Adaptive and Autonomous Aerospace Systems

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Part 1: Analysis of nonlinear and time-varying systems

Lecture 1: Introductory examples in Adaptive Control

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Outline

- Some notation

- Examples
  - Model Reference (Adaptive) Control of aircraft roll rate dynamics
  - Estimation of an uncertain parameter in a scalar algebraic system

Notation

- The real coordinate space of dimension *n*, denoted as $\mathbb{R}^n$, is the set consisting of all ordered n-tuples of real numbers of the form

$$(x_1, x_2, \ldots, x_n)$$

- With component-wise addition and scalar multiplication, it is a real vector space.
- The elements of the standard basis of $\mathbb{R}^n$ are denoted as

$$e_i = (0, \ldots, 1, \ldots, 0), \qquad i \in \{1, 2, \ldots, n\}$$

- In standard matrix notation, each element of $\mathbb{R}^n$ is written as a column vector

$$x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^\top$$

- $\mathbb{R}^{m \times m}$ denotes the set of all $n \times m$ matrices with real elements.

N.B: lower case letters are typically used for vectors, capital case letters for matrices.

Norms on finite-dimensional are denoted by a single bar $|\cdot|:\mathcal{X}\to\mathbb{R}_{\geq 0}$

- Norms on finite-dimensional vector spaces are all equivalent, i.e.,

$$c_1|x|_a \leq |x|_b \leq c_2|x|_a, \quad \forall x \in \mathcal{X}$$

Euclidean norm and induced 2-norm:

$$|x|_2 := \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x^\top x} \qquad |A|_2 := \sup_{x\in\mathbb{R}^n\setminus\{0\}} \frac{|Ax|_2}{|x|_2} = \lambda_{max}(A^\top A) = \bar{\sigma}(A)$$

Point-set distance:

$$|x|_{\mathcal{A}} := \inf_{p\in\mathcal{A}} |x-p|$$

Functional spaces

The set of continuously differentiable functions of order *n* is denoted as $\mathcal{C}^n(\mathcal{D})$

Norms on functional spaces

- Infinity norm (bounded signals)

$$\|u(t)\|_\infty = \sup_{t\in\mathbb{R}_{\geq 0}} |u(t)|$$

- *p*-norm (“energy” bounded signals)

$$\|u(t)\|_p = \left(\int_0^\infty |u(\tau)|^p d\tau\right)^{\frac{1}{p}}$$

- *L*-spaces

$$\mathcal{L}_p := \{u : \mathcal{D}_t \mapsto \mathcal{D}_u \text{ piecewise continuous} : \|u(t)\|_p < \infty\}$$

Inner product spaces

The inner product is denoted with $\langle \cdot, \cdot \rangle : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_{\geq 0}$

Given an inner product space $(\mathcal{X}, \langle \cdot, \cdot \rangle)$, the function $|\cdot| = \sqrt{\langle \cdot, \cdot \rangle}$ defines a norm.

Schwartz inequality: let x, y belong to $(\mathcal{X}, \langle \cdot, \cdot \rangle)$. Then, $|\langle x, y \rangle| \leq |x||y|$ and $|\langle x, y \rangle| = |x||y|$ if and only if x, y are linearly dependent.

For $\mathcal{X} = \mathbb{R}^n$, the application $\langle x, y \rangle = x^\top y$ defines the standard inner product in $\mathbb{R}^n$.

Model Reference Control (MRC): motivational example

In the design of flight control systems, it is essential to provide:

- closed-loop stability;
- good reference tracking performance
- robustness to model uncertainties, control failures and environmental disturbances.

In the framework of LTI systems, the main control design approaches are:

- Loop shaping (SISO systems)
- State (output) feedback (eigenvalue assignment/Linear Quadratic Regulator (LQR)) (+integral action)
- H_∞ synthesis (structured/unstructured)

It can be shown that

- With a proper selection of the weight matrices (R, Q), the closed loop system has gain margin $[-6dB, +\infty]$ and at least 60deg phase margin at each plant input control channel.

- LQR control is robust to "matched uncertainties", i.e., uncertainties that are present in the system control channels.

$$\dot{x} = Ax + B(u + f(x,t))$$

N.B.: LQG control, i.e., combined optimal state estimation (Kalman filter) and optimal state feedback (LQR), does not possess <u>any</u> guaranteed stability margins (*Doyle, "Guaranteed Margins for LQG regulators", 1978*).

