Lançadores Espaciais
Space Launchers

Outline of Lecture 6:

- Thrust Coefficient
- Under- and Over-expanded Nozzles
- Nozzle contour

Thrust Coefficient

We have already seen that:

$$\mathcal{T} = \dot{m}u_e + (p_e - p_a)A_e$$

Substituting in this expression the values for the mass flow rate and exit velocity, and normalising the equation, it is possible to obtain:

$$C_T \equiv \frac{\mathcal{T}}{A^* p_0} = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right]} + \left(\frac{p_e}{p_0} - \frac{p_a}{p_0}\right)\frac{A_e}{A^*}$$

where p_0 is the stagnation pressure at the thrust chamber.

The parameter $\mathcal{T}/(A^* p_0)$ is an non-dimensional parameter which is called the <u>Thrust Coefficient</u>. This parameter does not depend on the stagnation temperature at the thrust chamber or the average molecular mass of the exhaust gases. Nevertheless, it depends on the stagnation pressure at the thrust chamber.

Thrust Coefficient

The thrust coefficient depends on the nozzle dimensions (characterized by the throat area). For the same thrust, the above relation shows that the higher is p_0 the smaller will be the nozzle size. However there are limits for the value of p_0 because for higher values of p_0:

— The required structural resistance increases. The thrust chamber walls must be stronger and heavier;

— The heat transfer rate to the nozzle walls increases, leading to higher temperatures at the nozzle walls. These higher temperatures require the use of expensive heat resistance materials;

— The required mechanical power at the fuel and oxidizer pumps increase, requiring more mechanical power from the turbine. This higher power means that the pumps and turbine will be larger, heavier and more expensive.

Thrust Coefficient

The thrust coefficient depends on the nozzle geometry since it depends on $(p_e - p_a)/p_0$, which is an indication of how well the nozzle is adapted to the ambient pressure. Usually, the thrust coefficient has values within the range 0.8 to 1.9.

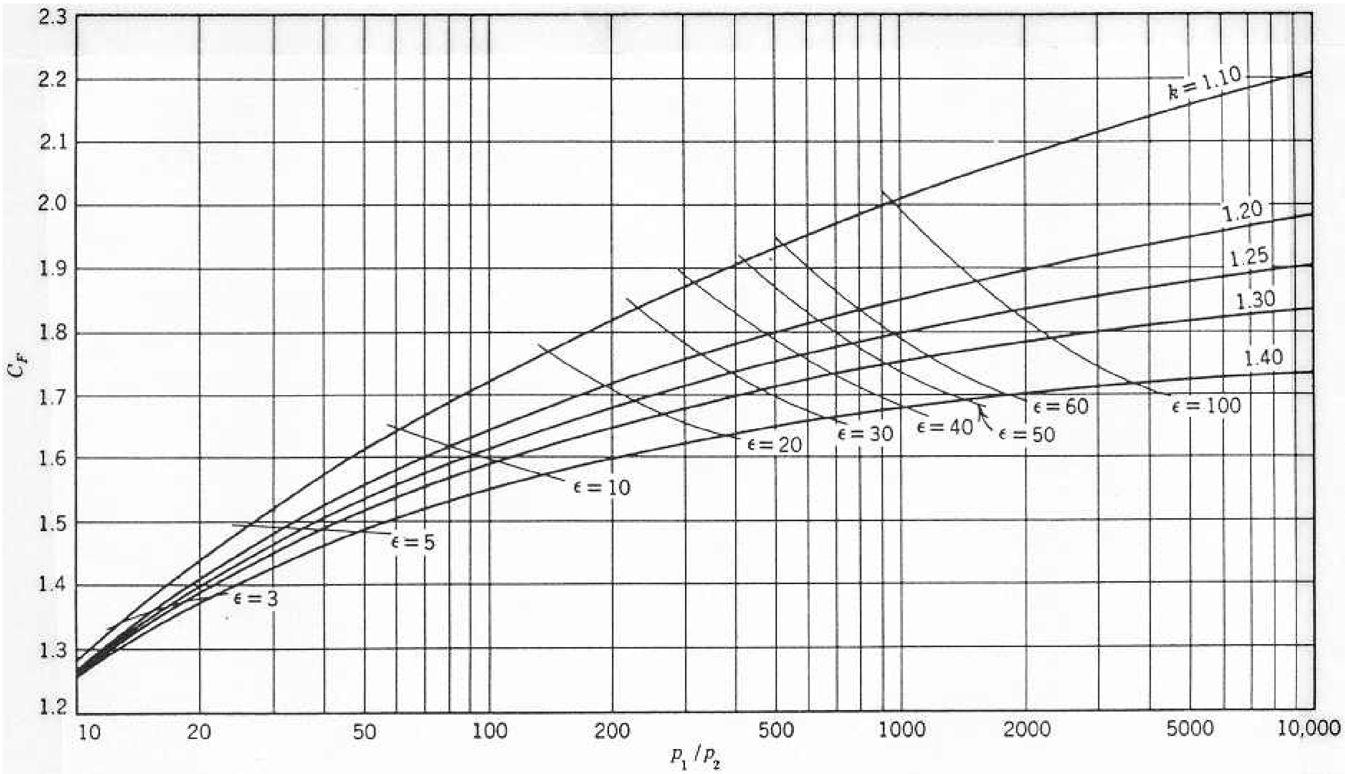
Multiplying the two coefficients just defined the following relation is reached:

