}$$

From equation (12) we know that when the climate change leads to an increase of variance X_k, the insurance company must adjust their premium reserves u and premium price c accordingly. Otherwise, the company will almost surely go bankrupt for some day. Therefore, we should consider the sustainability of company in finite time. To achieve this purpose, we define the pair of premium price and premium reserves (u, p) as "k-year sustainable", if $\mathbf{P}(\tau \leq k) < 0.1$, which means the probability of bankruptcy in the next k years is less than 0.1.

$$\sum_{t=1}^{k} \mathbf{P}\Big(\sum_{i=0}^{t} n_s S_i > u + \alpha t, \sum_{i=0}^{t-1} n_s S_i < u + \alpha(t-1)\Big) \leq 0.1 \tag{13}$$

4.5 Company Optimization Model

We set one decision-making period of the insurance company as k years. Suppose that the company has a total fund of M initially. The manager should divide the fund into two parts, u for the premium reservation and the remaining $M - u$ for investment. Assume that the investments cannot be converted to cash in the next k years, and that the company will gain investment revenue of $(M - u)((1 + i)^k - 1)$, where i is the annual investment return rate. Additionally, we assume that the manager can set their insurance product at price p for the purpose of profit optimization.

However on the other hand, the company must contend the potential risk for a total of $a - kp$ applicants. If the premium reserves of the company is not enough to cope with the future compensation, the company will face high risk of bankruptcy, rendering its development unsustainable.

To ensure the company's ability to cover the cost of future claims while also maintaining long-term health of insurance, we establish an optimization model for the company to determine the maximal profit under a controllable bankruptcy risk. The constraints of the optimization model include

- Financial Constraint: u, p, c, n_s and $M - u$ must be positive
- Sustainability Constraint : the premium price and premium reserves (u, p) is k-year sustainable, which means the company has little probability to undergo bankruptcy.

The objective of the optimization model is to maximize its profit

$$P = (M - u)\big((1 + i)^k - 1\big) + \sum_{t=1}^{k} n_s[p - \mathbf{E}(S_t)] \tag{14}$$

4.6 Model Simulation - The Flood in Zhejiang Province

We collect historical precipitation data in Zhejiang, China, where floods cause dramatic economic damage due to the monsoon climate and uneven precipitation. From government website we collect monthly data of 7 different areas in Zhejiang from 2011 to 2019.[1][2] Employing MLE, we get the parameters of the seven region as shown in the following table.

Table 2: The Precipitation Prediction of Zhejiang Province

Area	1	2	3	4	5	6	7
(μ_1, σ_1)	(844,362)	(724,374)	(901,380)	(732,326)	(707,336)	(525,326)	(525,310)
(μ_2, σ_2)	(931,682)	(783,528)	(899,555)	(929,632)	(1046,752)	(699,375)	(664,374)
$\cdots$	$\cdots$	.	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
(μ_{12}, σ_{12})	(778,427)	(630,467)	(769,426)	(780,365)	(770,387)	(689,472)	(558,197)

We set the increasing rate c in equation (2) to be 10. Take area 1 (Ningbo) for example, we may assume the loss distribution $S(k) \sim f(X_k)$, where X_k is the precipitation of year k, which is subject to $N(2156, 474 + 10k)$. Determine a, b, c in loss function f in equation (3) as $a = 1$, $b = 2600$, $c = 300$ and $k = 10$.

Following that, we conduct a simulation to explore feasible combinations of pairs (u, p) under financial and sustainability constraints. We randomly sample 1000 different loss cases and count the cases where the company goes bankrupt. If the number of bankruptcy cases is less than 10, then we regard the region is k-year sustainable. The bankruptcy probability under different choice (u, p) and the feasible region (blue part) are determined as shown in following figures. The x-axis in Figure 3 represents the premium reserve u and the y-axis represents the insurance price p.

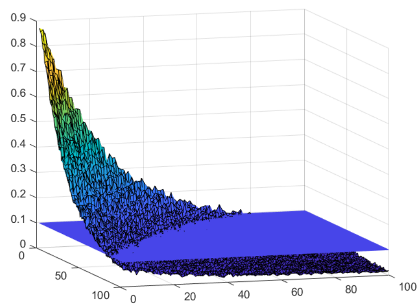

Figure 2: Relationship Between (u, p) and Ruin Probability

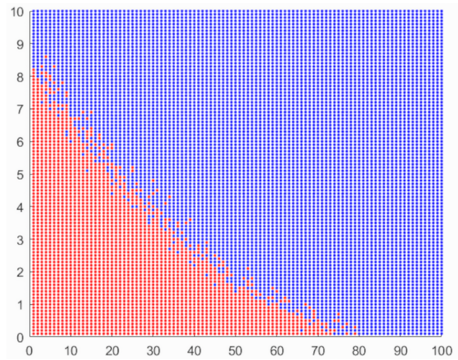

Figure 3: Feasible Region of (u, p)

It can be roughy estimated that the feasible region is the area above the line $p = 7 - \frac{1}{10}u$.

Then we apply optimization model in the feasible region. After investigating the average annual profit of investments owned by insurance companies, we set $i = 0.05$ as a proper rate. We also set $F = 100$ as a numerical example. Now, we search for parameters within the feasible region that maximize the profit function. It is determined that

$$(u, p) = (41, 2.9)$$

is the optimal premium reserves and insurance price. And the maximal profit of the insurance company is $P_{optimal} = 57.73$.

