POLITECNICO
MILANO 1863

DEPARTMENT OF MECHANICAL
ENGINEERING

Control and actuating devices for agriculture

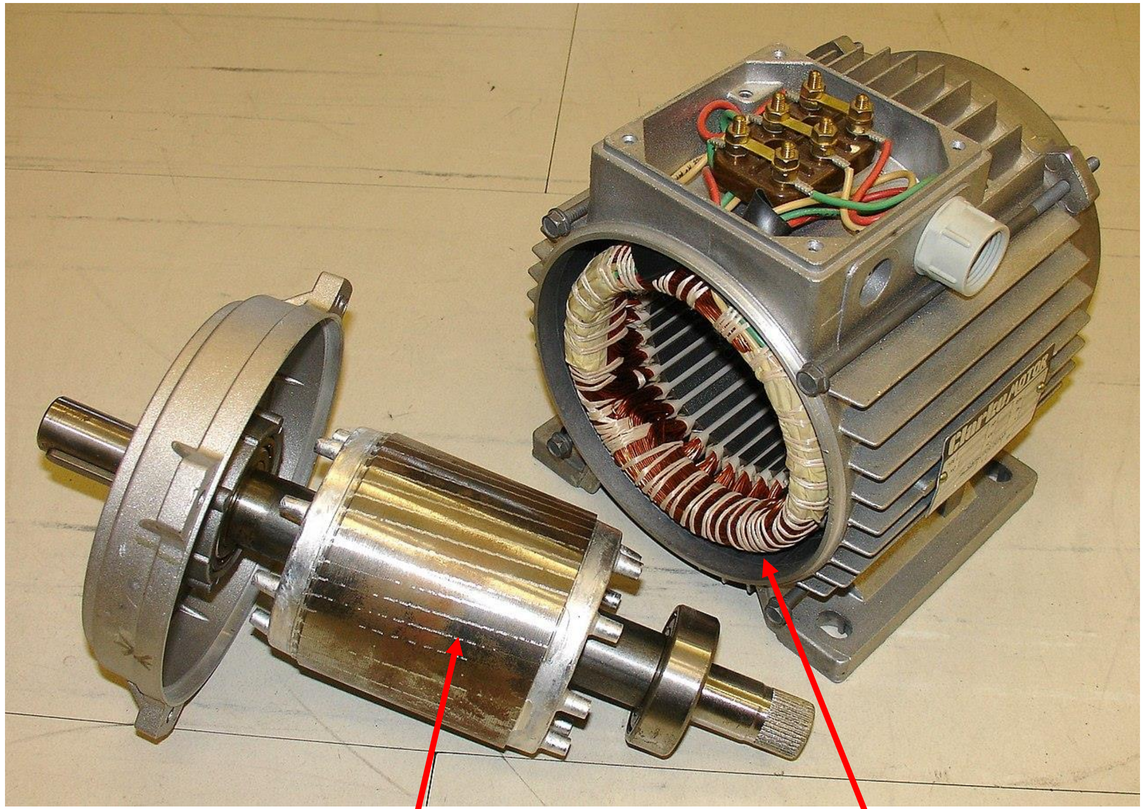
Electric motors

Michele Vignati

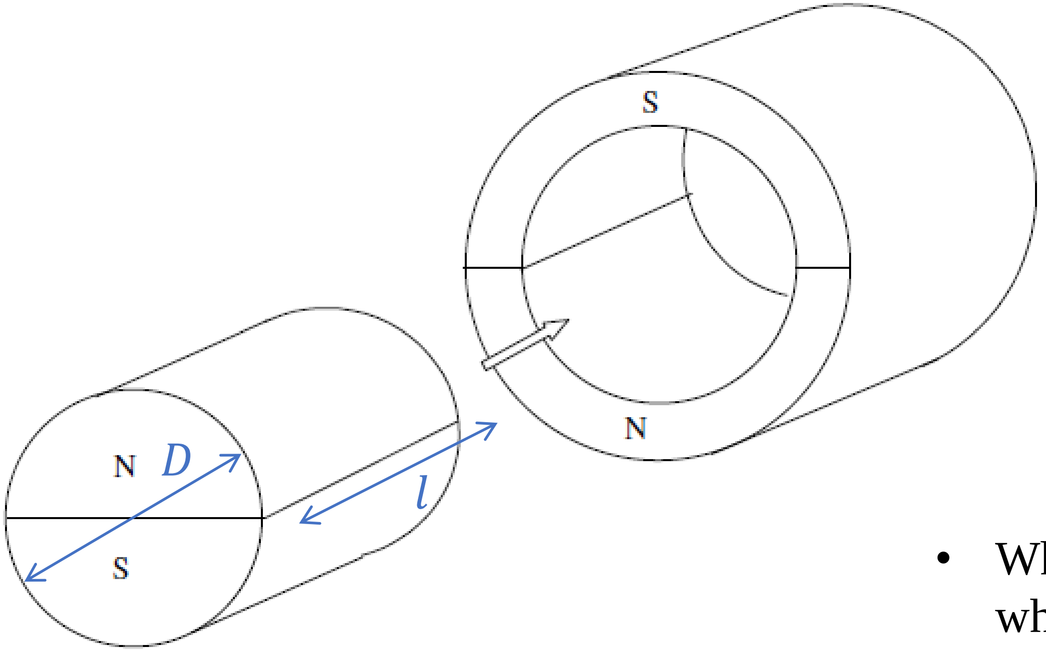
ELECTRIC MOTOR

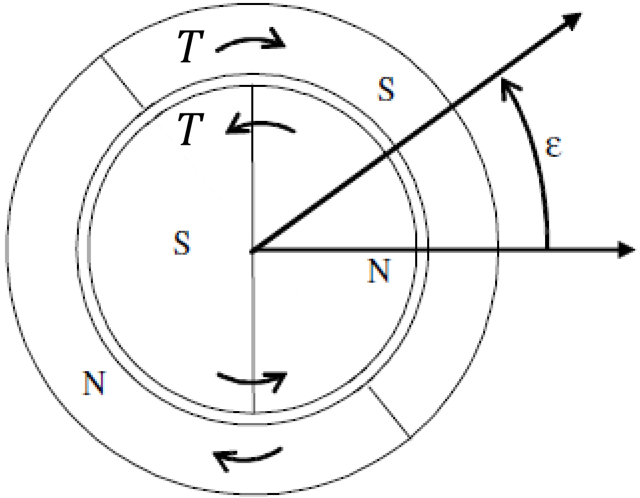
Torque is generated by the interaction between two magnetic fields

- Stator magnetic field $\vec{B}_s$
- Rotor magnetic field $\vec{B}_r$



ELECTROMAGNETIC JOINT





- When two axial symmetric magnets interact, a torque is generated which tends to align the N and S poles
- The torque is proportional to the sinus of the angle, the magnetic flux density, and the volume of the airgap.

$$T = -V_0 \frac{B_{iM} B_{eM}}{2\mu_0} \sin \varepsilon$$
$$V_0 = \pi D l t$$

MAGNETIC FIELD

$$\vec{B} = \mu\,\vec{H}$$

magnetic flux density
[*tesla=weber/meter*^{2}] → [T=Wb/m^2]

permeability of the medium
[*henry/meter*] → [H/m]

magnetic field intensity
[*ampere/m*] → [A/m]

$$[\mathrm{T}] = \left[\frac{\mathrm{Wb}}{\mathrm{m}^2}\right] = \left[\frac{\mathrm{H}}{\mathrm{m}} \cdot \frac{\mathrm{A}}{\mathrm{m}}\right] = \left[\frac{\Omega \mathrm{sA}}{\mathrm{m}^2}\right] = \left[\frac{\mathrm{Vs}}{\mathrm{m}^2}\right]$$

Where can we find magnetic fields?

a) magnetic materials (the magnetic field is originated by the spin of an orbital electron, i.e. at atomic level)

b) electric circuits (the magnetic field is originated by the motion of the electric charges along a conductor)

Note

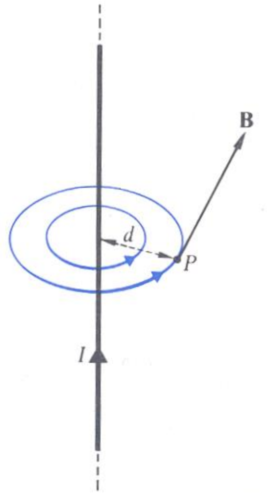
$$\mathrm{Wb} = \mathrm{V} \cdot \mathrm{s},\;\; \mathrm{H} = \mathrm{W} \cdot \mathrm{s}$$

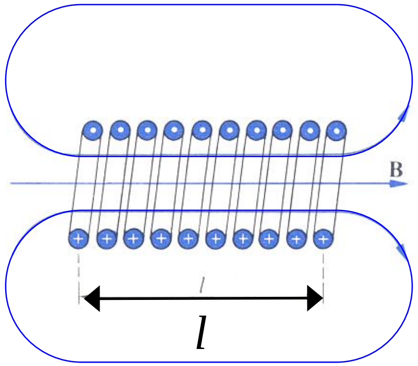
MAGNETIC FIELDS CREATED BY ELECTRIC CURRENTS

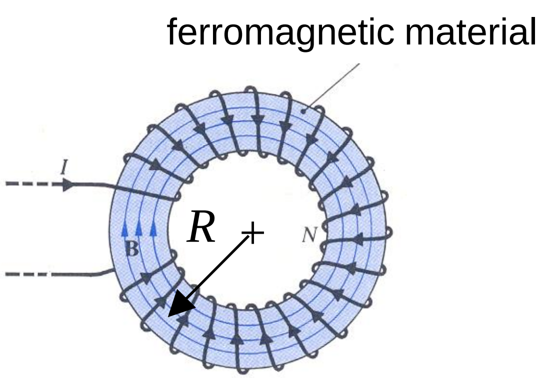
$$B = \mu \frac{I}{2\pi d}$$

$$B = \mu \frac{NI}{l}$$

$$B = \mu \frac{NI}{2\pi R}$$

a) 

b) 

c) 

field around a long straight conductor

field of a solenoid coil

field inside a toroidal coil

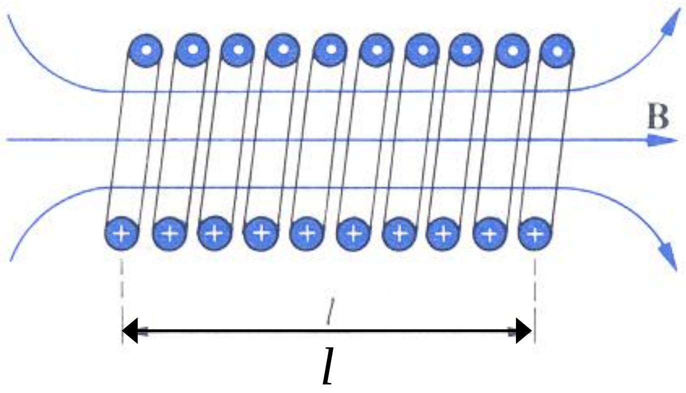
The magnetic flux lines are always closed lines.
The magnetic flux lines are linked with the electric circuit which originates them.
In case a), if the conductor is grasped in the right hand with the thumb extending in the direction of *I*, the fingers curl in the direction of *B*. In case b) and c) the same rule applies. Note that *I* and *B* are exchanged.

MAGNETIC FLUX AND MAGNETIC CIRCUITS

Magnetic flux through a surface *S*:

$$\Phi = \int_S \vec{B} \times \vec{n}\, dS \qquad [Wb]$$

Example of magnetic circuit: the solenoid coil



$$\Phi = \mu \frac{NI}{l} S = BS$$

$$NI = \frac{l}{\mu S}\Phi \quad \Rightarrow \quad NI = \Re \cdot \Phi$$

magnetomotive force

reluctance $[henry^{-1}] \rightarrow [H^{-1}]$

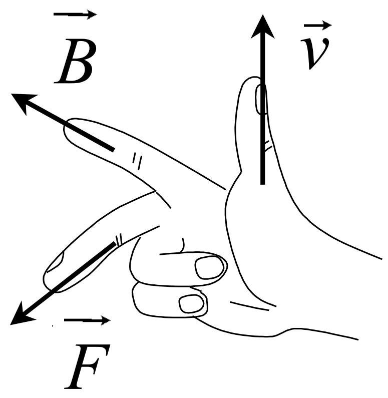
Equivalent to the Ohm's law: $V = R \cdot I$

MAGNETIC FORCES ON ELECTRIC CONDUCTORS WITH FLOWING CURRENT

<u>Lorentz force</u>:

force on a positive electric charge q, moving with the velocity $\vec{v}$ in a magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$



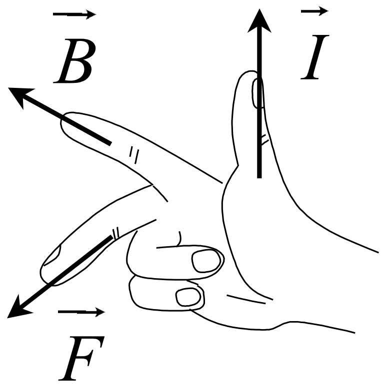
right hand rule

Elementary force acting on a short element of conductor $\mathrm{d}l$ with a current $\vec{I}$ flowing in it, as a result of a magnetic field

$$\mathrm{d}\vec{F} = \mathrm{d}l\,\vec{I} \times \vec{B}$$

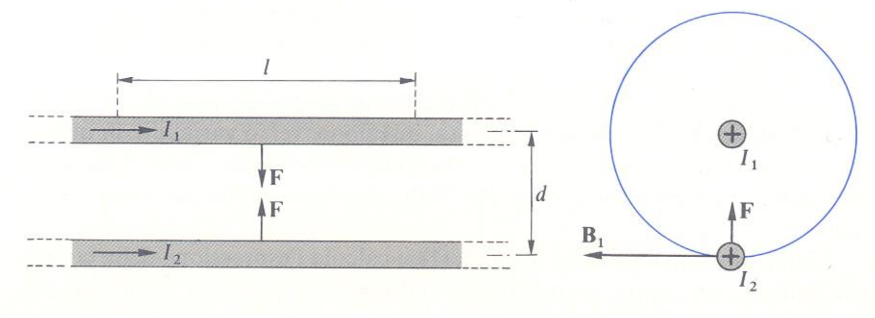
Force acting on a long straight conductor with a current $\vec{I}$ flowing in it, as a result of a uniform magnetic field

$$\vec{F} = l\,\vec{I} \times \vec{B}$$



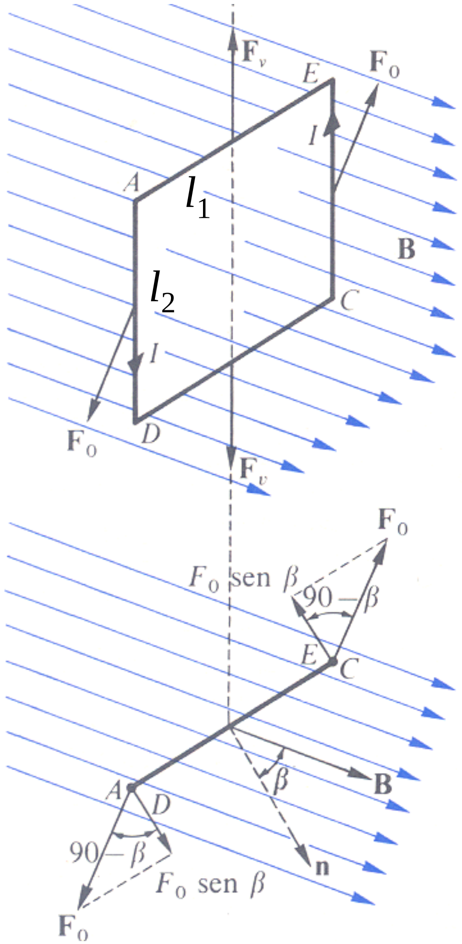
Magnetic forces on electric conductors with flowing current

1) Force exchanged by two parallel and long straight conductors



$$F = \frac{\mu l}{2\pi d} I_1 I_2$$

attraction when the two currents have the same direction



2) Torque acting on a rectangular coil immersed in a uniform magnetic field

$$F_0 = I\, l_2\, B$$

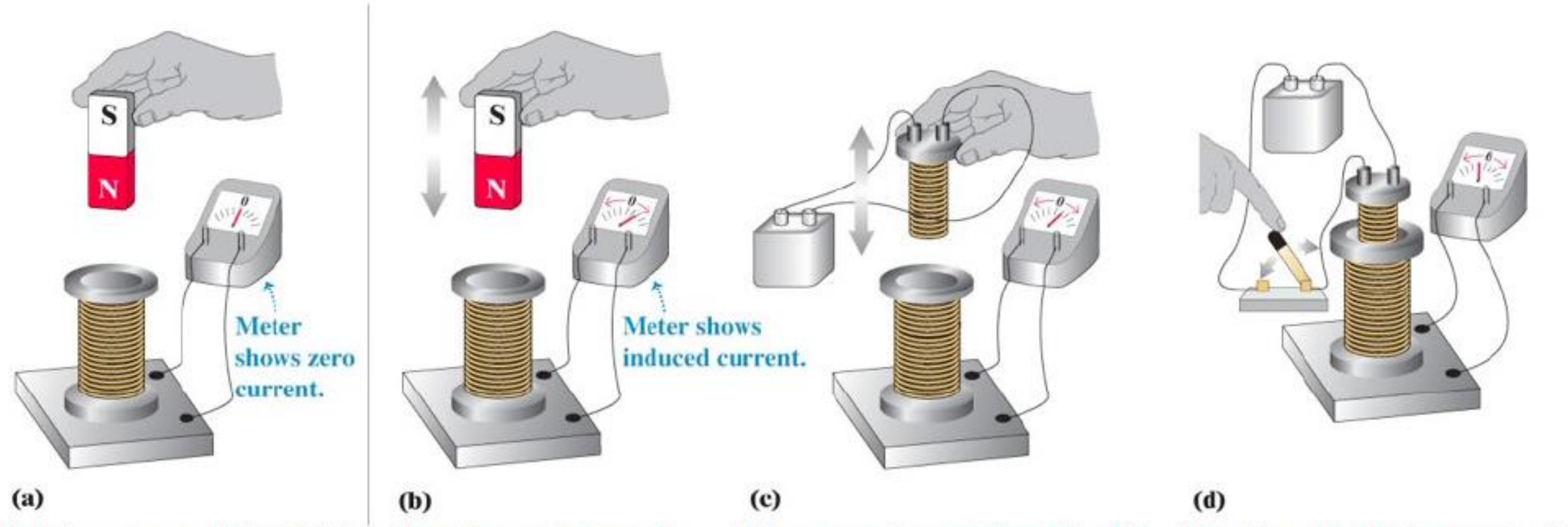
$$T = F_0 \sin\beta\, l_1 = I\, l_1 l_2\, B \sin\beta = I \Phi_{max} \sin\beta$$

the torque is zero when vector $\vec{B}$ is normal to the coil plane, that is in correspondence with the maximum flux

ELECTROMAGNETIC INDUCTION

Let’s consider an electric circuit linked with a variable magnetic flux.

The flux variation can be associated either to a change in the magnetic field strength/direction or to a modification of the surface within the coil.



A stationary magnet does NOT induce a current in a coil.

Move the magnet toward or away from coil.

Move a second, current-carrying coil toward or away from the coil.

Vary the current in the second coil (by closing or opening a switch).

ELECTROMAGNETIC INDUCTION

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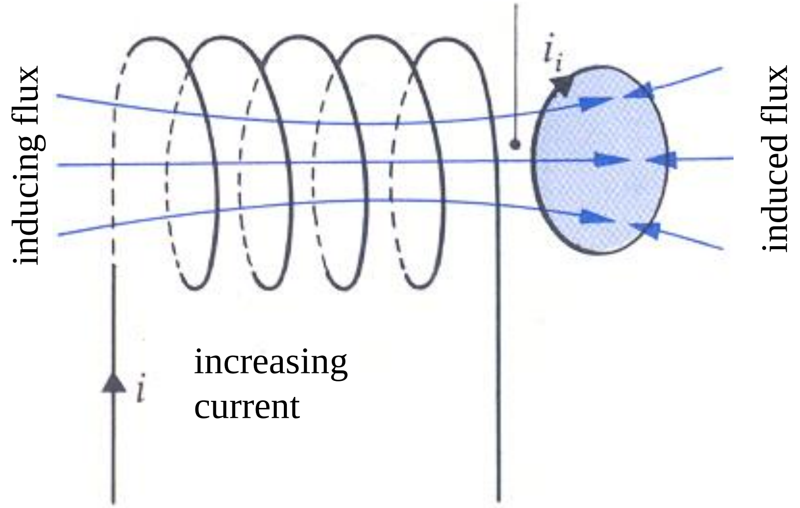


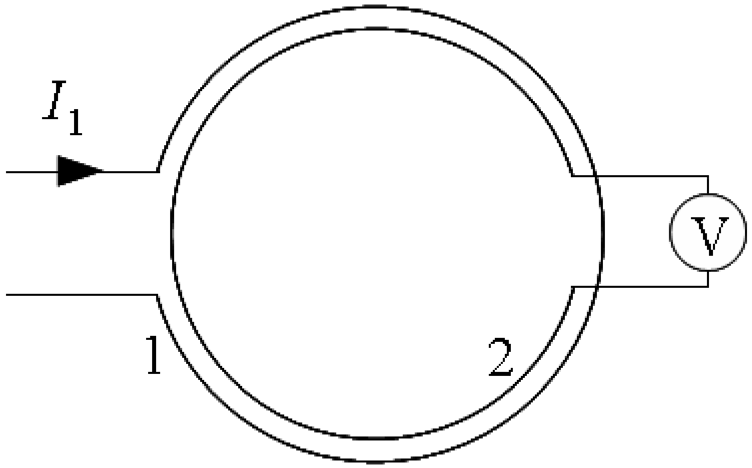
Faraday's Law: $$e = -\frac{d\Phi}{dt}$$

electromotive force (emf)

If the circuit experiencing the emf is made of N turns, the emf is equal to $N \cdot e$

Any change in the magnetic flux linked with a coil will cause a voltage (emf) to be "induced" in the coil itself, no matter how the change is produced.

ELECTROMAGNETIC INDUCTION



1 = inducing circuit

2 = induced circuit

I_1 = variable current

emf on the circuit 2 due to a variation of the flux generated by the circuit 1:

$$e_{21} = -\frac{d\Phi_{21}}{dt} = -L_{21}\frac{dI_1}{dt}$$

L_{21} = mutual-inductance [H]

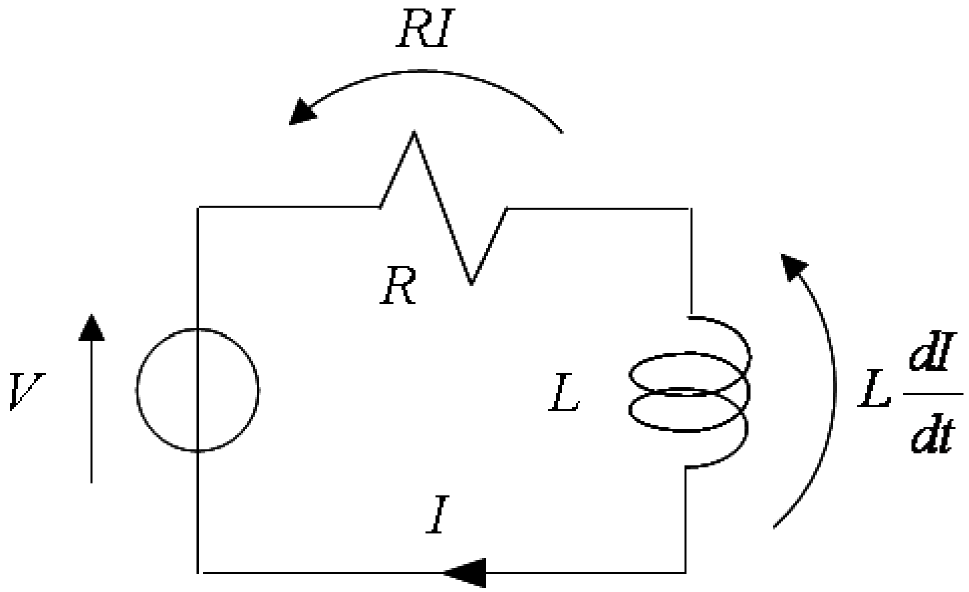
emf on the circuit 1 due to a variation of the flux generated by the circuit 1 itself:

$$e_{11} = -\frac{d\Phi_{11}}{dt} = -L_{11}\frac{dI_1}{dt}$$

L_{11} = self-inductance [H]
or simply inductance (indicated with L)

ELECTROMAGNETIC INDUCTION

$$e_{11} = -\frac{d\Phi_{11}}{dt} = -L\frac{dI}{dt} \longrightarrow \text{opposed to the current}$$



$$V = RI + L\frac{dI}{dt}$$

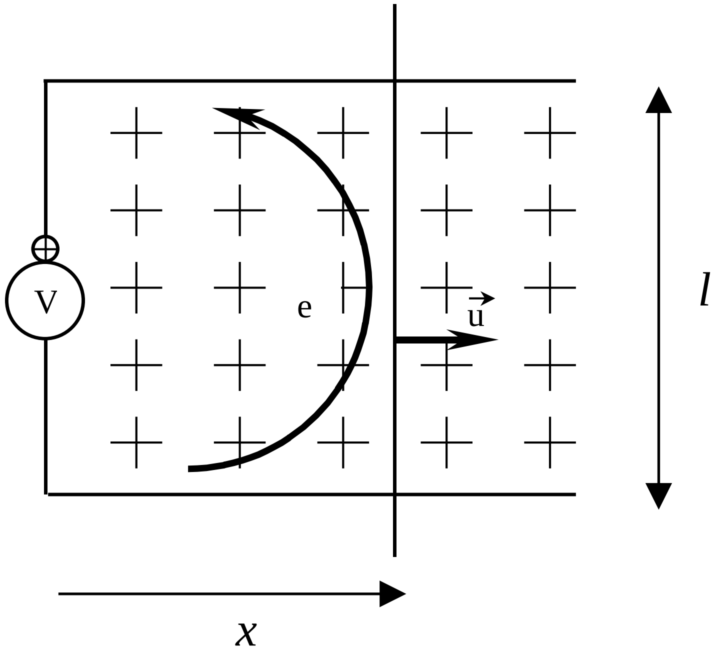
When an electric current flows through a circuit, a magnetic field is generated whose flux lines are linked with the circuit itself. If the current is variable, the magnetic flux varies accordingly and a self-induced emf arises.

PRINCIPLES OF ELECTROMECHANICS

A device to convert mechanical energy into electrical or vice versa is called an electromechanical transducer.

Let's consider a conductor of length l, moving to the right at velocity u normal to a constant magnetic field (whose flux lines are entering the figure below), that closes a circuit on which a voltmeter is inserted.

Because of the motion, the flux linked with the circuit increases and an emf is induced on the conductor and revealed by the voltmeter.



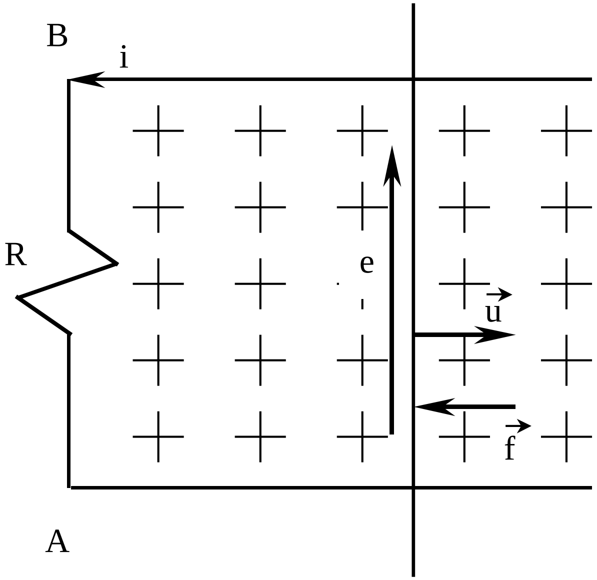
$$\Phi = B \cdot S = B \cdot lx$$

$$\Rightarrow \quad e = \frac{d\Phi}{dt} = Bl\frac{dx}{dt} = Blu$$

Each charged particle in the moving conductor experiences a force (Lorentz force): positive charges are forced upward and negative charges downward. Thus a difference of potential (emf) across the conductor arises.

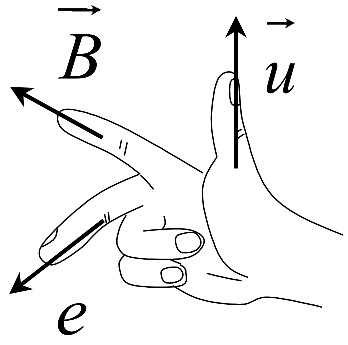
By closing the circuit with a resistance positioned between the pins A-B, it is possible to point out the conversion from mechanical power to electrical power.

A current i is circulating and a consequent energy dissipation is taking place:



$$i = \frac{e}{R} \qquad P = e \cdot i = R \cdot i^2$$

$$e = Blu$$



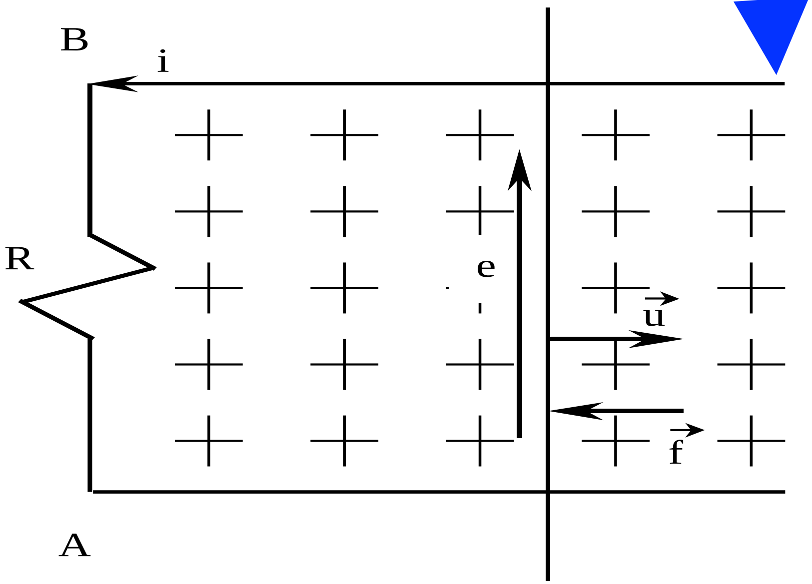
The emf produces a current (with equal direction of emf) and a consequent induced flux which is opposite to the inducing one.

PRINCIPLES OF ELECTROMECHANICS

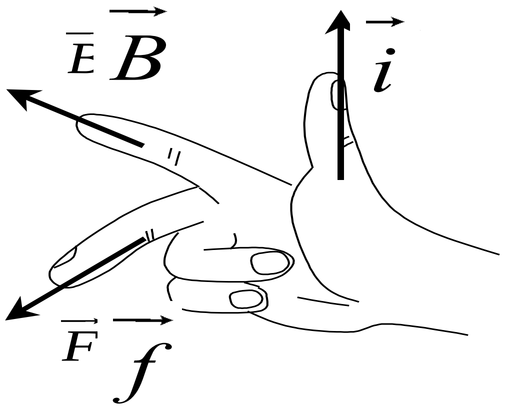
BY SPENDING MECHANICAL ENERGY, OPPOSITE TO THE LORENT FORCE WE CAN CONVERT IT INTO ELECTRICAL ENERGY

Since there is no other energy source in the circuit, the energy that is dissipated on the resistor must come from the mechanical force applied to the moving conductor.

In steady-state conditions (the conductor moves at constant speed) the electric system produces a resisting force f (opposite to the velocity u) which is equal and opposite to the driving force.

