



# Adaptive and Autonomous Aerospace Systems

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Part 2: Adaptive Control

Lab experience on adaptive control



**POLITECNICO**  
MILANO 1863

**ASCL**  
AEROSPACE SYSTEMS & CONTROL LABORATORY

Fundamental system  
and control research



**ASCL**

AEROSPACE SYSTEMS & CONTROL LABORATORY

Applied aerospace systems  
and control research:

Guidance, navigation and control  
(GNC) for aircraft/spacecraft

Experimental activities:

UAV design, prototyping and  
integration

## Fundamental system and control research

- Model identification
- Data-driven controller design
- Nonlinear attitude determination
- Nonlinear and geometric control for multibody systems
- Anti-windup design
- Adaptive control

## Applied aerospace systems and control research

- Rotorcraft and tiltrotor attitude control
- Helicopter vibration and noise – identification and control
- UAV guidance, navigation and control
- Attitude dynamics and control of spacecraft
- Combined spacecraft control

## Experimental activities

- Study of estimation and control problems for multirotor UAVs
- Experimental validation of new modelling, identification and control technologies
- Development of innovative multirotor platforms

# Introduction to UAVs: Main configurations and capabilities



# Unmanned Aerial Vehicles

Emergence of **Unmanned Aerial Vehicles (UAVs)** capable of agile autonomous maneuvering in possibly unknown or cluttered environments (*Advanced Air Mobility & Delivery*).



## Fixed wings UAVs

- high endurance (long time of flight)
- high payload capabilities (advanced sensors and high computational power).
- a runway is needed to take off and land (small models can be launched/caught).
- non-zero forward velocity is needed to fly (due to aerodynamic constraints)



MQ-4C Triton (Northrop Grumman)



AirSTAR (NASA)



## Rotary wings UAVs

- high maneuverability
- Vertical Take Off and Landing (can land on very small areas)
- stationary/slow flight (useful to perform long time tasks in the same position).
- can easily fly in small and cluttered environment (e.g., by performing hovering and slow motion).



ROG-1 (ANT-X)



V-150 (UMS SKELDAR)

# Unmanned Aerial Vehicles

## Mixed configurations



Fixed-wing tiltrotor (Bell Eagle Eye)



VTOL + fixed wing (Wing (Google))

## Unconventional configurations:

### Tilt-arm/rotor UAVs

propellers + servo-actuators

The thrust vectoring capability increases significantly the operation range:

- Improved maneuverability (partial or full decoupling of tracking objectives)
- Augmented disturbance rejection capabilities



Tilt-X (ASCL)