4.7 Influence of Climate Change on the Model

In our Flood Model, climate change is manifested by the increase of precipitation variance as $\sigma_k = \sigma_0 + ck$. To estimate the impact of climate change on our model, we adjust the initial variance σ_0 and increasing rate c. Subsequently, we compare the difference of maximal profit, optimal price and optimal preserves under different weather conditions.

The case where $\sigma_0 = 474, \mu = 30$ is a simulation with faster speed of climate change. Our results show that the company must increase its premium reserves and insurance price to cope with larger risk. This necessitates a trade-off, leading to a reduction in investment, an increase of insurance price and a decrease of sales volume. Moreover, rising prices of insurance will impose heavier economic burdens on the property owners and put them in the dilemma of increasing flood risk and rising insurance price.

Table 3: Influence of Climate Change on the Model

	σ_0	c	$u_{optimal}$	$p_{optimal}$	P_{max}
1	474	10	41	2.9	57.53
2	474	30	73	4.0	41.25
3	474	5	35	3.1	60.46
4	574	10	64	3.3	52.38
5	974	10	/	/	/

In contrast, the case where $\sigma_0 = 474, \mu = 5$ demonstrates that when the pace of climate change slows down, which leads to a relatively lower risk of flood, the company can reduce premium reserves and allocate more money on investment.

Besides, even if the trend of climate change remains unchanged, the company must increase its premium reserves and its insurance price after 10 years. This adjustment, however, will probably result in a decrease in profit.

To make matters worse, if σ increases at a constant speed, then after 50 years(case where $(\sigma_0 = 974, \mu = 5)$), (u, p) is always not $10 - yearsustainable$ regardless of the strategies the company take. This means the insurance company will take at least 10 % risk of bankruptcy, indicating that the insurance company must shoulder uncontrollable risk if they continue to provide insurance products under worsened weather conditions. For insurance companies, therefore, there are no alternatives but to withdraw from these high-risky areas. And the property owners in that region must decide between moving away or seeking government funding assistance to compensate the expected net loss of the insurance company.

5 Wildfire Model

In contrast to our Flood Model where we use a macro-mechanism based on precipitation to forecast the property loss, the disaster model regarding wildfire can be established on the basis of micro-mechanism. We design this specially-tailored disaster model of wildfire, based on historical, geographical and economic data in California, U.S., aiming at modelling the mechanism of ignition, spread and suppression of a wildfire. Our model is an application of cellular automata, which is commonly used in the simulation of spatial-temporal evolution of complex systems.

5.1 Preparations for the Model

We divide the map of California into 55×29 grid, generating what we call geographic units. Each unit is further divided into 50×50 basic units. Each basic unit is about 74.43 acres, which is appropriate for modeling wildfires that are over medium size. (For clarity, in the following paragraphs, we always use subscript (p, q) when discussing geographic unit and use subscript (i, j) when discussing basic unit.) We shall see that geographic unit is mainly used for information collection while basic unit is used for simulation of wildfire.

We summarize the historical data of temperature, precipitation, forest coverage, wildfire frequency and wildfire property loss for 10-15 years.[4][5] Using temperature , precipitation and forest coverage as predictor variables and acres burned for each

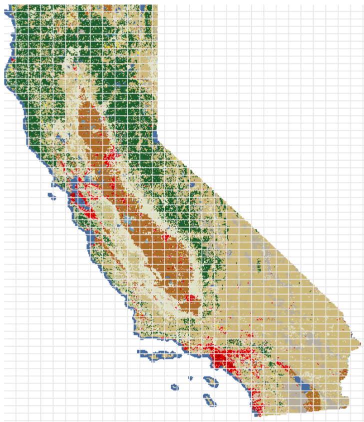

Figure 4: Geographic Unit 55×29

county as the response variable, we can perform multiple linear regression. Since the acres burned of different counties and in different years vary by orders of magnitude, we make the following transformation.

$$F = ln(S + 1)$$

where S is the total acres burned of a certain county in a certain year and F is defined to be the Fire Risk Indicator.

Table 4: Multiple Linear Regression

Predictor Variable	Coefficients	T value	Significance
Temperature	0.043	4.325	<0.000
Forest Coverage	-0.719	-3.136	0.002
Total	0.567	2.398	0.017

The ANOVA results show that the significance level of the regression model is <0.000. Also note the significance levels of three predictor variables shown in Table 4, which clearly indicate that the temperature and forest coverage have significant positive impact on F and precipitation has significant negative impact on F.

Now that we have confirmed the important factors that contribute to wildfire, we are going to characterize the risk level of each geographic unit. We employ the Random Forest Method, which is based on ensemble learning, bagging and decision tree. We take the average value of temperature, precipitation and forest coverage of the last 10 years as predictor variables and use the historical data above as the training set. Then, the Random Forest algorithm generate Fire Risk Indicator F_{pq} of all geographic units at present, as a measure of the risk level.