$$\mathcal{T} = \dot{m}\, c^* C_T$$

In this equation, the characteristic velocity, c^*, describes the thrust chamber performance, and the thrust coefficient, C_T, qualifies the exhaust nozzle performance. Values calculated from experimental measurements can be compared with the ideal, theoretical values just presented to assess whether the corresponding components have good performances.

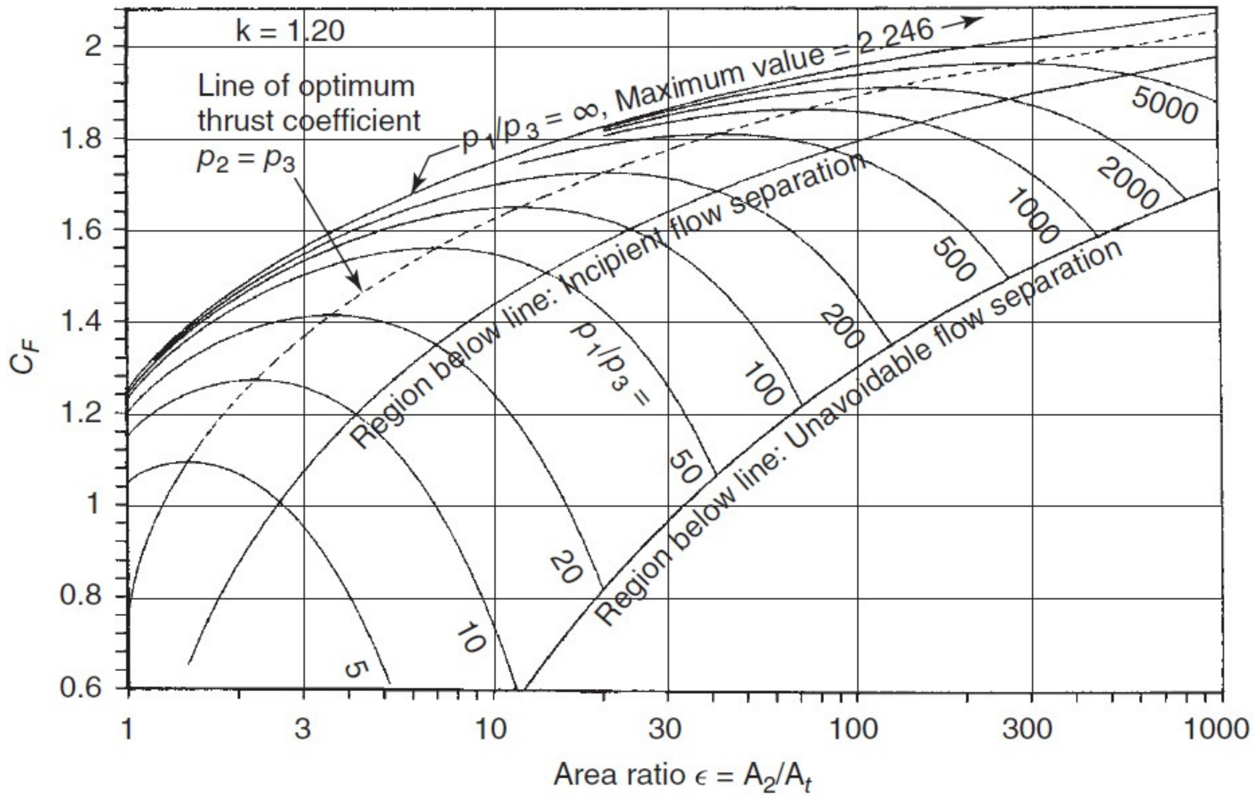
