Adaptive and Autonomous Aerospace Systems

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Part 1: Analysis of nonlinear and time-varying systems

Lecture 1: Introductory examples in Adaptive Control

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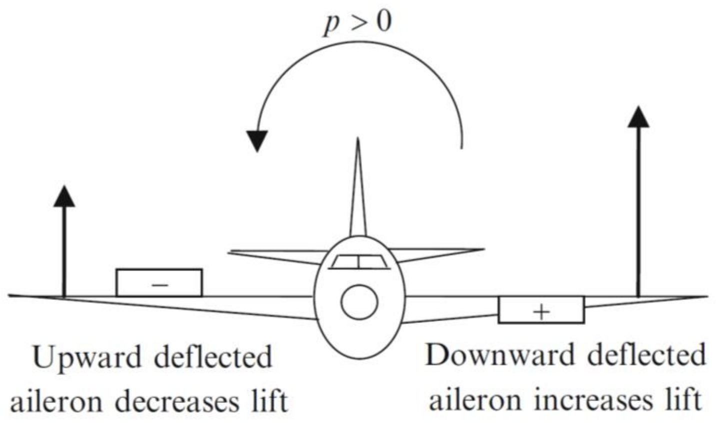
Outline

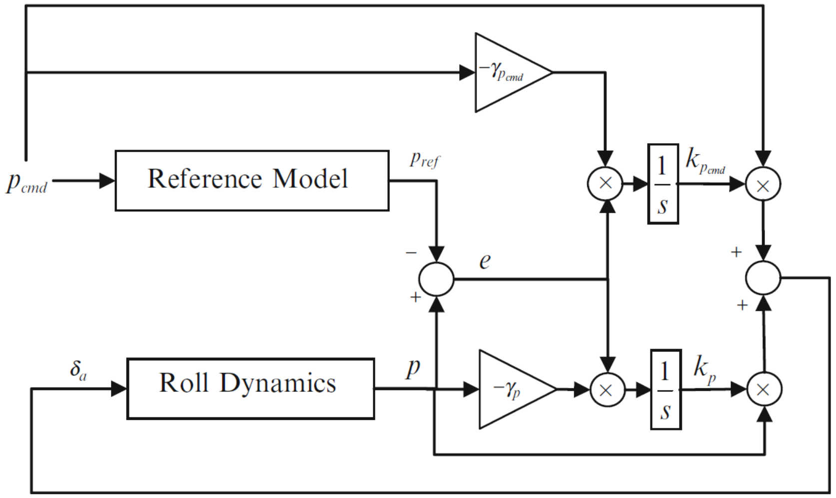
- Recap on example “Model Reference (Adaptive) Control of aircraft roll rate dynamics”.
- Second example: estimation of an uncertain parameter in a scalar algebraic system.
- Plan for Friday lecture

Recap: MRAC for aircraft roll rate dynamics

Direct MRAC law

$$\begin{aligned}
\dot{p}_{ref} &= a_{ref} p_{ref} + b_{ref} p_{cmd} \\
\dot{\hat{k}}_p &= -\gamma_p \operatorname{sign}(L_{\delta_a}) p (p - p_{ref}) \\
\dot{\hat{k}}_{p_{cmd}} &= -\gamma_{p_{cmd}} \operatorname{sign}(L_{\delta_a}) p_{cmd} (p - p_{ref}) \\
\delta_a &= \hat{k}_p p + \hat{k}_{p_{cmd}} p_{cmd}
\end{aligned}$$





Lesson learned

- The *direct* Model Reference Adaptive Controller operates using only available (online measured). signals in the system.
  The adaptive control law makes use of:
  (a) the system state p
  (b) the state of the reference model p_{ref},
  (c) The commanded signal p_{cmd}
  (d) the tracking error $e = p - p_{ref}$
  (e) the sign of the control effectiveness $sign(L_{\delta_a})$.

- All signals in the closed-loop system remain uniformly bounded in time.

Introductory examples in Adaptive Control

- The system state p tracks the state of the reference model p_{ref} globally and asymptotically. However, a characterization of the system transient dynamics in model reference adaptive control remains an open problem.

- The adaptive parameters are <u>not</u> guaranteed to converge to their true (unknown) values nor are they assured to converge to constant values in any way.
    - All that is known is that these parameters remain uniformly bounded in time.