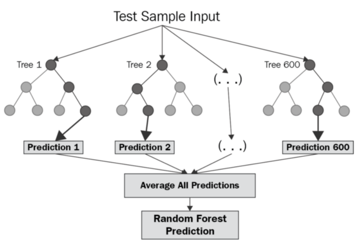

Figure 5: Random Forest Method

Next, we divide the total property loss (dollar amount) of each county by the total acres burned in that county[6], and take average of 10 years, then we obtain the Property Loss Multiplier PLM_{pq}.

$$PLM = \frac{1}{10}\sum_{y=1}^{10}\frac{PLy}{AB_y} \tag{15}$$

It reasonable to believe that a wildfire of the same scale will result in different levels of property loss in different region. Hence, we use PLM_{pq} to reflect the heterogeneous impact of a potential wildfire in geographic unit (p, q).

Furthermore, we introduce Season Multiplier $SM(t)$. The wildfire in California has apparent seasonal characteristics, with most wildfires occurring in July, August and September. Therefore, it is reasonable to specify different risk levels for different seasons. We take the average acres burned for each month in the last 10 years and use spline interpolation to obtain $SM(t)$ that changes smoothly in 360 days of one year.

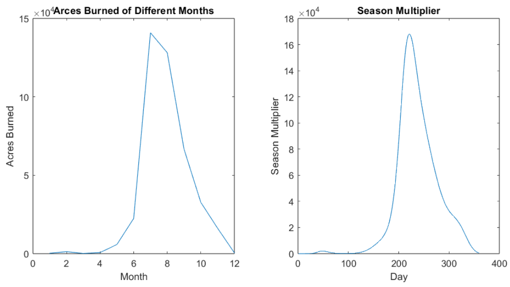

Figure 6: Season Multiplier

After modified by $SM(t)$, the wildfire risk level of geographic unit (p, q) at time t is

$$R_{pq}(t) = F_{pq} \times SM(t) \tag{16}$$

Moreover, we distinguish two kinds of risks

- R^i: the risk that a unit is ignited as the source of a wildfire
- R^s: the risk that a unit is ignited by a fire in the neighborhood

Hence we have

$$\begin{cases} R^i_{pq}(t) = F_{pq} \times SM(t) \times M_i \\ R^s_{pq}(t) = F_{pq} \times SM(t) \times M_s \end{cases} \tag{17}$$

where M_i and M_s are two parameters that remain to be determined. Intuitively, M_s is much bigger than M_i.

One last parameter we need to introduce is $Supre$, which represents the suppression efforts of firefighters. Historical data in last ten years shows that it take on average 8 days to suppress a wildfire, hence we set $Supre = 0.125$. We shall also consider stochastic factors in the suppression of wildfire.

5.2 Cellular Automata

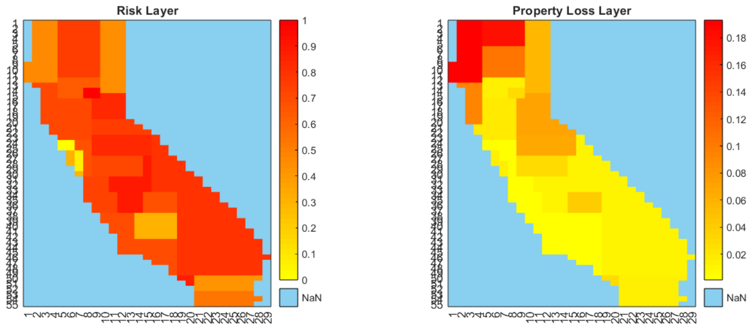

Figure 7: Spatial Layers

As shown above, we have now constructed two spatial layers in our grid: F_{pq} and PLM_{pq} and one temporal layer: $SM(t)$. We now focus on basic unit, which inherits F, PLM and $SM(t)$ from corresponding geographic unit. We specify the mechanism and dynamics of a wildfire as shown in Algorithm 1:

Algorithm 1: Wildfire Ignition, Spread, and Loss Emergence

Input: F_{ij}, $SM(t)$, PLM_{ij}: the 3 layers at basic unit (i, j)
Output: L_{pqt}: the Loss incurred at day t in geographic unit (p, q)
, $1 \leq t \leq 360$

Calculate R^i_{ij} and R^s_{ij} based on equation (17).
for *t=1:360* **do**
  if *Basic unit (i,j) is not burning* ($fire(i, j) = 0$) **then**
    generate a random number *rand* $\sim$ uniform[0,1]
    if $rand < R^i_{i,j}$ **then**
      Basic unit (i, j) starts burning (Set $fire(i, j) = 1$);
      The fire leads to property loss PLM_{ij} in basic unit (i, j), and this amount is added to corresponding L_{pqt}, the total loss of geographic unit (p, q) at day t.
  else
    for n *basic unit which is not burning and in the neighborhood of the burning unit*, **do**
      generate a random number *rand* $\sim$ uniform[0,1]
      if $rand < R^s_{i,j}$ **then**
        The fire spreads to unit n, say $unit(i, j+1)$, (Set $fire(i, j+1) = 1$);
        The fire leads to property loss $PLM_{i,j+1}$ in basic unit $(i, j+1)$, and this amount is added to corresponding L_{pqt}, the total loss of geographic unit (p, q) at day t.
  for *Each burning unit* (i, j) **do**
    $fire(i, j) = max\Big\{0, fire(i, j) - Supre - Supre \times Normal(0, 1)\Big\}$

Recall that there remain two parameters to be determined: M_i and M_s. They are determined by trails: we choose the best combination of parameters to make our simulative path resemble the historical data of wildfire size, frequency and property loss.