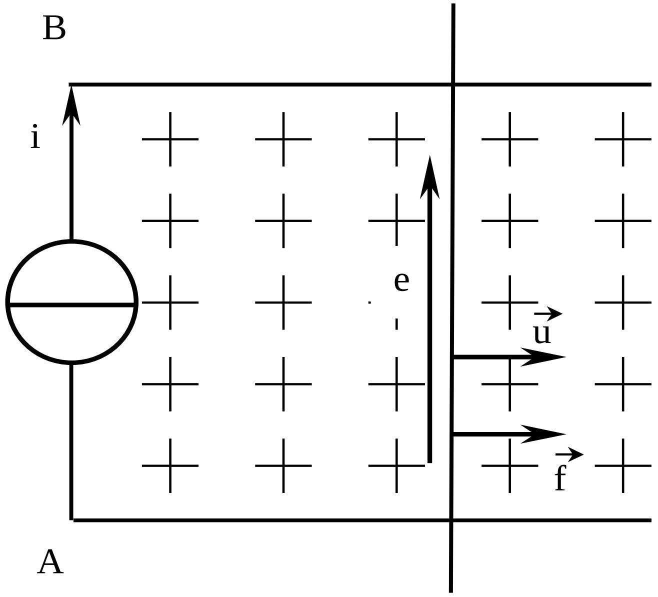


Lorentz force: $\vec{f} = l\vec{i} \times \vec{B}$



The elementary system described is working like an <u>electric generator</u>. The electric current produced by the emf e is dissipated by the circuit resistance.

PRINCIPLES OF ELECTROMECHANICS



Lorentz force: $\vec{f} = \vec{l}i \times \vec{B}$

AND IT'S VALID ALSO IN THE OPPOSITE SENSE WHERE WE CONVERT ELECTRICAL ENERGY INTO MECHANICAL ENERGY

The described electric machine (electric generator) is perfectly reversible, in that it can become an <u>electric motor</u>: in this case the energy is provided by a current generator and is transformed into mechanical energy.
The electric power is now opposite in sign and, for energy conservation, also the Lorentz force f is in the opposite direction with respect to the previous case: it now becomes a driving force (in the same direction of u).
In steady-state conditions (the conductor moves at constant speed) the mechanical system produces a resisting force (opposite to the velocity u) which is equal to the driving electrical force.

For constant magnetic flux density B:

- <u>the driving force f is proportional to the current i</u>
- <u>the emf e is proportional to the velocity u</u>

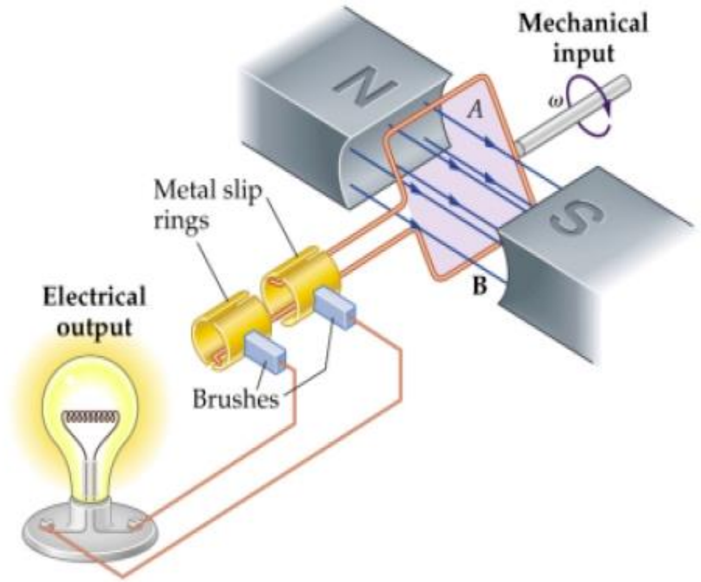
$$\boxed{\begin{aligned} f &= l\,Bi \\ e &= Blu \end{aligned}}$$

$$Bl = K_{\Phi}$$

DC MOTOR

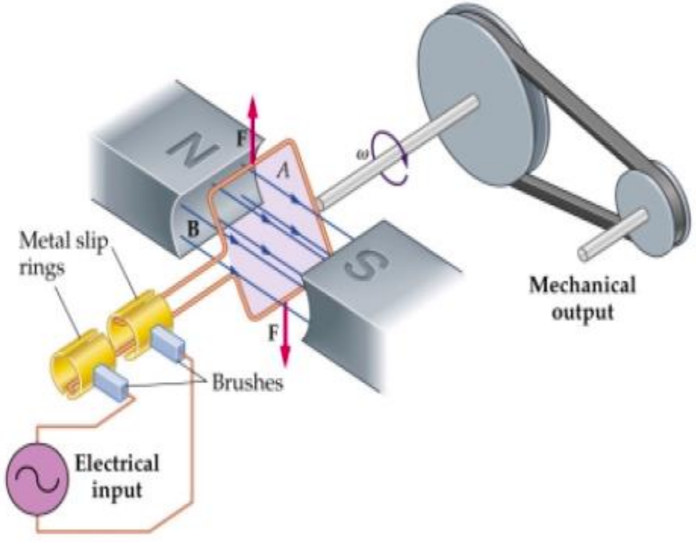
THE DOUBLE POSSIBILIY: MECHANICAL → ELECTRICI AND ELECTRIC → MECHANICAL IS EXACTLY HOT IT WORKS THE DUALITY OF THE ELECTRICAL GENERATOR AND THE ELECTRICAL MOTOR.

Electric generator



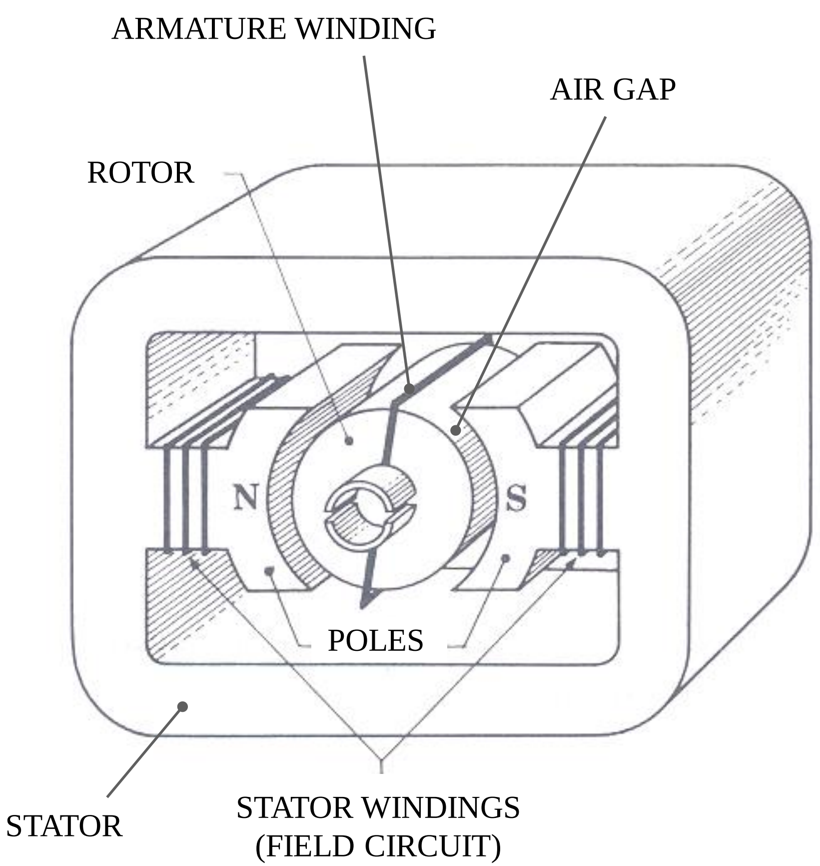
Energy is provided by a mechanical source of power, making the coil rotate. As the coil rotates, an emf is produced, which can be used to power an electric circuit.

Electric motor

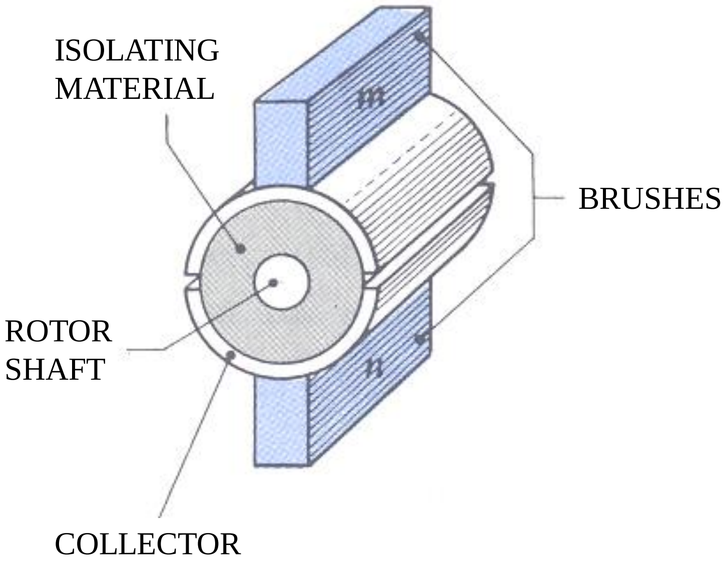


Energy is provided by a current generator, and it is transformed into mechanical energy.

DC MOTOR



MECHANICAL COMMUTATOR

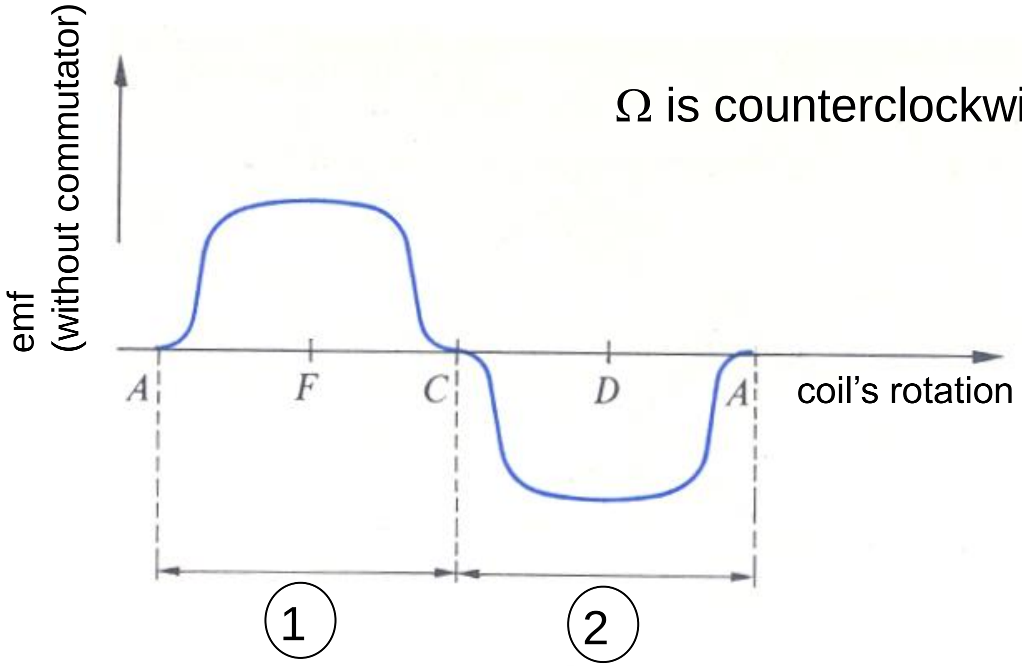


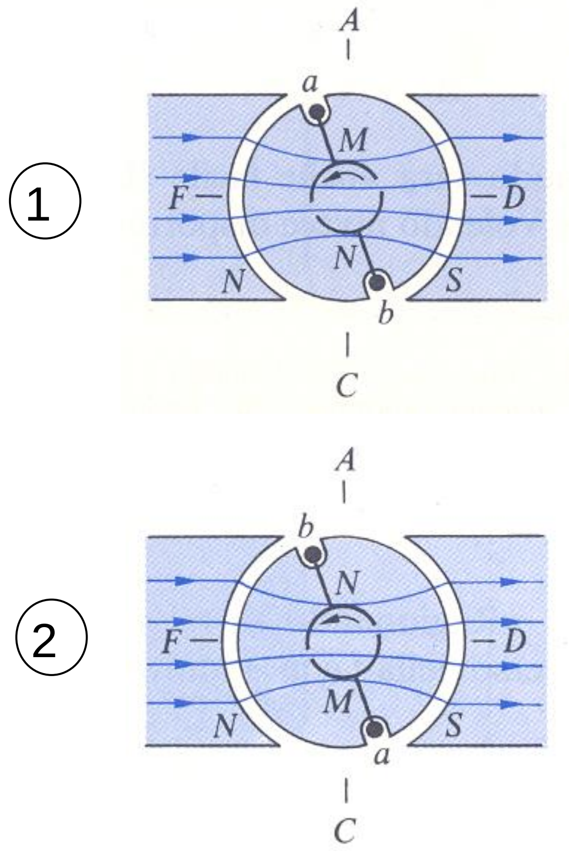
The stator's magnetic field can be produced by means of either permanent magnets or an excitation circuit

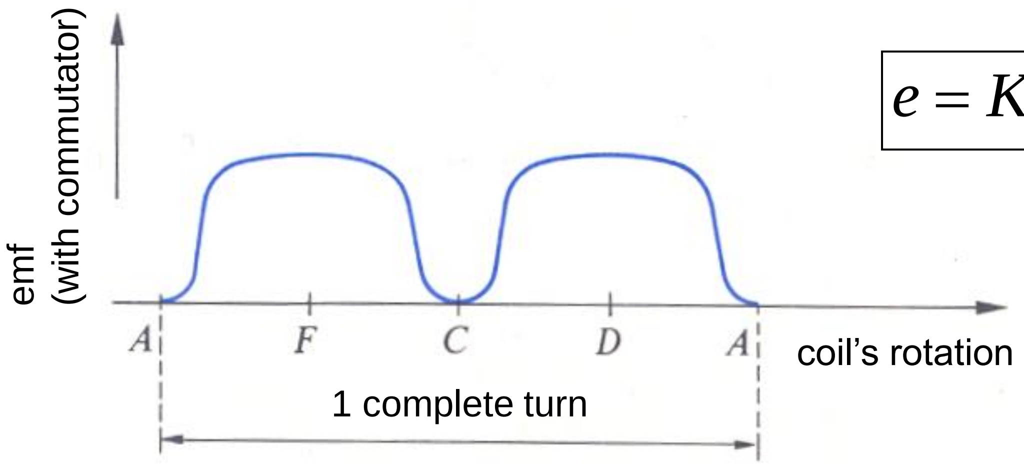
The commutator is designed for producing a polarity inversion in one rotor's half-turn with respect to the other

DC MOTOR

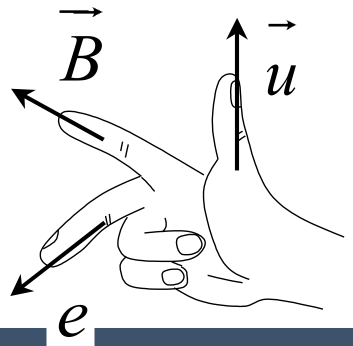
Ω is counterclockwise



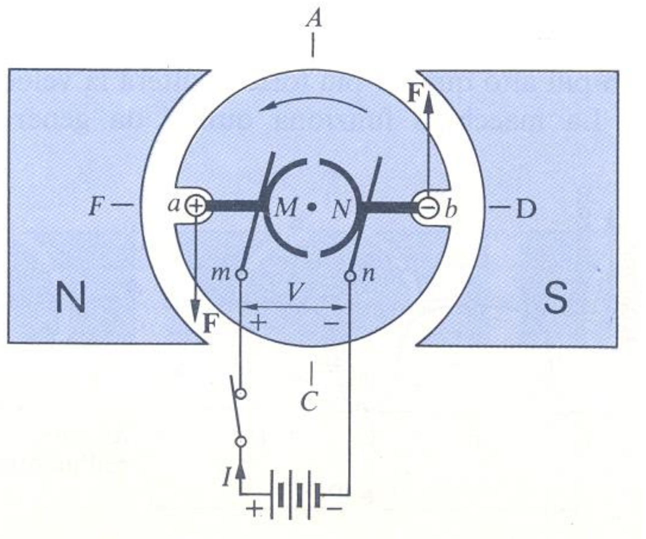


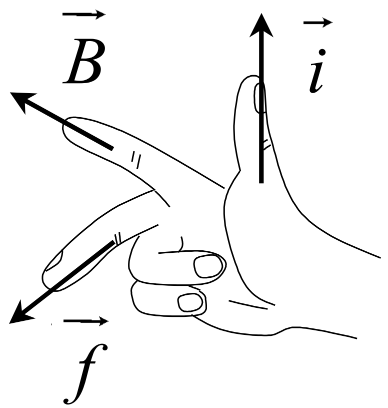


$$\boxed{e = K_{\Phi}\Omega}$$

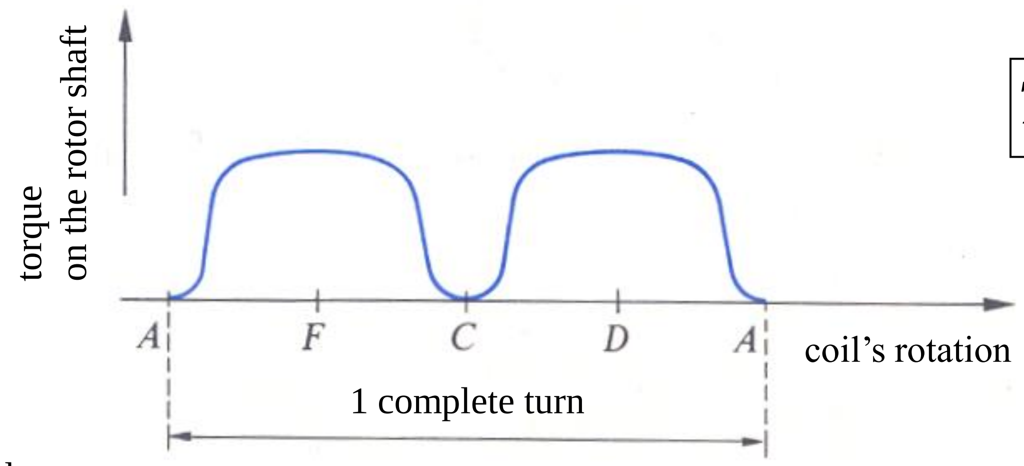


DC MOTOR



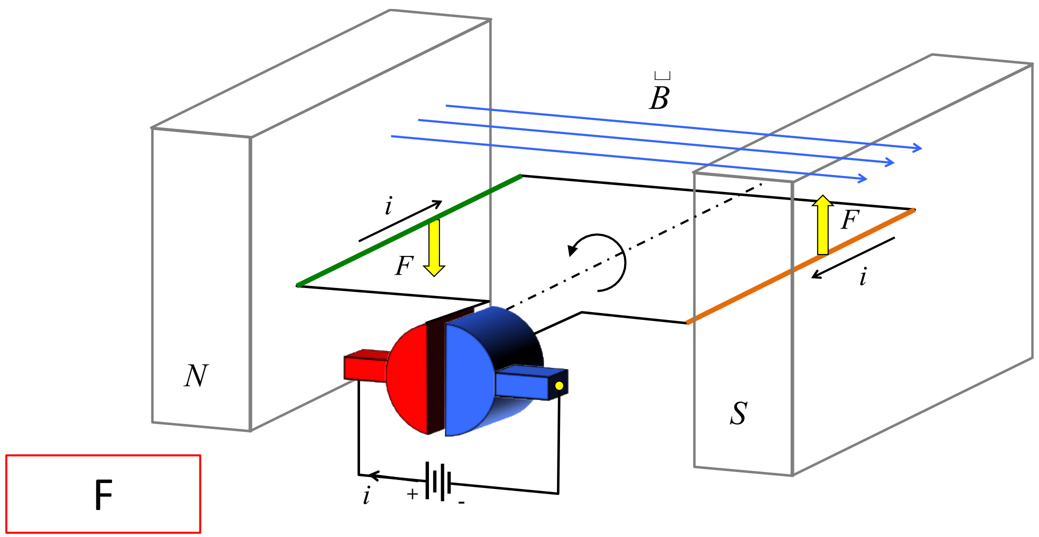


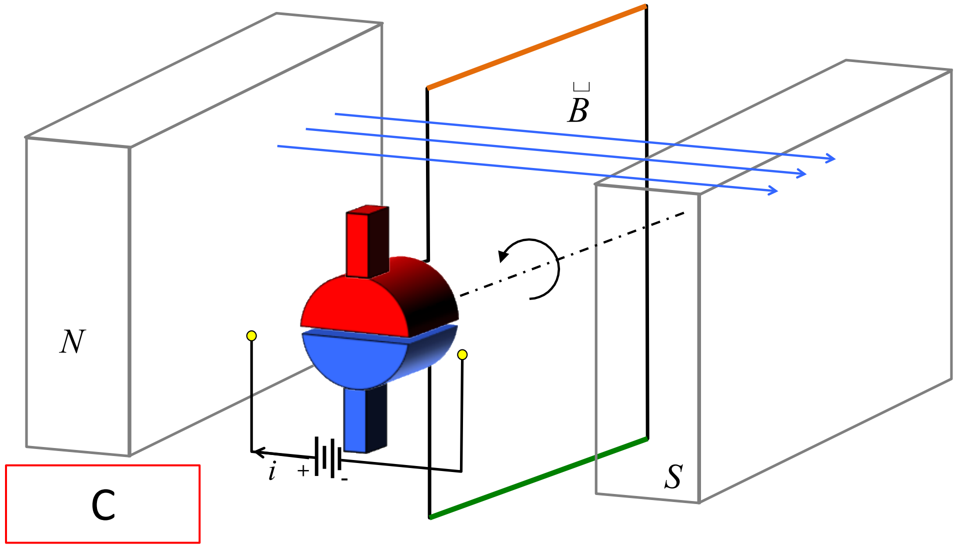
constant voltage *V* → direct current *I* on the rotor's winding → forces *f* acting on the conductors *a* and *b* → driving torque on the rotor shaft