Thrust Coefficient

The following figure shows the thrust coefficient as a function of the pressure ratio, area ratio, and specific heat ratio, γ, for a correct or optimum expansion.



Thrust Coefficient

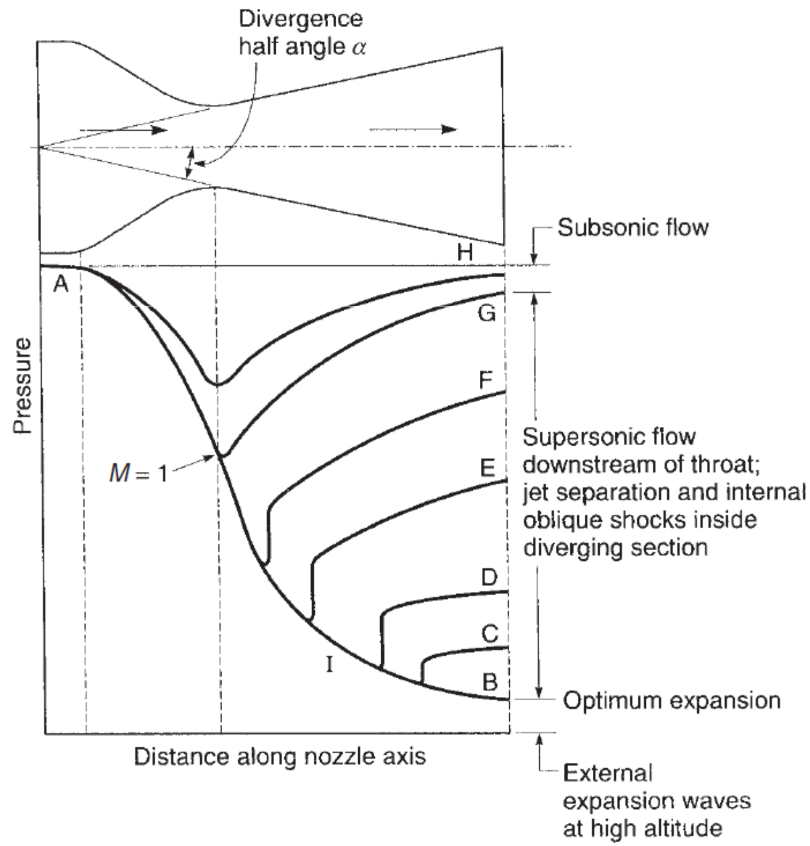
The next figure presents the thrust coefficient as a function of the pressure ratio and area ratio, for a value of $\gamma = 1.2$.



Under- and overexpansion are considered in this figure.