Error variables

$$e = e - p_{ref}$$ Reference model mismatch

$$\Delta k_p = \hat{k}_p - \bar{k}_p$$
$$\Delta k_{p_{cmd}} = \hat{k}_{p_{cmd}} - \bar{k}_{p_{cmd}}$$ Gain estimation errors

Error dynamics for MRAC: nonlinear, time-varying, perturbed system Ordinary Differential Equation (ODE):

$$\dot{e} = (a_{ref} + L_{\delta_a}\Delta k_p)e + L_{\delta_a}(\Delta k_p p_{ref} + \Delta k_{p_{cmd}} p_{cmd})$$
$$\dot{\Delta k_p} = -\gamma_p sign(L_{\delta_a})(e + p_{ref})e$$
$$\dot{\Delta k_{p_{cmd}}} = -\gamma_{p_{cmd}} sign(L_{\delta_a}) p_{cmd} e$$
$$\dot{p}_{ref} = a_{ref} p_{ref} + b_{ref} p_{cmd}$$

Estimation of an uncertain parameter in a scalar algebraic system

$$y(t) = \bar{\theta} u(t)$$

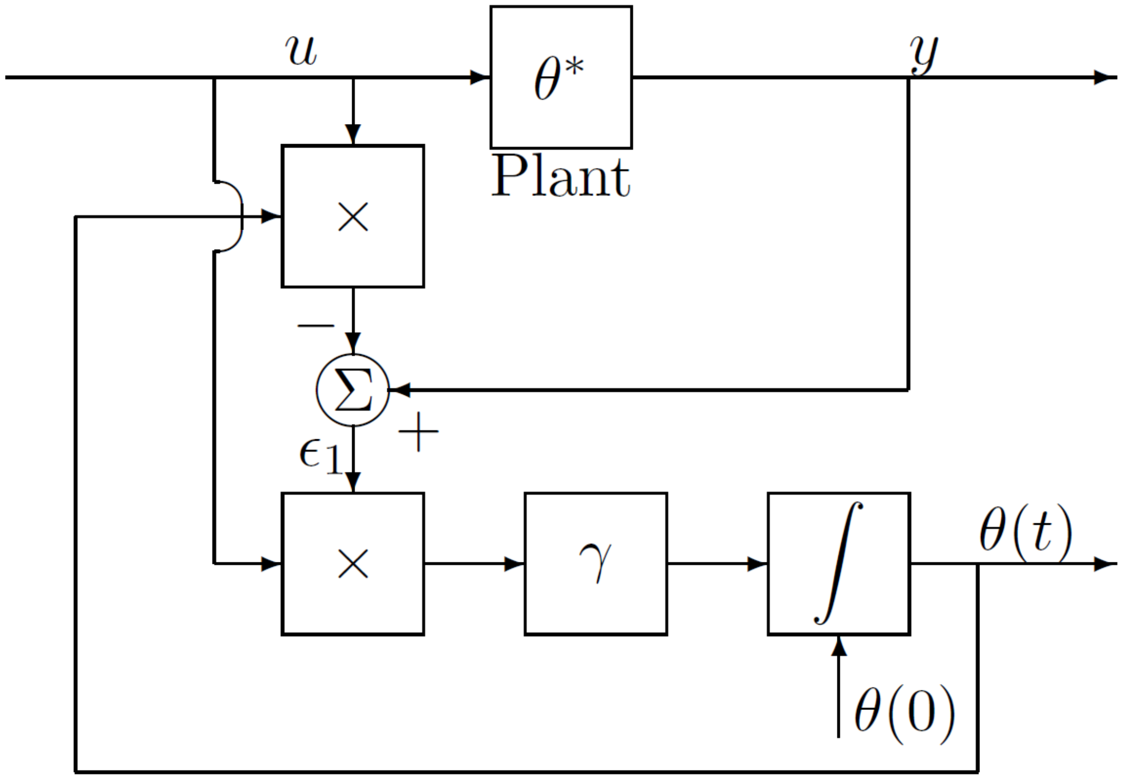
- $y \in R$ (measured)
- $u \in R, u(t) \in L_\infty$ (measured)
- $\bar{\theta} \in R$ unknown constant

Problem: to develop an online estimation algorithm for $\bar{\theta}$ given measurement of u and y at each time instant.

Estimation algorithm

$$\dot{\hat{\theta}}(t) = \gamma u(t)(y(t) - \hat{\theta}(t)u(t))$$

$\gamma \in R_{>0}$ ≔ adaptation gain



Error equation

$$\dot{e}_\theta = -\gamma u(t)^2 e_\theta$$

Solution

$$e_\theta(t) = e^{-\gamma \int_{t_0}^{t} u(\tau)^2 d\tau} e_\theta(t_0)$$

Lesson learned

- Under some regularity assumptions on the input signal, the adaptive law guarantees that the estimation error e_y converges to zero (globally).
- The parameter estimation error e_θ converges to a constant (that might be different from zero).
- When the input $u(t)$ is PE, the parameter estimation error e_θ converges to zero.
- Under PE conditions, the convergence to zero of e_θ is not necessarily exponential.

What happens with disturbances? $y(t) = \bar{\theta}u(t) + d(t)$

Plan for Friday lecture

- Familiarization with Simulink (bring your own laptop)

- Coding session
  - Implementation of the adaptive estimator for the algebraic system example.
  - Implementation of MRAC laws for the aircraft roll rate example.