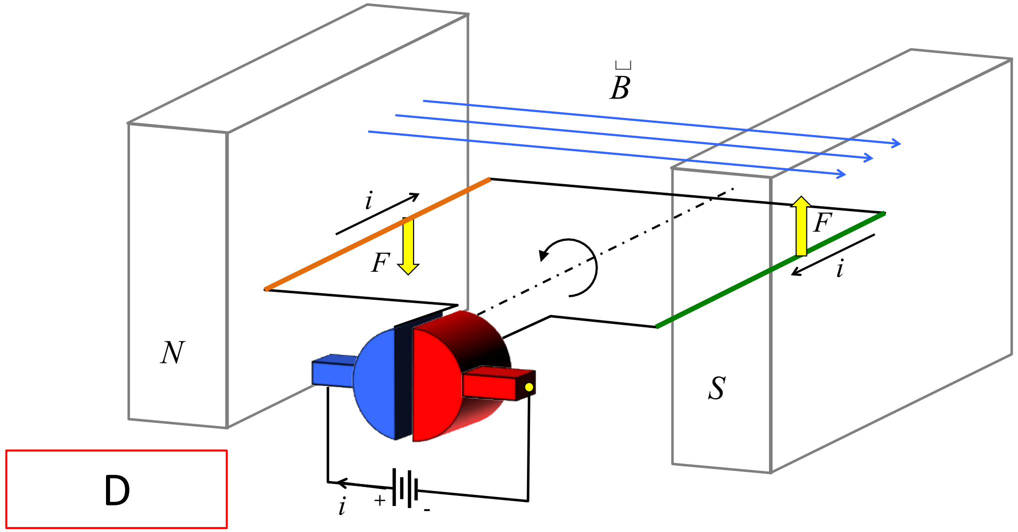


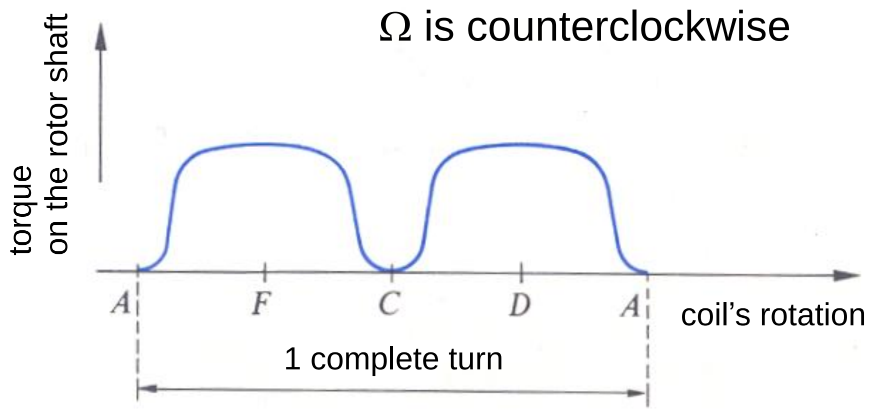
$$\boxed{T = K_{\Phi} i}$$

DC MOTOR-COMMUTATOR

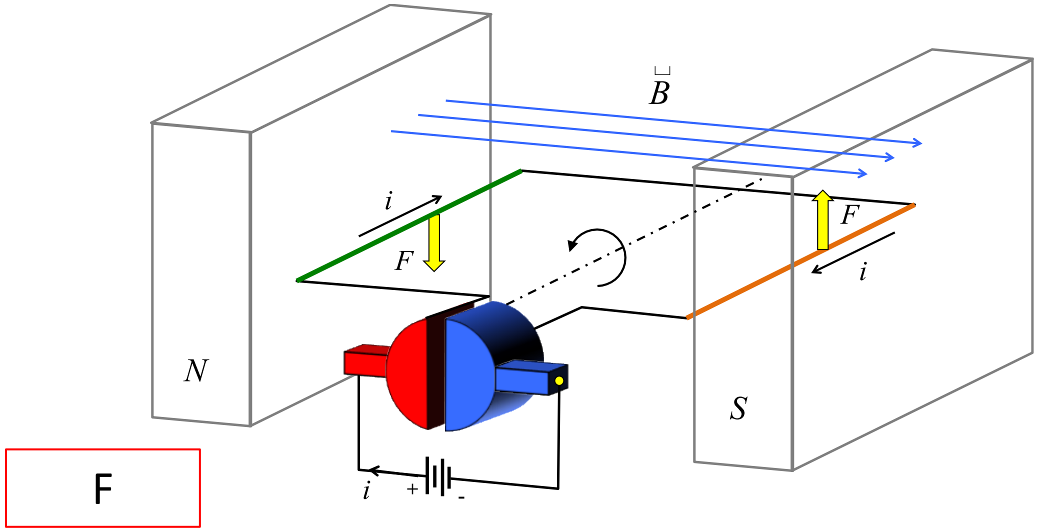


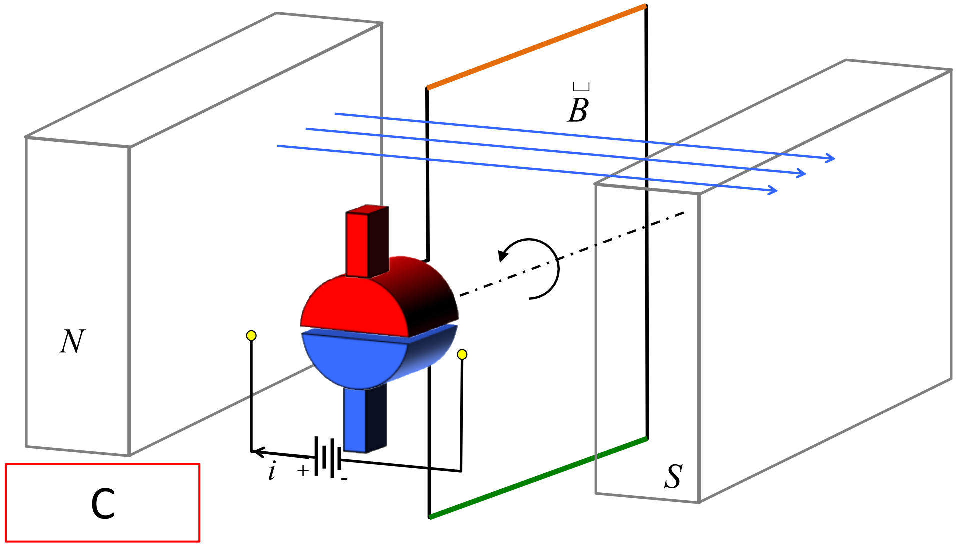


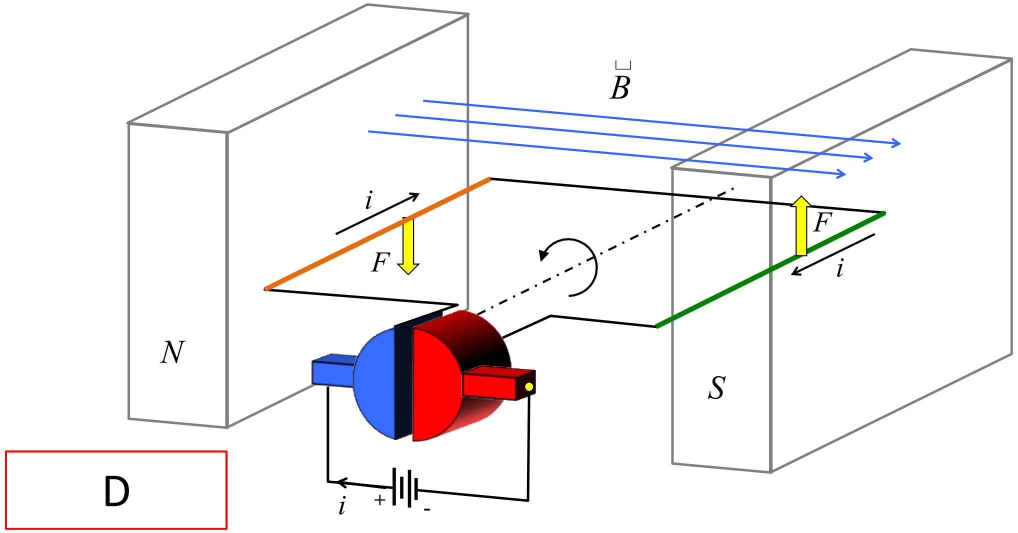


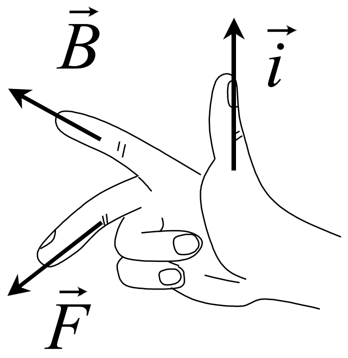


DC MOTOR-COMMUTATOR

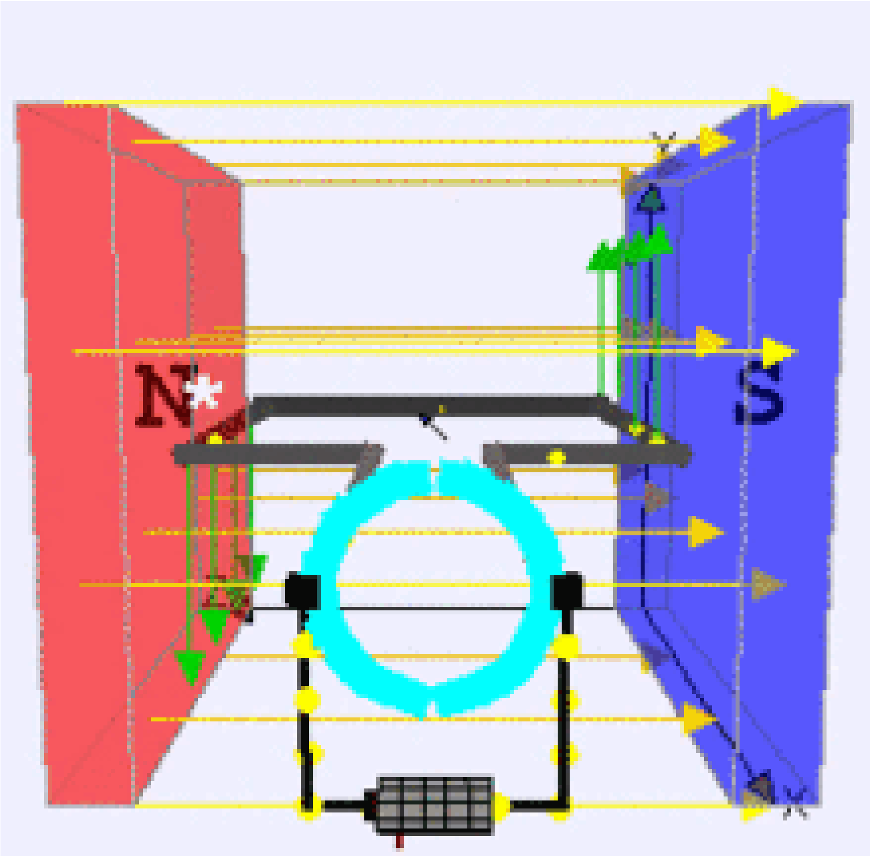




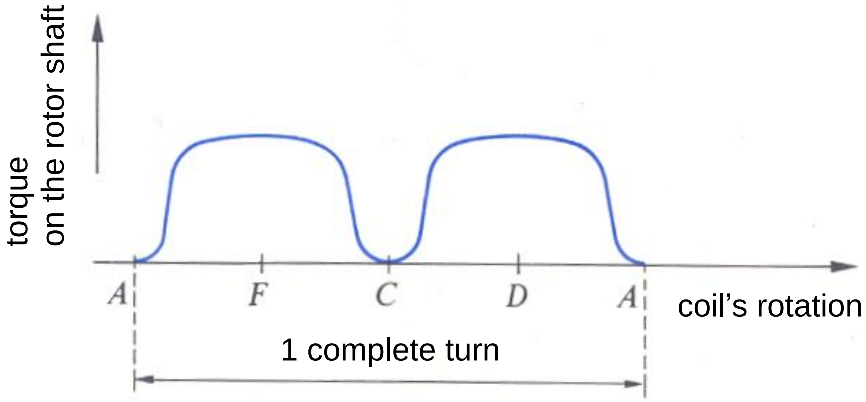




DC MOTOR

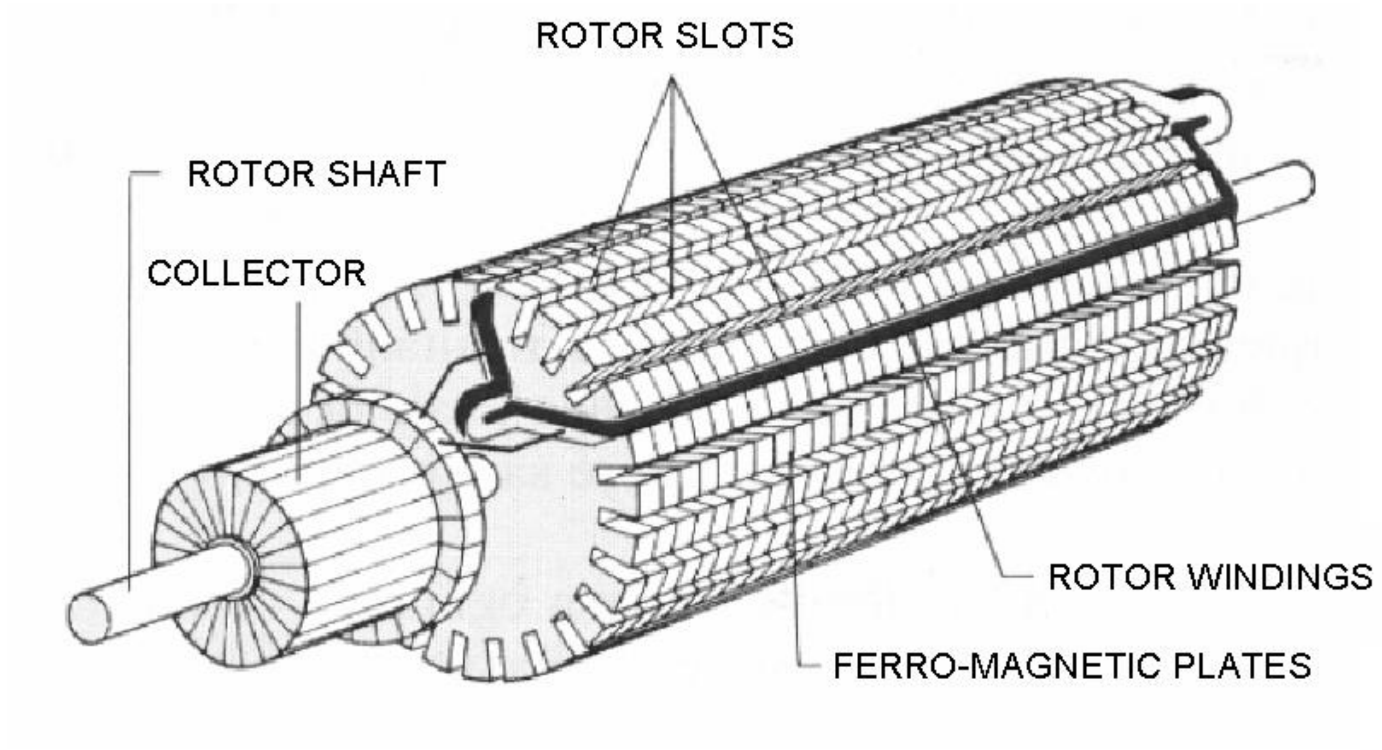


Ω is counterclockwise



DC MOTOR

ROTOR CONSTRUCTION

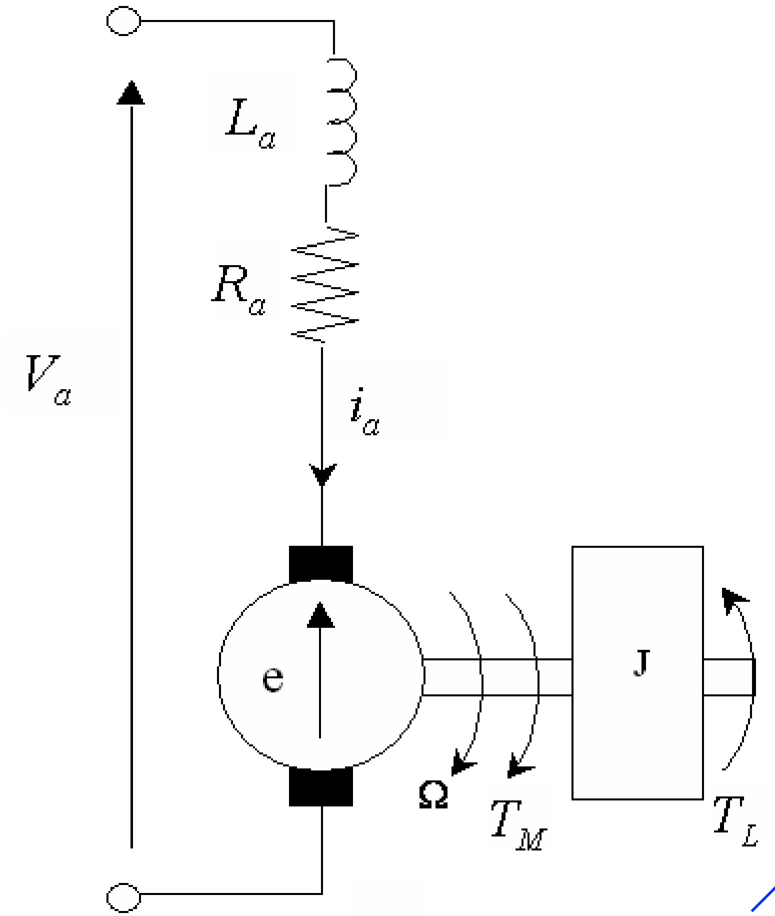


A more regular torque can be obtained by means of more rotor windings, with proper collector’s design

In any case, the overall rotor+load inertia reduces the fluctuations in the angular speed (similarly to a flywheel)

DC MOTOR: DYNAMIC MODEL

PERMANENT MAGNET DC MOTOR



let's consider the total power:
total power electrical circut =

will be equal to the mechanical power

Electric circuit equation

$$V_a = R_a I_a + L_a \frac{\mathrm{d}I_a}{\mathrm{d}t} + e$$

inertia* acc + viscus damping* speed=
torque motor - torque user

Mechanical system equation

$$J\dot{\Omega} + c\Omega = T_m - T_r$$

flux constant, B*l, will be our torque constant that relate the torque to the current but also the viceversa with the toorque speed!!

Electro-mechanical equations

$$T_m = K_\Phi i_a$$

$$e = K_\Phi \Omega$$

Power equation

$$P = V_a \cdot I_a = R_a \cdot I_a{}^2 + L_a \frac{\mathrm{d}I_a}{\mathrm{d}t} \cdot I_a + e \cdot I_a$$

the only dissipation we have!

dissipated power

power stored in the system

power delivered to the rotor

$$e \cdot I_a = T_M \cdot \Omega$$

DC MOTOR STEADY-STATE TORQUE

we obtained a model
that's exactly the same
of the hydraulic actuator

Torque characteristics

$$T_m = K_\Phi \frac{V_a}{R_a} - \frac{K_\Phi^2}{R_a}\Omega$$

Electric circuit equation

$$V_a = R_a I_a + L_a \frac{\mathrm{d}I_a}{\mathrm{d}t} + e$$

Electro-mechanical equations

$$T_m = K_\Phi I_a$$
$$e = K_\Phi \Omega$$

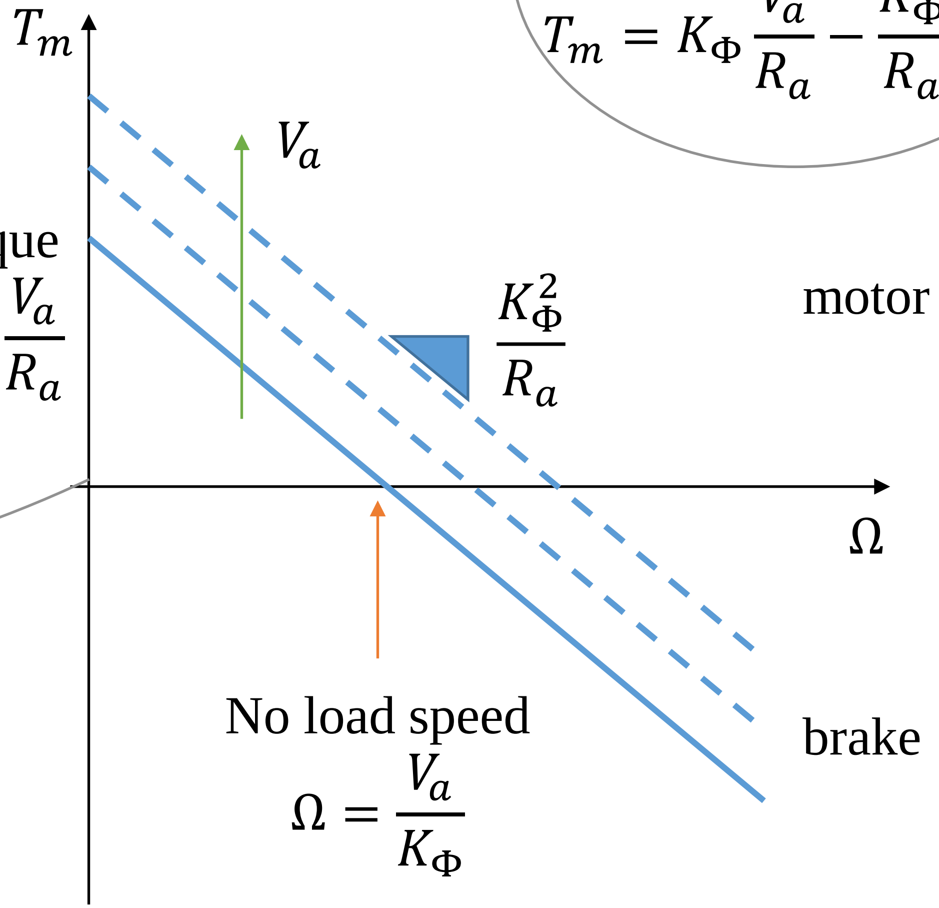
At steady-state

$$I_a = \frac{V_a}{R_a} - \frac{K_\Phi}{R_a}\Omega$$

when Ω = 0 → e=0

Stall torque

$$T_0 = K_\Phi \frac{V_a}{R_a}$$



No load speed

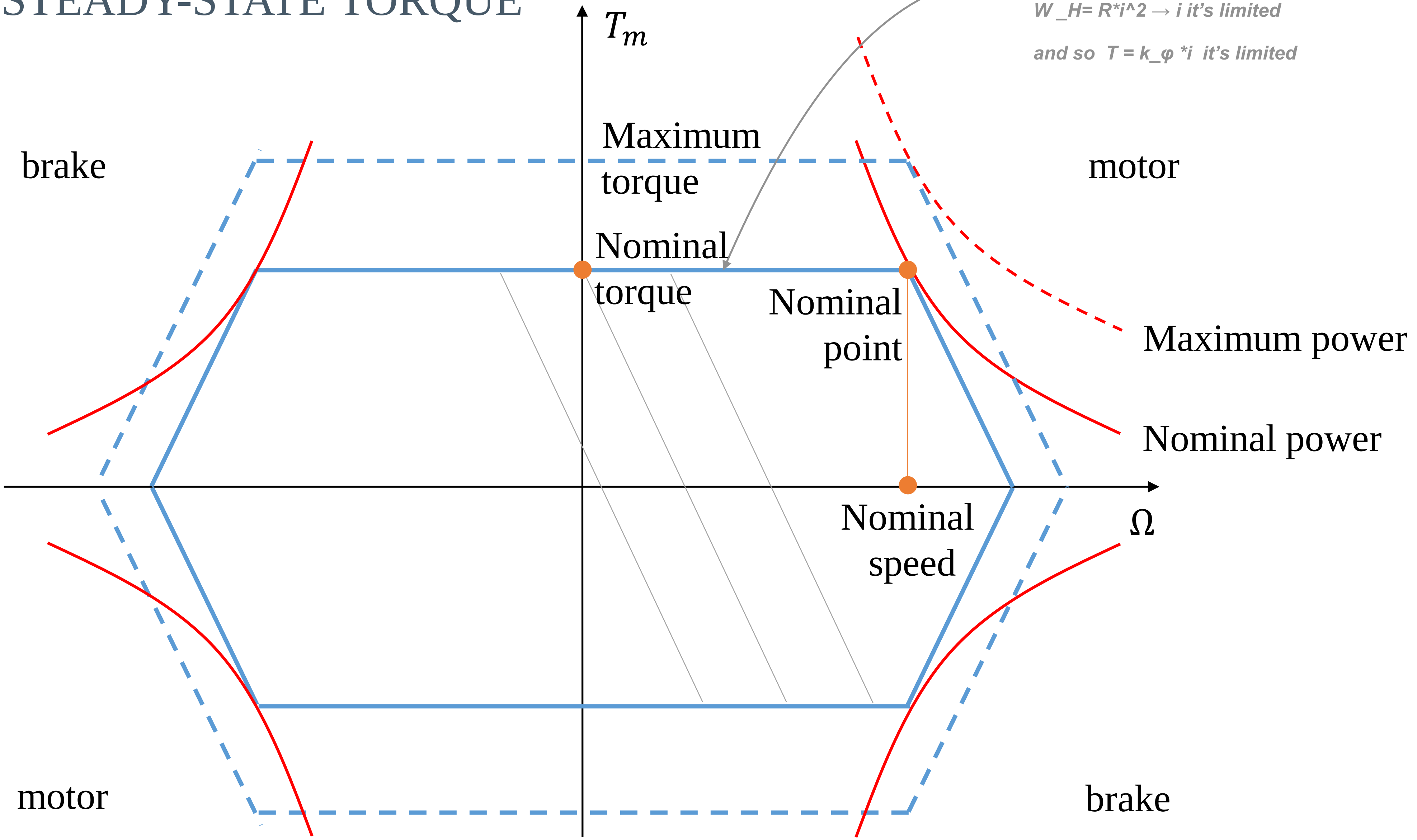
$$\Omega = \frac{V_a}{K_\Phi}$$

DC MOTOR STEADY-STATE TORQUE

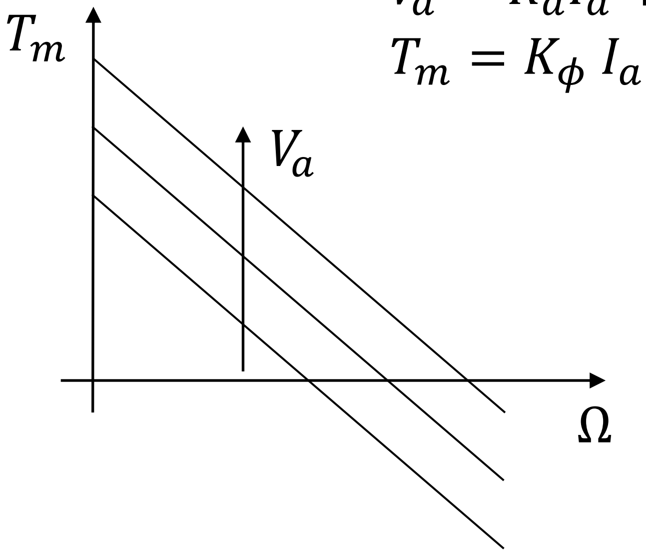
torque upper bound due to
the limit of the current due to the limit
of the material otherwise we burn everrything

W _H= R*i^2 → i it's limited

and so T = k_φ *i it's limited



DC MOTOR EXAMPLE DATA 1



$$V_a = R_a I_a + K_\phi \Omega$$
$$T_m = K_\phi \, I_a$$

$$T_m = \frac{K_\phi}{R_a}\left(V_a - K_\phi \Omega\right) = bV_a - a\Omega$$

Controller input

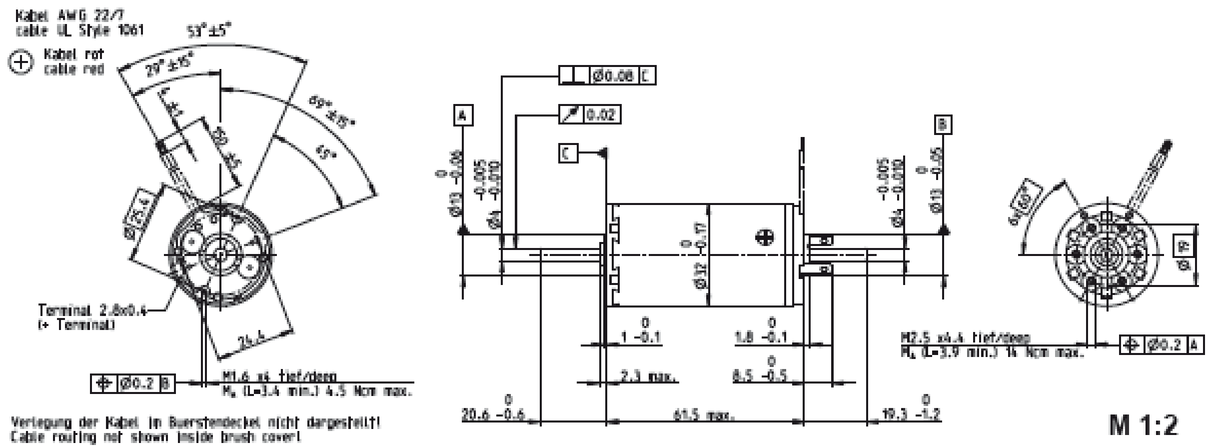
a=2.03e-4 Nm s/rad

b=0.0095 Nm/V

$$\tau_m = \frac{L_a}{R_a} = \frac{1}{6875} \text{ s}$$

maxon A-max

A-max 32 ∅32 mm, Graphite Brushes, 15 Watt

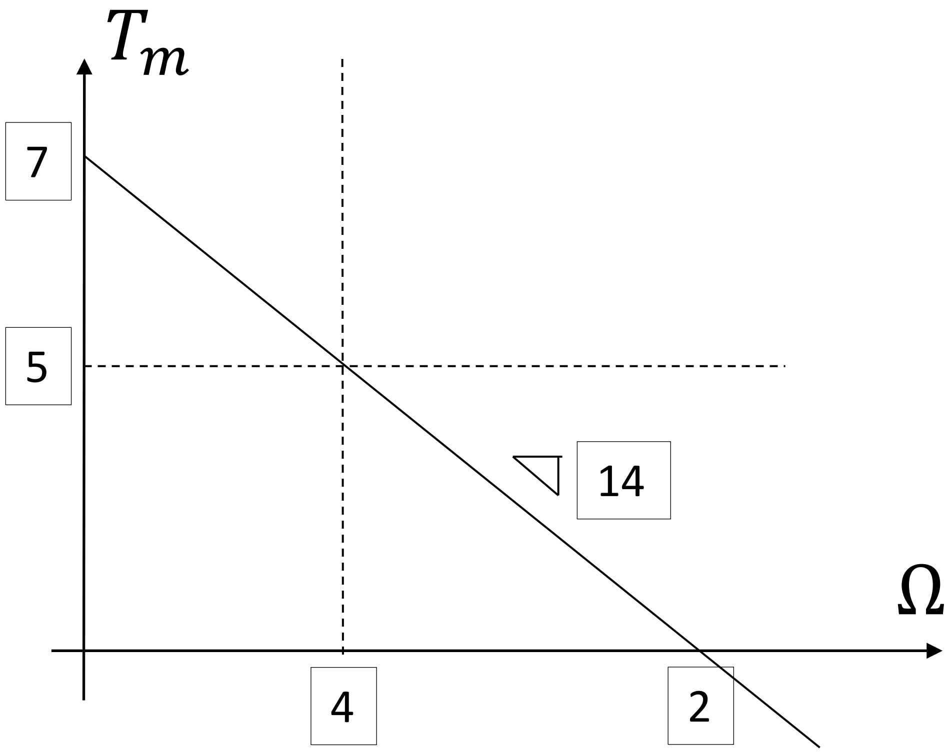


Motor Data					
	Values at nominal voltage				
1	Nominal voltage	V	6	9	12
2	No load speed	rpm	5870	4940	4680
3	No load current	mA	154	83.5	58.6
4	Nominal speed	rpm	4110	3090	2920
5	Nominal torque (max. continuous torque)	mNm	36.5	35	37.2
6	Nominal current (max. continuous current)	A	3.95	2.12	1.6
7	Stall torque	mNm	127	95.3	101
8	Stall current	A	13.2	5.58	4.19
9	Max. efficiency	%	78	76	77
	Characteristics				
10	Terminal resistance	Ω	0.454	1.61	2.86
11	Terminal inductance	mH	0.066	0.209	0.416
12	Torque constant	mNm/A	9.58	17.1	24.1
13	Speed constant	rpm/V	996	559	396
14	Speed / torque gradient	rpm/mNm	47.2	52.8	47
15	Mechanical time constant	ms	21.9	21.7	21.4
16	Rotor inertia	gcm²	44.2	39.2	43.5

so i can overload at max x3

DC MOTOR EXAMPLE DATA 1

$$V_a = L_a \dot{I}_a + R_a I_a + K_\phi \Omega$$
$$T_m = K_\phi I_a$$



	Motor Data			
		Values at nominal voltage		
	1	Nominal voltage	V	12
	2	No load speed	rpm	4680
	3	No load current	mA	58.6
	4	Nominal speed	rpm	2920
	5	Nominal torque (max. continuous torque)	mNm	37.2
	6	Nominal current (max. continuous current)	A	1.6
	7	Stall torque	mNm	101
	8	Stall current	A	4.19
	9	Max. efficiency	%	77
		Characteristics		
	10	Terminal resistance	Ω	2.86
	11	Terminal inductance	mH	0.416
K_ϕ	12	Torque constant	mNm/A	24.1
K_ϕ^{-1}	13	Speed constant	rpm/V	396
$\frac{R_a}{K_\phi^2}$	14	Speed / torque gradient	rpm/mNm	47
	15	Mechanical time constant	ms	21.4
	16	Rotor inertia	gcm²	43.5

$$T_m = \frac{K_\phi}{R_a}\left(V_a - K_\phi \Omega\right) = bV - a\Omega$$

Controller input
a = 2.03e-4 Nm s/rad
b = 0.0095 Nm/V

DC MOTOR EXAMPLE DATA 2

DC030C Series Salient Characteristics

MOTOR DATA	SYMBOL	UNITS	DC030C-1	DC030C-2	DC030C-3
Max DC Terminal Voltage	V_T	V	48	48	48
Max Speed (Mechanical)	ω_{MAX}	rpm	10000	10000	10000
Continuous Stall Torque[1]	T_{cs}	Nm	0.019	0.041	0.060
		oz-in	2.7	5.8	8.5
Peak Torque (Maximum)[1]	T_{pk}	Nm	0.068	0.22	0.36
		oz-in	9.6	31	51
Coulomb Friction Torque	T_f	Nm	0.0035	0.0042	0.0049
		oz-in	0.50	0.60	0.70
Viscous Damping Factor	D	Nm s/rad	3.0E-06	3.4E-06	3.7E-06
		oz-in/krpm	0.045	0.050	0.055
Thermal Time Constant	τ_{th}	min	10	13	16
Thermal Resistance	R_{th}	°C/W	17	14	11
Max. Winding Temperature	Θ_{MAX}	°C	155	155	155
Rotor Inertia	J_r	kg m^2	2.0E-06	3.7E-06	5.8E-06
		oz-in-s^2	2.9E-04	5.2E-04	8.2E-04
Motor Weight	W_m	g	130	170	210
		oz	4.7	6.1	7.6

OPTIONS	
Gearboxes	G30A, G35A, G51A
Encoders	E22A, E30C/D
Brakes	B30A

[1] Recorded at maximum winding temperature at 25°C ambient and without heatsink.

DC030C Series

The DC030C series brush commutated DC motor is a 30 mm diameter, high performance unit offered in 3 lengths with continuous output torques of 0.0177 to 0.0586 Nm.

Features:

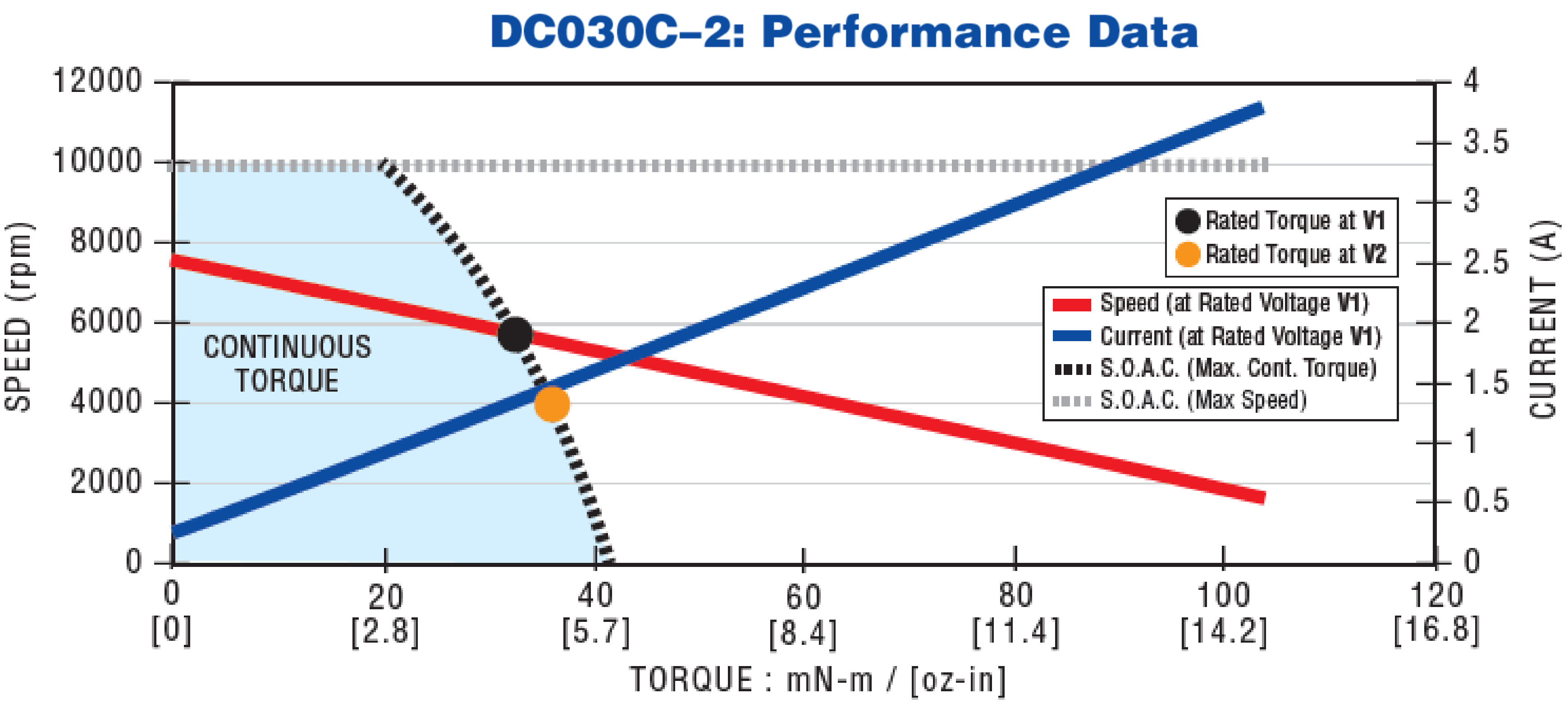
Speeds up to 10,000 RPM possible
DC bus voltage up to 48 VDC
Eight standard windings - special windings available
2 pole stator with magnetic cogging reduction
Bonded neodymium magnets
7 slot armature
Heavy duty ball bearings
Long life graphite brushes – RFI suppression available

Assembly Options:

• Encoder: E22A, E30C/D • Gearbox: G30A, G35A, G51A • Brake: B30A

DC MOTOR EXAMPLE DATA 2

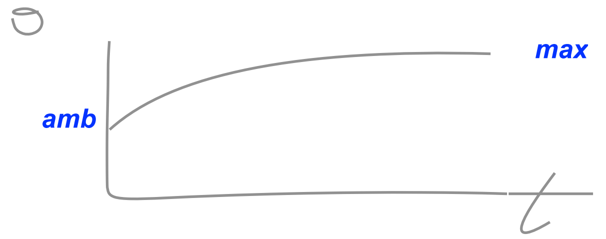
MOTOR DATA	SYMBOL	UNITS	
Rated Voltage V1	V_r	V	12.0
Rated Torque [1] ●	T_r	Nm	0.036
		oz-in	5.2
Rated Speed [1]	ω_r	rpm	4180
Rated Current [1]	I_r	A	2.9
Rated Power [1]	P_r	W	16
No Load Speed	ω_{nl}	rpm	6940
No Load Current	I_{nl}	A	0.43
Rated Voltage V2	V_r	V	9.55
Rated Torque [1] ●	T_r	Nm	0.039
		oz-in	5.5
Rated Speed [1]	ω_r	rpm	2410
Rated Current [1]	I_r	A	3.0
Rated Power [1]	P_r	W	9.7
No Load Speed	ω_{nl}	rpm	5480
No Load Current	I_{nl}	A	0.40
Motor Constant	K_M	Nm/√W	0.014
		oz-in/√W	2.0
Torque Constant	K_T	Nm/A	0.0158
		oz-in/A	2.23
Voltage Constant	K_E	V s/rad	0.0158
		V/krpm	1.65
Terminal Resistance	R_{mt}	Ω	1.29
Inductance	L	mH	0.79
Peak Current	I_{pk}	A	9.3
Electrical Time Constant	τ_e	ms	0.61
Mechanical Time Constant	τ_m	ms	19



[1] Recorded at maximum winding temperature at 25°C ambient and without heatsink.