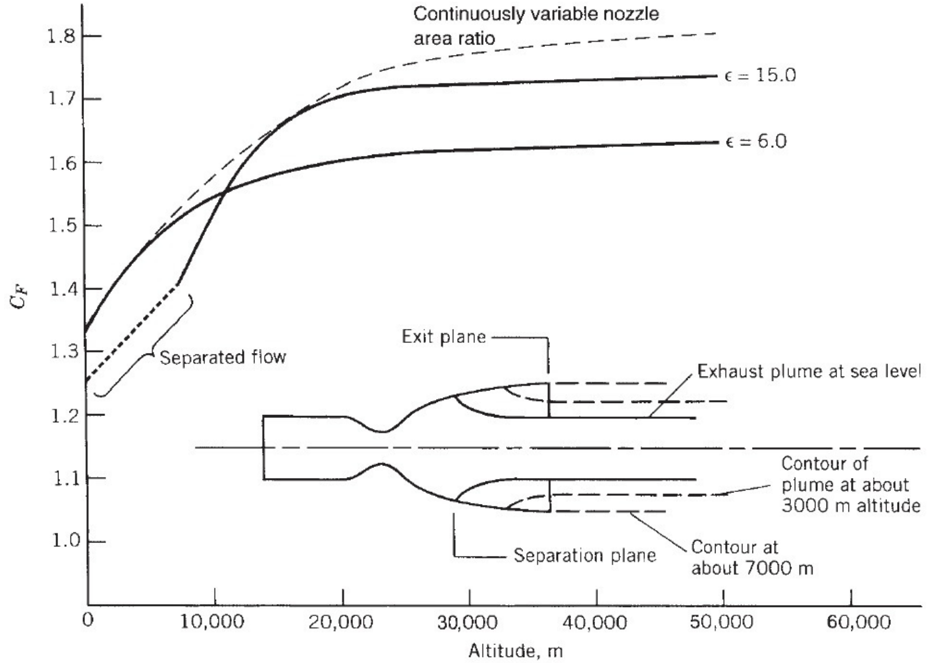
Under-and Overexpanded Nozzles

The evolution of pressure along a nozzle with fixed area ratio is shown in the next figure:



Under-and Overexpanded Nozzles

When the altitude varies, the operation can change from over-expanded operation to an under-expanded operation, see next figure:



The area ratio is chosen so that the nozzle has correct or optimum expansion for an intermediate altitude, by optimizing the global performance (for example, the total impulse, the specific impulse, range or, ...).

Nozzle contour

Usually the thrust chamber and nozzle have a circular cross-section. Previous arguments are one-dimensional and do not allow the design of the contour of the nozzle. In the following some topics concerning the design of the nozzle contour will be presented.

In practice, it is found that the contour of the converging portion of the nozzle is not important, provided it is a reasonably smooth contour with no sharp edges. The reason is that this part of the nozzle is subjected to a large favourable pressure gradient, resulting in a low loss subsonic flow.

The diverging part of the nozzle requires more care since it is subjected to supersonic flow and any surface disturbance may cause the appearance of shock waves resulting in the introduction of some losses. The figure in the next slide presents some geometries proposed for the diverging part of the nozzle.