# Unmanned Aerial Vehicles

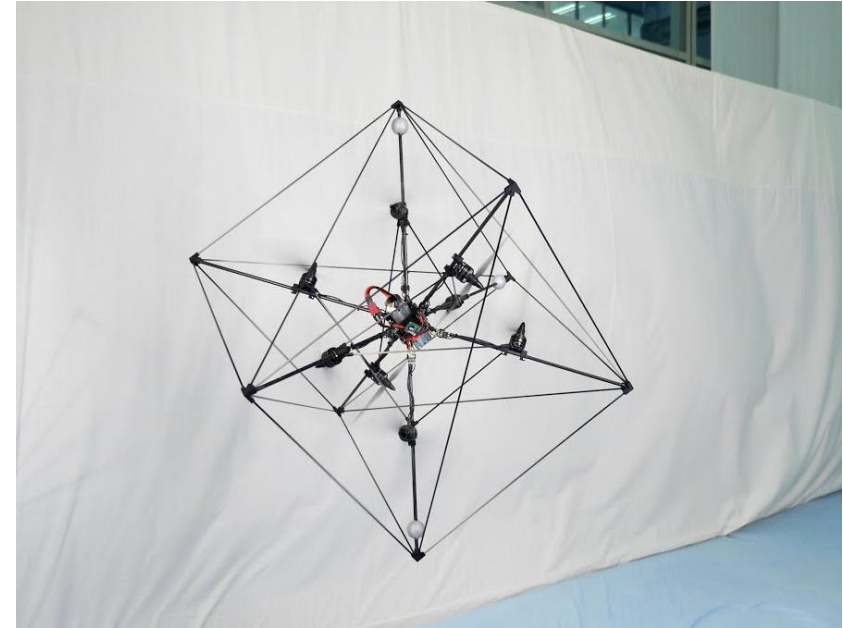


## Unconventional configurations:

### Fixed-tilted multirotor UAVs

6+ propellers inclined with respect to the airframe

Partial or full thrust-vectoring capabilities



Omnicopter (ETH Zurich)



## Multirotor UAVs

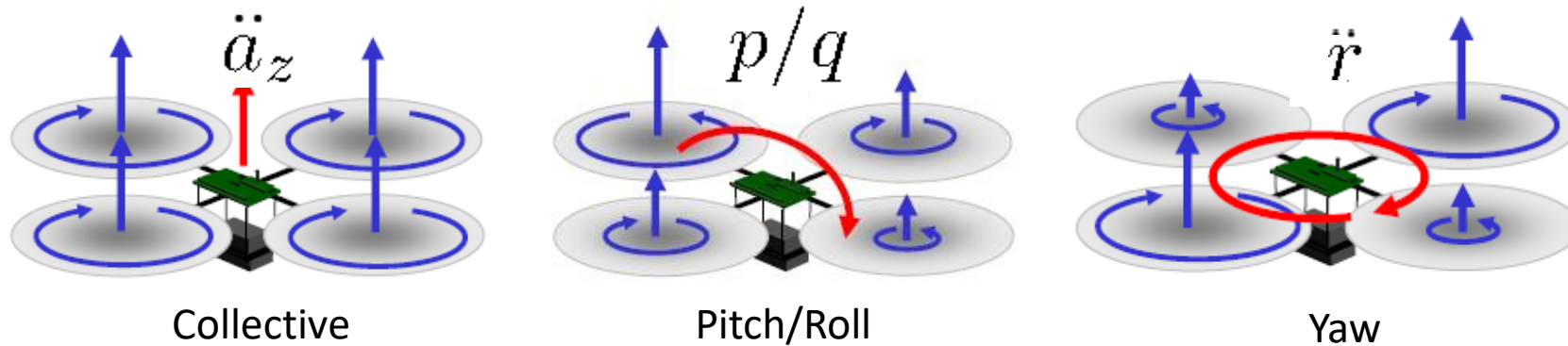
- Quadrotor (and in general multirotor) UAVs are the most flexible and adaptable platforms for undertaking aerial research.
- Small and safe (well operated in an indoor laboratory environment).
- Can carry sensor and computer payload to function autonomously.
- Capable of hover flight or forward flight.
- Sensing and control problems are ubiquitous in aerial flight control systems.



ANT-X quadrotor

# Unmanned Aerial Vehicles

A quadrotor consists of 4 individual rotors attached to a rigid cross airframe in a coplanar fashion.



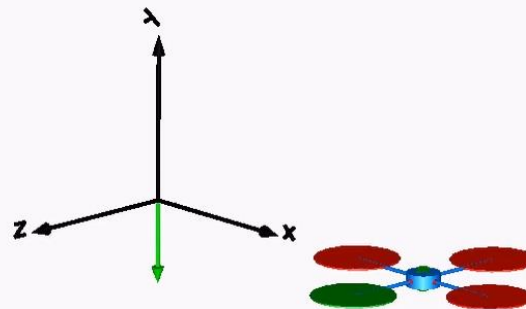
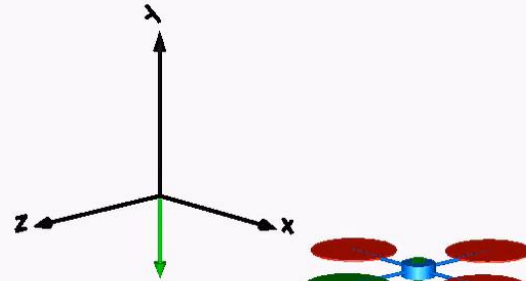
Motion control in quadrotors is achieved by **differential control** of the thrust generated by each rotor:

