MIE100S: Dynamics – April 25, 2022

Final Exam – 2.5 Hours

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Problem	**Marks**	**Page**
1a-c	10	2-4
2a-c	10	5-7
3a-b	10	8,9
4a-c	10	10-12
5a-b	10	13-14
Spare Pages		15-16

- **You must *neatly* show *all* rough work to earn credit for each answer**.
- This answer booklet contains 12 pages, including this cover page, **printed two-sided.**
- If you run out of room answering any of these questions, then ***clearly indicate*** that your solution is continued on page 15 and 16.
- **Do not** tear any pages from this booklet.
- Avoid folding or tearing any of the sheets to the extent possible.
- **Place your student ID at the front, right-hand corner of your desk**.

Permitted Aids:

- Non-communicating/non-programmable calculator: Casio FX-991EX or Casio FX-991ES PLUS or Casio FX-991MS or Sharp EL-520X or Sharp EL-520W
- One 8 ½" x 11" aid sheet, any colour, brought to the test by the student. You may write on both sides of the sheet.
- Ruler, Protractor

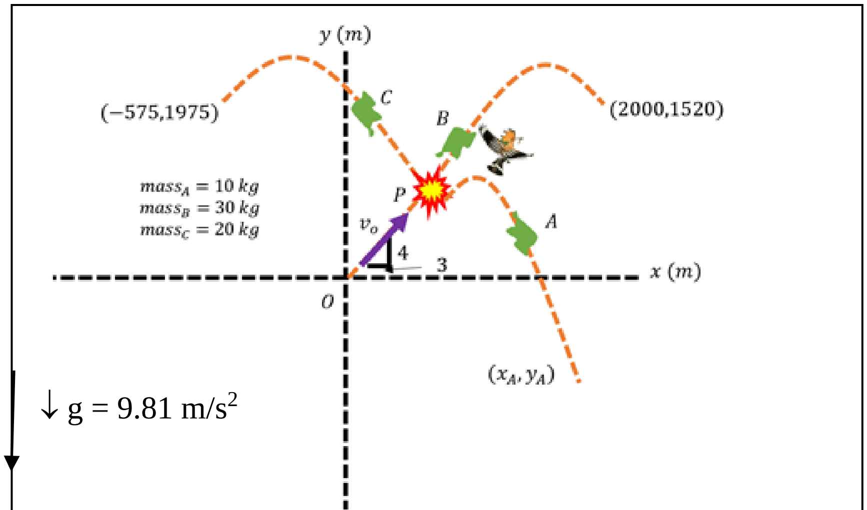
Present your complete solution in the space provided

Question 1

A 60 kg unpowered projectile is launched from O with an initial speed of $v_o = 125\frac{m}{s}$.

The projectile breaks up 7 seconds after launch at point P into 3 separate components – A, B and C with masses of 10kg , 30kg and 20 kg respectively.

B and C land on hills at the coordinates shown in the diagram.

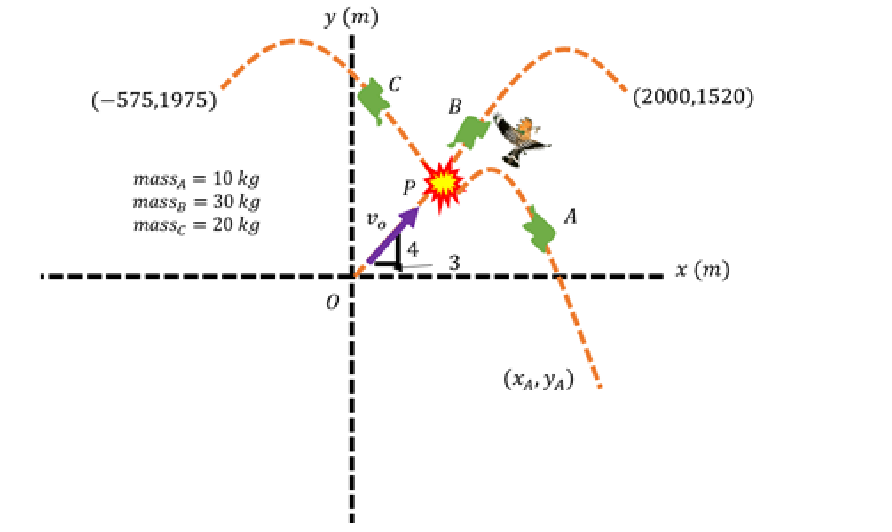


Question 1

Pieces A, B and C remain airborne for $t_A = 2.87\ sec$, $t_B = 32.1\ sec$ and $t_C = 6\ sec$ respectively after the explosion.