In the presence of matched uncertainties, a deterioration of the system baseline closed-loop performance is inevitable.

LQR controllers are designed to be robust to the entire class of matched uncertainties, they are not tuned to handle any specific uncertainty from the class.

H_∞ controllers can be robustly tuned to guarantee closed-loop stability and a specified minimum level of performance for a given class of uncertainties.

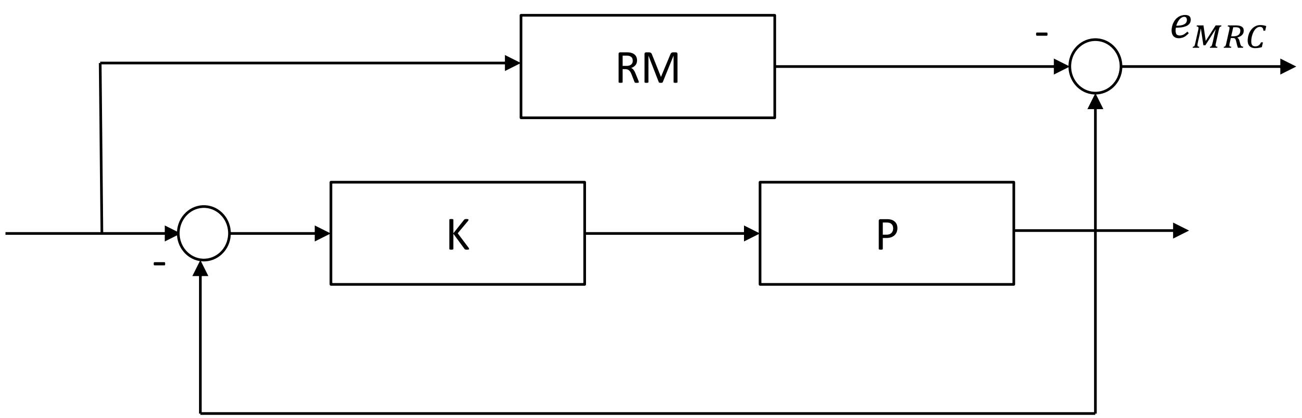
These approaches, however, might be too conservative when uncertainties are large.

For a recap about LQR control, see Lavretsky's book (ch. 2, sec. 2.3) or Astrom's book (ch. 7, sec. 7.4)

Can we restore a given baseline level of performance while operating under matched uncertainties?

A useful control design approach is based on the concept of Model Reference (MR).

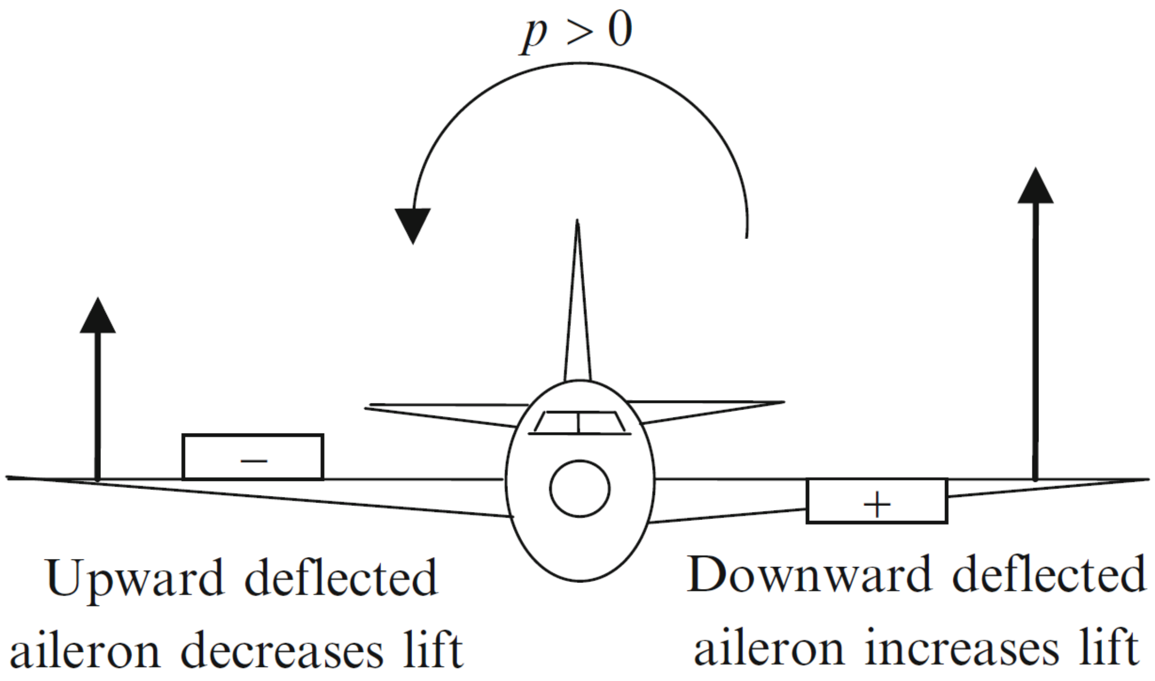
Both fixed-gain and adaptive controllers can leverage the MR concept.