- the force component (along the vertical axis of the airframe) is generated by the sum of the thrust of the individual rotors.
- pitch and roll torques are obtained by differential thrust along the NS axis or EW axis.
- yaw torque is obtained by differential control of the NS rotors compared to the EW rotors such that the total thrust is constant.

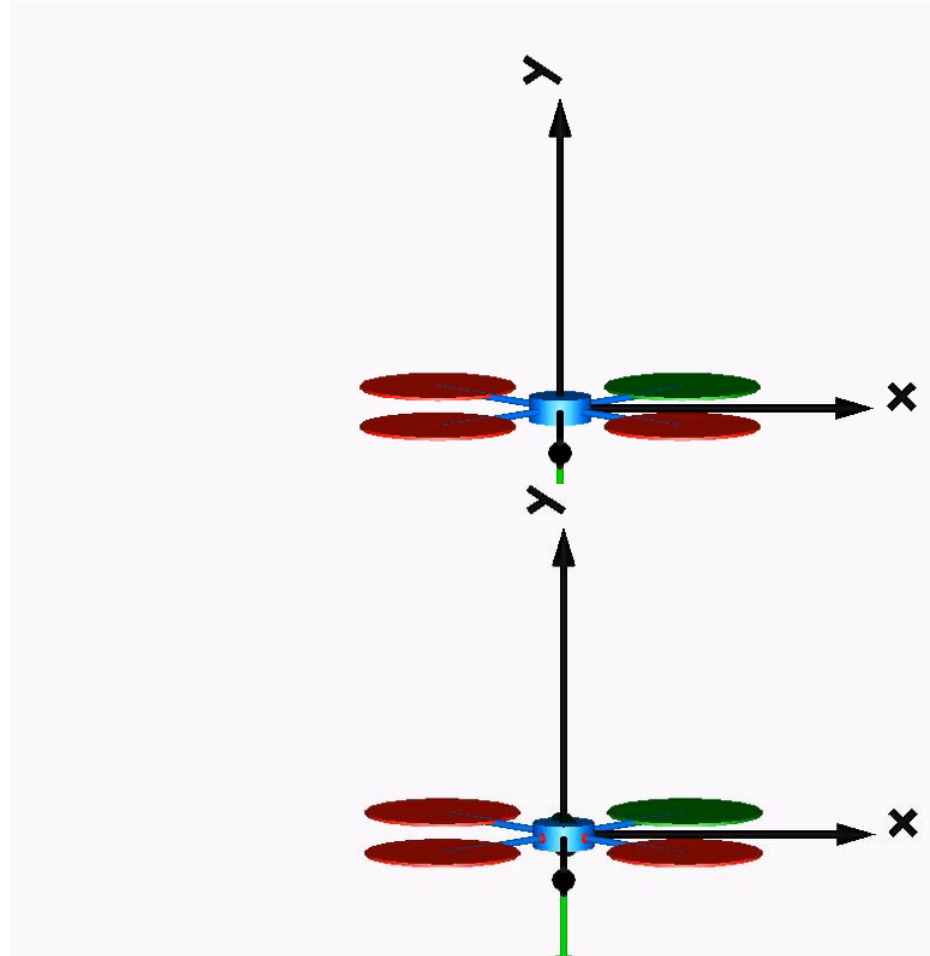
A quadrotor is an **underactuated** platform

- 4 inputs – 6 degrees of freedom (attitude and position)
- No force component can be actively generated in the plane of the rotors.
- Translational degrees of freedom in the horizontal plane can be controlled through the attitude dynamics.

