Adaptive and Autonomous Aerospace Systems

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Part 2: Adaptive Control

Lect 5: a quick introduction to L1 adaptive control

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Outline

- Introduction and background on L1 adaptive control
- From predictor-based MRAC to L1 control
- L1 adaptive control of a SISO uncertain system

Introduction and background on L1 adaptive control

Introduction and background on L1 adaptive control

L1 adaptive control theory is motivated by the emerging need to certify advanced adaptive flight critical systems with a more affordable Verification&Validation process.

This objective requires the development of a control architecture with a priori quantifiable transient and steady-state performance specifications and robustness margins.

The key feature of L1 adaptive control architectures is the guaranteed robustness and transient performance in the presence of fast adaptation.

This presentation is a very short introduction to L1 adaptive control: for further details see the paper:

N. Hovakimyan, C. Cao, E. Kharisov, E. Xargay and I. M. Gregory, "L1 adaptive control for safety-critical systems," in IEEE Control Systems Magazine, vol. 31, no. 5, pp. 54-104, Oct. 2011, doi: 10.1109/MCS.2011.941961.

Fast adaptation

The adaptation rate in L1 architectures can be selected so that the time scale of the adaptation process is faster than the time scales associated with plant parameter variations and the underlying closed-loop dynamics.

Robust adaptation

Despite fast adaptation in L1 architectures, the robustness properties of the closed-loop adaptive system can be adjusted independently of the adaptation rate.

This is possible to achieve by appropriate formulation of the control objective

- uncertainties in any feedback loop can be compensated for only within the available bandwidth of the control channel.

L1 adaptive control was suggested as an advancement to MRAC.

Ideally, adaptive controllers are expected to correctly respond to all changes in initial conditions, reference inputs, and uncertainties by quickly identifying a set of control parameters that provide a satisfactory system response.

In standard MRAC schemes fast adaptation leads to high frequency oscillations in the control signals and increased sensitivity to time delays (poor robustness characteristics).

Fundamental question: what is the upper bound on the adaptation rate that does not result in poor robustness characteristics?

Introduction and background on L1 adaptive control

L1 adaptive control theory addresses this question by setting in place an architecture for which the estimation loop is decoupled from the control loop

This decoupling allows for

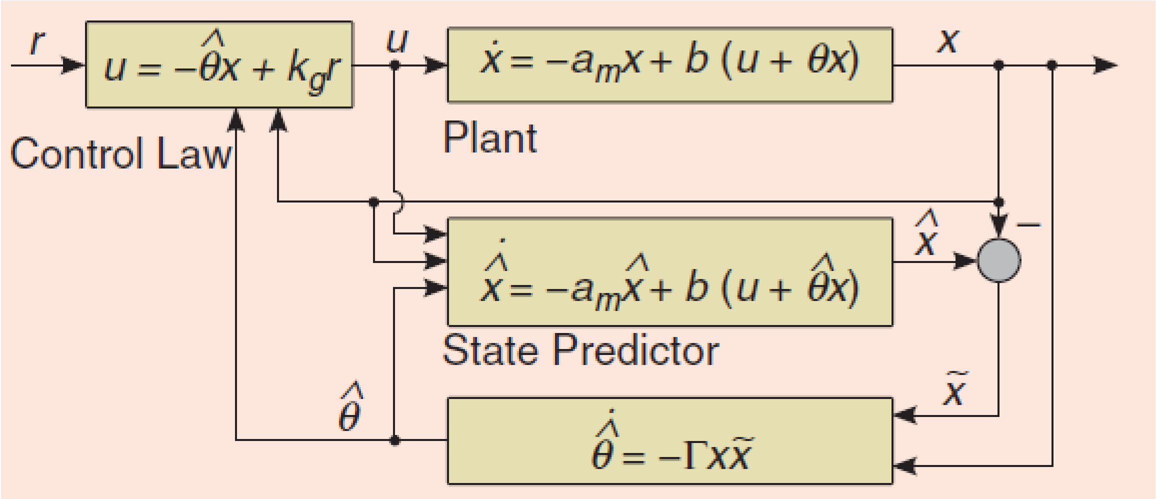
- an arbitrary increase of the estimation rate (limited by the available hardware)
- guaranteed robustness and transient performance (limited by the available control channel bandwidth)