The reference model embeds the desired closed-loop command tracking performance.

- The goal of MRC is to make the model error mismatch e_{MRC} as small as possible.

Fixed-Gain Model Reference Control of aircraft roll rate dynamics



The roll dynamics of a conventional aircraft is controlled using *differential* motion of ailerons and spoilers.

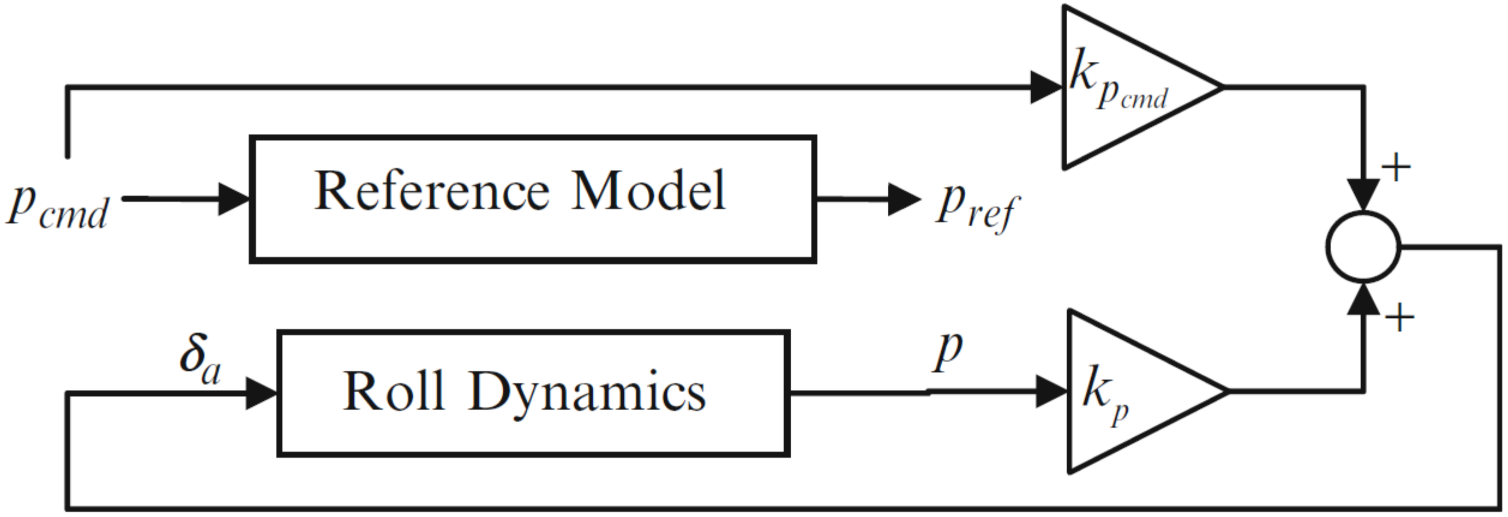
The approximated dynamics for control design is