a) Calculate the point $P(x, y)$ and speed of the projectile at the time of the explosion. **[2 marks]**

b) The bird in the diagram is at coordinates (775m, 792.9m). **[4 marks]**



Question 1

i. What is the distance between the bird and projectile at $t = 7\ sec$?

ii. What would the initial speed have to be to ensure a direct collision with the bird at 7 seconds?

Part A) and B)

Locate the point of explosion:

$$x_p = x_o + V_{x_o}t = \left(\frac{3}{5}\right)(125)(7) = \mathbf{525\ m}$$

$$y_p = y_o + V_{y_o}t - \frac{1}{2}gt^2 = \left(\frac{4}{5}\right)(125)(7) - \frac{1}{2}(9.81)(7^2) = \mathbf{459.7\ m}$$

0

$$Distance = \sqrt{(792.9 - 459.7)^2 + (775 - 525)^2} = \mathbf{416.6\ m}$$

Find velocity of rocket at explosion:

$$V_x = V_{x_o} = \left(\frac{3}{5}\right)(125) = \mathbf{75\ m/s}$$

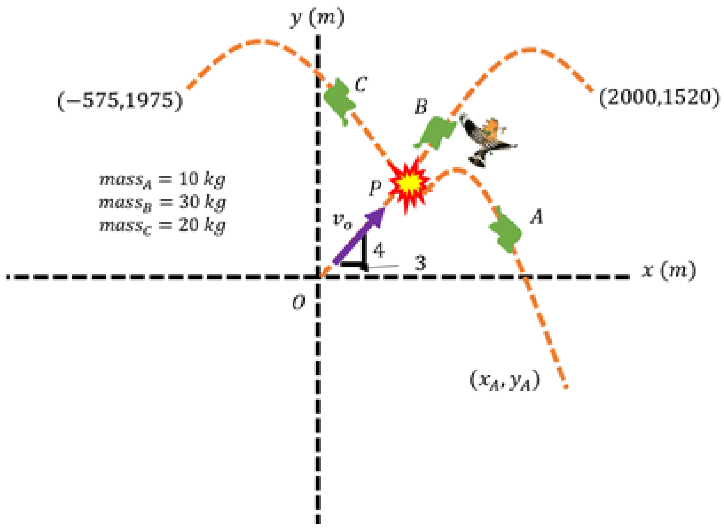
$$V_y = V_{y_o} - gt = \left(\frac{4}{5}\right)(125) - (9.81)(7) = \mathbf{31.3\ m/s}$$

Find what initial velocity is needed to collide with bird

$$775 = V_{x_o}t = \left(\frac{3}{5}\right)(?)(7) = \mathbf{184.5\frac{m}{s}},\ \boldsymbol{verify\ in\ other\ coordinate\ equation}$$

Pieces A, B and C remain airborne for $t_A = 2.87\ sec$, $t_B = 32.1\ sec$ and $t_C = 6\ sec$ respectively after the explosion.

c) Evaluate the coordinates of the valley at which A lands (x_A, y_A) . **[4 marks]**



Question 1

Analyse motion of C after explosion:

$$x_B = x_P + V_{x_B} t_B \ ,\ 2000 = 525 + V_{x_B}(32.1)$$

$$y_B = y_P + V_{y_B} t_B - \frac{1}{2} g t_B^2 \ ,\ 1520 = 459.7 + V_{y_B}(32.1) - \frac{1}{2}(9.81)(32.1^2)$$

$$B\ parameters: \begin{cases} V_{x_B} = 45.95\ m/s \\ V_{y_B} = 190.5\ m/s \end{cases}$$