$V = RI + L\,dI + k\_\varphi\,\Omega$
in laplace: $V - k\_\varphi\,\Omega = (R + L s)\,I$
$(V - k\_\varphi\,\Omega) / (1+\tau s)\;R = R + \tau I$

THERMAL MODEL OF THE ELECTRIC MOTOR



A simple model can be established

Calling $\boxed{\Theta \text{ the motor windings temperature,}}$ it can be model as a first order system

In Laplace domain

$$\Theta = \Theta_{amb} + \frac{R_{th}}{\tau_{th}s + 1} W_{th}$$

In Time domain

$$\tau_{th}\dot{\Theta} + \Theta = \Theta_{amb} + R_{th}W_{th}$$

$\boldsymbol{W_{th} = R_a I_a^2}$ is the heat generated by joule effect

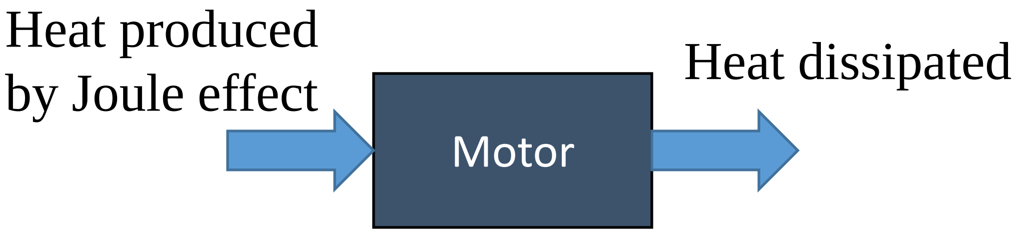
R_{th} is the thermal resistance of the motor (°C/W)

τ_{th} is the thermal time constant (s)

This simple model allows us to compute the windings temperature.

nominal working condition

in simulink you can iinsert this model, integrate and find the temperature at which it's working



MOTOR DATA	SYMBOL	UNITS	DC030C-1	DC030C-2
Max DC Terminal Voltage	V_T	V	48	48
Max Speed (Mechanical)	ω_{MAX}	rpm	10000	10000
Continuous Stall Torque[1]	T_{cs}	Nm	0.019	0.041
		oz-in	2.7	5.8
Peak Torque (Maximum)[1]	T_{pk}	Nm	0.068	0.22
		oz-in	9.6	31
Coulomb Friction Torque	T_f	Nm	0.0035	0.0042
		oz-in	0.50	0.60
Viscous Damping Factor	D	Nm s/rad	3.0E-06	3.4E-06
		oz-in/krpm	0.045	0.050
Thermal Time Constant	τ_{th}	min	10	13
Thermal Resistance	R_{th}	°C/W	17	14
Max. Winding Temperature	Θ_{MAX}	°C	155	155
Rotor Inertia	J_r	kg m^2	2.0E-06	3.7E-06
		oz-in-s^2	2.9E-04	5.2E-04
Motor Weight	W_m	g	130	170
		oz	4.7	6.1

so basically to know the working temperature of a dc motors, it's sufficient to know the current.-

but keep in mind that the parameters are given in 25 degree and no heat sink. so it's usu
ally to cinsider if there is a fan or not fan

THERMAL MODEL OF THE ELECTRIC MOTOR

In Laplace domain

$$\Theta = \Theta_{amb} + \frac{R_{th}}{\tau_{th}s + 1} W_{th}$$

$W_{th} = R_a I_a^2$ is the heat generated by joule effect
R_{th} is the thermal resistance of the motor (°C/W)
τ_{th} is the thermal time constant (s)

Maximum continuous power can be computed as

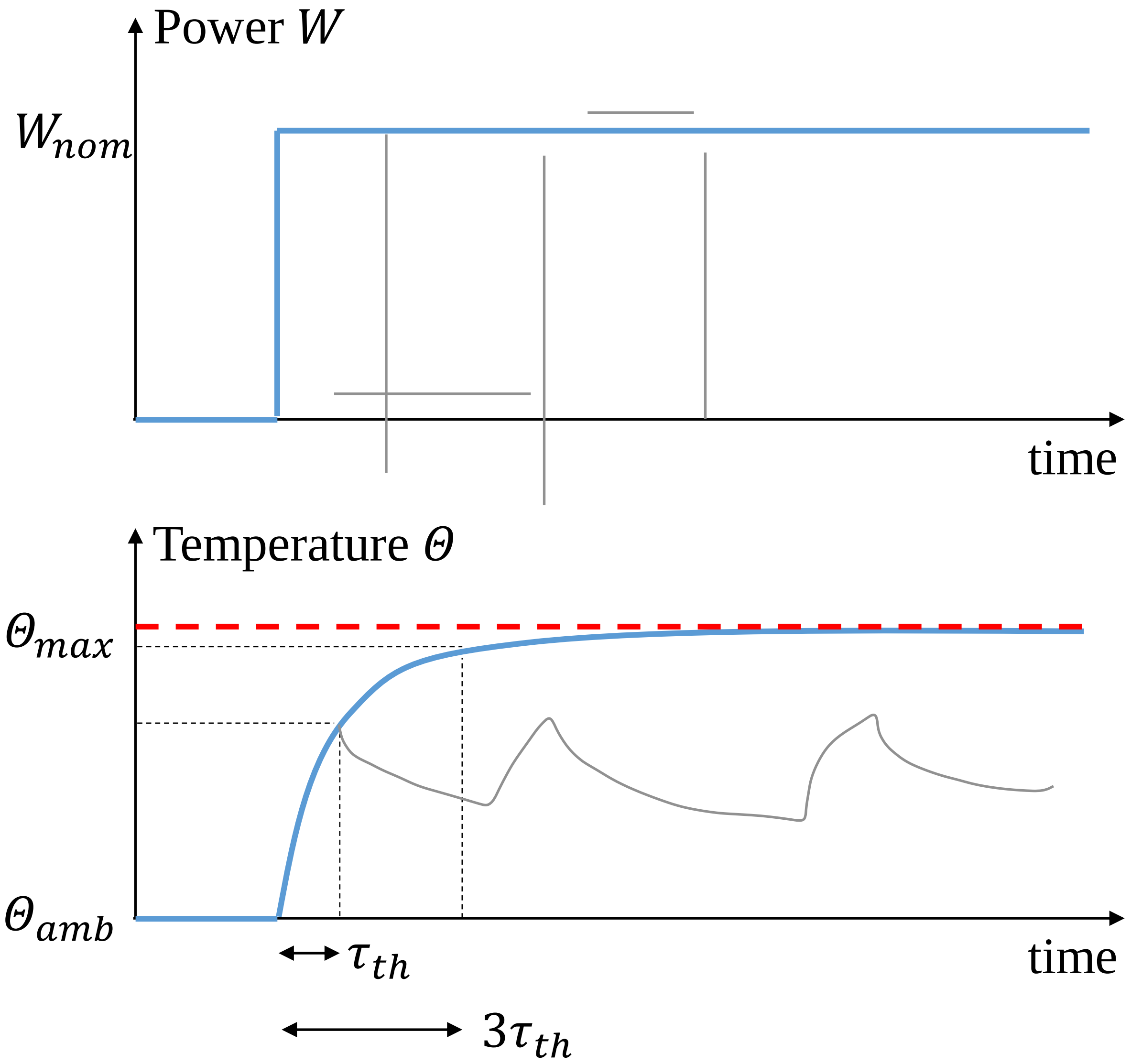
$\Theta_{amb} = 25°\text{C}$, $\Theta_{max} = 155°\text{C}$

$$\Delta\Theta = \Theta_{max} - \Theta_{amb} = R_{th} W_{th}$$

$$W_{th,nom} = \frac{\Delta\Theta}{R_{th}} = 9.3\ \text{W} = R_a I_a^2$$

$$I_{nom} = \sqrt{\frac{W_{th,nom}}{R_a}} = 2.68\ \text{A}$$

$$T_{nom} = K_\Phi I_{nom} = 0.042\ \text{Nm}$$



SERVICE FACTOR

The **service factor** - SF - is a measure of periodically overload capacity at which a motor can operate without damage.

A motor operating continuously at a service factor greater than 1 will have a reduced life expectancy compared to operating at its rated nameplate power.

$$SF = \frac{Power_{required}}{Power_{nominal}}$$

From **CEI EN 60034-1**

SF affects the thermal life of the motor. To operate at higher power (SF >1) means to thermal stress the winding insulation.

To preserve motor life, maximum temperature that can be accepted decreases when SF increases

	Maximum Winding Temperature Rise °C						
	1.0 Service Factor				1.15 Service Factor		
Insulation Class	A	B	F	H	A	B	F
	60	80	105	125	70	90	115

DUTY TYPE (EN 60034-1)

Duty: the statement of the load(s) to which the machine is subjected, including, if applicable, starting, electric braking, no-load and rest and de-energized periods, and including their durations and sequence in time.

It is the responsibility of the purchaser to declare the duty. The purchaser may describe the duty by one of the following:

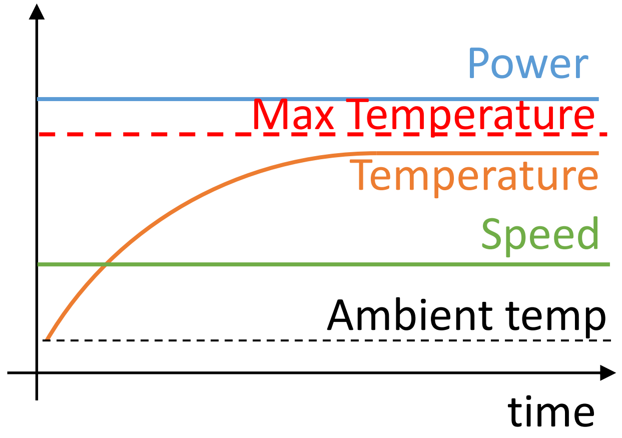
a) numerically, where the load does not vary or where it varies in a known manner;
b) as a time sequence graph of the variable quantities;
c) by selecting one of the duty types S1 to S10 that is no less onerous than the expected duty.

Duty types S2, S3 and S6 share a time to rest (no external load is applied) during which the motor lowers its temperature, thus affording to overload such motor with respect to the same motor when loaded with a S1 duty cycle.

DUTY TYPE

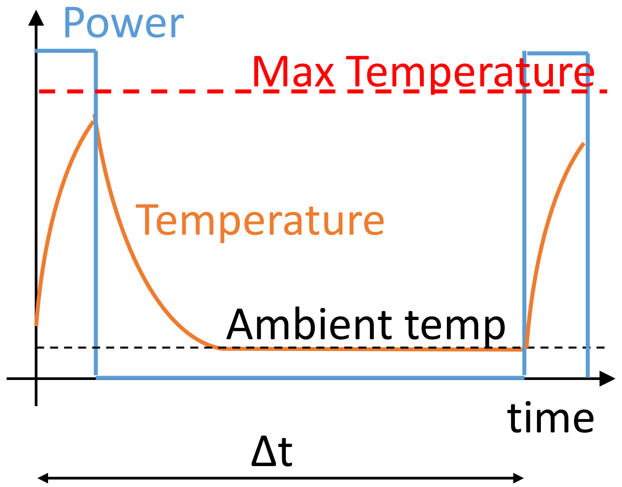
for example a fan

Code	S1: Continuous running duty
Description	Operation at a constant load maintained for sufficient time to allow the machine to reach thermal equilibrium.
Typical applications	Hydraulic pumps, fans, blowers
Abbreviation	S1



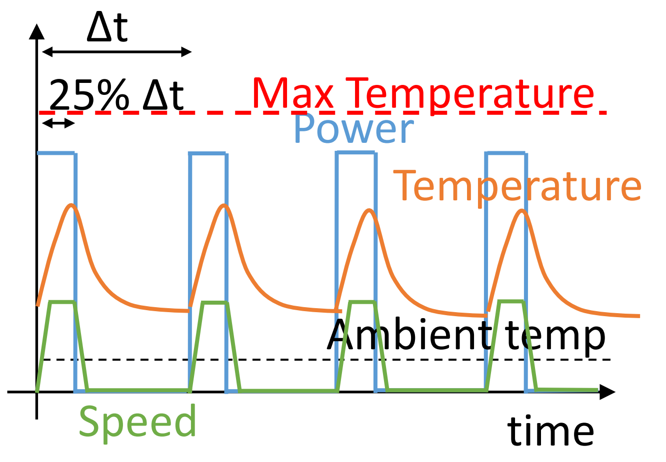
blow dries, usually they actually can reach the max temperatuure, but you turn off before hey reach it. and of courese you also have to decleare the time because it's relevant

Code	S2: Short-time duty
Description	Operation at constant load for a given time, less than that required to reach thermal equilibrium, followed by a time de-energized and at rest of sufficient duration to re-establish machine temperatures within 2 K of the coolant temperature
Typical applications	Household appliances like eggbeaters, hairdryer and so on
Abbreviation	S2 + duration of the duty , S2 30 min

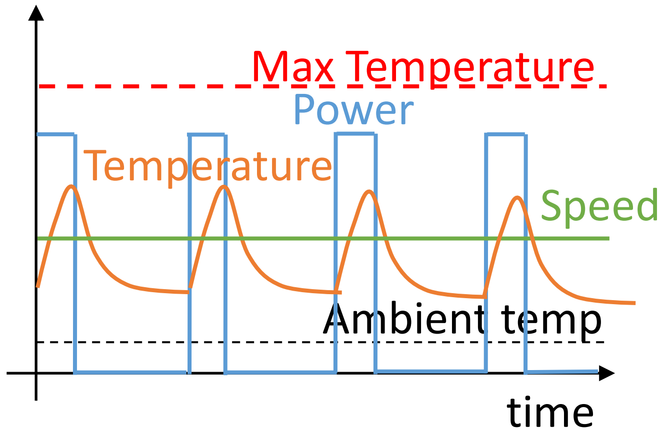


DUTY TYPE

Code	S3: Intermittent periodic duty
Description	A sequence of identical duty cycles, each including a time of operation at constant load and a time de-energized and at rest. In this duty, the cycle is such that the starting current does not significantly affect the temperature rise.
Typical applications	Lifting motors
Abbreviation	S3 + cyclic duration factor*, S3 25%



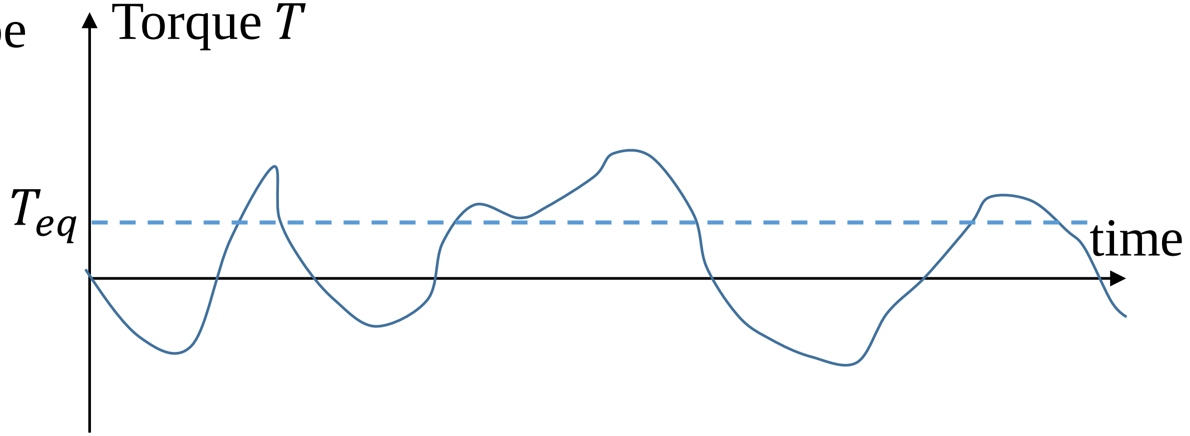
Code	S6: Continuous-operation periodic duty
Description	A sequence of identical duty cycles, each cycle consisting of a time of operation at constant load and a time of operation at no-load. There is no time de-energized and at rest.
Typical applications	Saw machines for wood, oil pressure pumps
Abbreviation	S6 + cyclic duration factor* , S6 40%



DUTY CYCLE FOR A GENERIC TORQUE TIME HISTORY

If the duty cycle is not clearly defined (e.g. traction motor for a vehicle), then the equivalent motor torque can be identified.

$$T_{eq} = \sqrt{\frac{1}{T}\int_{t}^{t+T} [T(t)]^2 \mathrm{d}t}$$



Or, in case of discrete signal

$$T_{eq} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} T_i^2}$$

for example of a car, you do not know the torque travel.
to compute the equivalent torque you use the area and then you compare it with the contiuuouus usage

Then a S1 duty cycle can be considered with a constant torque equal to the equivalent torque.

This is true if the torque fluctuation are much faster then the thermal time constant

ELECTRIC MOTORS CLASSIFICATION

DC motors
There are no phases in direct current that is why DC electric motors only uses 2 wires to run. They are the first motors to be invented.

AC motors

you need AC to generate a rotating magnetic field.

The AC electric motor converts AC (Alternating Current) electrical energy into mechanical energy. These electric motors are powered using a single-phase or three-phase alternating current. The basic working principle of AC motor is the **rotating magnetic field** (RMF) generated by the stator winding when an alternating current is passed through it. The rotor (having its own magnetic field) follows the RMF & starts rotation.

Special motors
There are several types of special electric motors that are the modified versions of other motor designed for special purposes.

ELECTRIC MOTORS CLASSIFICATION

DC motors

There are no phases in direct current that is why DC electric motors only uses 2 wires to run. They are the first motors to be invented.

- Brushed DC Motor
- Brushless DC Motor
- Coreless or Ironless DC Motors

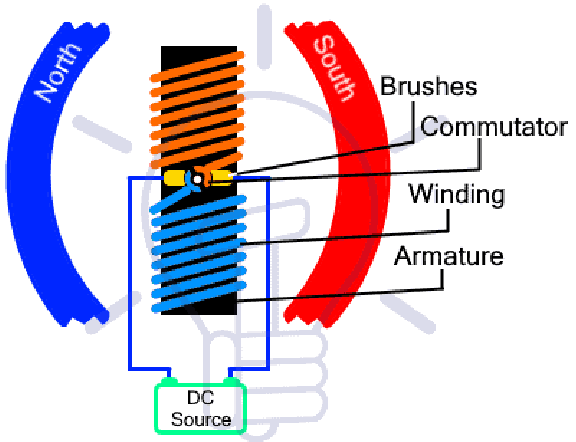
ELECTRIC MOTORS CLASSIFICATION

DC motors

Brushed DC Motor have brushes & commutators. They are used for connecting a stationary circuit with a rotating circuit. In such case, the rotor winding of the motor is energized through conductive brushes. The disadvantage of any brushed motor is that they require frequent maintenance due to continuous sliding of brushes & the sparks generated between them. However, they are quite simple in design & are in expensive.

The brushed DC electric motors are further classified into

- Permanent Magnet DC Motor:
- Separately Excited Motor
- Self-Excited DC Motor



Brushed Motor

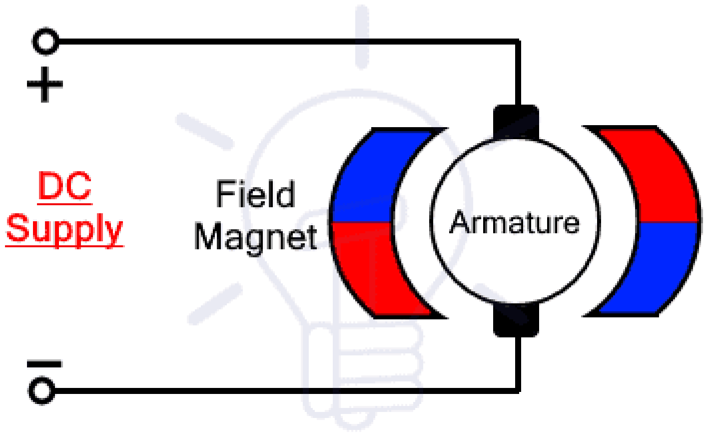
ELECTRIC MOTORS CLASSIFICATION

DC motors

Brushed DC Motor have brushes & commutators.

The brushed DC electric motors are further classified into

- **Permanent Magnet DC Motor:** the stator magnetic field is generated using a permanent magnet placed in the stator.
- Separately Excited Motor
- Self-Excited DC Motor



Permanent Magnet DC Motor

the problem is that you have brushes and obviusly you have to keep switching between one and the other. so that the voltage keeps going on to +220 to 0 to -220 so that you have vey high current in the switching that can generate spark. so if you have to work in enviorment where there is flame risk you cannot use this motors.

for example with flavour, they take flames veryr easy.

ELECTRIC MOTORS CLASSIFICATION

we can change the φ parameters in :
$V - Ri = k \varphi \Omega$
so that we are also able to change the slope of the curve.

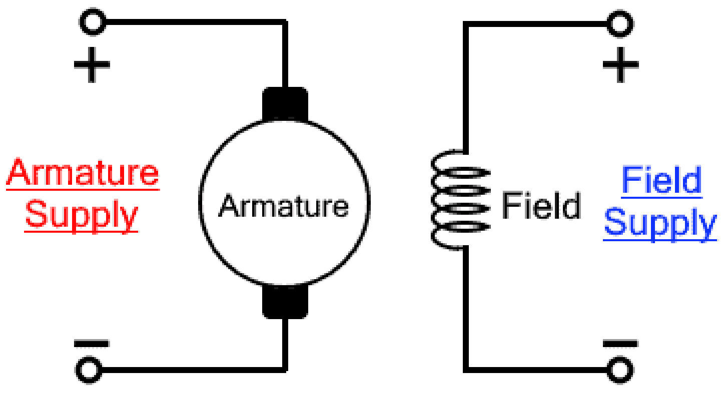
obviusly you pay this in control because you need a somehow of controlling it

DC motors

Brushed DC Motor have brushes & commutators.

The brushed DC electric motors are further classified into

- Permanent Magnet DC Motor
- **Separately Excited Motor:** The excitation refers to the energizing of the field windings also known as the stator's windings. Both windings i.e. field windings and armature windings are connected with a separate power supply.
- Self-Excited DC Motor



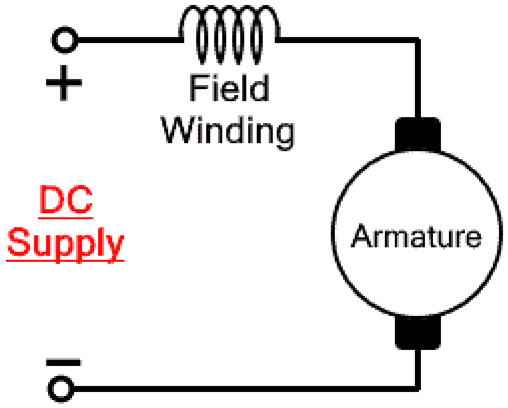
Separately Excited DC Motor

DC motors

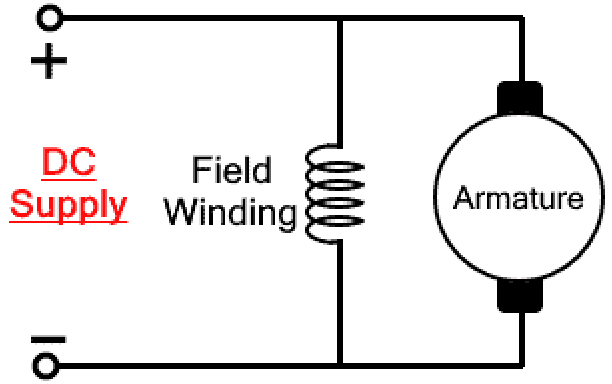
Brushed DC Motor have brushes & commutators.

The brushed DC electric motors are further classified into

- Permanent Magnet DC Motor
- Separately Excited Motor
- **Self-Excited DC Motor:** The field winding is electrically connected with the armature windings. A single power source energizes both windings.
  - **Series Wound**: the field winding is connected in series with the armature windings
  - **Shunt Wound**: the field winding is connected in parallel with the armature winding
  - **Compound Wound**: it combines both the parallel combination as well as the series combination of the field & armature windings



Series Wound DC Motor

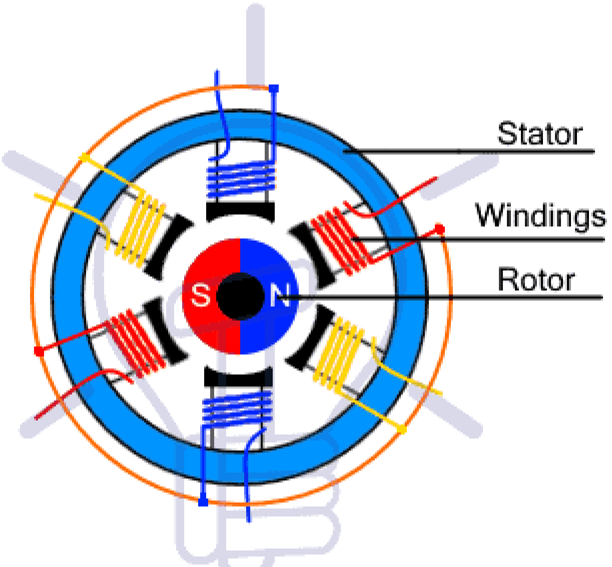


Shunt Wound DC Motor

DC motors

Brushless DC Motor (BLDC Motor) does not have any carbon brushes and commutators assembly.
Rotor magnetic field is produced by permanent magnets.
Stator magnetic field must rotate. This is done by powering different windings, phased, in sequence. The input is switched between the stator windings to generate a magnetic field that pushes and pulls on the rotor magnetic field causing it to rotate in its direction.
A Hall Effect sensor is used to detect the position of the rotor and switch the input to the correct stators winding respectively.

The brushless motor speed depends on the frequency of the AC power supplied by the controller. Which is why it is also called **synchronous motor**



Brushless DC Motor

while you do not have wear of the brushes, and not have the sparks problem. yuo need to be very precise in switching on / off the phasee. also permament magnets are very expensive.

ELECTRIC MOTORS CLASSIFICATION

AC motors

The AC electric motor converts AC (Alternating Current) electrical energy into mechanical energy. These electric motors are powered using a single-phase or three-phase alternating current. The basic working principle of AC motor is the **rotating magnetic field** (RMF) generated by the stator winding when an alternating current is passed through it. The rotor (having its own magnetic field) follows the RMF & starts rotation.

Asynchronous Motor

The type of AC motor that never runs at synchronous speed is called asynchronous speed. Its rotor speed is always less than the synchronous speed. It does not require separate rotor excitation.

Synchronous Motor

AC motor has a constant speed called synchronous speed that only depends on the frequency of the supply current. The speed of such electric motors only varies with variation in supply frequency and remains constant upon varying loads. It is used for constant speed application and precision control.

A synchronous motor has the same stator design as asynchronous motor and it generates a rotating magnetic field when supplied with input alternating current. While the rotor design may vary i.e. it uses a separate DC excitation to generate its own magnetic field.

ELECTRIC MOTORS CLASSIFICATION

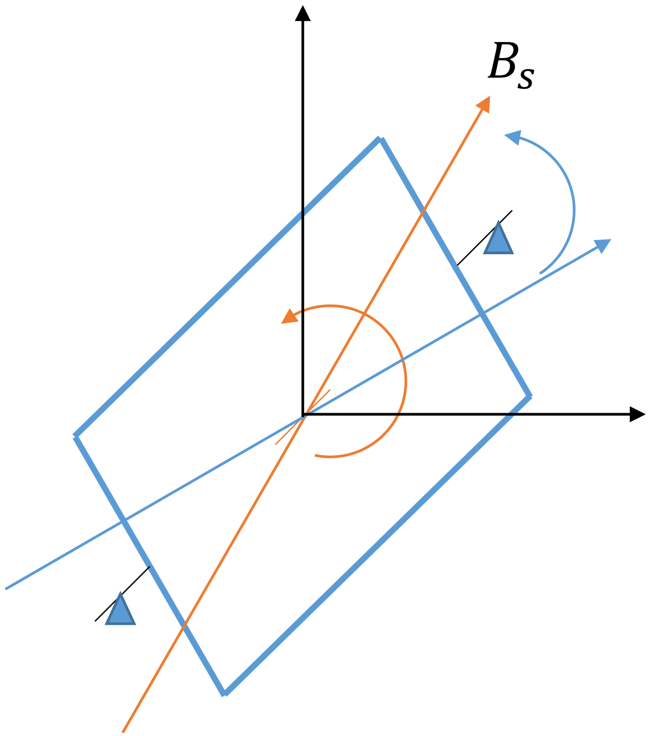
AC motors

Asynchronous Motor

The type of AC motor that never runs at synchronous speed is called asynchronous speed. Its rotor speed is always less than the synchronous speed. It does not require separate rotor excitation.