$$\dot{p} = L_p p + L_{\delta_a} \delta_a + d_p(t, \cdot)$$

L_p := roll damping derivative
L_{δ_a} := dimensional rolling moment derivative

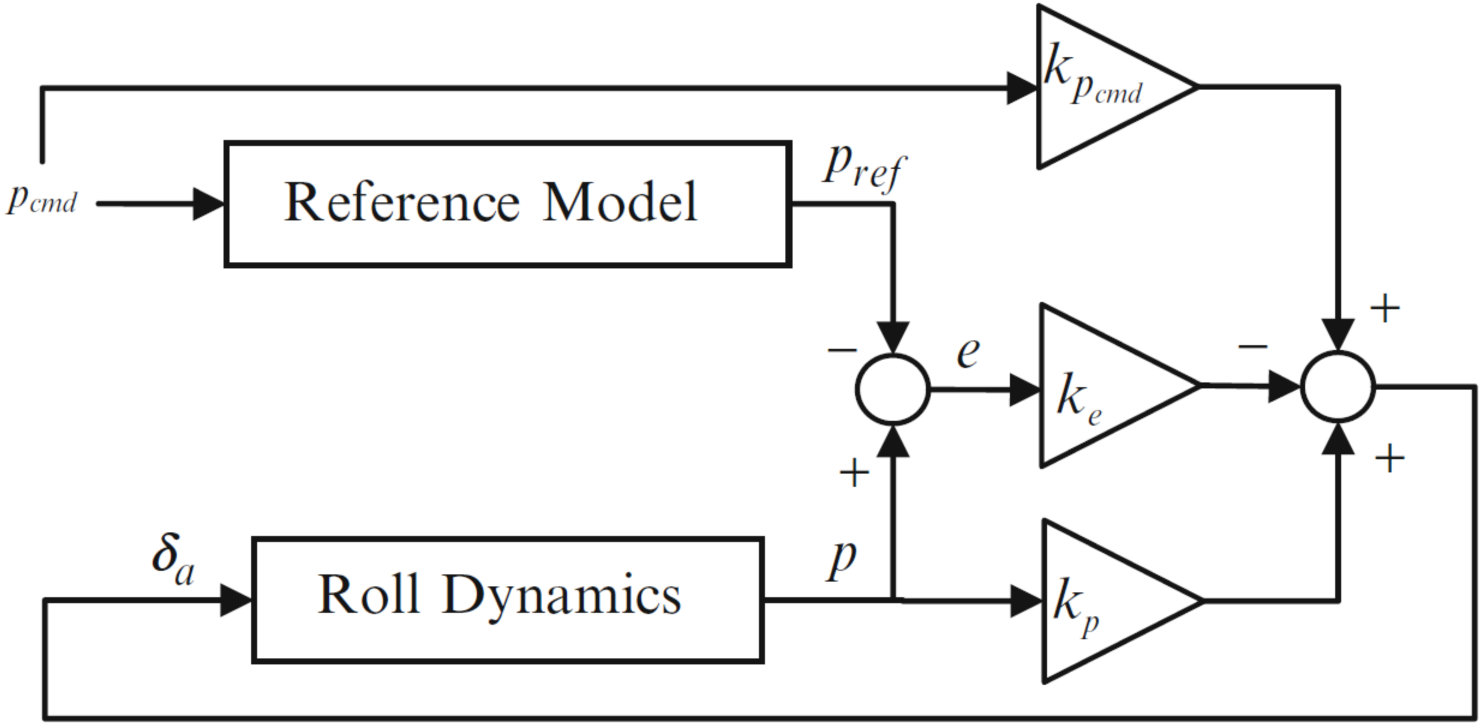
Assumptions: "small" p and δ_a; aircraft yawing motion suppressed by the rudder.