Analyse Motion of C after explosion:

$$x_C = x_P + V_{x_C} t_C \ ,\ -575 = 525 + V_{x_C}(6)$$

$$y_C = y_P + V_{y_C} t_C - \frac{1}{2} g t_C^2,\ 1975 = 459.7 + V_{y_C}(6) - \frac{1}{2}(9.81)(6^2)$$

$$C\ Parameters: \begin{cases} V_{x_C} = -183.3 m/s \\ V_{y_C} = 281.98\ m/s \end{cases}$$

Apply conservation of momentum:

$$mv_x = \sum m_i V_{x_i},\ 60(75) = 10V_{x_A} + 30(45.95) + 20(-183.3),\ V_{x_A} = 678.8\ m/s$$

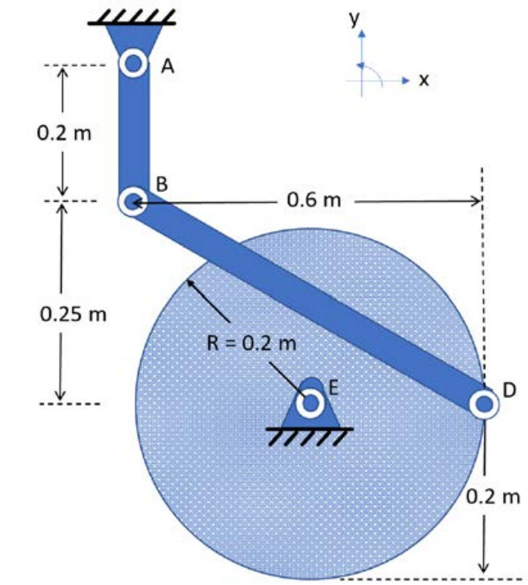
$$mv_y = \sum m_i V_{y_i},\ 60(31.3) = 10V_{y_A} + 30(190.5) + 20(281.98),\ V_{y_A} = -947.7\ m/s$$

Analyse motion of A after explosion:

$$x_A = x_P + V_{x_A} t_A \ = 525 + 678.8(2.87) = \mathbf{2473.2m}$$

$$y_A = y_P + V_{y_A} t_A - \frac{1}{2} g t_A^2 = 459.7\ - 947.7(2.87) - \frac{1}{2}(9.81)(2.87^2) = \mathbf{-2300.5\ m}$$

$$Distance = \sqrt{(-2300.5 - (-2300.5))^2 + (2473.2 - 2473.2)^2} = \mathbf{0\ m}$$



Question 2

Question 2

At the instant shown, bar *AB* is in the vertical position and has an angular velocity of $10\,\frac{rad}{s}$ counterclockwise and it is **slowing** down at a rate of $2\,\frac{rad}{s^2}$.

Bar BD is pinned to a circular disc which can rotate about a fixed point E.

a) Determine the acceleration of point B. **[3 marks]**

$$\boldsymbol{\omega}_{AB} = 10 \text{ rad/s} \circlearrowleft$$

$$v_B = (AB)\omega_{AB}$$
$$= (0.200)(10)$$
$$= 2 \text{ m/s}$$

$$\mathbf{v}_B = v_B \longrightarrow$$

$$\mathbf{v}_D = v_D \downarrow$$

$$\omega_{BD} = \frac{v_B}{\ell_{BC}} = \frac{2\,\frac{m}{s}}{0.25\,m} = 8\,\frac{rad}{s}\ (CW)$$

$$v_D = \ell_{CE}\omega_{BD} = 0.6\text{ m}\left(8\,\frac{\text{rad}}{\text{s}}\right) = 4.8\,\frac{\text{m}}{\text{s}}\,(\text{down})$$

$$\omega_{DE} = \frac{v_D}{\ell_{DE}} = \frac{4.8\,\frac{m}{s}}{0.2\,m} = 24\,\frac{rad}{s}\,(CW)$$