## Maneuverability limitations



## Disturbance rejection capabilities





# Mathematical model of quadrotor UAVs

To develop a mathematical model suitable for flight control system design, the following is needed:

- a model for the «bare aircraft»: for multicopter UAVs the equations of motion for a rigid body suffice;
- a model for the actuators, i.e., the ESC + motor + propeller groups;
- a model for the sensors, i.e., the devices through which we can sense the vehicle's motion.

In this presentation we will focus on the first bullet.

## Rigid body modeling: attitude dynamics

The model describing the attitude dynamics is given by Euler's equation

$$J \dot{\omega} + \omega \times J\omega = \tau_c + \tau_e.$$

- $J = J^T \in R^{3 \times 3}$  is the inertia matrix referred to the Center of Mass (CoM) of the quadrotor
- $\omega = [p \quad q \quad r]^T \in R^3$  is the vector of the body components of the angular velocity;
- $\tau_c = [L_c \quad M_c \quad N_c]^T \in R^3$  is the control torque acting at the CoM of the quadrotor delivered by the propellers.
- $\tau_e = [L_e \quad M_e \quad N_e]^T \in R^3$  is an exogenous torque acting at the CoM of the quadrotor collecting the effect of exogenous forces and torques (gravity, aerodynamics,...)

# Modeling the dynamics of quadrotors

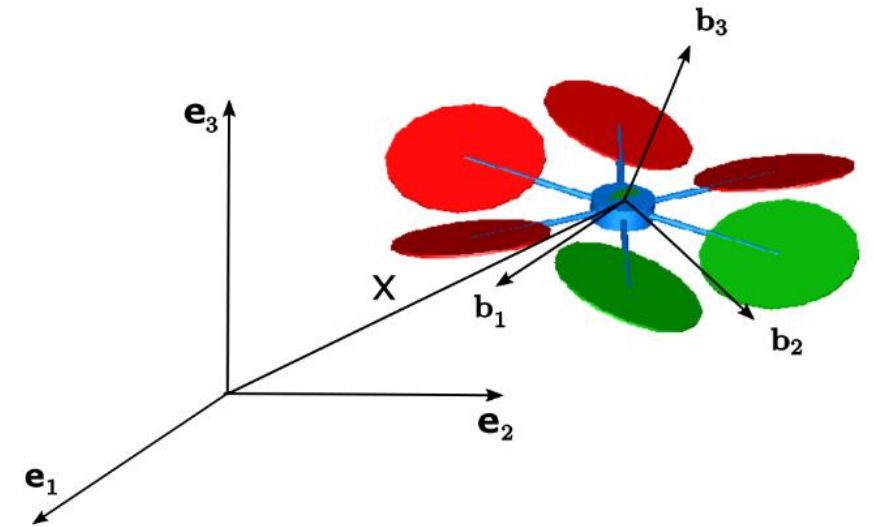
The attitude motion of the quadrotor is fully characterized by **Euler's equation** and the **kinematic equation**

$$\dot{R} = RS(\omega)$$

where  $R = [b_1 \ b_2 \ b_3]$  is the **rotation matrix** describing the orientation of the body fixed frame w.r.t. to the inertial frame.

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

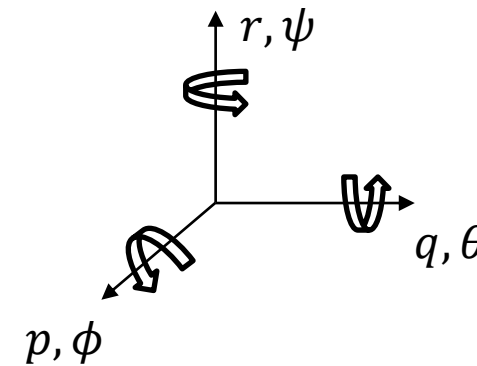
is a skew symmetric matrix such that  $S(\omega)y = \omega \times y \ \forall \omega, y \in R^3$ .