MRC for aircraft roll rate dynamics



$$\delta_a = k_p p + k_{p_{cmd}} p_{cmd}$$

MRC with error feedback for aircraft roll rate dynamics



$$\dot{p}_{ref} = a_{ref} p_{ref} + b_{ref} p_{cmd}$$

$$\delta_a = k_p p + k_{p_{cmd}} p_{cmd} - k_e (p - p_{ref})$$

MRAC for aircraft roll rate dynamics

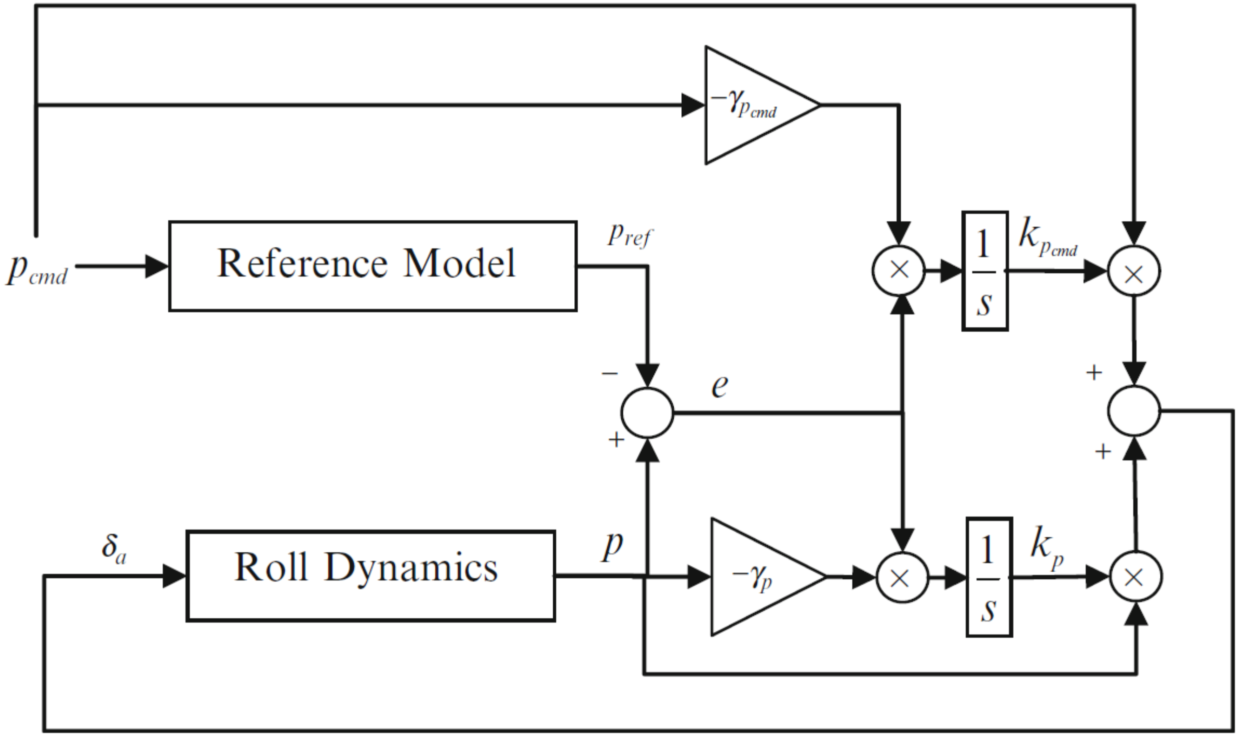
Direct MRAC law

$$\dot{p}_{ref} = a_{ref} p_{ref} + b_{ref} p_{cmd}$$

$$\dot{\hat{k}}_p = -\gamma_p \mathrm{sign}(L_{\delta_a}) p (p - p_{ref})$$

$$\dot{\hat{k}}_{p_{cmd}} = -\gamma_{p_{cmd}} \mathrm{sign}(L_{\delta_a}) p_{cmd} (p - p_{ref})$$

$$\delta_a = k_p p + k_{p_{cmd}} p_{cmd} - k_e (p - p_{cmd})$$



A few remarks:

- The *direct* Model Reference Adaptive Controller operates using only available (online measured). signals in the system.
  The adaptive control law makes use of:
  (a) the system state p
  (b) the state of the reference model p_{ref},
  (c) The commanded signal p_{cmd}
  (d) the tracking error $e = p - p_{ref}$
  (e) the sign of the control effectiveness $sign(L_{\delta_a})$.

- All signals in the closed-loop system remain uniformly bounded in time.

- The system state p tracks the state of the reference model p_{ref} globally and asymptotically.
  However, a characterization of the system transient dynamics in model reference adaptive control remains an open problem.

Introductory examples in Adaptive Control

- The adaptive parameters are <u>not</u> guaranteed to converge to their true (unknown) values nor are they assured to converge to constant values in any way.
  - All that is known is that these parameters remain uniformly bounded in time.

- Sufficient conditions for parameter convergence are known as Persistency of Excitation.

- It turns out that for a first-order linear system, persistent excitation is guaranteed if the commanded signal contains at least one sinusoidal component.
  - In this case, the two adaptive gains will converge to their true constant unknown values, exponentially fast.

Estimation of an uncertain parameter in a scalar algebraic system

$$y(t) = \bar{\theta}u(t)$$

- $y \in R$ (measured)
- $u \in R, u(t) \in L_\infty$ (measured)
- $\bar{\theta} \in R$ unknown constant

Problem: to develop an online estimation algorithm for $\bar{\theta}$ given measurement of u and y at each time instant.