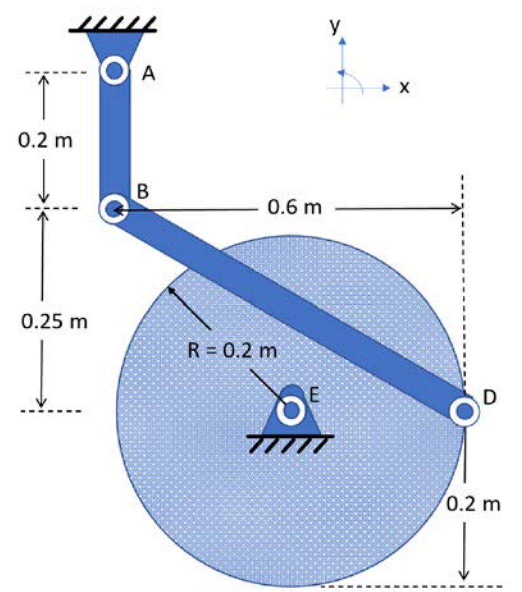
$$\alpha_{AB} = 2\,\frac{rad}{s^2} \qquad \omega_{AB} = -10\,\frac{rad}{s}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times (-0.2\,m\,\vec{j}) - \omega_{AB}^2\,\vec{r}_{AB}$$

$$\vec{a}_B = 0 + \left(-2\,\vec{k}\,\frac{rad}{s^2}\right) \times (-0.2\,m\,\vec{j}) - \left(10\,\frac{rad}{s}\right)^2 (-0.2\,m\,\vec{j})$$

$$\vec{a}_B = -0.4\,\frac{m}{s^2}\,\vec{i} + 20\,\frac{m}{s^2}\,\vec{j}$$

b) Determine the angular velocity of the circular disc. **[3 marks]**

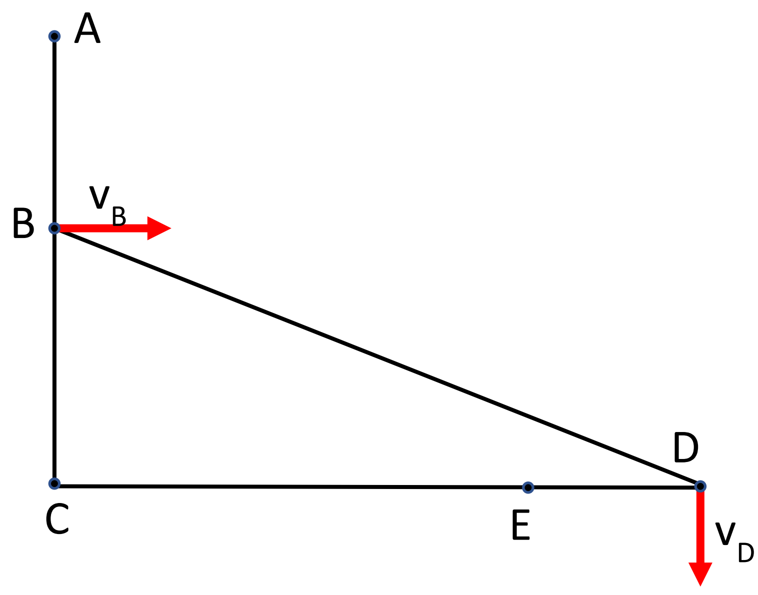


Question 2

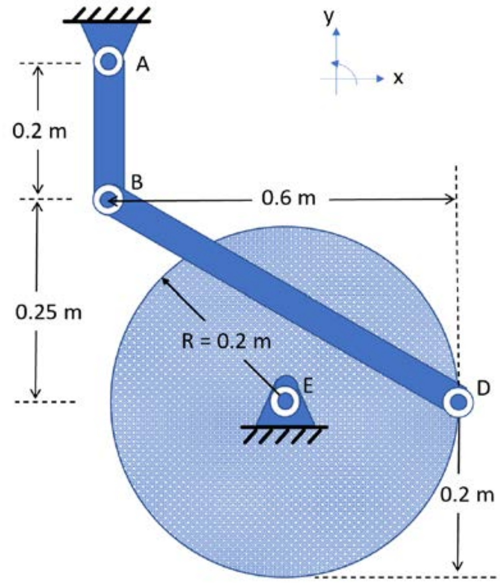
$$\vec{\omega}_{BD} = -8\frac{rad}{s}$$

$$\vec{v}_D = -4.8\frac{m}{s}$$

$$\vec{\omega}_{DE} = -24\frac{rad}{s}$$



c) Determine the angular acceleration of the circular disc. **[4 marks]**



Question 2

$$\alpha_{AB} = 2\frac{rad}{s^2} \qquad \omega_{AB} = -10\frac{rad}{s}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times (-0.2\,m\,\vec{j}) - \omega_{AB}^2\,\vec{r}_{AB}$$