With an L1 adaptive controller in the feedback loop, the response of the closed-loop system can be predicted a priori, thus reducing the amount of Monte Carlo analysis required to verify and validate these systems

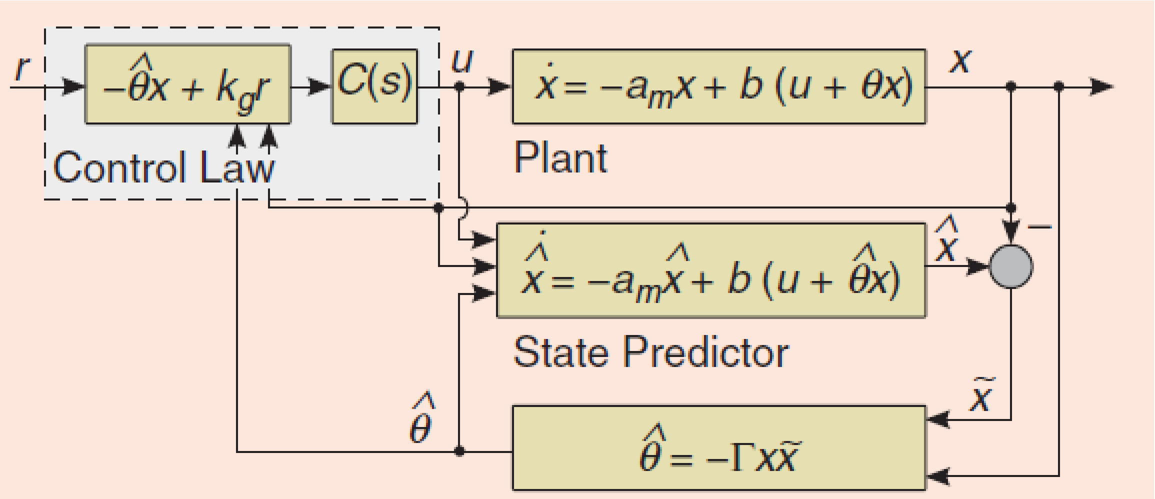
From predictor-based MRAC to L1 adaptive control

From predictor-based MRAC to L1 control

Architectural differences



Predictor MRAC



L1 adaptive control

From predictor-based MRAC to L1 control

Scalar system with known input gain

$$\dot{x} = a_{ref} x + bu + \theta^\top x$$

MRAC with state-predictor

$$\dot{\hat{x}} = a_{ref} \hat{x} + bu + \hat{\theta}^\top x$$

$$\dot{\hat{\theta}} = \gamma_\theta x \hat{e} \qquad \hat{e} = \hat{x} - x$$

$$u = -\hat{\theta} x + k_r r$$

N.B. Changed convection w.r.t. to slides on MRAC

Closed-loop prediction error dynamics

$$\dot{\hat{e}} = a_{ref} \hat{e} + b \Delta\theta x \qquad \Delta\theta = \hat{\theta} - \theta$$

From predictor-based MRAC to L1 control

Leveraging Lyapunov analysis, we can show that

$$V(e, \Delta\theta) = \frac{1}{2}\hat{e}^2 + \frac{1}{2\gamma_\theta}\Delta\theta^2$$

$$\dot{V}(t) = a_{ref}\hat{e}(t)^2, \quad a_{ref} < 0$$

Since the predictor with the specified control input is a stable system

$$\dot{\hat{x}} = a_{ref}\hat{x} + bu + \hat{\theta}^\top x = a_{ref}\hat{x} + bk_r r$$

then all the signals are uniformly bounded and $\hat{e}(t) \to 0$

From predictor-based MRAC to L1 control

Moreover, $\dot{V} \leq 0$ means that

$$\frac{1}{2}\hat{e}(t)^2 \leq V(t) \leq V(0) = \frac{1}{2\gamma_\theta}\Delta\theta^2(0)$$

$(\hat{e}(0) = 0)$ which in turn allows us to write the uniform bound

$$|\hat{e}(t)| \leq \frac{|\Delta\theta(0)|}{\sqrt{\gamma_\theta}} \quad \forall t \geq 0$$

Assuming $|\theta| \leq \theta_{MAX}$, we have $|\Delta\theta| \leq 2\theta_{MAX}$ and the above bound can be written as

$$|\hat{e}(t)| \leq \frac{2\theta_{MAX}}{\sqrt{\gamma_\theta}} \quad \forall t \geq 0$$

From predictor-based MRAC to L1 control

Result

The MRAC design allows to bound the transient performance by increasing the adaptive gain.