# Modeling the dynamics of quadrotors

By parametrizing the rotation matrix with **roll-pitch-yaw** angles  $(\psi, \theta, \phi)$ , the previous equation becomes

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix},$$



The kinematics in terms of Euler angles suffers from singularity issues.

- other representations are often employed for (nonlinear) control purposes (e.g., quaternions, rotation matrix).



# Modeling the dynamics of quadrotors

The model describing the position kinematics and dynamics of a quadrotor is described by:

$$\begin{aligned}\dot{p} &= v \\ m\dot{v} &= mge_3 - T_c b_3(\phi, \theta, \psi) + f_e\end{aligned}$$

- $p = [x \ y \ z]^\top$  is the position of the quadrotor CoM with respect to the inertial frame origin (**North-East-Down** convection);
- $v = [v_x \ v_y \ v_z]^\top$  is the velocity of the quadrotor centre of mass (resolved in the inertial frame);
- $mge_3 = mg[0 \ 0 \ 1]^\top$  is the force associated with gravity (resolved in the inertial frame);
- $f_e = [f_{e_x} \ f_{e_y} \ f_{e_z}]^\top$  is a disturbance force (aerodynamic drag);
- $T_c$  is the **total thrust** delivered by the propellers, which is directed along the negative direction of the vertical axis of the quadrotor frame, represented in the inertial frame by the unit vector  $b_3$ .

# Modeling the dynamics of quadrotors

The most common operating scenario of quadrotors corresponds to the so-called «hovering» condition, in which the quadrotor is moving at low speed with small deviations with respect to a fixed position  $\bar{p}$ :

$$p = \bar{p} + \Delta p$$

$$v = \Delta v$$

$$\omega = \Delta \omega$$

$$(\phi, \theta, \psi) = (\Delta \phi, \Delta \theta, \Delta \psi)$$

where the  $\Delta$  terms represent small deviations.

The thrust in hovering conditions is the sum of the weight of the drone plus a small deviation  $\Delta T_c$ :

$$T_c = mg + \Delta T_c$$

# Modeling the dynamics of quadrotors

Linearizing the equations of motion presented in the previous slides with respect to such an equilibrium point, one obtains:

$$\begin{aligned}\Delta\dot{\phi} &= \Delta p \\ \Delta\dot{\theta} &= \Delta q \\ \Delta\dot{\psi} &= \Delta r \\ J_{\phi}\Delta\dot{p} &= L_c + L_e \\ J_{\theta}\Delta\dot{q} &= M_c + M_e \\ J_{\psi}\Delta\dot{r} &= N_c + N_e\end{aligned}$$

(attitude)

$$\begin{aligned}\Delta\dot{x} &= \Delta v_x \\ \Delta\dot{y} &= \Delta v_y \\ \Delta\dot{z} &= \Delta v_z \\ m\Delta\dot{v}_x &= -mg\Delta\theta + f_{xe} \\ m\Delta\dot{v}_y &= mg\Delta\phi + f_{ye} \\ m\Delta\dot{v}_z &= -\Delta T_c + f_{ze}\end{aligned}$$

(translation)

# Modeling the dynamics of quadrotors

- The linearized attitude dynamics is described by three sets of independent equations, one for each axis, e.g., for the pitch axis:

$$\begin{aligned}\dot{\Delta\theta} &= \Delta q \\ J_{\theta}\dot{\Delta q} &= M_c + M_e\end{aligned}$$

- The in-plane linearized translational dynamics is influenced by the attitude dynamics. For instance, the translation along the x-axis is affected by the pitch rotation:

$$\begin{aligned}\dot{\Delta x} &= \Delta v_x \\ m\dot{\Delta v_x} &= -mg\Delta\theta + f_{e_x}\end{aligned}$$

- By properly controlling the **pitch** and **roll** dynamics one can indirectly control the **position** dynamics.