The asynchronous motors are briefly classified into two types;

- **Induction Motor**: is a type of AC asynchronous motor that works on the principle of electromagnetic induction between the stator & the rotor. The revolving magnetic flux induces a current in the rotor due to electromagnetic induction which produces torque in the rotor. It is the most used electrical motor in industries.
- **Commutator Motor**



$$\theta_r = \Omega_r t$$

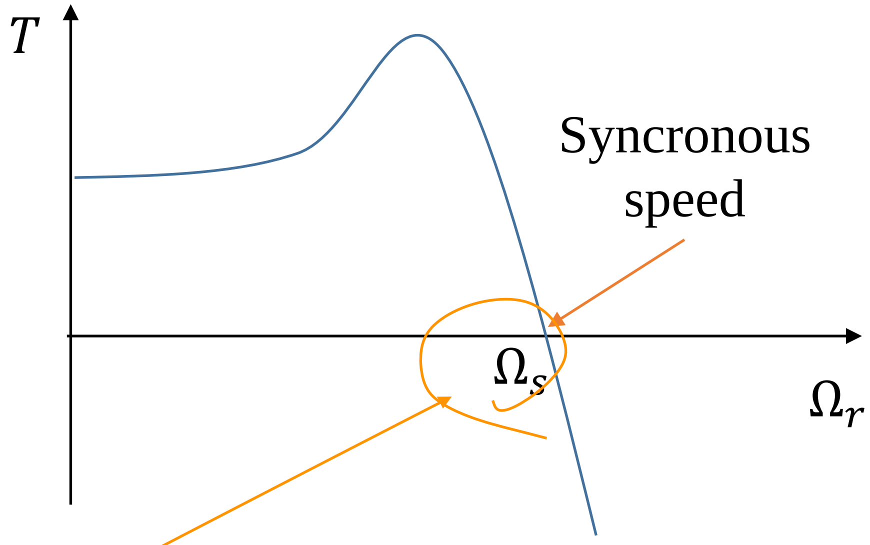
$$\theta_s = \Omega_s t$$

$$s = \frac{\Omega_s - \Omega_r}{\Omega_s}$$

$$\Phi_{rs} = B_s S = BS_0 \cos(\theta_s - \theta_r) = BS_0 \cos[(\Omega_s - \Omega_r)t]$$

$$e_{rs} = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = -BS_0(\Omega_s - \Omega_r)\sin[(\Omega_s - \Omega_r)t]$$

ELECTRIC MOTORS CLASSIFICATION



AC motors

Asynchronous Motor

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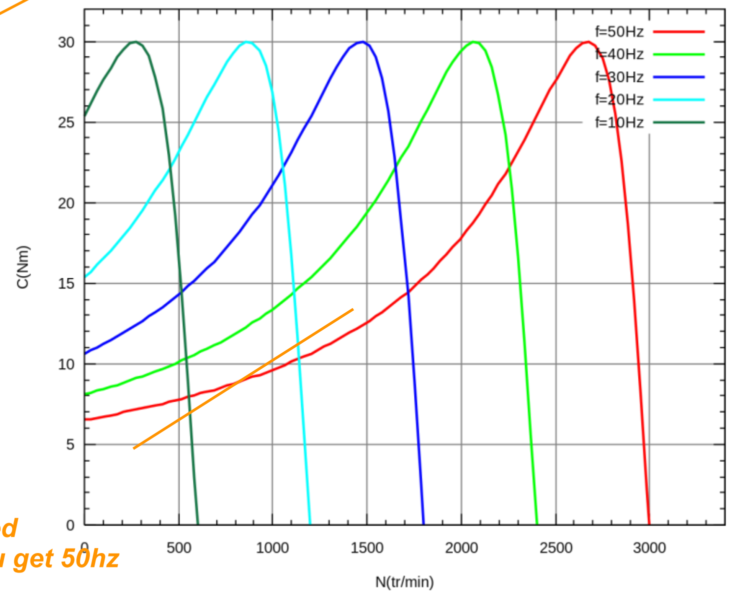
The asynchronous motors are briefly classified into two types;

- **Induction Motor**: is a type of AC asynchronous motor that works on the principle of electromagnetic induction between the stator & the rotor. The revolving magnetic flux induces a current in the rotor due to electromagnetic induction which produces torque in the rotor. It is the most used electrical motor in industries.
- **Commutator Motor**



this is the synch speed at which the torque is rotating

the higher the slope, the higher the heighen values.

the problem is that tif the initial load is higher than the inial ttroqe

so by chanign the frequency of the rotor you can have the full working motors
for changing the frequency you need to have dedicated electronics. because jsut connecting it to the plug you get 50hz

ELECTRIC MOTORS CLASSIFICATION

AC motors

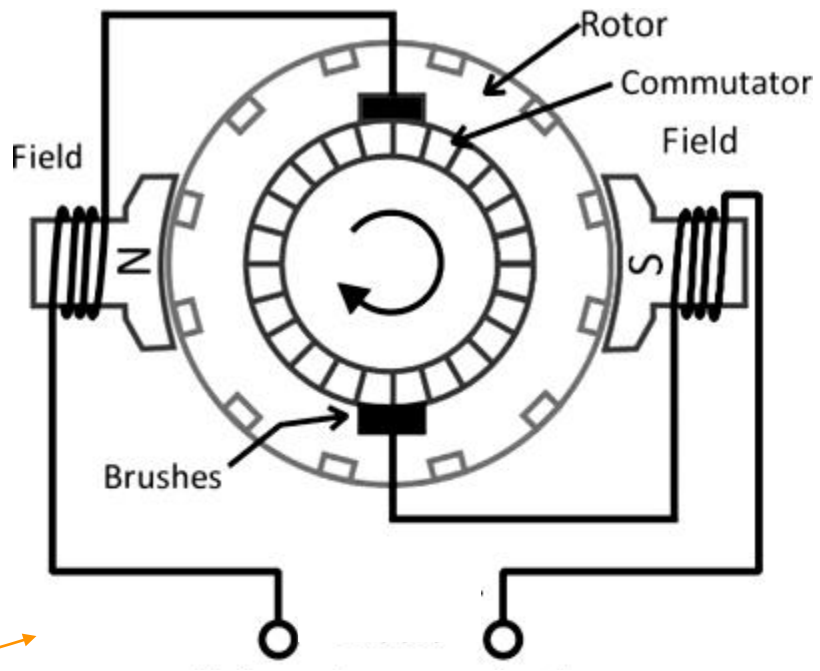
Asynchronous Motor

The type of AC motor that never runs at synchronous speed is called asynchronous speed. Its rotor speed is always less than the synchronous speed. It does not require separate rotor excitation.

The asynchronous motors are briefly classified into two types;

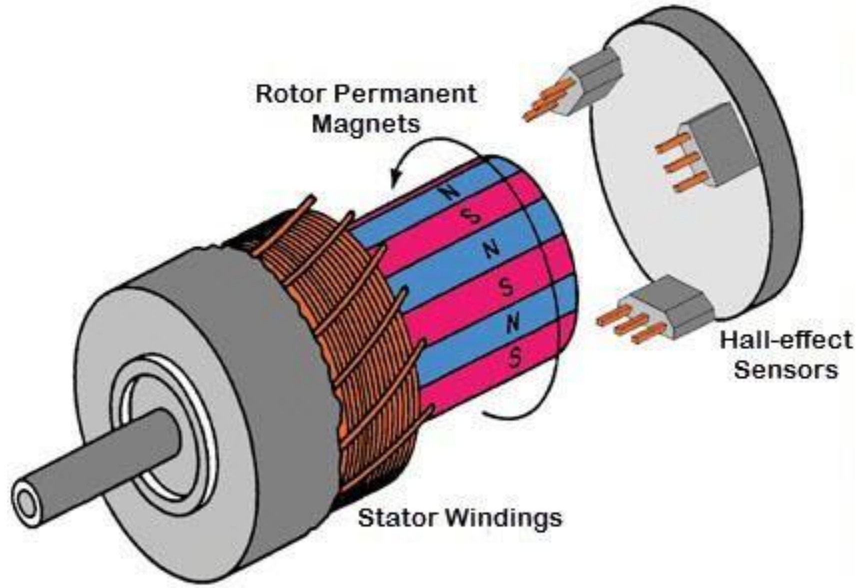
- **Induction Motor**
- **Commutator Motor:** utilizes commutator and brush assembly to supply power to its rotor. Such electric motors have wound-type rotor. AC Series Motor, the stator and rotor windings are in series.

anello di pacinott.
this is equal to the brushed dc motoro wth an alternat



Series wound AC asynchronous motor

ELECTRIC MOTORS CLASSIFICATION



AC motors

Synchronous Motor

AC motor has a constant speed called synchronous speed that only depends on the frequency of the supply current. The speed of such electric motors only varies with variation in supply frequency and remains constant upon varying loads. It is used for constant speed application and precision control.

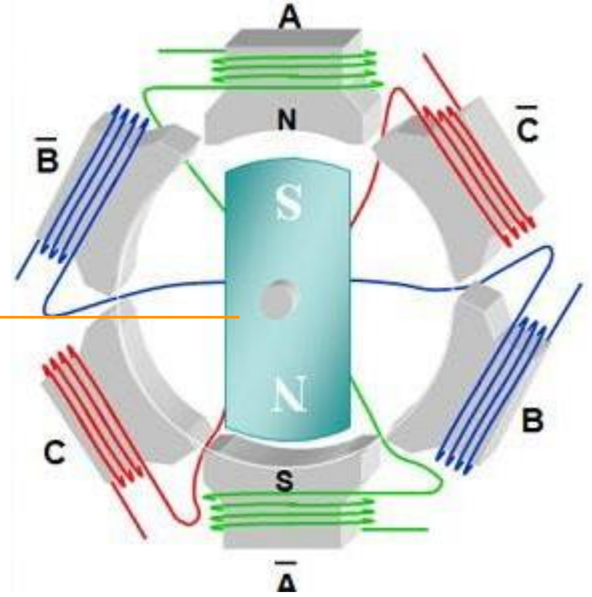
A synchronous motor has the same stator design as asynchronous motor and it generates a rotating magnetic field when supplied with input alternating current. While the rotor design may vary i.e. it uses a separate DC excitation to generate its own magnetic field.

by connecting tis to the plkug. we have 50hz of the
so we cannot start the motors just by connecting to the motors.

if the electornic is not fast enough they suffer of isntability so the electronic is very important. they are the most expensive.

you need to control the position, if the rotor is lost becaus ef a distirvance you have a very strng vibratioon in the overall system

while an induction motor is enough to attach it to



ELECTRIC MOTORS CLASSIFICATION

Special motors

There are several types of special electric motors that are the modified versions of other motor designed for special purposes.

Servo Motors: used for pushing/Pulling or lifting or rotating an object at some specific angle. Servo motor can be designed to run on AC as well as DC power supply.

Stepper Motors: or a stepping motor is a brushless DC motor whose full rotation is divided into a number of equal steps. Such motor rotates in steps (fixed degrees) instead of rotating continuously. Such stepping movement offers great precision which is utilized is robotics.

Universal Motor: is a special type of motor that can run on AC as well DC power supply. it is a brushed series-wound motor where the field windings is connected in series with the armature windings.

ELECTRIC MOTORS CLASSIFICATION

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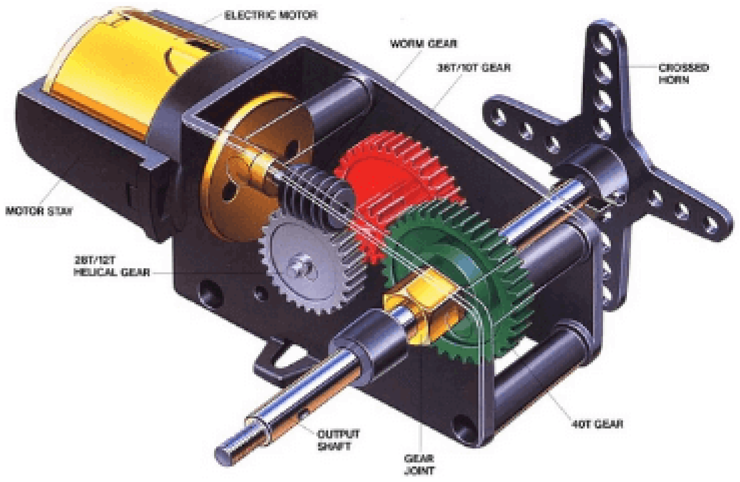
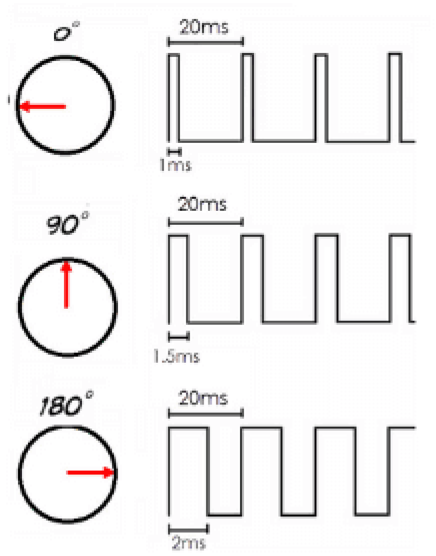
Stepper Motors

Universal Motor





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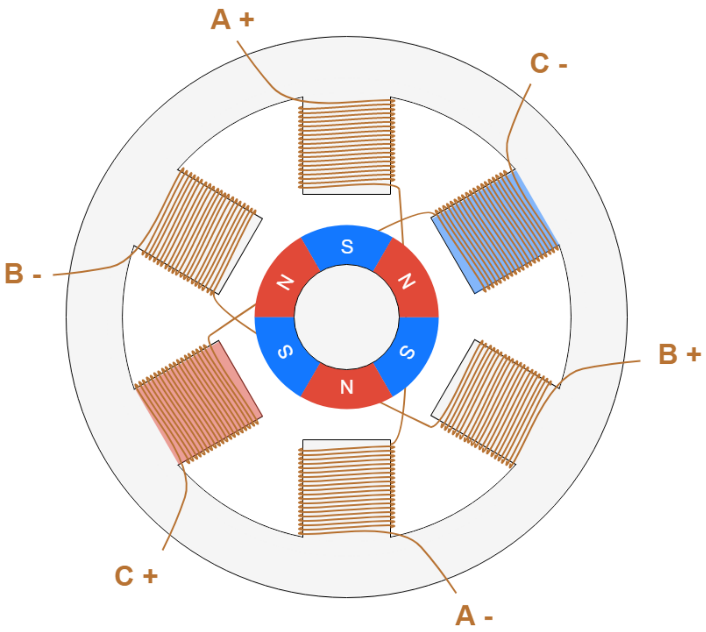

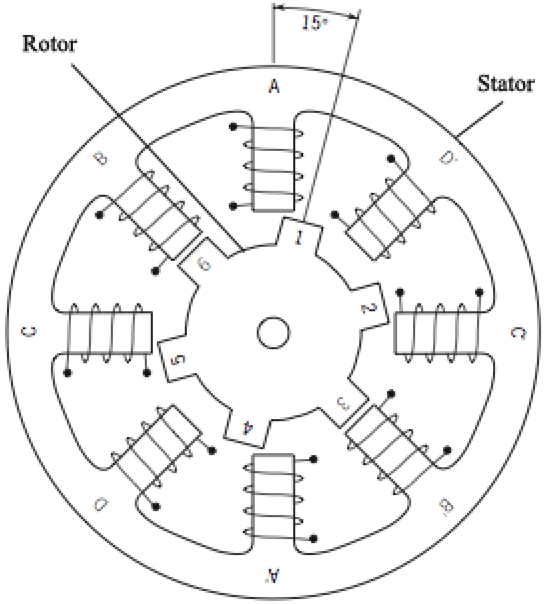

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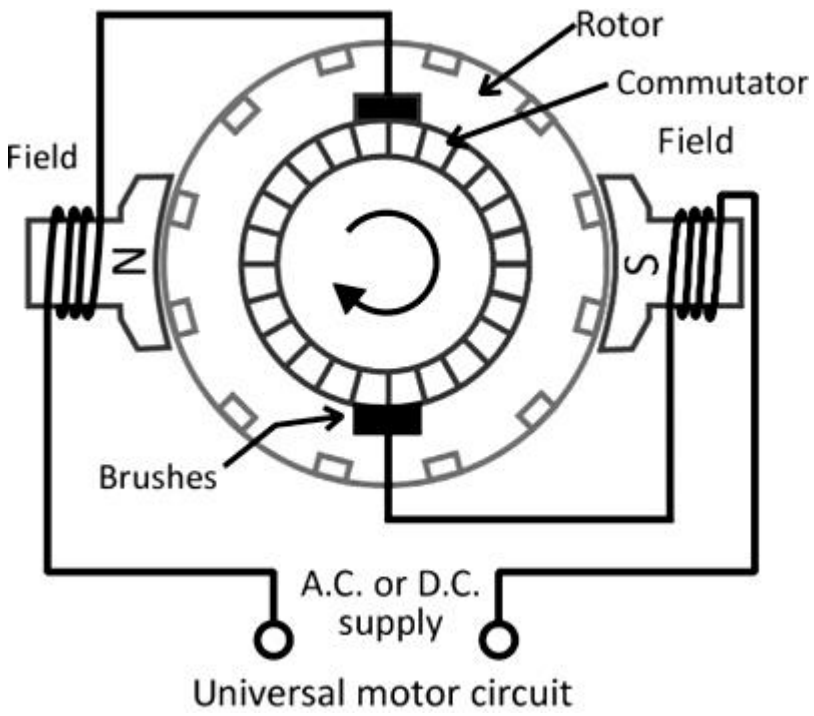
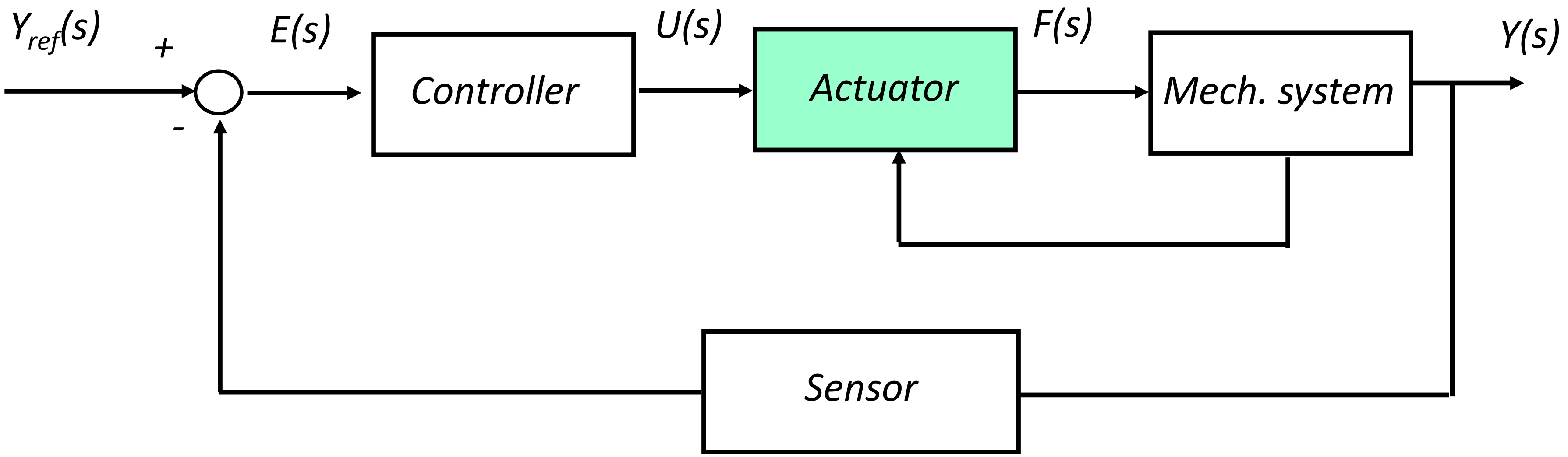


Control system overview phi



- As for the hydraulic actuator, a natural (i.e. intrinsic) feedback exists which couples the two subsystems (mechanical system and electric actuator)
- It is not possible to model an electric actuator separately from the mechanical system that receives the force produced by the actuator itself

Main components of a DC drive

DC electric motor

electronic power converter *

hardware

logic

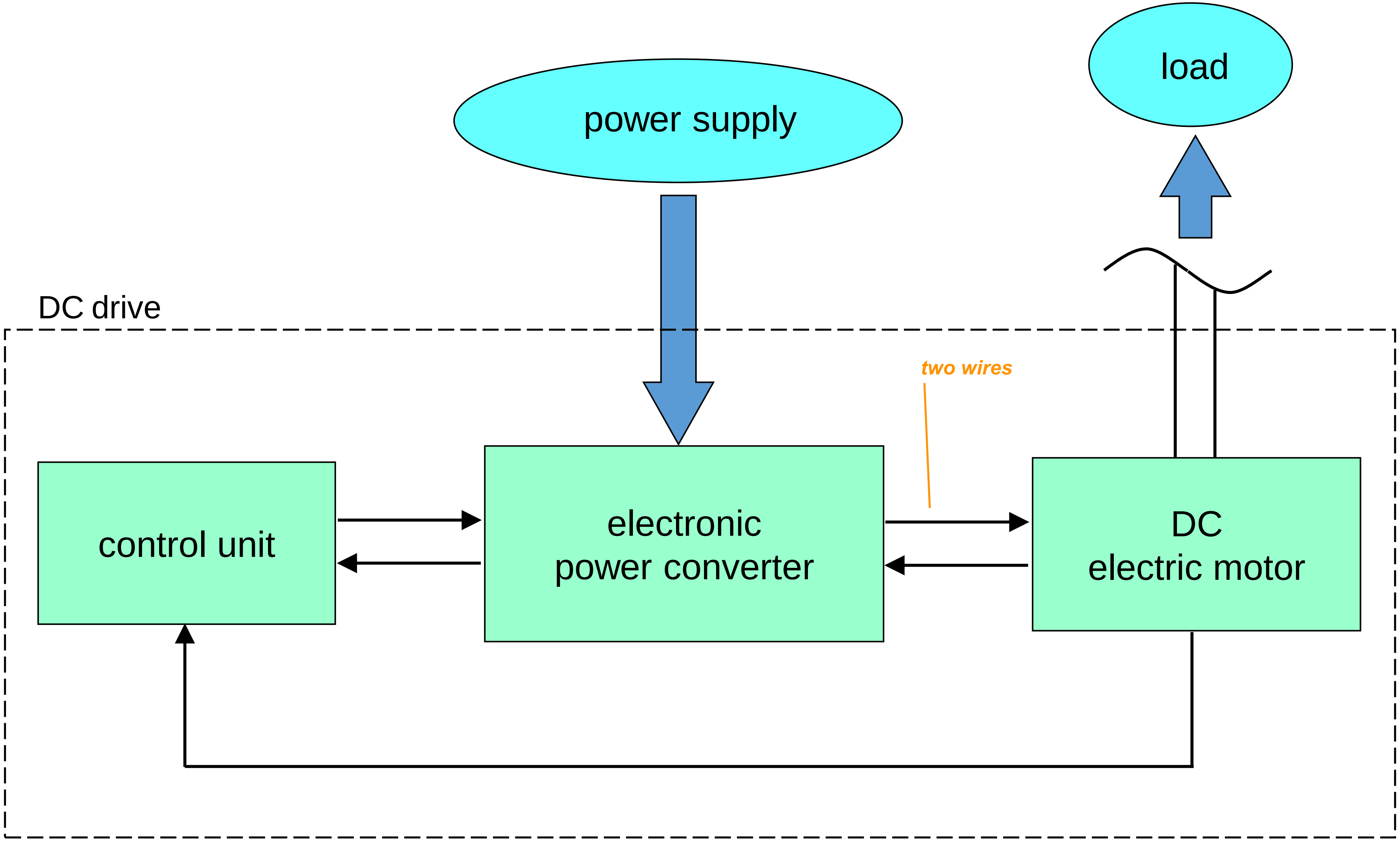
control unit

the first approximation is that we neglet
the dynamic of the electronic power convert
as we have done with the spulve valve

+ sensors (speed, current, voltage, etc.)

* We will neglect the dynamics of this component since the switching frequency of the power semiconductor valves is generally higher than 1 kHz

Main components of a DC drive



DC motor

we are talking about brush DC motor

- Advantages of DC motor:
  - Ease of control
  - Deliver high starting torque
  - Near-linear performance
- Disadvantages:
  - High maintenance
  - Large and expensive (compared to induction motor)
  - Not suitable for high-speed operation due to commutator and brushes
  - Not suitable in explosive or very clean environment

because of the sparks

DC motor

• The DC drive is relatively simple and cheap (compared to induction motor drives). But DC motor itself is more expensive.

so in many cases like drones we use brushless (DC synchronous motors)

• Due to the numerous disadvantages of DC motor (esp. maintenance), it is getting less popular, particularly in high power applications.

• For low power applications the cost of DC motor plus drives is still economical.

• For servo application, DC drives is still popular because of good dynamic response and ease of control.

DC motor: dynamic model

PERMANENT MAGNET DC MOTOR

electric circuit

mecchanical system
with viscius bearing and the
inertia momentum

$$\begin{cases} V_a = R_a i_a + L_a \dfrac{di_a}{dt} + e \\ J\dot{\Omega} + c\Omega = T_M - T_L \end{cases}$$

$$T_M = K_\Phi i_a$$

$$e = K_\Phi \Omega$$

1 irder

$$\begin{cases} V_a = R_a i_a + L_a \boxed{\dfrac{di_a}{dt}} + K_F \Omega \\ \boxed{J\dot{\Omega}} + c\Omega = K_F i_a - T_L \end{cases}$$

1 order

so two first order differential rrquation

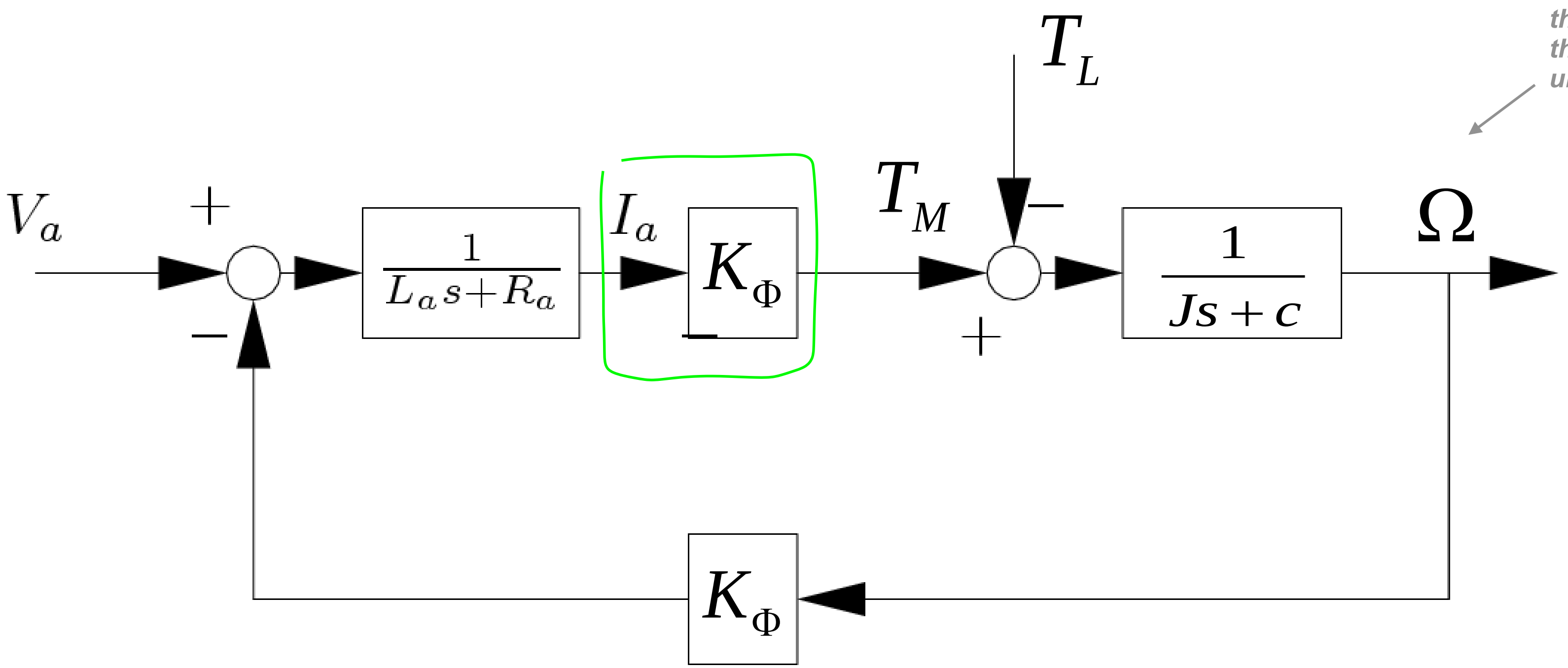
that now we write in a block diagram
1. laplace domain

DC motor: dynamic model

PERMANENT MAGNET DC MOTOR

$$
\begin{cases}
(L_a s + R_a) I_a = V_a - K_\Phi \Omega \\
(Js + c)\Omega = \boxed{K_\Phi I_a} - T_L
\end{cases}
$$

this is speed. is the actual speed that the motors manages to produce under load



DC motor: steady-state behavior

For fixed stator flux and armature voltage:

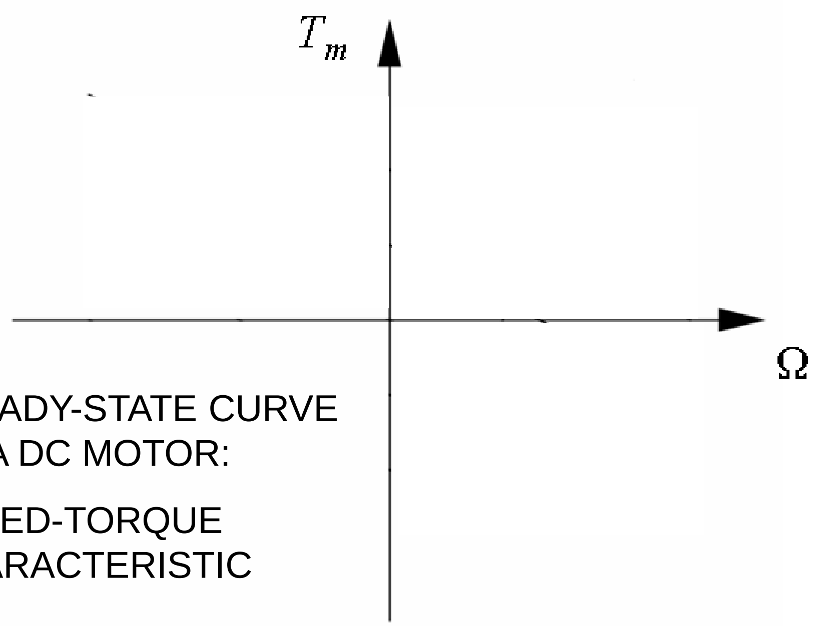
$$V_a = R_a i_a + e$$

$$e = K_\Phi \Omega$$

$$\Rightarrow \; i_a = \frac{V_a - K_\Phi \Omega}{R_a}$$

$$\boxed{T_M = K_\Phi i_a = \frac{K_\Phi}{R_a} V_a - \frac{K_\Phi{}^2}{R_a} \Omega}$$

$$\boxed{T_M = \frac{K_\Phi^2}{R_a} \left(\Omega_0 - \Omega \right)}$$

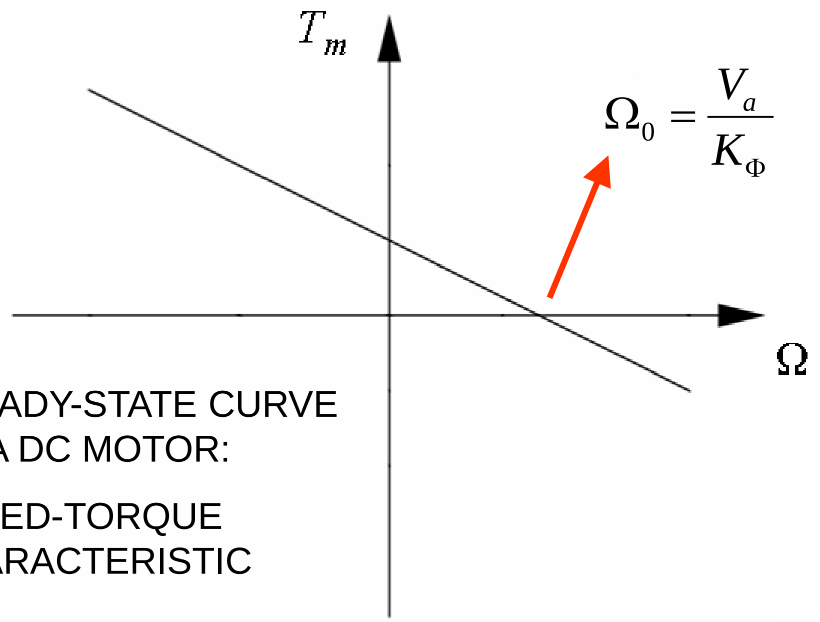


STEADY-STATE CURVE OF A DC MOTOR:

SPEED-TORQUE CHARACTERISTIC

DC motor: steady-state behavior

For fixed stator flux and armature voltage:



STEADY-STATE CURVE OF A DC MOTOR:

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$$V_a = R_a i_a + e$$

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$$\boxed{T_M = K_\Phi i_a = \frac{K_\Phi}{R_a} V_a - \frac{K_\Phi^{\ 2}}{R_a} \Omega}$$

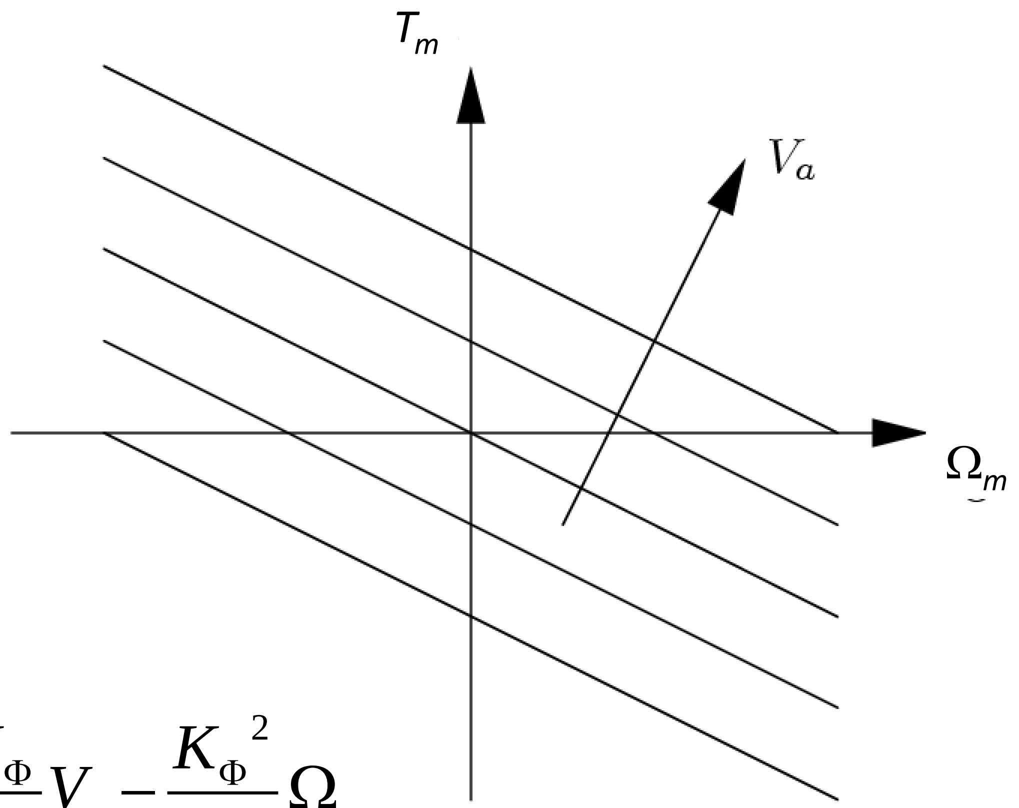
$$\boxed{T_M = \frac{K_\Phi^2}{R_a} \left(\Omega_0 - \Omega \right)}$$

Which is the physical meaning of the negative slope?