However,

- the control input is not protected from the fast oscillations coming from the estimation algorithm when the gain is large (instability, reduced tolerance to time-delays);

- There is a strong interaction between estimation and control loop which makes the tuning difficult (Monte Carlo or trial and error procedures).

L1 adaptive control of a SISO uncertain system

L1 adaptive control of a SISO uncertain system

The L1 adaptive controller is obtained from the predictor-based MRAC using the control input

$$u(s) = C(s)\hat{\eta}(s)$$

where $C(s)$ is a stable, strictly proper low pass filter and $\eta(s)$ is the Laplace transform of the signal

$$\eta(t) = -\hat{\theta}(t)x(t) + k_r r(t) = u_{MRAC}$$

Unlike the predictor-based MRAC, the closed-loop system with the L1 input does not behave as the reference model due to the limited bandwidth of the control channel enforced by the filter.

L1 adaptive control of a SISO uncertain system

To derive the dynamics of the reference model for the L1 controller, we consider

$$\dot{x}_{ref} = a_{ref} x_{ref} + b u_{ref} + \theta^\top x_{ref}$$

where the reference input is defined as

$$u_{ref}(s) = C(s)\hat{\eta}_{ref}(s)) = C(s)(-\theta x_{ref(s)} + k_g r(s))$$

The L1 reference model in the Laplace domain is

$$x_{ref}(s) = H(s)C(s)k_r r(s) + H(s)(1 - C(s))\theta x_{ref}(s) + x_{in}(s)$$

where

$$H(s) := \frac{b}{s - a_{ref}}, \quad x_{in}(s) := \frac{1}{s - a_{ref}} x_0$$

The transfer function $(1 - C(s))$ is a high-pass filter by construction which attenuates the low frequency content of the uncertainty θx_{ref}

$$x_{ref}(s) = H(s)C(s)k_r r(s) + H(s)(1 - C(s))\theta x_{ref}(s) + x_{in}(s)$$

The approach differs from MRAC schemes, where the ideal controller tries to compensate for the uncertainty in all the frequency range.

The L1 adaptive controller pursues a less ambitious, yet practically achievable, objective, namely, compensation of only the low-frequency content of the uncertainty within the bandwidth of the control channel.

As the name suggests, L1 control exploits the L1-norm of a system.

The L1 norm sets the relation between the peak values of the system input and output.

It is also called the peak-to-peak gain of a system.

Let G(s) be a proper and stable system and assume null initial conditions. Then, for the bounded input $u(t)$, its output can be written as

$$y(t) = g(t) * u(t) = \int_0^t g(t-\tau)u(\tau)d\tau$$

where $g(t)$ is the impulse response of $G(s)$

L1 adaptive control of a SISO uncertain system

Letting

$$\|y\|_{\mathcal{L}_\infty} \triangleq \sup_{t \geq 0} |y(t)|$$

we obtain the uniform bound

$$\begin{aligned} |y(t)| &= \left| \int_0^t g(\tau) u(t-\tau) d\tau \right| \leq \int_0^t |g(\tau)| |u(t-\tau)| d\tau \\ &\leq \int_0^\infty |g(\tau)| d\tau \|u\|_{\mathcal{L}_\infty} = \|G(s)\|_{\mathcal{L}_1} \|u\|_{\mathcal{L}_\infty} \end{aligned}$$

from which the L1-norm is defined. Since the bound is uniform, we can take the supremum of the left and side

$$\|y\|_{\mathcal{L}_\infty} \leq \|G(s)\|_{\mathcal{L}_1} \|u\|_{\mathcal{L}_\infty}$$

N.B: for unstable or improper systems, the L1 norm is unbounded.

L1 adaptive control of a SISO uncertain system

The consequence of the low-pass filter in the control channel is that the stability of the L1 reference system is not guaranteed a priori as it is for the ideal system (5).