Nozzle contour

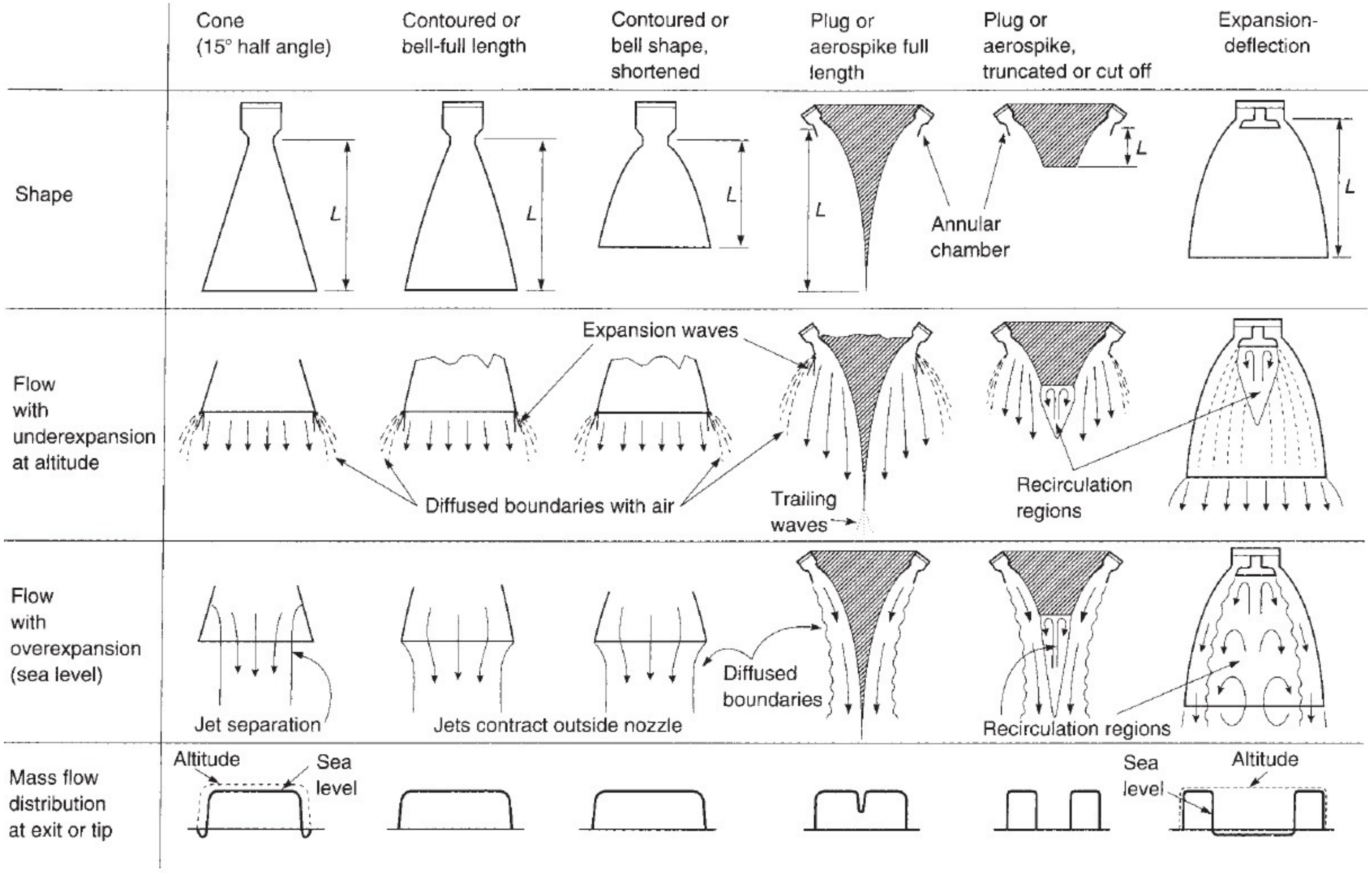
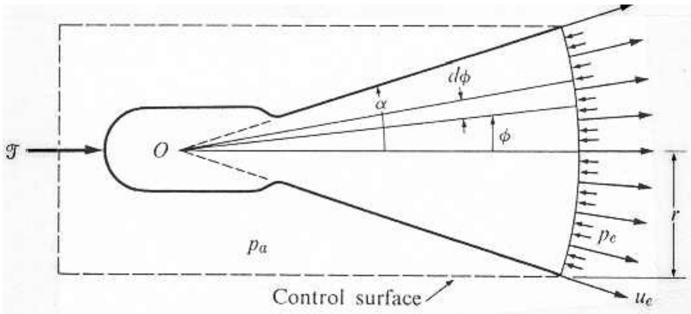


FIGURE 3–11. Simplified diagrams of different generic nozzle configurations and their flow effects.

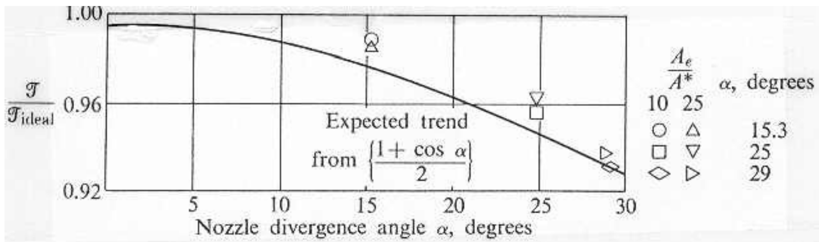
Nozzle contour

A nozzle with the shape of a cone is a simple geometry, easy to manufacture and was one of the first geometries being used. Due to its simplicity tends to be used in small-size rocket engines.