If the required torque T_m increases, the current i_a must increase too.
As a consequence, the rotor will slow down and the emf e must decrease .

Control of a DC drive

ARMATURE-CONTROLLED DC MOTOR



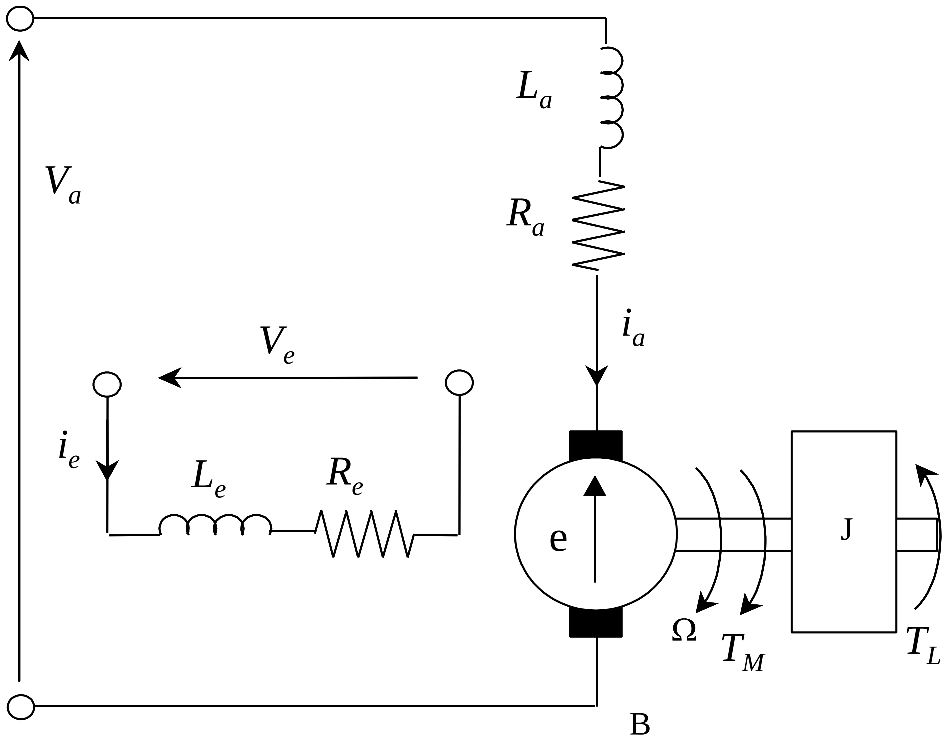
By varying the input voltage V_a through the electronic power converter, the speed-torque characteristic can be adjusted

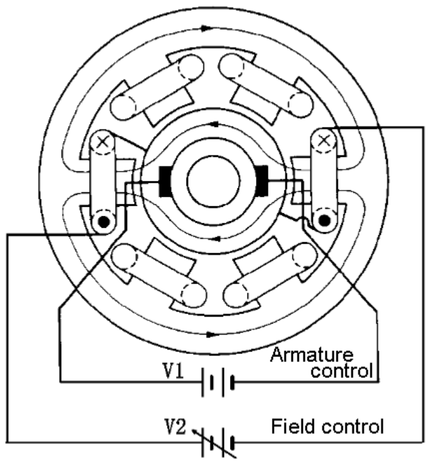
$$T_M = \frac{K_\Phi}{R_a} V_a - \frac{K_\Phi{}^2}{R_a} \Omega$$

All speed-torque characteristics have the same slope

DC motor: dynamic model

SEPARATELY EXCITED DC MOTOR





$$\Phi_e = L_e i_e$$

$$\begin{cases} V_a = R_a i_a + L_a \dfrac{di_a}{dt} + e \\ V_e = R_e i_e + \dfrac{d\Phi_e}{dt} = R_e i_e + L_e \dfrac{di_e}{dt} \\ J\dot{\Omega} + c\Omega = T_M - T_L \end{cases}$$

$$T_M = K\Phi_e i_a$$

$$e = K\Phi_e \Omega$$

DC motor: dynamic model

SEPARATELY EXCITED DC MOTOR

$$\begin{cases} V_a = R_a i_a + L_a \dfrac{di_a}{dt} + e \\ V_e = R_e i_e + \dfrac{d\Phi_e}{dt} = R_e i_e + L_e \dfrac{di_e}{dt} \\ J\dot{\Omega} + c\Omega = T_M - T_L \end{cases}$$

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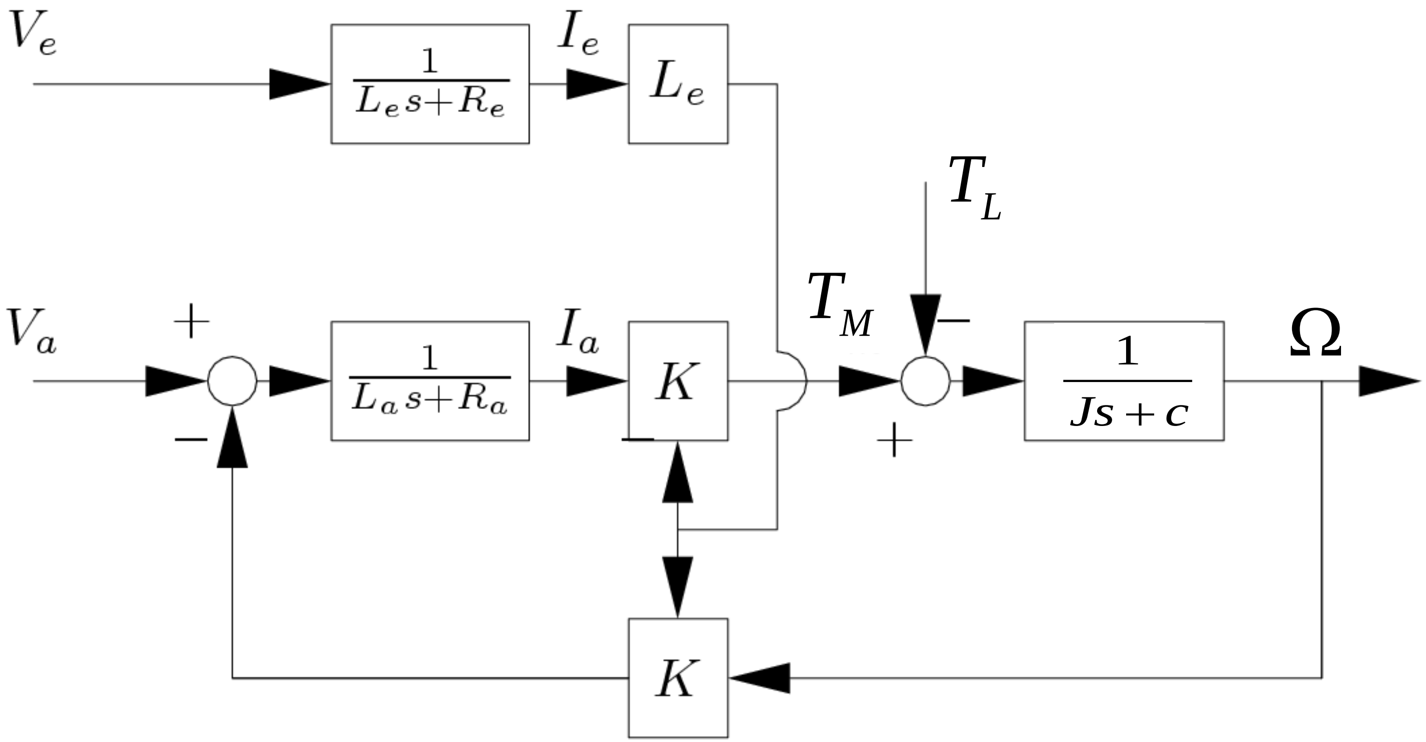
DC motor: dynamic model

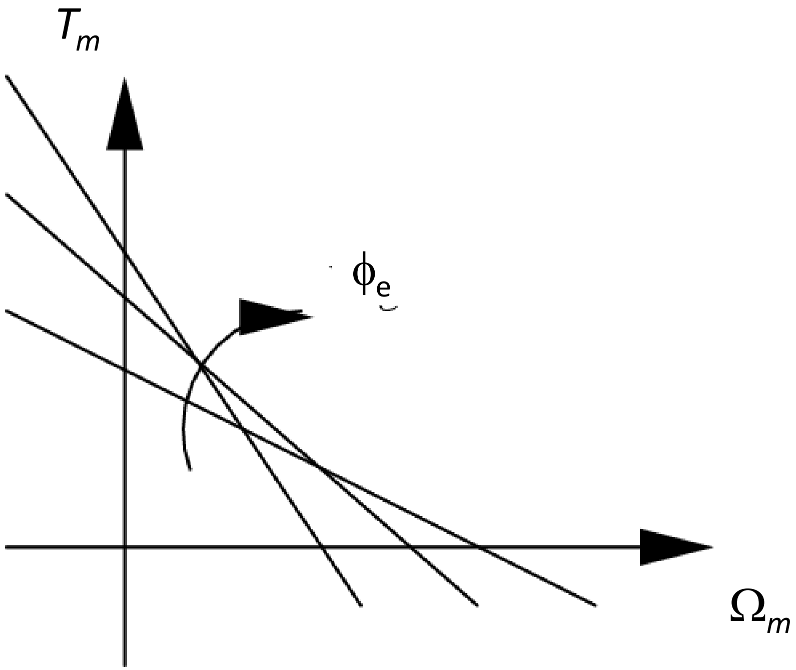
SEPARATELY EXCITED DC MOTOR

$$\begin{cases} (L_a s + R_a) I_a = V_a - K\Phi_e \Omega \\ V_e = (L_e s + R_e) I_e \\ (Js + c)\Omega = K\Phi_e I_a - T_L \end{cases}$$

$\Phi_e = L_e I_e$



FIELD-CURRENT CONTROLLED DC MOTOR



Separately excited DC motors allow further regulation, by acting on the stator's flux Φ_e produced by the excitation circuit

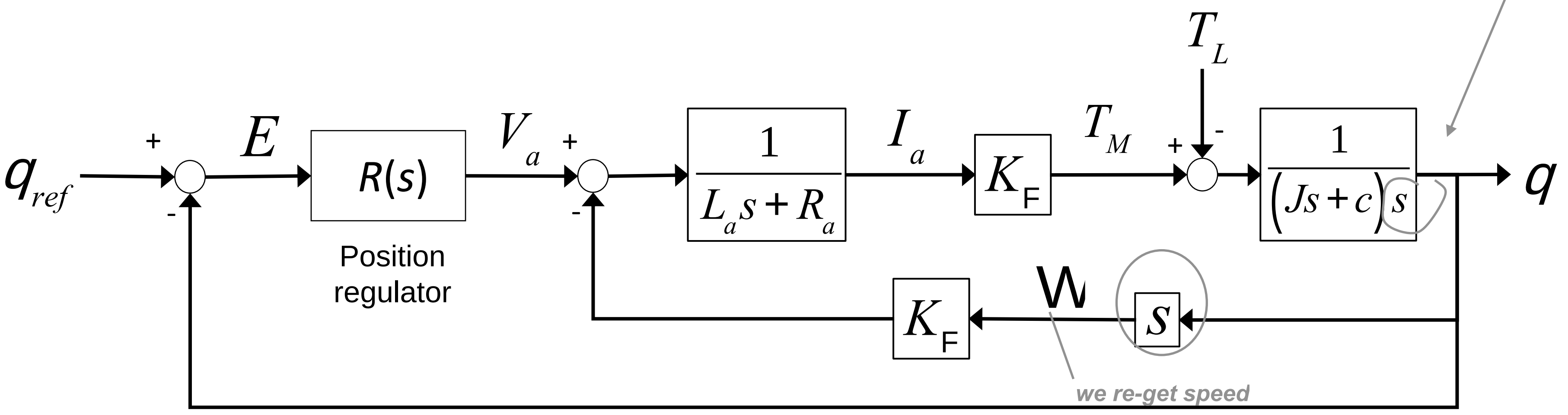
By varying the excitation current I_e and consequently Φ_e, the slope of the speed-torque characteristic can be adjusted

$$T_M = \frac{K\Phi_e}{R_a}\left(V_a - K\Phi_e\Omega\right)$$

1

PERMANENT MAGNET DC MOTOR-Position control

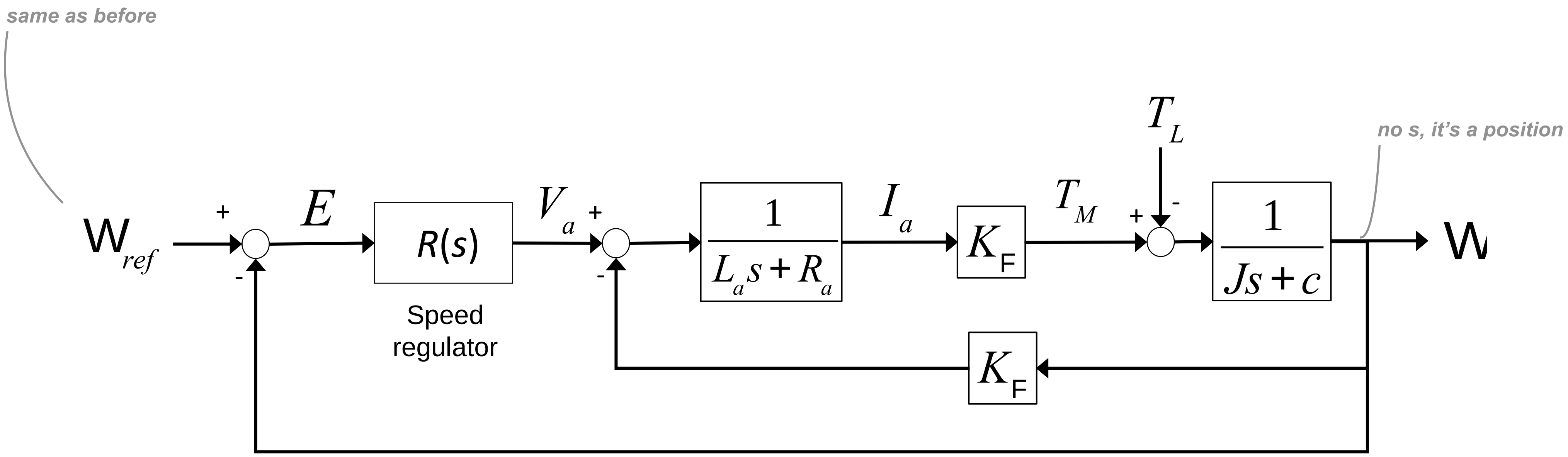
if we want the position we no more need the speed but its integral, here the s



we re-get speed

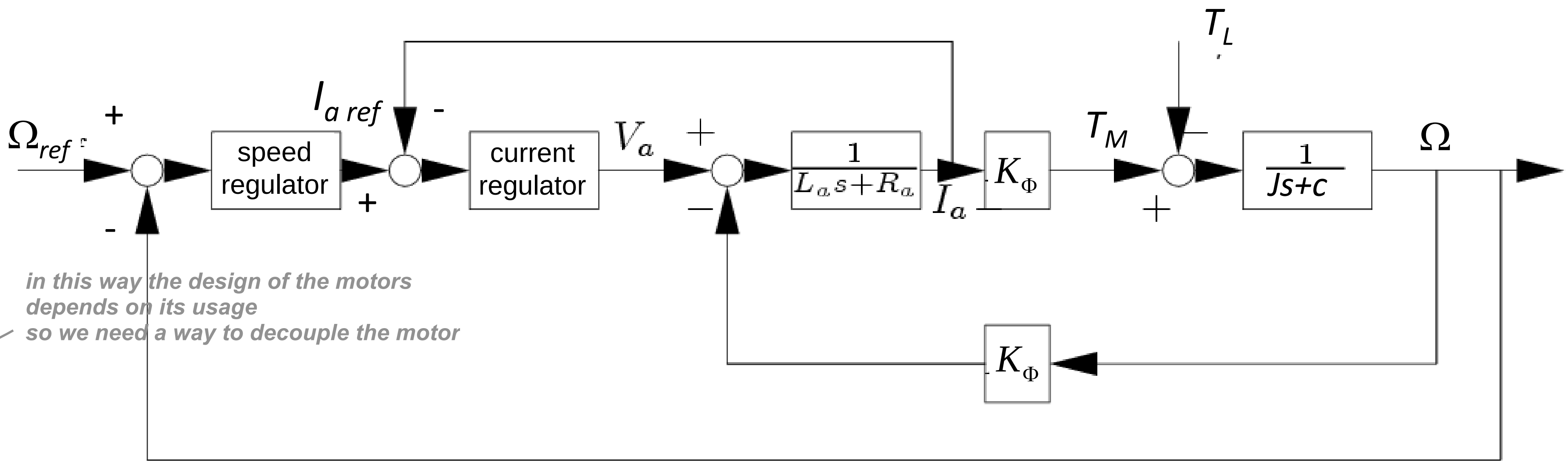
Control of a DC drive

PERMANENT MAGNET DC MOTOR-Speed control



PERMANENT MAGNET DC MOTOR-Speed control

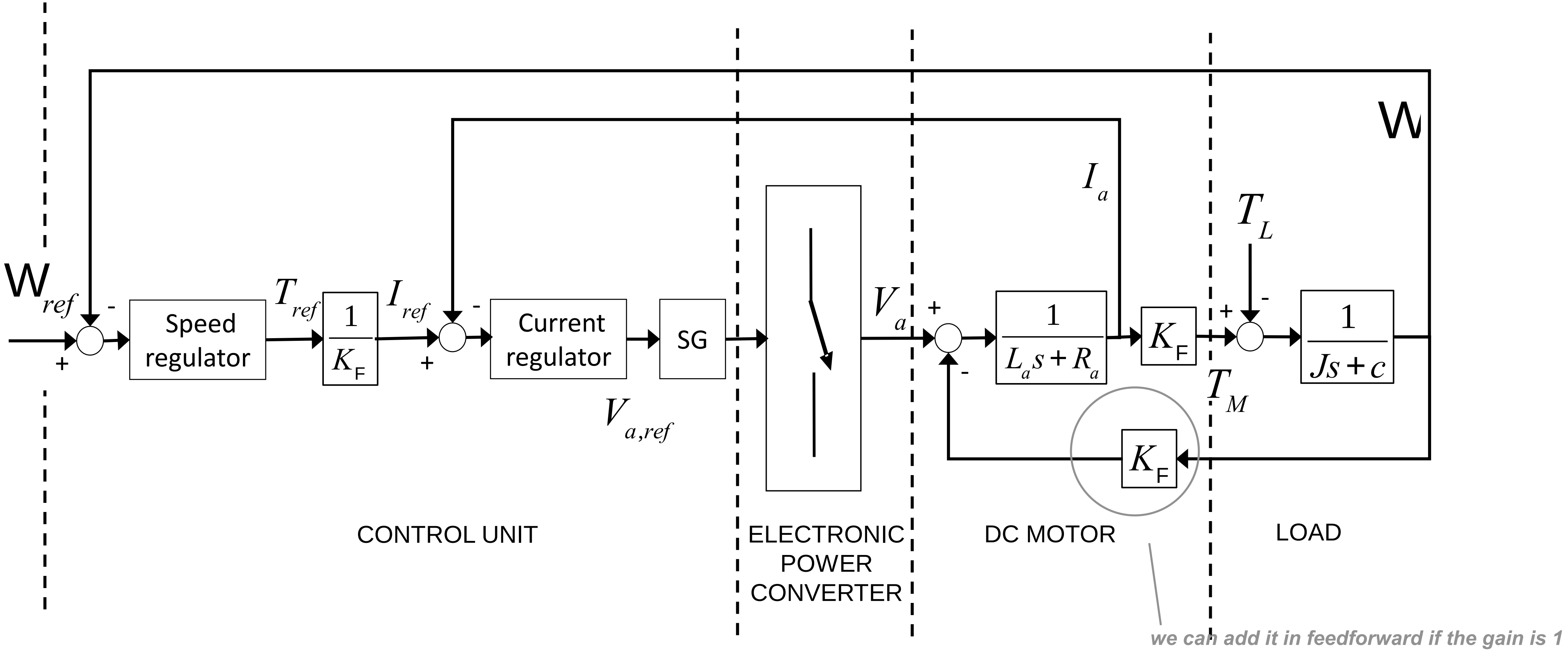
The control system is based on 2 feedback control loops: the inner one provides current regulation, while the outer loop is devoted to speed control



in this way the design of the motors depends on its usage
so we need a way to decouple the motor

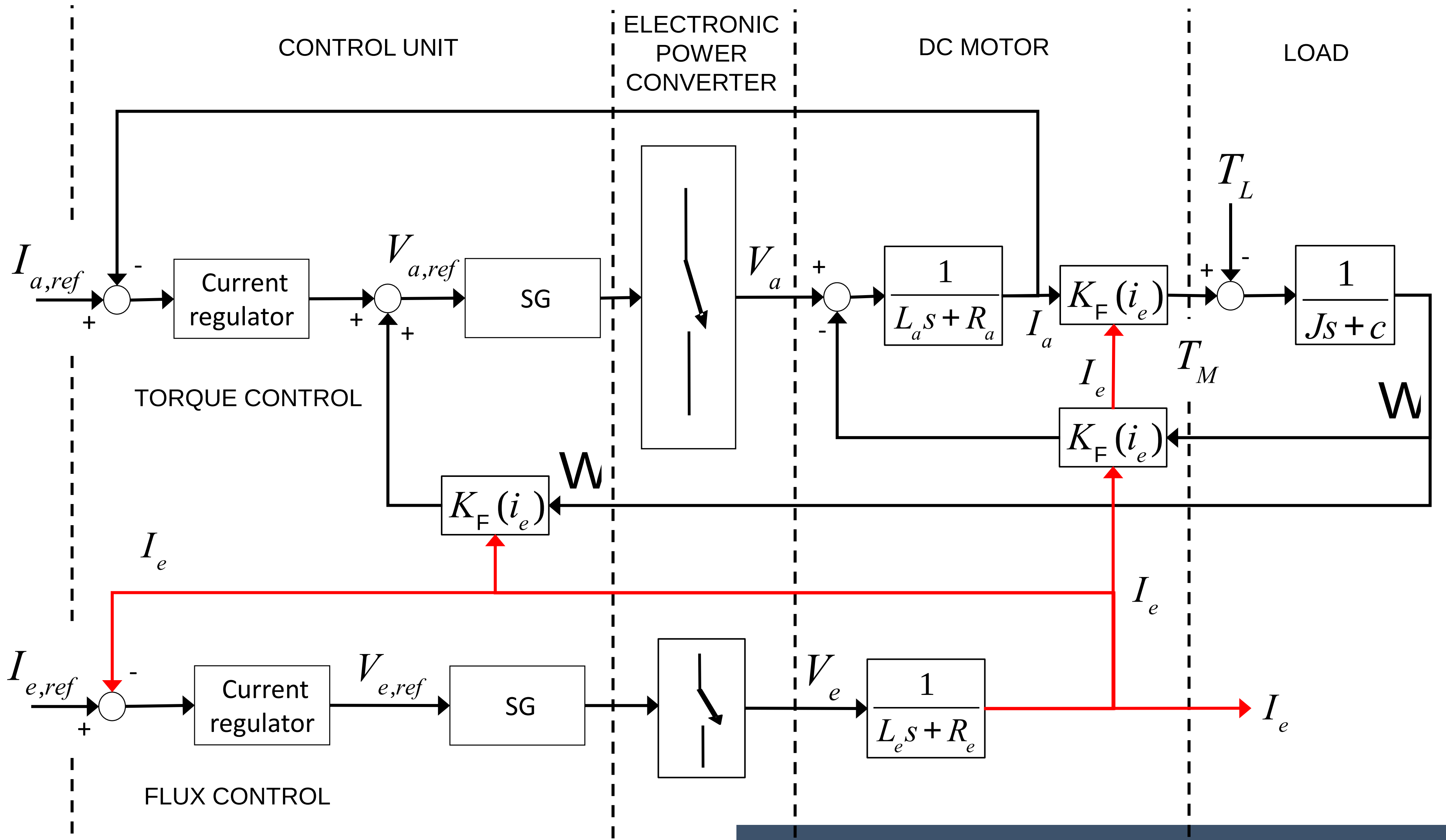
Control of a DC drive

PERMANENT MAGNET DC MOTOR-Speed control



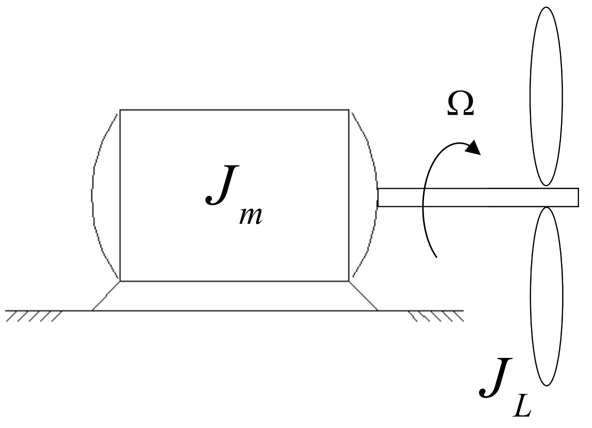
Control of a DC drive

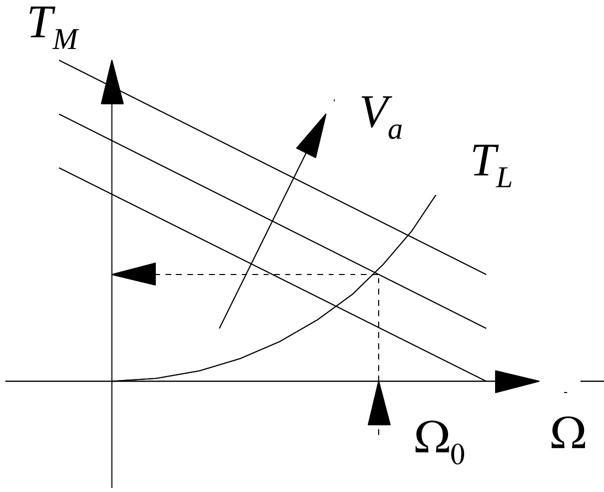
SEPARATELY EXCITED DC MOTOR-Torque control



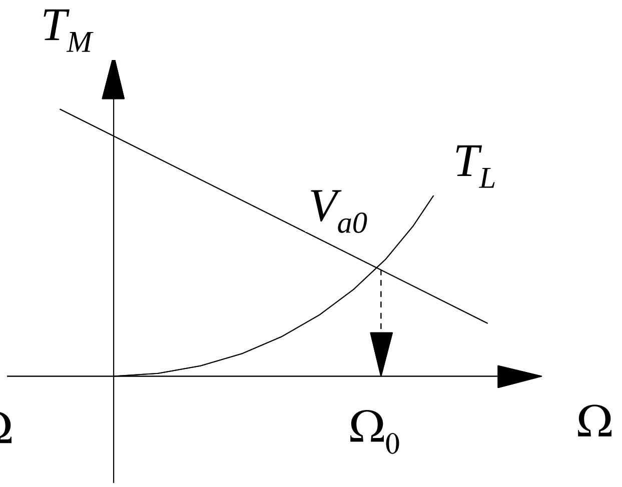
Example: speed regulation of an air compressor driven by a Permanent Magnet (PM) DC motor

The load torque is proportional to the square of the angular speed: $T_L = c_L \cdot \Omega^2$





inverse dynamics



direct dynamics

$$J\dot{\Omega} = T_M - T_L = K_\Phi i_a - c_L \Omega^2$$

$$\left(J = J_m + J_L\right)$$

Speed control in DC drives

<u>Example</u>: speed regulation of an <u>air compressor</u> driven by a PM DC motor

$$\begin{cases} L_a \dfrac{di_a}{dt} + R_a i_a + K_\Phi \Omega = V_a \\ J\dot{\Omega} = K_\Phi i_a - c_L \Omega^2 \end{cases} \qquad \begin{cases} \dfrac{di_a}{dt} = -\dfrac{R_a}{L_a} i_a - \dfrac{K_\Phi}{L_a}\Omega + \dfrac{1}{L_a} V_a \\ \dot{\Omega} = -\dfrac{c}{J}\Omega^2 + \dfrac{K_\Phi}{J} i_a \end{cases}$$

$$\underline{x} = \begin{Bmatrix} \Omega \\ i_a \end{Bmatrix} \quad \Rightarrow \quad \dot{\underline{x}} = \underline{f}\left(\underline{x}, u\right)$$

nonlinear differential equation

$$u = V_a$$

Assuming small deviations form the steady-state operating condition, the nonlinear differential equations can be linearized

Stability analysis will be performed on the new set of linear equations

Computation of the steady-state operating condition:

$$\underline{f}\left(\underline{x}_0, u_0\right) = 0 \qquad \Rightarrow \qquad \underline{x}_0 = \begin{Bmatrix} \Omega_0 \\ i_{a0} \end{Bmatrix}$$

in steady-state conditions

$$\begin{cases} V_{a0} = R_a i_{a0} + K_\Phi \Omega_0 \\ T_M = T_L \end{cases} \qquad \Rightarrow \qquad c_L \Omega_0^2 + \frac{K_\Phi^2}{R_a}\Omega_0 = \frac{K_\Phi}{R_a} V_{a0} \quad \Rightarrow \quad \Omega_0$$

$$\begin{aligned} T_M &= T_{M0} = K_\Phi i_{a0} \\ T_L &= T_{L0} = c_L \Omega_0^2 \end{aligned} \qquad \Rightarrow \qquad i_{a0} = \frac{c_L}{K_\Phi}\Omega_0^2$$

Speed control in DC drives

Perturbation of steady-state condition:

$$\underline{\bar{x}} = \underline{x} - \underline{x}_0 \Rightarrow \begin{cases} \bar{\Omega} = \Omega - \Omega_0 \\ \bar{i}_a = i_a - i_{a0} \end{cases} \qquad \bar{u} = \bar{V}_a = V_a - V_{a0}$$