Introductory examples in Adaptive Control

Estimation algorithm

$$\dot{\hat{\theta}}(t) = \gamma u(t)(y(t) - \hat{\theta}(t)u(t))$$

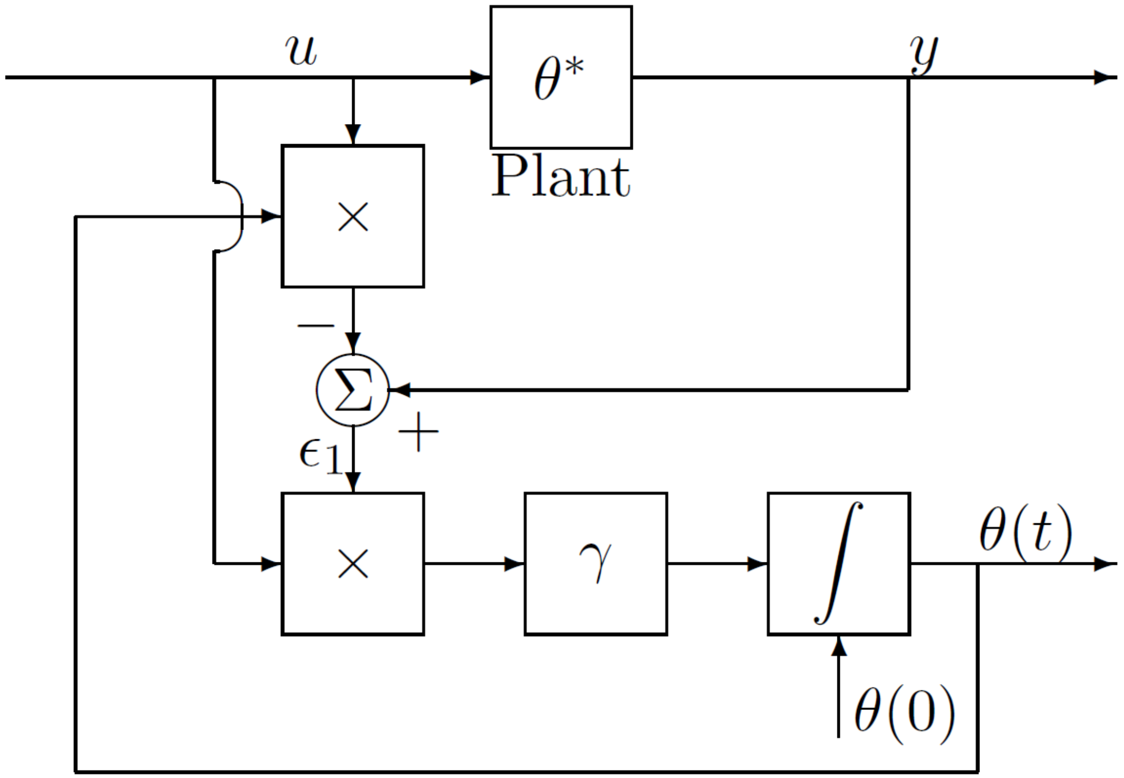
$\gamma \in R_{>0}$ ≔ adaptation gain

Error equation

$$\dot{e}_\theta = -\gamma u(t)^2 e_\theta$$

Solution

$$e_\theta(t) = e^{-\gamma \int_{t_0}^{t} u(\tau)^2 d\tau} e_\theta(t_0)$$



Lesson learned

- Under some regularity assumptions on the input signal, the adaptive law guarantees that the estimation error e_y converges to zero (globally).
- The parameter estimation error e_θ converges to a constant (that might be different from zero).
- When the input $u(t)$ is PE, the parameter estimation error e_θ converges to zero.
- Under PE conditions, the convergence to zero of e_θ is not necessarily exponential.

What happens with disturbances? $y(t) = \bar{\theta}u(t) + d(t)$

Homework

- Familiarize yourself with the Matlab/Simulink code for the roll rate dynamics example (uploaded in webeep).
- Extend the code to implement the MRAC laws that have been developed in this lecture.
- Compare results between MRC and MRAC.
- Develop a Matlab/Simulink code to simulate the adaptive estimator of the algebraic system.