This simulation algorithm is the key component of our cellular automata, which displays how fire is ignited, spreads, incurs property loss and is suppressed. Stochastic factors are emphasized in the whole process of ignition, spread and suppression, which captures the essence of insurance and disaster modeling. We visualize our simulation as shown below, where we use colors to represent the area that is burning on a specific day.

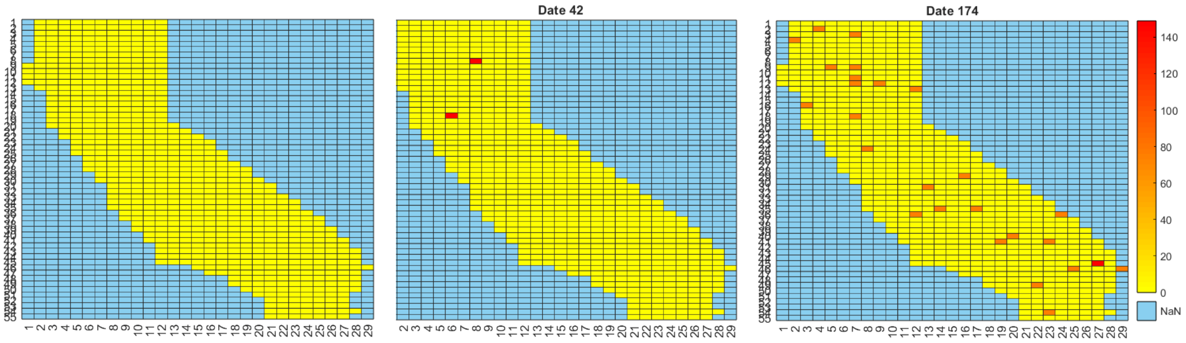

Figure 8: Wildfire Simulation

It can be seen that our simulation have good performance.

Using this simulation model, we obtain L_{pqt}, the loss incurred at day t in geographic unit (p, q). Therefore, when we fix any geographic unit (p, q), we have one simulative path over 360 days that depicts the loss flow in geographic unit (p, q). Based on this path (and repetitive simulations), we can forecast underwriting liabilities for any geographic unit (p, q).

We demonstrate the total acres burned in each month in California (average of 2010-2023) and the results in one simulative path of our model.

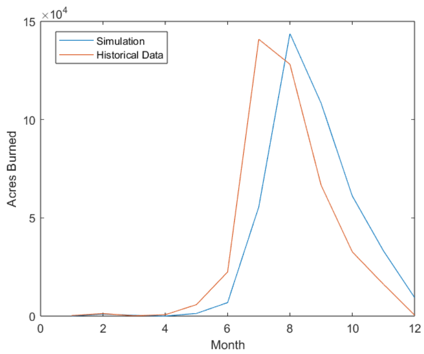

Figure 9: Comparison between Simulation and Historical Data

It can be seen that our simulation have good performance.

We also compare the annual property loss caused by wildfire in California (average of 2010-2023) and the corresponding result in one simulative path. The former number is \$$1.4362 \times 10^9$ and the latter is \$$1.6791 \times 10^9$, which also supports the validity of our simulation model.

5.3 Insurance Pricing Model and Analysis of Bankruptcy Risk

Finding a balance between reducing the bankruptcy risk and maximizing the profit is the core to enhance resilience of insurance companies. In wildfire model, our approach is to minimize premium reserves under the constraint of the company's bankruptcy probability. We assume that the insurance coverage persists for one year. We use surplus process to evaluate the bankruptcy risk:

$$U_t = U_0 + ct - \sum_{i=1}^{t} S_i \tag{18}$$

Here U_t equals to the cash of company at time t, U_0 equals to premium reserves, $c > 0$ is the revenue from insurance fee per unit of time, and S_i equals to the property loss at time i. If $U_t < 0$, the company undergoes bankruptcy. In this model, we don't take the demand curve of market into consideration and the insurance fee determined by this model is actually the pure premium.

According to Efficient Market hypothesis in financial economics, the annual insurance fee should be consistent with the present value of annual property loss, we can thereby determine the value of c as follows:

$$c = \frac{1}{360}\mathbf{E}\Big(\sum_{i=1}^{360} S_i\Big) \tag{19}$$

In practice, we use our wildfire model to determine c. For each geographic unit, we perform simulation for $n = 200$ times, and replace the expectation in equation (19) with average, then we obtain c_{pq} in geographic unit (p, q).

$$c_{pq} = \frac{1}{200 \times 360}\big(\sum_{n=1}^{200}\sum_{i=1}^{360} S^n_{pqi}\big) \tag{20}$$

where the superscript n represents the n^{th} simulative path.

Recall that when $U_t < 0$, the company undergoes bankruptcy. For one predetermined amount of premium reserves of a geographic unit, we can obtain the probability of bankruptcy within one year. Conversely, by limiting the probability of bankruptcy of a geographic unit, we can obtain the minimum premium reserves that could ensure the sustainability of the insurance company.