As indicated in the next figure, at exit there is a divergence of the flow leading to an exit flow which is not parallel and axial. This flow suffers a penalty in terms of the axial exit momentum.



This penalty, called divergence loss, is quantified in the next figure, assuming an ideal rocket with a conical exhaust flow.

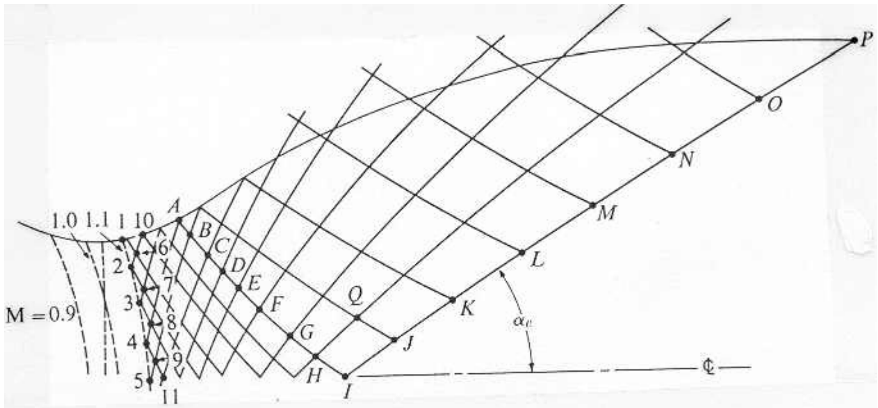


Nozzle contour

Divergence cone exit angles of around 15° are generally used. This value is a compromise between divergence loss and the nozzle length (the greater the nozzle length the heavier and bulkier will be the nozzle).

The bell-shaped or contour nozzle shows large divergence angles immediately after the throat, with a gradual decrease of this angle towards the exit. The divergence angle at exit should be zero or close to zero in order to obtain a parallel, uniform, axial velocity profile.

The two-dimensional contour of this type of nozzle can be determined using the method of characteristics, see next figure. In this way it is possible to eliminate the divergence loss.



Nozzle contour

The next figure shows a practical example, with a large area ratio – notice that the exit divergence angle is not zero.

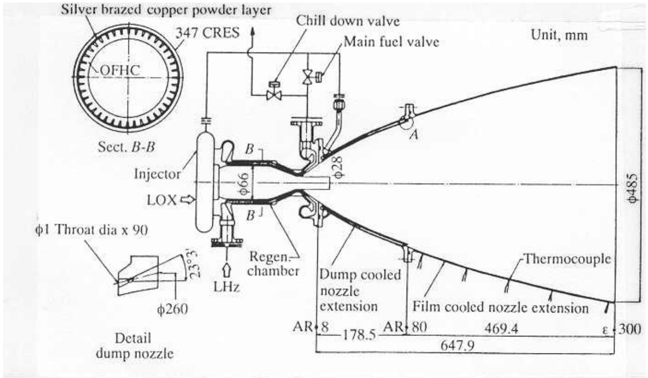
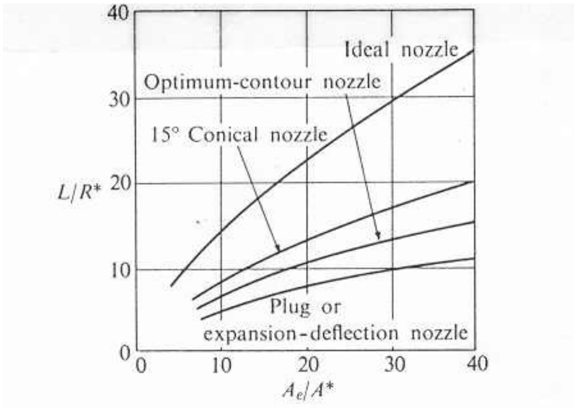