## Thrust-torque generation mechanism

- The control force  $T_c$  and the three torque components  $L_c, M_c, N_c$  are the resultant of the forces and torques delivered by the propellers.
- While  $T_c, L_c, N_c, M_c$  are typically considered as variables for control design, they are obtained through a mixing of the thrusts  $T_i$  ( $i = 1, 2, 3, 4$ ) of the propellers (cross configuration):

$$\begin{bmatrix} T_c \\ L_c \\ M_c \\ N_c \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & -1 & -1 & -1 \\ b & b & b & b \\ -\frac{\sqrt{2}}{2} & +\frac{\sqrt{2}}{2} & +\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ +\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & +\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ +\sigma & +\sigma & -\sigma & -\sigma \end{bmatrix}}_{\text{input matrix } W} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

where  $\sigma$  is an aerodynamic coefficient relating torque and thrust and  $b$  is the arm length.



# Modeling the dynamics of quadrotors

The thrust-mixing model presented in the previous slide is based on a simplified (quadratic) model of the rotor aerodynamics.

**Thrust:**

$$T_i = c_T \Omega_i^2$$

**Torque:**

$$\tau_{p_i} = -\sigma \xi_i T_i$$

According to the simplified model, no force is generated in the plane of the rotors.

➤ This is true only for hovering conditions.

**Secondary aerodynamics** effects are typically neglected for control design purposes (dominated with high gain control) but they might become important for **high-speed flight**.

# Modeling the dynamics of quadrotors


Drag is that frictional force that opposes the relative motion of an object in air.

At typical speeds for quadrotors

- parasitic drag associated with the airframe is almost negligible;
- the drag force models to be considered are primarily based on rotor aerodynamic effects.

Several sources of drag in rotors:

- Blade flapping
- Induced drag
- Translational drag
- Profile drag

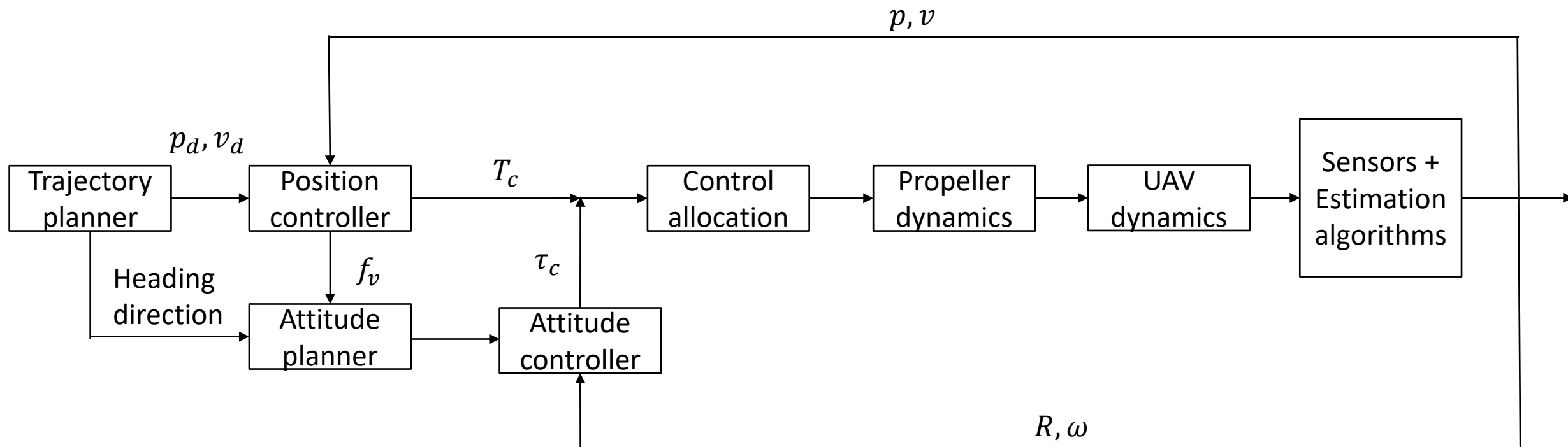


Typically neglected or modeled using a lumped approach for control design.