Specifically, if the largest risk of bankruptcy we can accept is r, we can temporarily set $U_0 = 0$ and obtain possible paths of U_t by simulation. Then we summarize the minima of the 200 paths and sort them from large to small to find the $[200 \times (1 - r)]^{th}$ number, denoted by U_0^*. Consequently, if we set $U_0 = U_0^*$ as the premium reserve, the possibility of bankruptcy is just $r\%$.

For demonstration, we show two simulative paths in geographic unit $(2, 2)$. The minimum premium reserve for to guarantee 5% bankruptcy risk in this unit is \$96471. We set the premium reserve as \$60000, and as anticipated, bankruptcy occurs.

Choosing the upper bound of the probability of bankruptcy within one year as 5%,

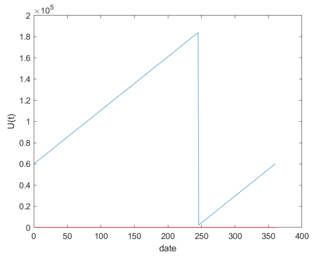

Figure 10: Normal Operation

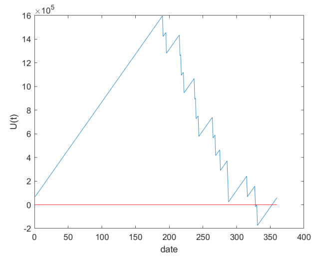

Figure 11: Bankruptcy

we find that the premium reserves vary greatly from 10,000 to 1,000,000. Results after normalization is shown below:

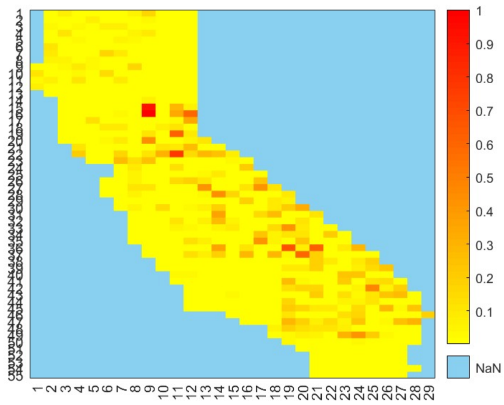

Figure 12: Minimum Premium Reserve Density Map

This figure indicates that premium reserve peaks at the mid-west region of California, where is in fact the region with highest rate of forest coverage. This observation justifies our simulation model.

Intuitively, higher requirement of reserves of geographic units results from higher wildfire risk of that unit. Since premium reserves earn no revenue, insurance companies might be discouraged from operating in such risky regions.

More concretely, we conduct a comprehensive and quantitative assessment of every geographic unit, considering both risk and revenue. We use the ratio of reserves(u) to revenue(c) as the basis for measurement of fund inefficiency in risky areas.

$$IE_{pq} = \frac{u_{pq}}{c_{pq}} \tag{21}$$

The higher this ratio, the less cost-effective it is for the company, as the company needs to retain more funds as reserves to undertake policies in this area, which decreases the amount of money available for alternative investments. On the contrary, a low ratio means that the insurance company can earn a lot from insurance fee without freezing a large amount of reserves. The results are shown in the following figure.

In geographic units with a high ratio, insurance companies are better off undertaking fewer policies or even completely withdraw form this area, whereas in units

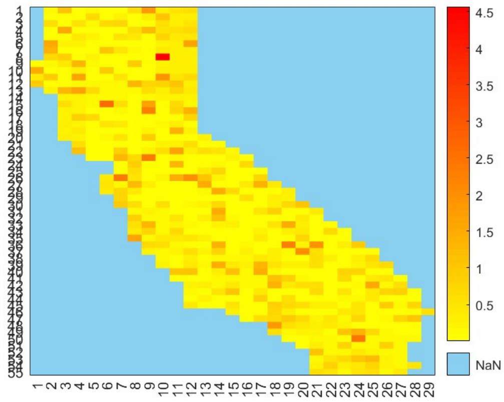

Figure 13: The Cost-effectiveness of Undertaking Business in a Geographic Unit

with a lower ratio, they can take on more business opportunities. However, people in risky units may protest against the action of insurance companies, and financial regulatory authorities might get involved in the seemly "immoral" strategies of insurance company. Therefore, alternative strategies for insurance companies could be setting a maximum limit for compensation or striving to reach a subsidy policy arrangement with the government.

On the other hand, communities and real estate practitioners are faced with the choice whether and where to develop. Given the fact that the global climate change is becoming increasingly severe, many places are facing unprecedented natural disaster risks with mounting volatility. Using our insurance model, we can gain insight into the cost-effectiveness of developing communities in those areas, which helps us in making informed decisions. More specifically, our insurance model implies that, the higher risk due to extreme weather events will not only increase the insurance fee, but also force some insurance companies out of risky areas due to low cost-effectiveness. If the insurance company deems it unwise to conduct business in a specific area, the developer should make a thoughtful decision about land development in that region, and vice versa.

5.4 Sensitivity Analysis

In our Wildfire Model, possible sensitivity problem may arise in the estimation of Fire Risk Indicator F_{pq} since our estimates of M_i and M_s are based on historical data instead of "future data". Therefore, if the patterns of temperature and precipitation variation change greatly in the future due to climate change, M_i and M_s must change accordingly, which make simulation results completely different.