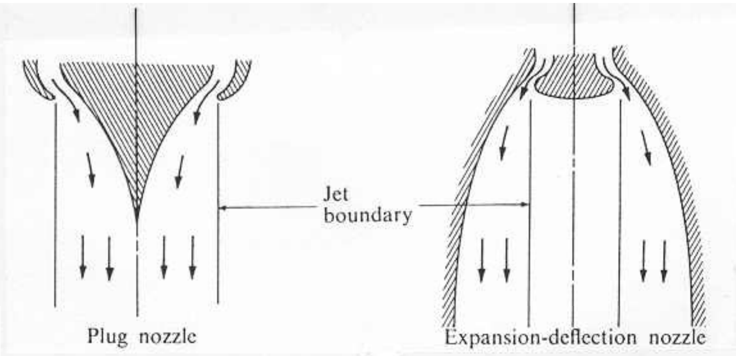
FIGURE 11.13 H_2-O_2 rocket nozzle; $A_e/A^* = 300$. (Courtesy AIAA [10]. © AIAA. Reprinted with permission.)

The nozzle length is an important variable that is shown in the figure below.



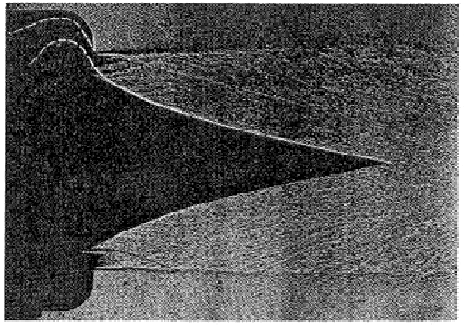
Nozzle contour

The previous geometries are affected by altitude variations, as already discussed. The plug and expansion-deflection nozzles are more insensitive to altitude variations due to the presence of a free, constant pressure flow boundary.

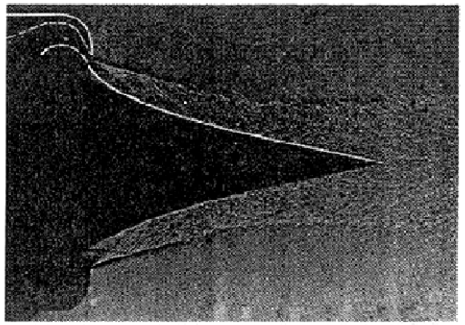


The figure below shows flow visualizations for different values of the pressure ratio.

https://www.youtube.com/watch?v=D4SaofKCYwo



(a) At design p_0/p_e



(b) Below design p_0/p_e

Real Nozzles

There are number of effects that lead to departures from the ideal performance, namely:

— Divergence losses;

— Pressure losses at the thrust chamber;

— Friction losses (boundary layers);

— Multiphase flow;

— Unsteady combustion processes;

— Chemical reactions during expansion;

— Transient pressure operation (start, stop or pulsing);

— Throat erosion;

— Non-uniform gas composition;

— Real gas properties;

— Altitude variations.

Problem

Problem: A rocket engine is to be designed with the following specifications:

— Propellants	Nitric acid and aniline (hypergolic)
— Mixture ratio	2.75
— Thrust	4448 N
— Thrust chamber pressure	20.7 bar
— Ambient pressure	1.01 bar
— Thrust chamber temperature	2994 K
— Mean molecular weight of exhaust gases	25 kg/kmol
— Ideal specific impulse	218 s
— Specific heat ratio	1.22

a) Estimate the overall dimensions of the nozzle. Assume that the real thrust is equal to 0.96 of the ideal thrust.

b) Estimate the thrust or combustion chamber dimensions.

Bibliography:

- Chapter 3

  "Rocket Propulsion Elements", G. P. Sutton and O. Biblarz, 9th ed. , John Wiley & Sons, 2016
- Chapter 11

  "Mechanics and Thermodynamics of Propulsion", P. Hill and C. Peterson, 2nd ed., Addison-Wesley Publishing Co., 1992