Equations of motion must be linearized in the neighborhood of steady-state condition to design feedback control of compressor speed. Aim of feedback control is to maintain compressor speed constant (and equal to steady-state) despite of disturbances/variations of the load:

$$\bar{T}_L = T_L - d$$

Linearization of load torque:

$$T_L \cong T_{L0} + \left.\frac{dT_L}{d\Omega}\right|_0 \bar{\Omega} = c_L \Omega_0^2 + 2 c_L \Omega_0 \bar{\Omega}$$

Speed control in DC drives

Perturbation of the steady-state condition:

$$\underline{\bar{x}} = \underline{x} - \underline{x}_0 \Rightarrow \begin{cases} \bar{W} = W - W_0 \\ \bar{i}_a = i_a - i_{a0} \end{cases} \qquad \bar{u} = \bar{V}_a = V_a - V_{a0}$$

Linearized equations of motion, valid for small deviations form the steady-state operating condition

$$\begin{cases} L_a \dfrac{d\bar{i}_a}{dt} + R_a \bar{i}_a + \cancel{R_a i_{a0}} + K_F \bar{W} + \cancel{K_F W_0} = \bar{V}_a + \cancel{V_{a0}} \\ J\dot{\bar{W}} = T_M - \left(T_L - d\right) = K_F \bar{i}_a + \cancel{K_F i_{a0}} - \left(\cancel{c_L W_0^2} + 2c_L W_0 \bar{W} - d\right) \end{cases}$$

Speed control in DC drives

Perturbation of the steady-state condition:

$$\bar{\underline{x}} = \underline{x} - \underline{x}_0 \Rightarrow \begin{cases} \bar{W} = W - W_0 \\ \bar{i}_a = i_a - i_{a0} \end{cases} \qquad \bar{u} = \bar{V}_a = V_a - V_{a0}$$

Linearized equation of motion, valid for small deviations form the steady-state operating condition

$$\begin{cases} L_a \dfrac{d\bar{i}_a}{dt} + R_a \bar{i}_a + K_F \bar{W} = \bar{V}_a \\ J\dot{\bar{W}} + 2c_L W_0 \bar{W} = K_F \bar{i}_a + d = \bar{T}_M + d \end{cases}$$

Perturbation of the steady-state condition:

$$\underline{\bar{x}} = \underline{x} - \underline{x}_0 \Rightarrow \begin{cases} \bar{\Omega} = \Omega - \Omega_0 \\ \bar{i}_a = i_a - i_{a0} \end{cases} \qquad \bar{u} = \bar{V}_a = V_a - V_{a0}$$

Linearized equation of motion, valid for small deviations form the steady-state operating condition

$$\dot{\underline{\bar{x}}} = [A]\underline{\bar{x}} + [B]\bar{u} + [D]\bar{d} = [A]\underline{\bar{x}} + [B]\bar{V}_a + [B_d]\bar{d}$$

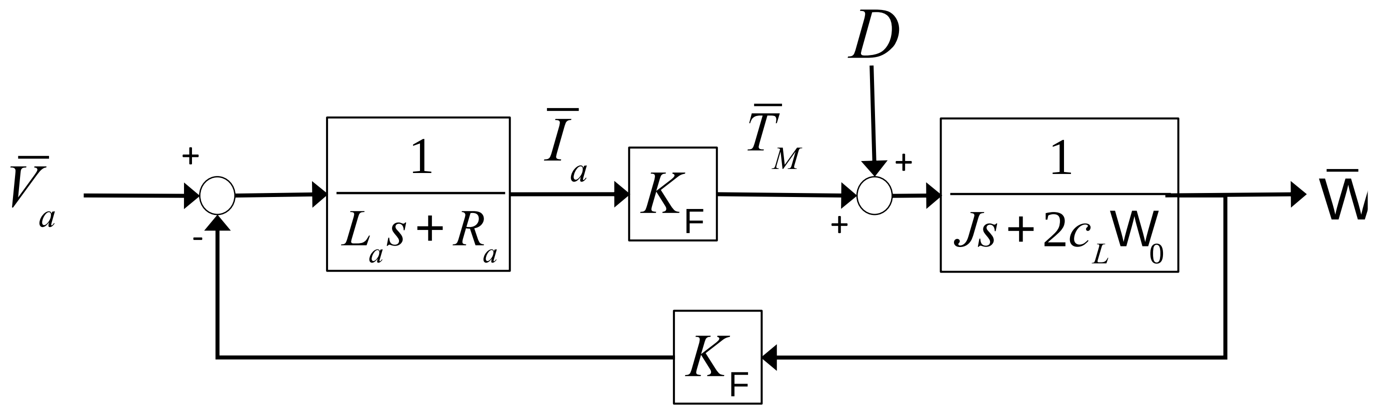
$$\dot{\underline{\bar{x}}} = \begin{bmatrix} -\dfrac{K\Phi}{L_a} & -\dfrac{R_a}{L_a} \\ -\dfrac{2c_L\Omega_0}{J} & \dfrac{K\Phi}{J} \end{bmatrix} \underline{\bar{x}} + \begin{bmatrix} \dfrac{1}{L_a} \\ 0 \end{bmatrix} \bar{V}_a + \begin{bmatrix} 0 \\ \dfrac{1}{J} \end{bmatrix} d \qquad \underline{\bar{x}} = \begin{bmatrix} \bar{\Omega} \\ \bar{i}_a \end{bmatrix}$$

TF model

Laplace transform

$$
\begin{cases}
(L_a s + R_a)\bar{I}_a = \bar{V}_a - K_F \bar{W} \\
(Js + 2c_L W_0)\bar{W} = K_F \bar{I}_a + D
\end{cases}
$$

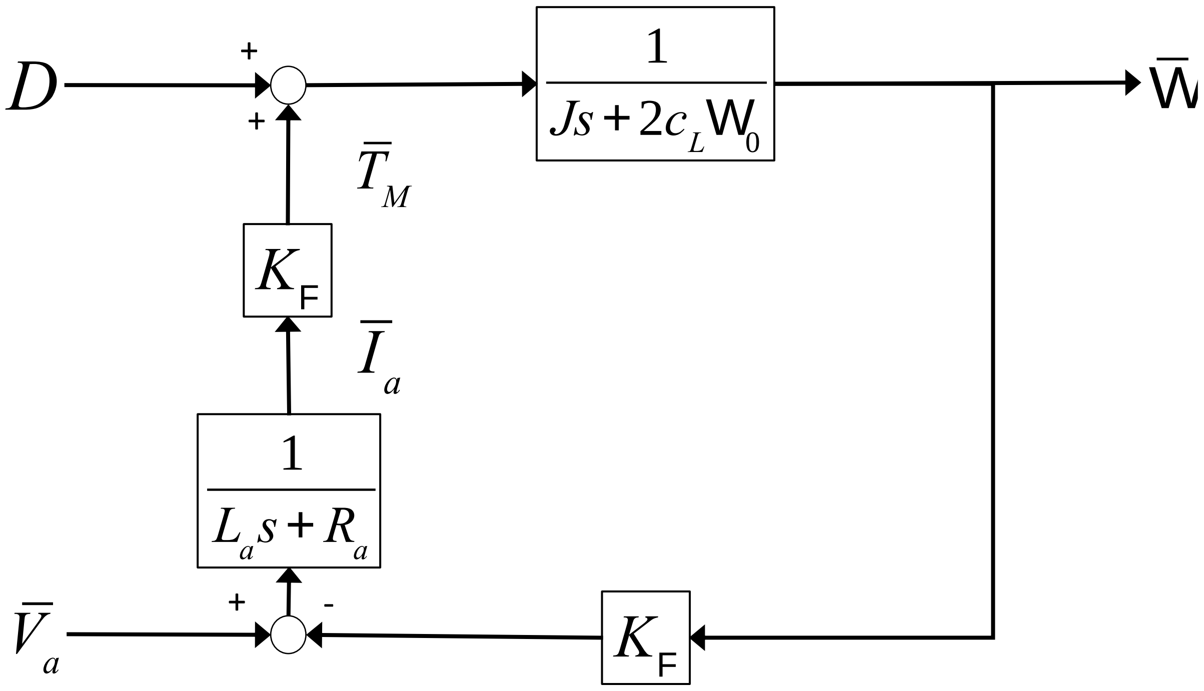
Two inputs: V_a and D



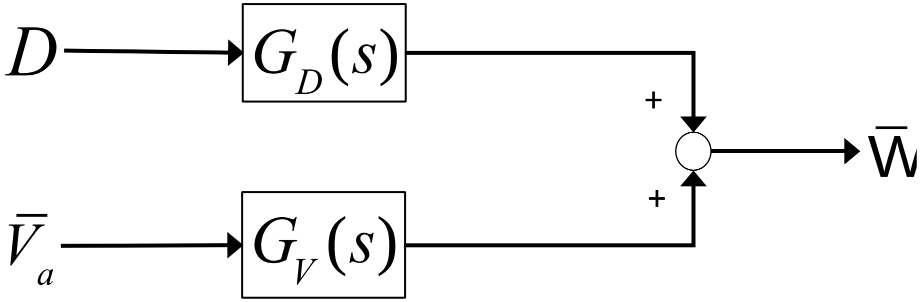
Laplace transform

$$\begin{cases}(L_a s + R_a)\bar{I}_a = \bar{V}_a - K_\Phi \bar{\Omega} \\ (Js + 2c_L\Omega_0)\bar{\Omega} = K_\Phi \bar{I}_a + D\end{cases}$$

Two inputs: V_a and D



Two-inputs system: V_a and D



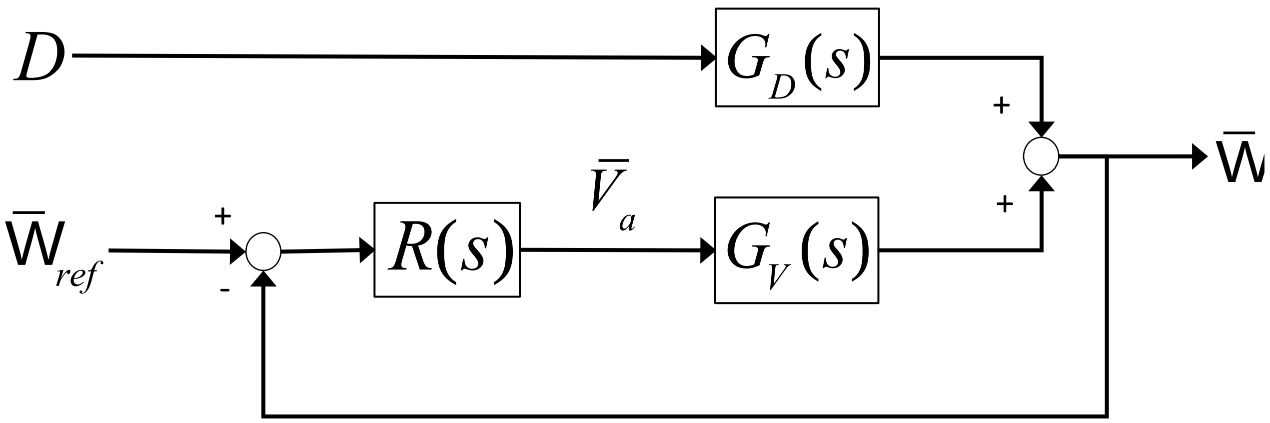
$$G_V(s) = \frac{\bar{\Omega}(s)}{V_a(s)} = \frac{K_\Phi}{(L_a s + R_a)(Js + 2c_L\Omega_0) + K_\Phi^2}$$

$$G_D(s) = \frac{\bar{\Omega}(s)}{D(s)} = \frac{L_a s + R_a}{(L_a s + R_a)(Js + 2c_L\Omega_0) + K_\Phi^2}$$

Feedback control of compressor speed-PD regulator

$$\bar{V}_a = \left(k_p + k_d s\right)\left(\bar{W}_{ref} - \bar{W}\right) = k_p\left(1 + T_d s\right)\left(\bar{W}_{ref} - \bar{W}\right)$$

$$R(s) = \left(k_p + k_d s\right) = k_p\left(1 + T_d s\right)$$



Two-inputs system: $D,\ \bar{W}_{ref}$

Case 1

Aim of PD controller is to make speed variations follow the reference as close as possible, in absence of disturbances on the load $\Rightarrow \ D(s) = 0$

$$\bar{V}_a = \left(k_p + k_d s\right)\left(\bar{\Omega}_{ref} - \bar{\Omega}\right) = k_p\left(1 + T_d s\right)\left(\bar{\Omega}_{ref} - \bar{\Omega}\right)$$

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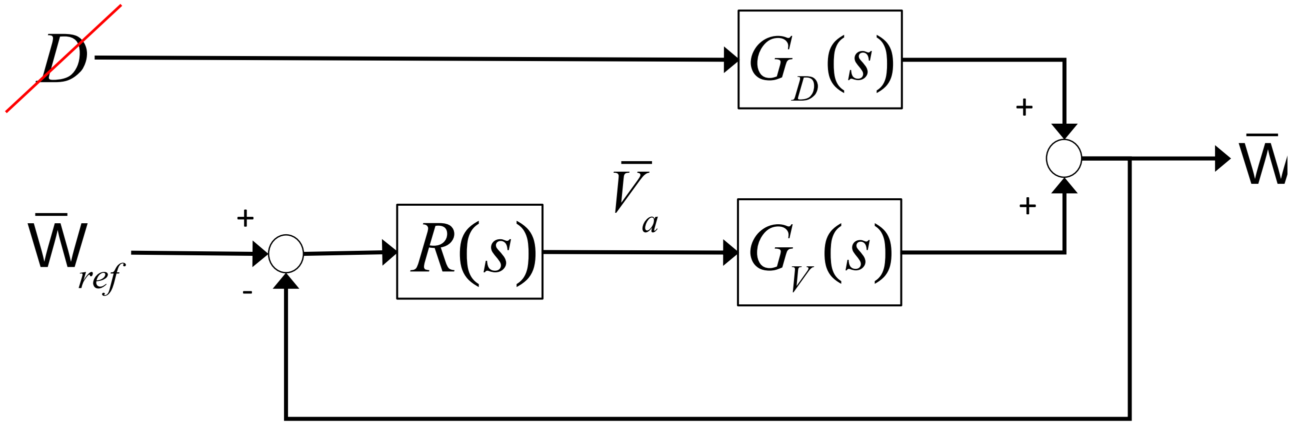

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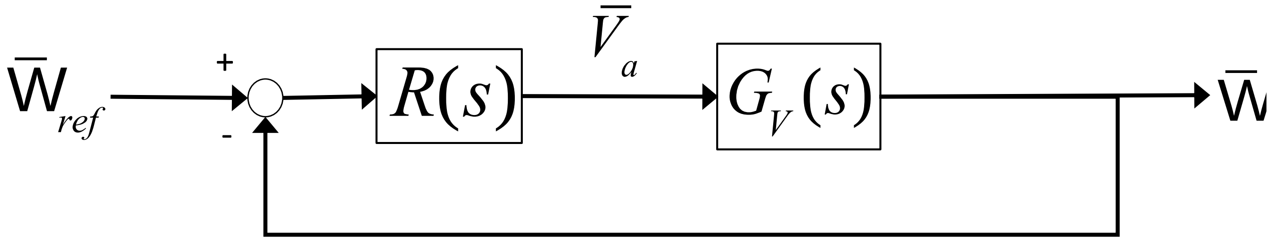


$$G_V(s) = \frac{\bar{\Omega}(s)}{\bar{V}_a(s)} = \frac{K_\Phi}{(L_a s + R_a)(Js + 2c_L\Omega_0) + K_\Phi^2}$$

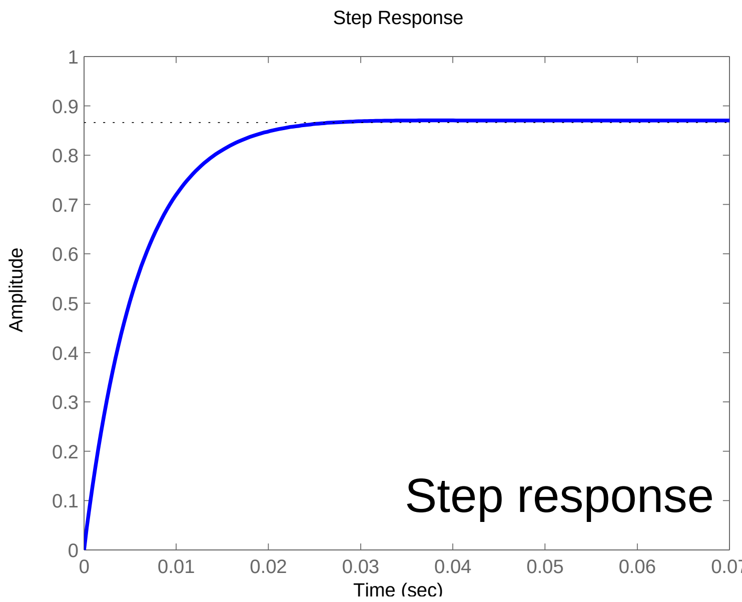
Speed control in DC drives

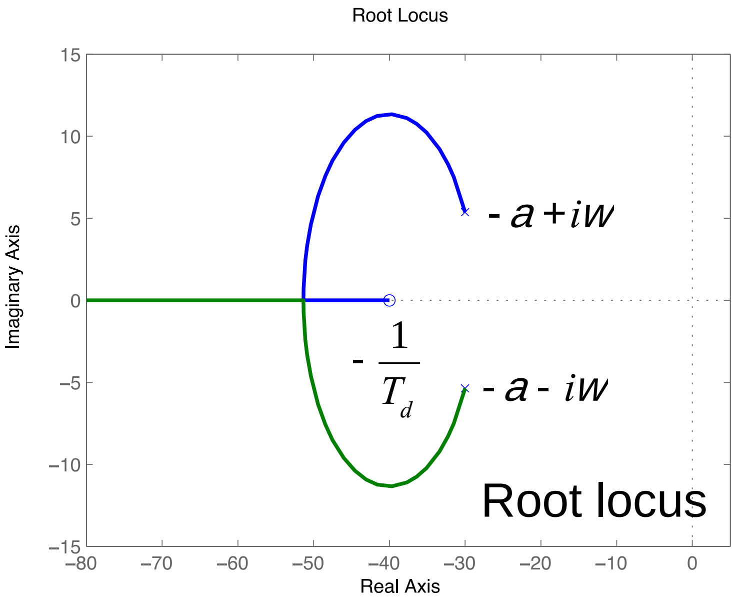
Closed-loop transfer function

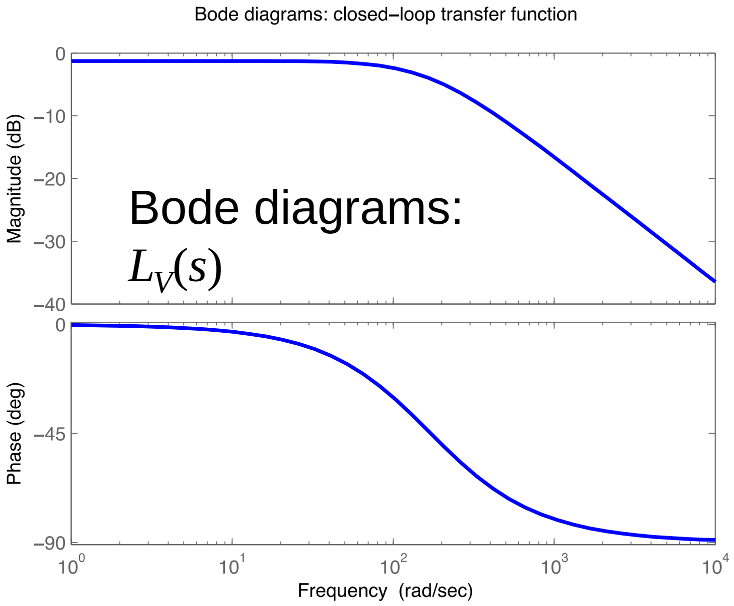
$$L_V(s) = \frac{\bar{W}(s)}{\bar{W}_{ref}(s)} = \frac{R(s)G_V(s)}{1+R(s)G_V(s)}$$

$$L_V = \frac{L_a s + R_a}{(Js + 2c_L W_0)(L_a s + R_a) + K_F\left(k_p + k_d s\right) + K_F^2}$$

Aim of the controller is to make compressor speed follow a given reference







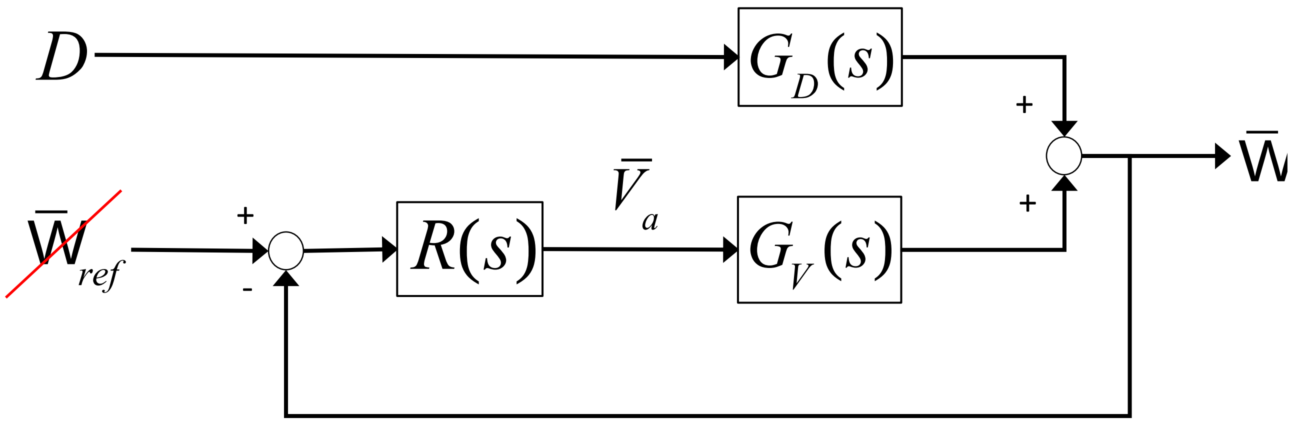
Case 2

Aim of PD controller is to maintain compressor speed constant (no speed variations), despite of disturbances on the load (i.e. variations of the aerodynamic load)

$$W = W_0 \Rightarrow \bar{W}_{ref} = 0$$

$$\bar{V}_a = \left(k_p + k_d s\right)\left(\bar{W}_{ref} - \bar{W}\right) = k_p\left(1 + T_d s\right)\left(\bar{W}_{ref} - \bar{W}\right) = - k_p\left(1 + T_d s\right)\bar{W}$$

$$R(s) = \left(k_p + k_d s\right) = k_p\left(1 + T_d s\right)$$



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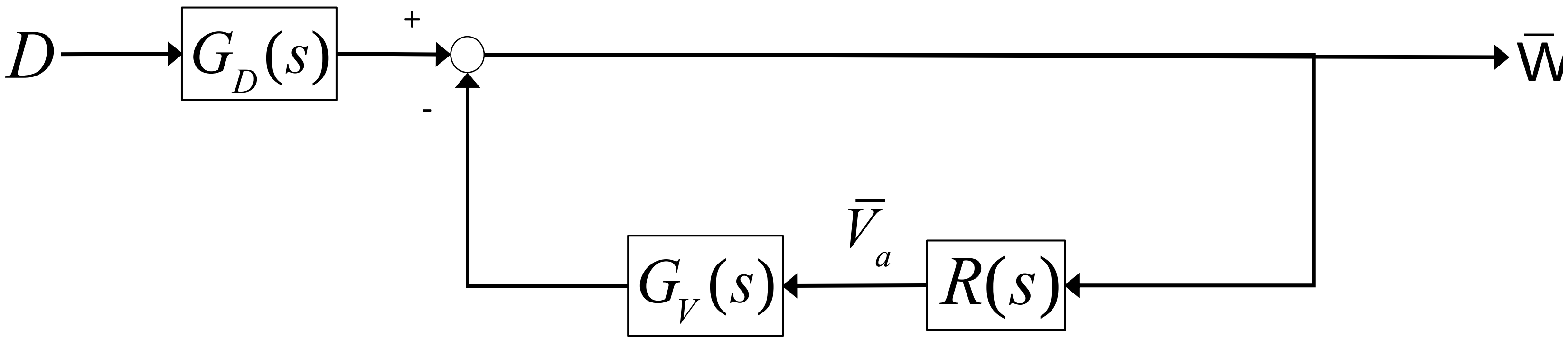
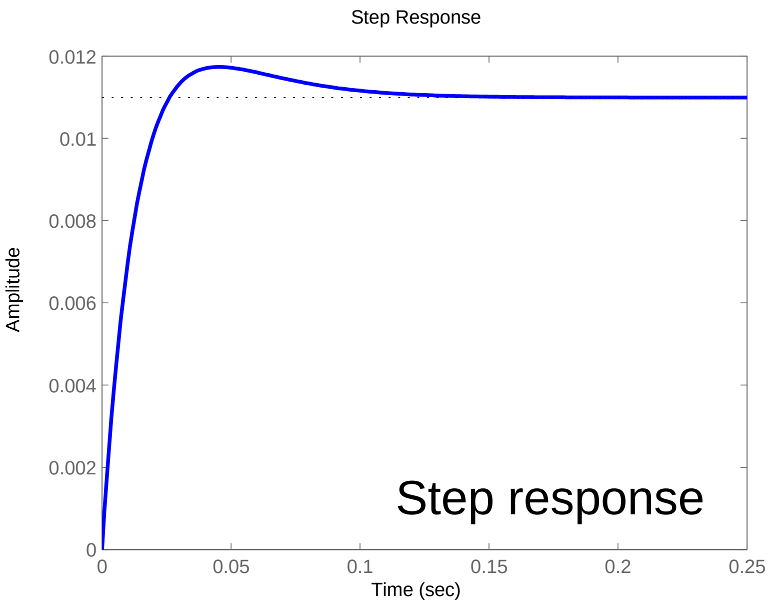


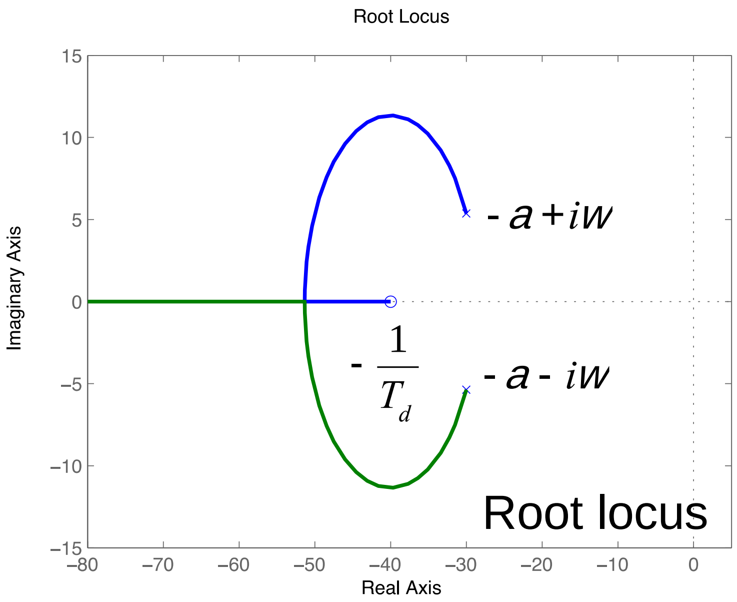
Closed-loop transfer function

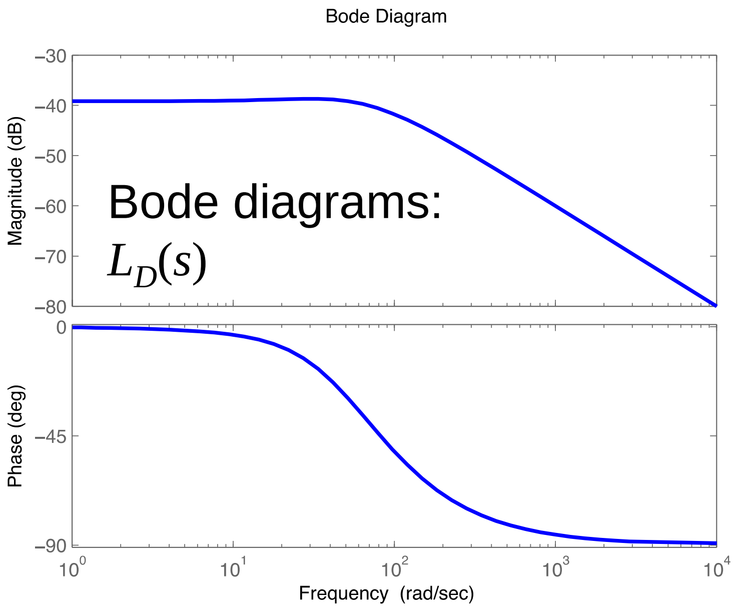
$$L_D(s) = \frac{\bar{\Omega}(s)}{D(s)} = G_D(s)\frac{1}{1+R(s)G_D(s)}$$

$$L_D = \frac{L_a s + R_a}{(Js + 2c_L\Omega_0)(L_a s + R_a) + K_\Phi\left(k_p + k_d s\right) + K_\Phi^2}$$



Aim of the controller is to maintain compressor speed constant, despite of disturbances on the load





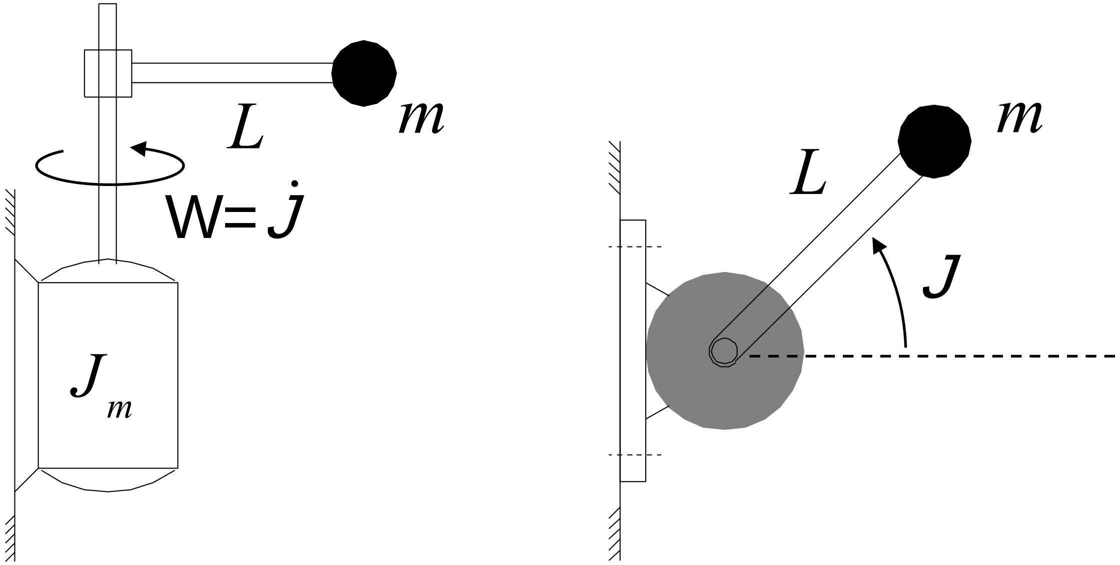
Position control in DC drives

$$\begin{cases} L_a \dfrac{di_a}{dt} + R_a i_a + K_F \dot{J} = V_a \\ \left(J_m + mL^2\right)\ddot{J} + c\dot{J} - K_F i_a = 0 \end{cases}$$

$$\boxed{W = \dot{J}}$$

Feedback control of point-mass position (PD regulator)

$$V_a = k_p\left(J_{rif} - J\right) + k_d\left(\dot{J}_{rif} - \dot{J}\right)$$



(a) vertical plane (b) horizontal plane

State-space representation

$$
\begin{cases}
L_a \dfrac{di_a}{dt} + R_a i_a + K_F \dot{J} = V_a = k_p \left(J_{ref} - J \right) + k_d \left(\dot{J}_{ref} - \dot{J} \right) \\
\dot{J} = \dot{J} \\
\left(J_m + mL^2 \right) \ddot{J} + c\dot{J} - K_F i_a = 0
\end{cases}
$$

$$
\begin{cases}
L_a \dfrac{di_a}{dt} + R_a i_a + \left(K_F + k_d \right) \dot{J} + k_p J = k_d \dot{J}_{ref} + k_p J_{ref} \\
\dot{J} - \dot{J} = 0 \\
J^* \ddot{J} + c\dot{J} - K_F i_a = 0
\end{cases}
$$