Here we set the Fire Risk Indicators as 1.05, 1.1 and 1.2 times of ones used in the previous model, for all geographic units. Then we perform simulation and obtain simulative loss flow in one year.

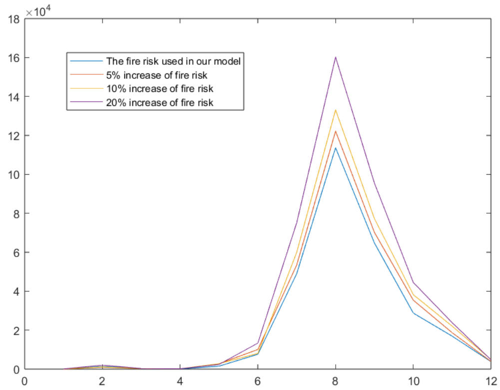

Figure 14: Sensitivity Analysis

We can also calculate the simulative annual loss for different levels of fire risk.

Risk Increase	0	5%	10%	20%
Total Annual Loss (10^9 dollar)	1.0126	1.1211	1.2094	1.4595
Rate of Loss Increase	/	10.71%	19.44%	44.13%

From the figure and table above, it can be concluded that the increase of fire risk indeed leads to non-negligible changes in the simulation. Due to time constraint (we need 5 hours to generate 200 paths for each set of parameters), detailed risk analysis is skipped. However, according to the analysis in section 5.3, we know that an increase of risk leads to an increase of insurance fee for the property owners and a decrease of cost-effectiveness for the insurance company, which might lead to their withdrawal. Therefore, as already demonstrated in the Flood Model, it must be worse off for both property owners and insurance companies if the climate change is more serious in the future.

At last, we should point out that the sensitivity of the Wildfire Model is not against its reliability. In practical use of this model, as long as we update all parameters needed to estimate the fire risk frequently and timely, we can obtain reliable forecasts, at least in the medium run.

6 Pricing Model of Special Buildings

6.1 Background and Problems

In communities which are likely to be affected by wildfire, real estate decisions must be considered carefully. In recent years, there are more and more news reports depicting the heart-broken fact that home owners in vulnerable communities of California are overwhelmed by skyrocketing insurance expenses.

For some communities, however, there is another non-negligible problem. There are some buildings of important historical, cultural and community significance. However, insurance coverage of these buildings may be inadequate for different reasons, one of which is the intrinsic problem of public goods. One example that reveals this problem is a church in local community. A church is of great significance for the community, providing spiritual guidance and support to community members. Moreover, some churches with a long history have valuable historical importance and have become an irreplaceable cultural symbol for the community. Hence, it can be concluded that churches provide important "public good" (or more accurately, public service). Nevertheless, since churches are non-profit and not operated by the government, their provision of public good may receive unfair and insufficient return. Some churches, despite their significance for the community, have inadequate funds, which lowers the priority of insurance affairs. Therefore, it is necessary to develop a model to assess the price for churches. Along with our insurance model developed in previous parts, the pricing model can offer helpful and practical preservation recommendations to owners of a church.

There are two core difficulties in the pricing of churches.

- Incomplete Market Problem: As a kind of real estate, churches have special purposes and are rarely seen in the real estate market.
- Implicit Value Problem: The value of a church not only depends on all material expenditures that assemble in this building, but also contains a "premium", which may originate from implicit value like historical, cultural and community significance.

The two problems make the pricing of churches much more difficult than ordinary houses. One method that is in current practice, is to measure the value of a church by its reconstruction expenses. However, it is rather difficult to apply to a particular church with unique characteristics. Besides, such method ignores the implicit value and the corresponding "premium". In sight of the weaknesses of traditional method, a brand-new approach, inspired by Clustering Algorithms, is proposed.

6.2 Pricing Algorithm

We first summarize the price offers of churches in the U.S.from real estate websites[8] and extract key features of a church, including size (S), lot size (LS), age of the church (A), local population density (PD), local real estate price level (median home price/Sq ft (HP^u) and median home price (HP)). We believe that the above six indices are main determinants of the value of a church. Due to time constraints, we only summarized 21 sets of information.

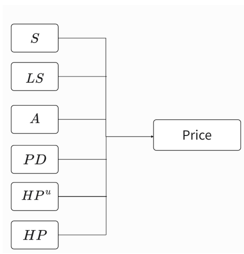

Figure 15: Price Determinants

Having established our database, we can choose any church, obtain the six indices, and use an evaluation algorithm to find churches in our database which are "the most similar" to the targeted church. Finally, we use the average price of "the most similar" churches as the value of the targeted church. The rationale behind this approach is as follows: in a market where transactions are rare, price is not determined by the equilibrium of demand and supply which requires a large number of transactions. Instead, the price is determined by value assessment of the buyer and seller. It is reasonable to argue that churches with similar physical and social conditions (as summarized by the six indices) will lead to similar value assessment and hence similar price.

Note that our approach has also tackled the problem of implicit value since the process of value assessment has already taken implicit value into account. In other words, in the value assessment of a church, the six indices (like history and local population density) contain information about the implicit value of a church, therefore, the price based on these six indices has included the "premium". To sum up, our pricing model resolves the two key difficulties of church pricing, hence, our price evaluation is both convincing and is able to reflect the historical, cultural and community significance of a church.