# (A quick introduction to) hierarchical control for quadrotors

# Hierarchical control design for quadrotors

Almost all control systems for small scale aerial robotics use a **hierarchical control architecture** for **position-heading control** according to the following scheme:



## Position control

For position control the quadrotor dynamics is rewritten as follows:

$$m\dot{v} = mge_3 - T_c b_3 + f_e = mge_3 + f_d + (f_a - f_d) + f_e$$

where  $f_a := T_c b_3$  is the actual delivered force while  $f_d$  is a “virtual” force input to be designed to achieve position control.

The term  $\Delta f := (f_a - f_d)$  represents the mismatch between the delivered and desired force

This mismatch can be regulated to zero by properly modulating the thrust  $T_c$  and by controlling the thrust-axis direction (**hierarchical design idea**).

$$r_1, \quad \rho, \quad \square, \quad n, \quad \square, \quad f,$$


## Attitude planner

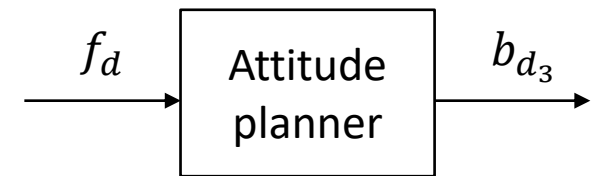
The objective is to specify a reference attitude trajectory to make the term  $\Delta f = (f_d + T_c b_3)$  converge to zero, namely, to have

$$T_c b_3 \rightarrow -f_d$$

The above objective can be achieved by selecting

$$T_c = |f_d|$$
$$b_{d_3} = -\frac{f_d}{|f_d|}$$

where  $b_{d_3}$  defines the desired direction of the third body-axis (the thrust-axis) to be tracked by the attitude controller.





# Hierarchical control design for quadrotors

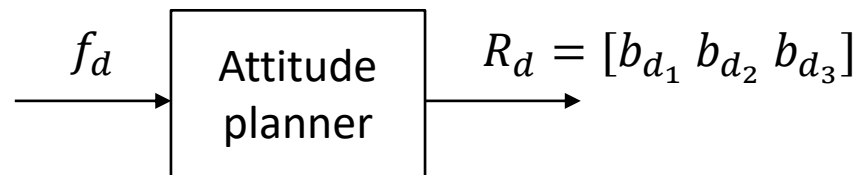
To fully specify the desired attitude, the desired heading direction