$$\vec{a}_B = 0 + \left(-2\,\vec{k}\frac{rad}{s^2}\right) \times (-0.2\,m\,\vec{j}) - \left(10\frac{rad}{s}\right)^2 (-0.2\,m\,\vec{j})$$

$$\vec{a}_B = -0.4\frac{m}{s^2}\,\vec{\imath} + 20\frac{m}{s^2}\vec{\jmath}$$

$$\vec{a}_D = \vec{a}_B + \vec{\alpha}_{BD} \times \vec{r}_{D/B} - \omega_{BD}^2 \vec{r}_{D/B}$$

$$\vec{a}_D = \; -0.4\frac{m}{s^2}\,\vec{\imath} + 20\frac{m}{s^2}\vec{\jmath} + \vec{\alpha}_{BD} \times (0.6\text{m}\,\vec{\imath} - 0.25m\,\vec{\jmath}) - \left(-8\frac{rad}{s}\right)^2 (0.6\text{m}\,\vec{\imath} - 0.25m\,\vec{\jmath})$$

$$\vec{a}_D = (-38.8 + 0.25\,\alpha_{BD})\frac{m}{s^2}\,\vec{\imath} + (36 + 0.6\,\alpha_{BD})\frac{m}{s^2}\vec{\jmath}$$

$$\vec{a}_D = \vec{a}_E + \vec{\alpha}_{DE} \times \vec{r}_{D/E} - \omega_{DE}^2 \vec{r}_{\frac{D}{E}}$$

$$\vec{a}_D = 0 + \vec{\alpha}_{BD} \times (0.2m\,\vec{\imath}) - \omega_{BD}^2 (0.2m\vec{\imath})$$

$$\vec{a}_D = -115.2\frac{m}{s^2}\,\vec{\imath} + 0.2\,\alpha_{DE}\frac{m}{s^2}\vec{\jmath}$$

$\vec{\imath}$: $\quad -115.2\frac{m}{s^2} = -38.8\frac{m}{s^2} + 0.25\,\alpha_{BD}$

$\vec{\jmath}$: $\quad 0.2\,\alpha_{DE} = 36 + 0.6\,\alpha_{BD}$

$$\alpha_{DE} = 305.6\frac{rad}{s^2}\,\vec{k}\,(CW)$$

$$\alpha_{DE} = 736.8\frac{rad}{s^2}\,\vec{k}\,(CW)$$

Therefore,

$$\boldsymbol{\alpha}_{BD} = 306 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_{DE} = 737 \text{ rad/s}^2$$