State-space representation

$$\underbrace{\begin{bmatrix} 0 & 0 & L_a \\ 0 & 1 & 0 \\ J^* & 0 & 0 \end{bmatrix}}_{[M]} \underbrace{\begin{Bmatrix} \ddot{\vartheta} \\ \dot{\vartheta} \\ \dfrac{di_a}{dt} \end{Bmatrix}}_{\underline{\dot{x}}} + \begin{bmatrix} (K_\Phi + k_d) & k_p & R_a \\ -1 & 0 & 0 \\ c & 0 & -K_\Phi \end{bmatrix} \underbrace{\begin{Bmatrix} \dot{\vartheta} \\ \vartheta \\ i_a \end{Bmatrix}}_{\underline{x}} = \begin{bmatrix} k_p & k_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \underbrace{\begin{Bmatrix} \vartheta_{ref} \\ \dot{\vartheta}_{ref} \end{Bmatrix}}_{\underline{u}}$$

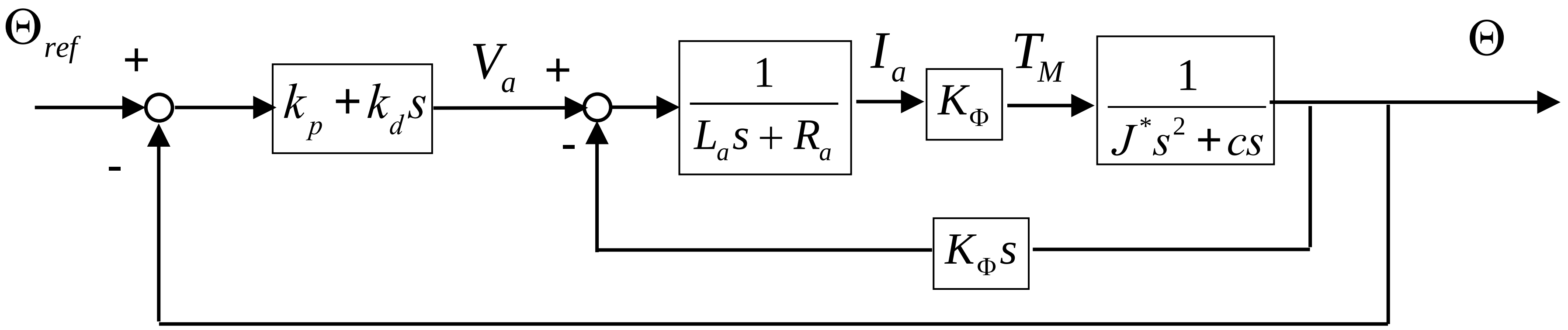
$$\begin{Bmatrix} \ddot{\vartheta} \\ \dot{\vartheta} \\ \dfrac{di_a}{dt} \end{Bmatrix} = \boxed{-[M]^{-1} \begin{bmatrix} (K_\Phi + k_d) & k_p & R_a \\ -1 & 0 & 0 \\ c & 0 & -K_\Phi \end{bmatrix}} \begin{Bmatrix} \dot{\vartheta} \\ \vartheta \\ i_a \end{Bmatrix} + \boxed{[M]^{-1} \begin{bmatrix} k_p & k_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \begin{Bmatrix} \vartheta_{ref} \\ \dot{\vartheta}_{ref} \end{Bmatrix}$$

$$\underline{\dot{x}} = [A]\underline{x} + [B]\underline{u}$$

Position control in DC drives

TF model

$$
\begin{cases}
L_a \dfrac{di_a}{dt} + R_a i_a + K_\Phi \dot{\vartheta} = V_a \\
\left(J_m + mL^2\right)\ddot{\vartheta} + c\dot{\vartheta} - K_\Phi i_a = 0 \\
V_a = k_p\left(\vartheta_{ref} - \vartheta\right) + k_d\left(\dot{\vartheta}_{ref} - \dot{\vartheta}\right)
\end{cases}
\xrightarrow{\mathit{Laplace}}
\begin{cases}
\left(L_a s + R_a\right) I_a = V_a - K_\Phi s \Theta \\
\left(J^* s^2 + cs\right)\Theta - K_\Phi I_a = 0 \\
V_a = (k_p + k_d s)\left(\Theta_{ref} - \Theta\right)
\end{cases}
$$

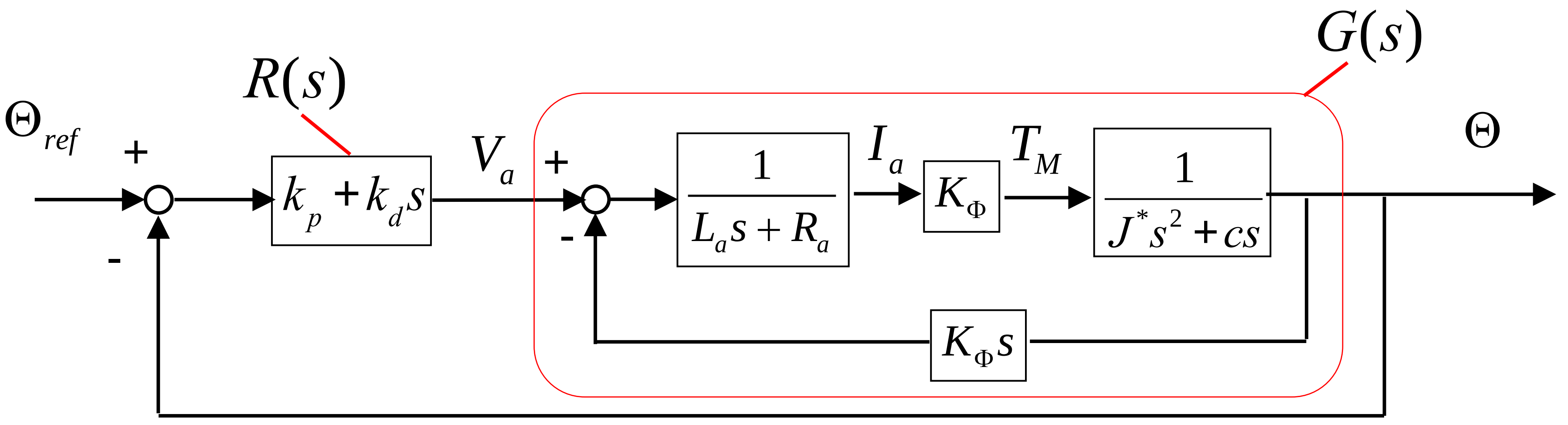


Position control in DC drives

TF model

$$\begin{cases} \left(L_a s + R_a\right) I_a = V_a - K_\Phi s \Theta \\ \left(J^* s^2 + cs\right)\Theta - K_\Phi I_a = 0 \\ V_a = (k_p + k_d s)\left(\Theta_{ref} - \Theta\right) \end{cases} \quad \Rightarrow \quad G(s) = \frac{K_\Phi}{\left(L_a s + R_a\right)\left(J^* s^2 + cs\right) + K_\Phi^2 s}$$

$$R(s) = k_p + k_d s$$ (PD regulator)



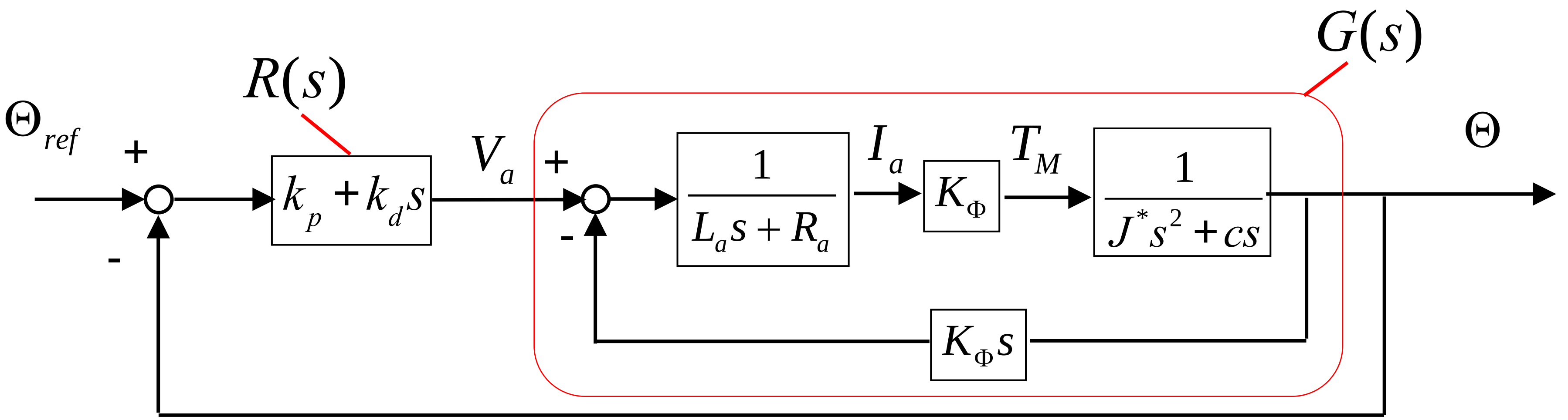
TF model

$$R(s)G(s) = \frac{V_a(s)}{\Theta_{ref}(s) - \Theta(s)} = \frac{\left(k_p + k_d s\right) K_\Phi}{\left(L_a s + R_a\right)\left(J^* s^2 + cs\right) + K_\Phi^2 s}$$

Loop transfer function

$$L(s) = \frac{\Theta(s)}{\Theta_{ref}(s)} = \frac{K_\Phi \left(k_p + k_d s\right)}{\left(L_a s + R_a\right)\left(J^* s^2 + cs\right) + \left(k_p + k_d s\right) K_\Phi + K_\Phi^2 s}$$

Closed-loop transfer function



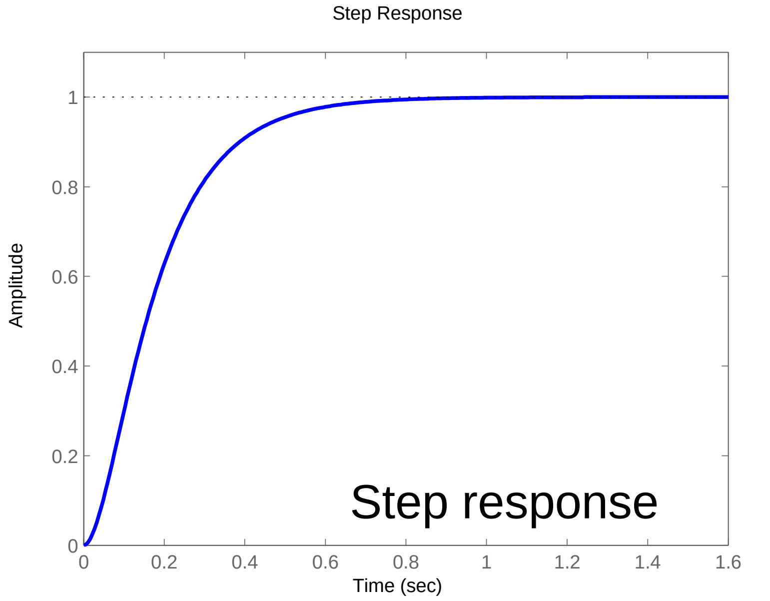
Speed control in DC drives

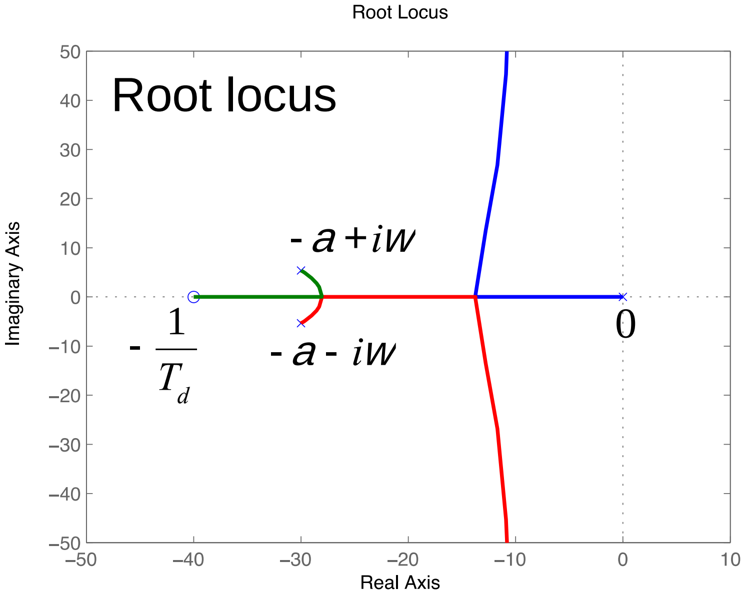
Closed-loop transfer function

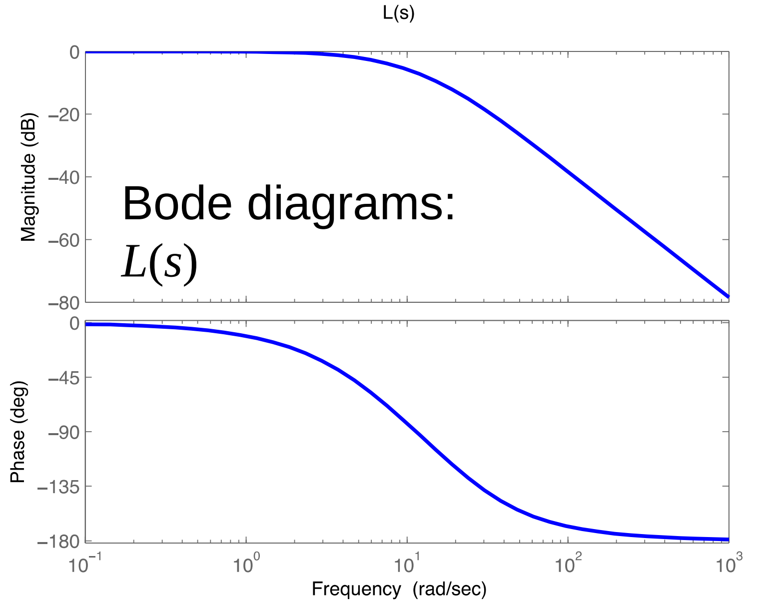
$$L(s) = \frac{Q(s)}{Q_{ref}(s)}$$

$$L(s) = \frac{K_F\left(k_p + k_d s\right)}{\left(L_a s + R_a\right)\left(J^* s^2 + cs\right) + \left(k_p + k_d s\right)K_F + K_F^2 s}$$

Aim of the controller is to make the point-mass follow a given reference trajectory







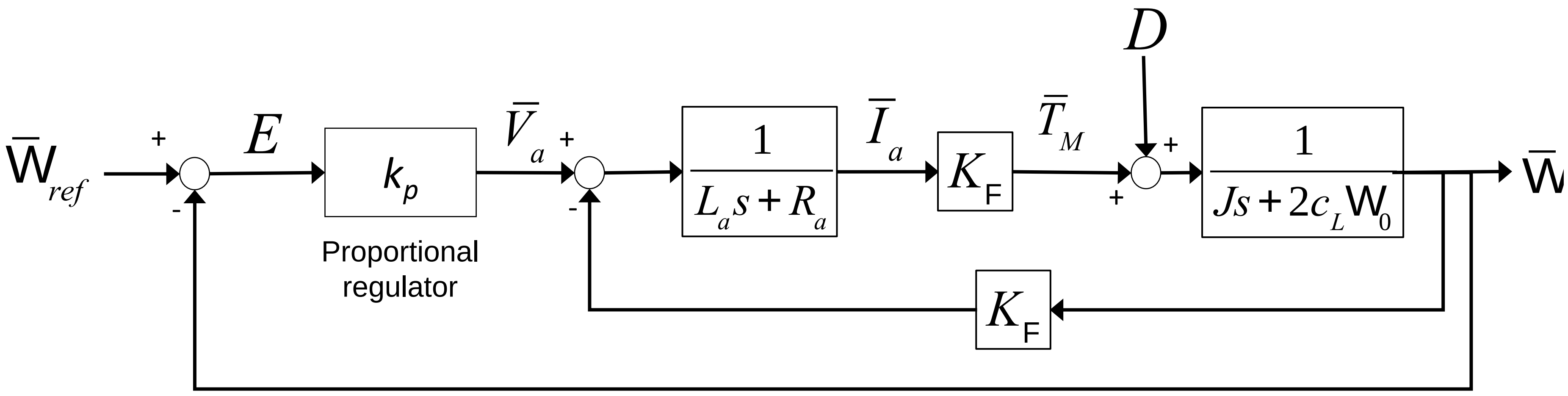
Laplace transform

$$\begin{cases} (L_a s + R_a)\bar{I}_a = \bar{V}_a - K_\Phi \bar{\Omega} \\ (Js + 2c_L\Omega_0)\bar{\Omega} = K_\Phi \bar{I}_a + D \end{cases}$$

Two inputs: V_a and D

Closed-loop feedback control system – proportional regulator

$$\bar{V}_a = k_p\left(\bar{\Omega}_{ref} - \bar{\Omega}\right)$$



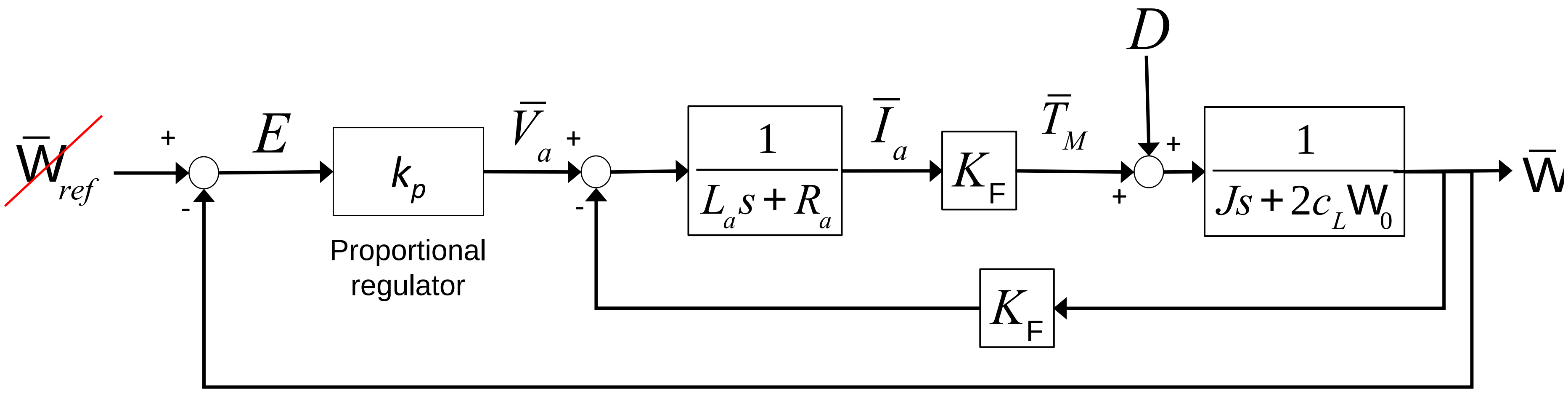
Speed control in DC drives

Laplace transform

$$\begin{cases} (L_a s + R_a)\bar{I}_a = \bar{V}_a - K_\Phi \bar{\Omega} \\ (Js + 2c_L \Omega_0)\bar{\Omega} = K_\Phi \bar{I}_a + D \end{cases}$$

Closed-loop feedback control system – proportional regulator

$$\bar{V}_a = k_p\left(\bar{\Omega}_{ref} - \bar{\Omega}\right) = -k_p \bar{\Omega} \qquad \Omega = \Omega_0 \Rightarrow \bar{\Omega}_{ref} = 0$$



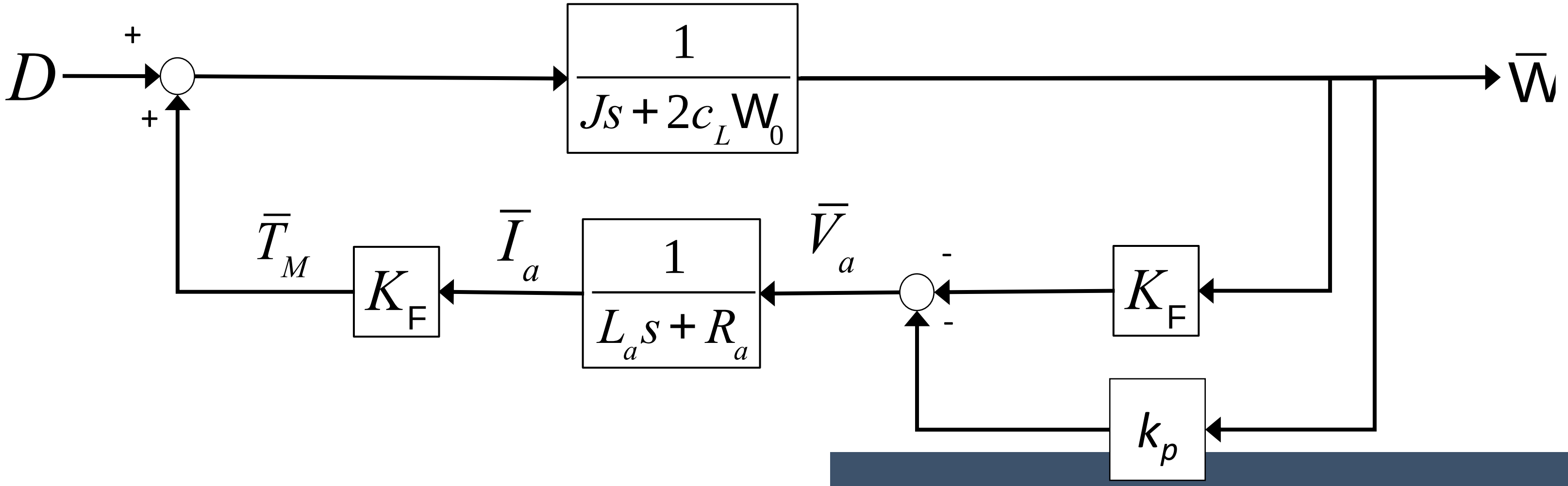
Speed control in DC drives

Laplace transform

$$\begin{cases} (L_a s + R_a)\bar{I}_a = \bar{V}_a - K_\Phi \bar{\Omega} \\ (Js + 2c_L\Omega_0)\bar{\Omega} = K_\Phi \bar{I}_a + D \end{cases}$$

Closed-loop feedback control system – proportional regulator

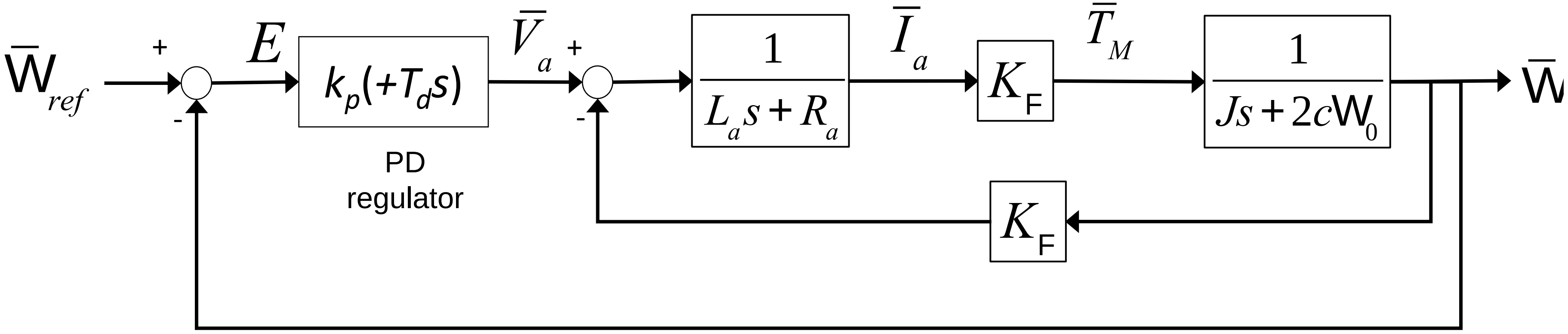
$$\bar{V}_a = k_p\left(\bar{\Omega}_{ref} - \bar{\Omega}\right) = -k_p\bar{\Omega} \qquad \Omega = \Omega_0 \Rightarrow \bar{\Omega}_{ref} = 0$$



Closed-loop feedback control system – Proportional-Derivative regulator

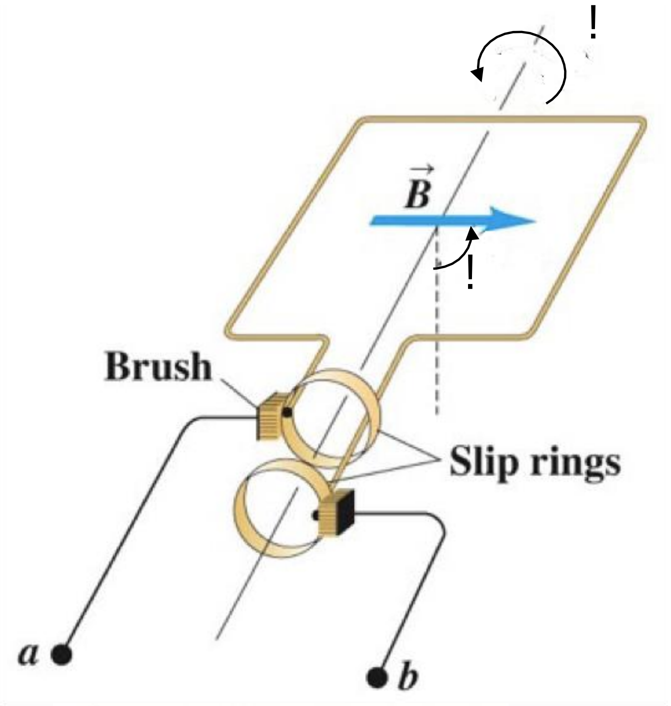
$$\begin{cases} (L_a s + R_a)\bar{I}_a = \bar{V}_a - K_F \bar{W} \\ (Js + 2cW_0)\bar{W} = K_F \bar{I}_a = \bar{T}_M \end{cases}$$

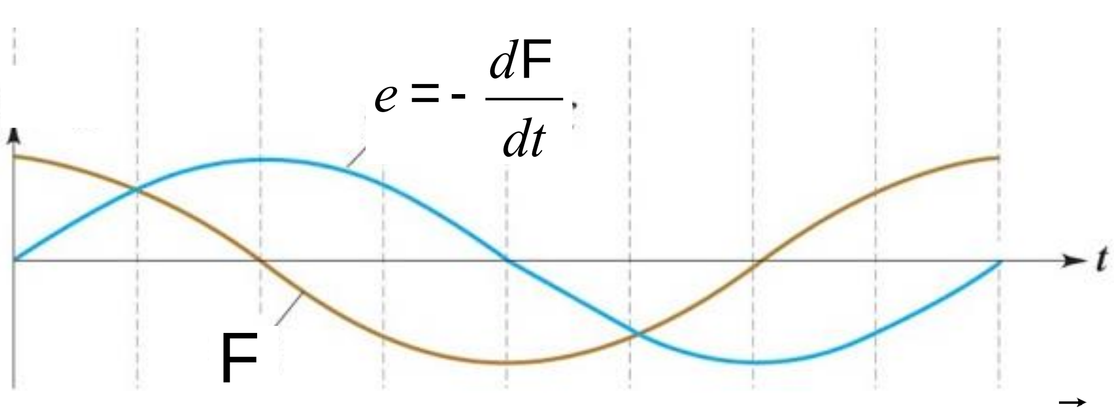
$$\bar{V}_a = \left(k_p + k_d s\right)\left(W_{ref} - W\right) = \left(k_p + T_d s\right)\left(W_{ref} - W\right)$$

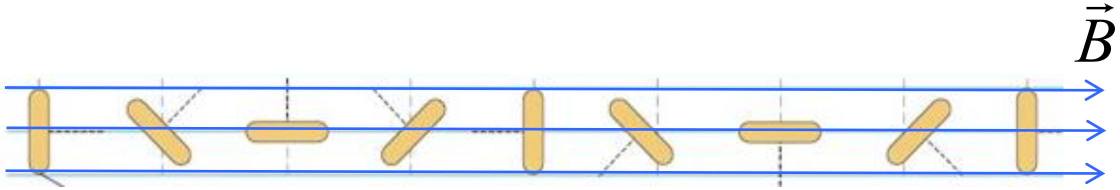


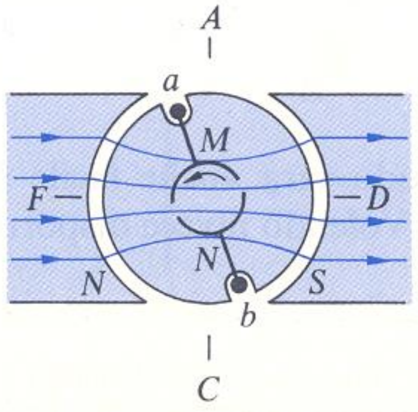
DC motor

Uniformly rotating rectangular coil in a constant magnetic field









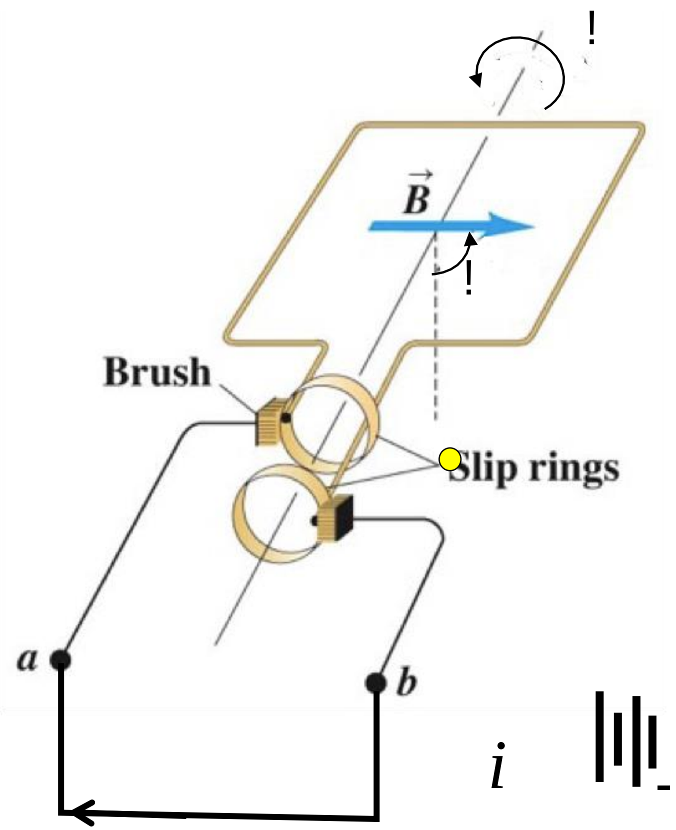
$$\Phi = \Phi_{max} \cos \Omega t$$

$$e = -\Omega \Phi_{max} \sin \Omega t$$

Without commutator, an alternate current is produced

DC motor

If no commutator is present, an alternate torque is produced by the



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