$$h_d = [\cos(\psi_d) \ \sin(\psi_d) \ 0]^\top$$

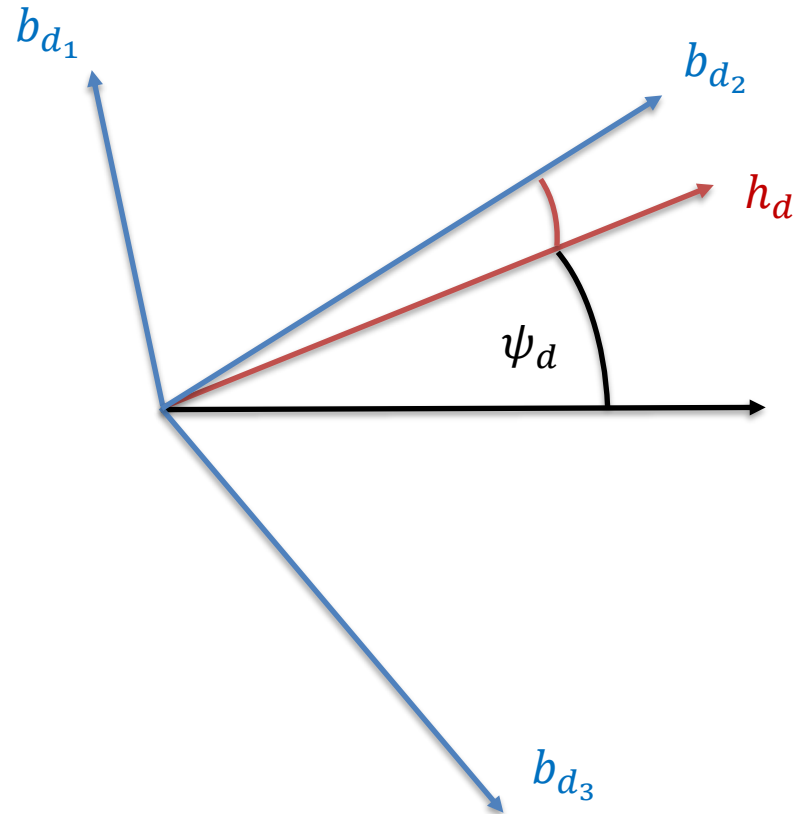
is merged together with the desired thrust-axis direction  $b_{d_3}$  to obtain

$$b_{d_1} = b_{d_2} \times b_{d_3}, \quad b_{d_2} = \frac{b_{d_3} \times h_d}{|b_{d_3} \times h_d|}$$

The output of the attitude planner is the corresponding rotation matrix (or quaternion)



## Construction of the reference attitude



$$b_{d_1} = b_{d_2} \times b_{d_3}$$
$$b_{d_2} = \frac{b_{d_3} \times h_d}{|b_{d_3} \times h_d|}$$

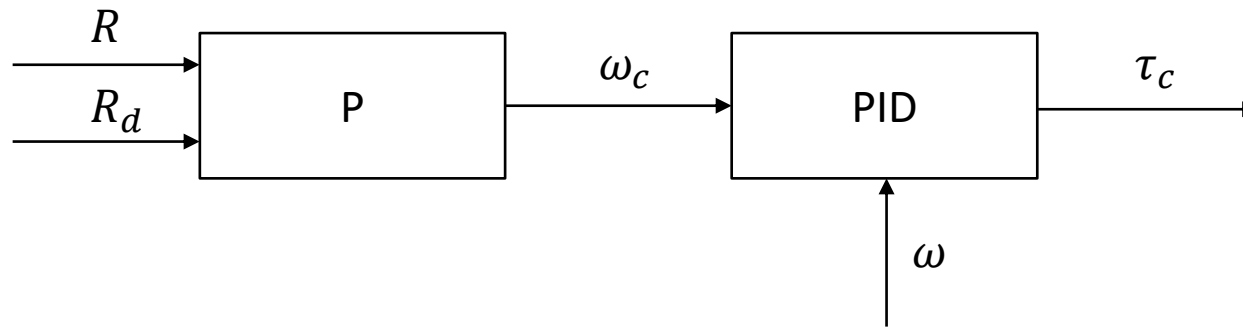
Thrust-axis direction

# Hierarchical control design for quadrotors

## Attitude control

The attitude subsystem is fully-actuated by the torque  $\tau_c$ , which can be used to track (ideally) any desired attitude trajectory.

Popular schemes used in autopilots are based on (nonlinear) PID loops (P/PID cascade)



Attitude error computation

- $R_e = R_d^T R$
- $q_e = q_d^T \otimes q^*$  ( $\otimes$  quaternion product,  $*$  conjugate quaternion)

## Control allocation

Since the input map  $W$  is invertible, the relationship between the rotor thrusts ( $T_i$ ) and the control variables ( $T_c, L_c, N_c, M_c$ ) can be inverted:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = W^{-1} \begin{bmatrix} T_c \\ L_c \\ M_c \\ N_c \end{bmatrix}.$$

The obtained rotor thrust commands are then converted into percentage throttles commands and sent to the ESC.