The techniques of determining the "most similar" churches is inspired by Clustering Algorithm, which typically defines a "distance" to measure the "similarity" of elements. Likewise, we standardize our data of the six indices and use the prevailing Euclidean distance to measure "similarity".

$$\begin{aligned} Distance = \Big[&(S_t - S_r)^2 + (LS_t - LS_r)^2 + (A_t - A_r)^2 + \\ &(PD_t - PD_r)^2 + (HP_t^u - HP_t^u)^2 + (HP_t - HP_r)^2\Big]^{\frac{1}{2}} \end{aligned} \tag{22}$$

where the subscript t represents targeted church, subscript r represents the church for reference. All data has been standardized such that they have zero mean and unit standard deviation.

Finally, we can choose one or several churches that have smallest $Distance$ and take the average price of these churches as our valuation.

We summarize our pricing model in the following flow chart:

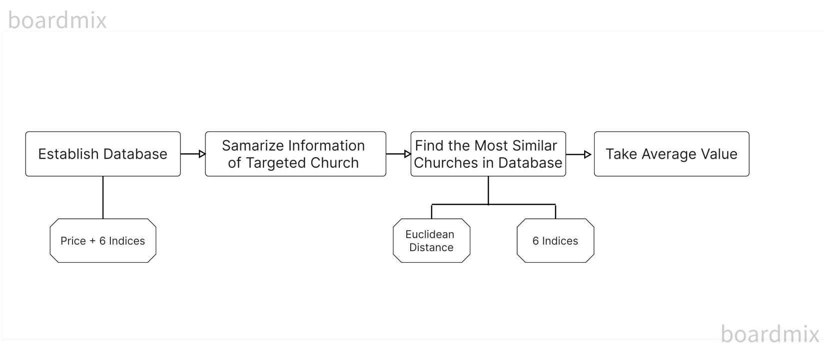

Figure 16: Pricing Model

At last, we point out that this method of church pricing would have better performance with the increase of the size of database. The essential idea and procedures of this method, however, remains unchanged when more data is added to our database.

7 Application of Insurance Model and Pricing Model

Now we have established the insurance model of wildfire and the pricing model of a church. We choose the Fort Dick Bible Church (FDBC) in Del Norte, California as an example and apply our models. Built in 1964, the historical church is the only church in the region of Fort Dick, making it significant and irreplaceable for local community. Unluckily, FDBC is faced with the threat of wildfire since it is located near a forest and is hence confirmed as COMMUNITY-IDENTIFIED ASSETS AT RISK in Del Norte County Fire Safe Council Community Wildfire Protection Plan[9]. We want to use our models to see whether FDBC should buy insurance or take alternative measures.

7.1 Application of Insurance Model

Since simulation results show that the minimum premium reserves and cost-effectiveness of this unit, i.e. $(2, 2)$, are both relatively low, insurance companies will continue their businesses here. Therefore, we only need to consider the impact of insurance fee. Based on the insurance model developed earlier, we can determine that the insurance fee for the basic unit $(2, 2)$ in which FDBC is located is $I^u = 594.49$ dollars per year.

Note that what we have obtained is the expected insurance fee for a basic unit, whose size is 74.43 acres. To estimate the insurance fee for FDBC, we need to make some reasonable conversion.

Firstly, we obtain the total housing unit of Del Norte, which is $N = 11193$, and median value of a housing unit, which is $P^u = 282500$ dollars, according to U.S. Census Bureau. Along with the total area of land of Del Norte, which is $AL = 643840$ acres, we can calculate the average property value of one acre.

$$V^a = \frac{N \times P^a}{AL}; \tag{23}$$

Besides, we can obtain the average insurance for one acre.

$$I^a = \frac{I^u}{74.43}; \tag{24}$$

Since the value of property and corresponding insurance fee have strong positive linear relation, it is reasonable to distribute the total insurance fee of a basic unit among all property value in that unit. Mathematically, we have the Property-Insurance ratio (PIR):

$$PIR = \frac{V^a}{I^a} \tag{25}$$

In our case, $PIR = 614.88$.

Finally, we can estimate the annual insurance fee of FDBC as follows:

$$I = \frac{V}{PIR} \tag{26}$$

where V will be obtain in the next part: the Pricing Model.

Table 5: The "Distance" Between FDBC and Churches in Database

Church ID	Distance	Price	Church ID	Distance	Price
7	2.43	70	6	3.75	195
11	2.79	80	18	4.26	99
16	2.79	99	15	4.27	99
9	2.80	275	8	4.28	274
21	2.84	190	19	4.39	65
20	2.95	100	10	4.42	164
17	3.25	281	14	4.78	119
3	3.41	90	1	4.80	799
5	3.69	199	13	4.85	140
4	3.69	80	2	6.34	125
12	3.74	281			

7.2 Application of Pricing Model

To apply our Pricing Model, we first collect the six indices of this church. Conducting the procedures introduced in section 6.2 , we obtain a list of $Distance$ between FDBC and churches in our database.