Taking the L1-norm of the transfer functions we have

$$\begin{aligned}\|x_{ref}\|_{\mathcal{L}_\infty} \leq & \|H(s)C(s)k_r\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty} \\ & + \|H(s)(1 - C(s))\theta\|_{\mathcal{L}_1} \|x_{ref}\|_{\mathcal{L}_\infty} + \|x_{in}\|_{\mathcal{L}_w}\end{aligned}$$

where

$$G(s) \triangleq H(s)(1 - C(s))$$

L1 adaptive control of a SISO uncertain system

The reference model output can be bounded

$$\|x_{\mathrm{ref}}\|_{\mathcal{L}_\infty} \leq \frac{\|H(s)C(s)k_g\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty} + \|x_{\mathrm{in}}\|_{\mathcal{L}_\infty}}{1 - \|G(s)\theta\|_{\mathcal{L}_1}}$$

Provided that

$$\|G(s)\theta\|_{\mathcal{L}_1} < 1$$

The filter $C(s)$ must be designed to ensure that the L1 bound above holds for all θ in the given uncertain set.

L1 adaptive control of a SISO uncertain system

By rewriting the control input as

$$u(s) = C(s)\left(k_r r(s) - \theta x(s) - \widetilde{\eta}(s)\right)$$

where $\tilde{\eta}(s) = \Delta\, \theta x$, the system dynamics in the Laplace domain becomes

$$\begin{aligned} x(s) &= \frac{b}{s - a_{ref}}(u(s) + \theta x(s)) + x_{in}(s) \\ &= H(s)\left(C(s) k_r r(s) + (1 - C(s))\theta x(s) - C(s)\widetilde{\eta}(s)\right) + x_{in}(s) \end{aligned}$$

Subtracting this expression from the reference model dynamics, we obtain:

$$x_{ref}(s) - x(s) = G(s)\theta\left(x_{ref}(s) - x(s)\right) + H(s)C(s)\tilde{\eta}(s)$$

$$x_{ref}(s) - x(s) = \frac{C(s)}{1 - G(s)\theta} H(s)\tilde{\eta}(s)$$

L1 adaptive control of a SISO uncertain system

Since the L1 adaptive controller uses the same state predictor and adaptation law as the MRAC with state predictor, the L1 controller has the same prediction error dynamics

$$\dot{\hat{e}} = a_{ref}\hat{e} + b\Delta\theta x \rightarrow \hat{e}(s) = \frac{b}{s - a_{ref}}\widetilde{\eta}(s)$$

The RM mismatch error becomes

$$e(s) = x_{ref}(s) - x(s) = \frac{C(s)}{1 - G(s)\theta}\hat{e}(s)$$

Hence, we arrive to the bound

$$\|e\|_{\mathcal{L}_\infty} \leq \left\| \frac{C(s)}{1 - G(s)\theta} \right\|_{\mathcal{L}_1} \|\hat{e}\|_{\mathcal{L}_\infty} \leq \left\| \frac{C(s)}{1 - G(s)\theta} \right\|_{\mathcal{L}_1} \frac{2\theta_{\max}}{\sqrt{\Gamma}}$$

L1 adaptive control of a SISO uncertain system

A similar analysis can be carried out by referring to the control input.

$$
\begin{aligned}
u_{\text{ref}}(s) - u(s) &= C(s)\left(-\theta\left(x_{\text{ref}}(s) - x(s)\right) + \widetilde{\eta}(s)\right) \\
&= -C(s)\theta\left(x_{\text{ref}}(s) - x(s)\right) + C(s)\frac{s + a_m}{b}\widetilde{x}(s)
\end{aligned}
$$

The infinity norm of the mismatch between the reference model input and the actual input decreases as the adaptation gain increases

$$
\|u_{ref} - u\|_{\mathcal{L}_\infty} \leq \|C(s)\theta\|_{\mathcal{L}_1} \|x_{ref} - x\|_{\mathcal{L}_\infty} + \left\|C(s)\frac{s + a_m}{b}\right\|_{\mathcal{L}_1} \frac{2\theta_{\max}}{\sqrt{\Gamma}}.
$$

Conclusions

According to the analysis, increasing the adaptation gain leads to improved performance (only limited by hardware capabilities and noise)

The design of the filter is a trade-off between performance and robustness

- Increasing the bandwidth of the filter allows one to cope with a larger set of uncertainty and to achieve higher performance (with $C(s) \to 1$ the MRAC algorithm is recovered).
- However, this could result in faster oscillations in the control channel (instability to unmodeled dynamics) and a reduced time-delay margin.