The Modified Booth Encoder multiplier

M. Martina

1 Introduction

In order to explain how the Modified Booth Encoder (MBE) multiplier works it is convenient to start from the Radix-2 multipliers. Let **a** and **b** be two unsigned values, each represented with n bits:

$$\mathbf{a} = \sum_{i=0}^{n-1} a_i 2^i, \tag{1}$$

$$\mathbf{b} = \sum_{j=0}^{n-1} b_j 2^j. \tag{2}$$

Let $\mathbf{c} = \mathbf{a} \cdot \mathbf{b}$, then $\mathbf{c}$ is represented with $2n$ bits. Stemming from (1) and (2) we can write $\mathbf{c}$ as:

$$\mathbf{c} = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_i b_j 2^{i+j}. \tag{3}$$

As it can be observed, the Radix-2 solution produces partial products which are in the form $p_j = a \cdot b_j$ or

$$p_j = \begin{cases} 0 & \text{if } \bar{b}_j \\ a & \text{if } b_j \end{cases} \tag{4}$$

where $a = [a_{n-1} a_{n-2} \dots a_1 a_0]$.

Table 1: Modified Booth Encoding.

$b_{2j+1} b_{2j} b_{2j-1}$	p_j
000	0
001	a
010	a
011	2a
100	-2a
101	-a
110	-a
111	0

2 Modified Booth Enconding (MBE)

MBE is an extension of the Radix-2 approach, namely instead of considering the multiplier on a bit-by-bit basis, more bits are analyzed simultaneously. Usually, MBE is a Radix-4 approach namely it produces half partial products with respect to the Radix-2 solution. This is achieved by dividing the multiplier in 3 bit slices (with $b_{-1} = 0$), where two consecutive slices feature a 1-bit overlap. If n is odd the multiplier must be sign extended to have "complete" triplets of bits. Then, each triplet of bits is exploited to encode the mutiplicand according to Table 1. As a consequence, the expression describing partial products, which can be derived from direct inspection of Table 1, is more complex in MBE than in Radix-2 solutions, namely $p_j = (b_{2j+1} \oplus q_j) + b_{2j+1}$, where

$$q_j = \begin{cases} 0 & \textbf{if } \left(\overline{b_{2j} \oplus b_{2j-1}}\right)\left(\overline{b_{2j+1} \oplus b_{2j}}\right) \\ a & \textbf{if } b_{2j} \oplus b_{2j-1} \\ 2a & \textbf{if } \left(\overline{b_{2j} \oplus b_{2j-1}}\right)\left(b_{2j+1} \oplus b_{2j}\right) \end{cases} \tag{5}$$

3 Adding partial products

The coverage of the dots can be made with any suited structure, including Wallace tree, Dadda tree, etc.