It can be shown that the first three churches for reference have small $Distance$ and we take their average price, which is $\frac{70+80+99}{3} = 83$ (k dollars).

$$V = 83000 \tag{27}$$

7.3 Analysis

According to equation (26), we obtain the insurance fee for FDBC.

$$I = 135.0$$

In order to judge whether this amount is high or low, we estimate the "rent" of FDBC using the Price to Rent Ratio in the U.S., which average 101.66 from 1970 until 2023[8]. The estimated "rent" is thereby $Rt = \frac{83000}{101.66} = 816.4$, which measures the value created by FDBC in one year in economic sense.

$$Rt = 816.4$$

Comparing the "rent" and insurance fee, we find that the insurance-rent ratio is 17%, which is rather high. Note that the insurance fee in our calculation only consider pure premium, which is equivalent to expected loss. The actual insurance fee, however, will also contain additional fee due to other expenditures in the operation of an insurance company. Typically, the pure loss ratio[10] of fire insurance is about 0.7, which means that the actual insurance fee in the market might be up to 192.9 dollars for FDBC, which is 24% of rent.

Our insurance model and pricing model give an estimate of the insurance-rent ratio. Based on this number, subjective decision can be made whether or not to insure FDBC. In our perspective, it seems sensible not to buy insurance due to the high insurance-rent ratio. Instead, other measures to prevent wildfire and protect the church can be taken. In fact, many measures like defensible space, brush clearing, water storage and employee training program of fire extinguishing can prevent or greatly mitigate the property loss even when there is a wildfire in the nearby forest. (Of course, in the buffer period where these measures are no in place, an insurance of short term is recommended). After all, the disaster model and the determination of insurance fee consider the average possibility of property loss of the whole region. However, by effective fire prevention measures, FDBC can make its individual possibility of property loss much lower than social average. In this way, FDBC can ensure its safety and continue to provide remarkable spiritual support to members of the Fort Dick community, even though without a commercial insurance.

8 Model Evaluation

8.1 Strength

- In the Flood Model, we employ random variable and stochastic process, which can appropriately reflect the increasing uncertainty caused by climate change.
- In the Wildfire model, the cellular automata can precisely simulate the micro-mechanism of wild fire, giving an accurate estimate of risk in different regions.
- In the pricing model, by comparing the church with already-priced churches, we convert the inestimable indicators into measurable distances, successfully tackling the problems of incomplete market and implicit value.

8.2 Weakness

- In the Flood Model, many parameters are roughly estimated, which can be determined in greater accuracy.
- In the Wildfire Model, limited by computing power, we only simulate the wild fire risk in 360 days, which may be not enough to manifest the influence of climate change.
- In the Pricing Model, we only consider 21 different churches in the database. So the "similarity" analysis can be more accurate if we collect more data.
- When our models are applied to FDBC, the evaluation of insurance premium is merely based on limited information, without considering other features such as fire protection conditions, which may cause error.

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A Appendix: A Community Letter to Fort Dick

Dear Sir or Madam,

As the climate change leads to increasing extreme weather events, questions concerning whether and how to protect historical landmarks emerges. In your community, such a famous landmark is Fort Dick Bible Church (FDBC), which is under mounting threat of wildfires. Receiving your inquiry about this issue, our team has been working on the establishment corresponding models to address your problem. Here is a letter to introduce our works and put forward our proposals.

We developed two main models - insurance model and preservation model, to assess the premium and value of the landmark FDBC. The insurance model was based on two sub-models: loss risk model and premium pricing model. We collected the data of precipitation, temperature, forest coverage and property loss in your area, and then analyzed them using the regression method. Through this methods, predictions were made for future property losses. Based on the assumption that the insurance fee is equal to the present value of the loss, we can also determine the premiums.

The preservation model is established to evaluate the value of FDBC. After comprehensively researching your church with multi-dimensional factors which include size, lot size, age of the church, local population density, local real estate price level and median home price, we compare the condition of your church with the already-priced church in our database and take the average value of several similar churches as the value of yours. The estimated value results in 83,000$. Noting that FDBC is public, it's very likely that individuals would be unwilling to insure it due to free-rider effect. Therefore, the government plays an important role in protecting FDBC. Upon application of our models, we observed that the proportion of fire insurance expense comparing to 'rent' will reach to 24%, which is slightly high and not entirely worth paying. Since the frequency of extreme weather events is expected to increase over time, the insurance premiums here, obtained from our insurance model, will also increase. Therefore, choosing long-term insurance is inadvisable for FDBC. The community should consider stopping buying the insurance after a few years.

One feasible method is that when the rate of fire insurance expense to 'rent' is lower than, for instance, 30%, the insurance can be continued./ Simultaneously, it is possible to construct fire isolation facilities to reduce the probability of wildfires of FDBC. Other strategies, including brush clearing, water storage and employee training program of fire extinguishing, are also highly useful to reduce the impact of wildfires. Moreover, the government can organize activities to raise the fire awareness of local residents. Given the fact that over 90% of wildfires are caused by human activities, regulating the fire safety habits of local residents is also a significant method to protect this area.

To ensure that the change from insurance to other measures is effective, we should measure the total present value of these alternative measures. However, from an economic perspective, this present value needs to be lower than that of insurance fees of areas that the new measures can affect. Otherwise, consideration should be given to relocating this landmark.

However, we must recognize that the climate change is the root cause of all issues. Only by protecting the ecological environment and alleviating the climate change can we embrace prosperity and sustainability of our community.

We hope that these strategies can assist in solving your problems. If the suggestions are not satisfactory enough, please feel free to contact us for help!

Best,

Team