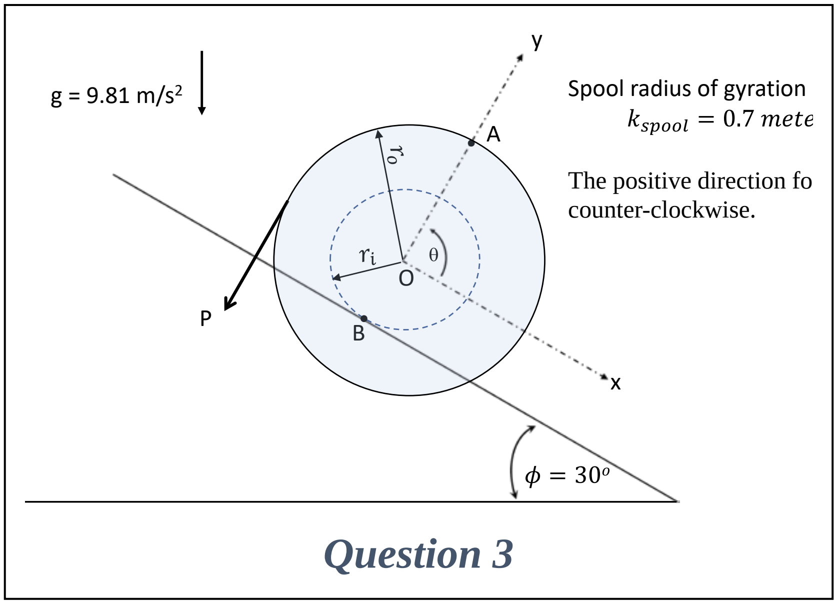
Question 3

A circular spool of mass 15 kg has an inner radius, $r_i = 0.4\ meters$ and outer radius, $r_o = 0.9\ meters$. Its radius of gyration is $k_{spool} = 0.7\ meters$.

The spool rolls without slipping on its inner hub along a rod slanted at 30 degrees above the horizontal, as shown in the diagram.

A rope is wound around its outer rim, and a tension P = 65 *Newtons* is applied to the rope and is acting normal to the ramp.

At the instant shown in the diagram, the spool has an angular velocity of $\omega = -5\frac{rad}{s}$.



(a) Determine the velocity of point "A" and the acceleration of point "B" at the instant shown in the diagram. Express your answer in the x-y coordinate system shown in the diagram. **[4 marks]**

$$\vec{v}_A = (|\omega|)(0.90 + 0.40\ meters)\vec{\imath}$$

$$= \left(5\frac{rad}{s}\right)(1.30m)\vec{\imath}$$

$$\vec{a}_B = 0\vec{\imath} + v_{B/O}^2/0.4\,\vec{\jmath}$$

$$= (0.40\ x\ 5)^2/0.4\,\vec{\jmath}$$

Alternate solution for $\vec{v}_A$

$$\vec{v}_A = \omega_{whl} \times \vec{r}_{G/B}$$

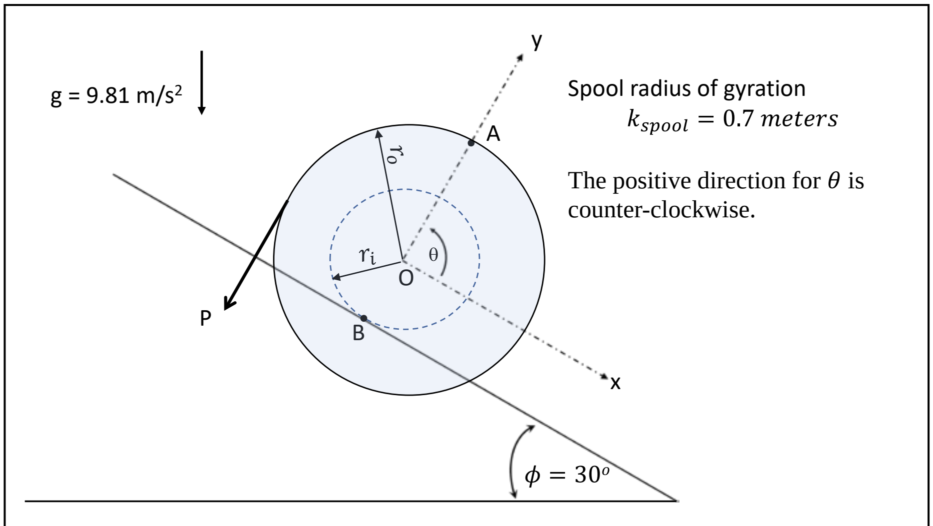
$$\vec{v}_A = \left(-5\,\vec{k}\frac{rad}{s}\right) \times (1.3\ m\,\vec{\jmath})$$

$$\vec{v}_A = 6.5\,\vec{\imath}\frac{m}{s}$$

$$\vec{v}_A = 6.5\,\vec{\imath}\frac{m}{s}$$

$$\vec{a}_B = 10\vec{\jmath}\ \frac{\mathrm{m}}{s^2}$$

(b) Determine the angular acceleration α of the spool at the instant shown in the diagram, with positive angles measured in the counterclockwise direction as shown in the diagram. **[6 marks]**



Question 3

$$I_G = \ (15\ kg)(0.7\ meters)^2 = 7.35\ kg \cdot m^2$$

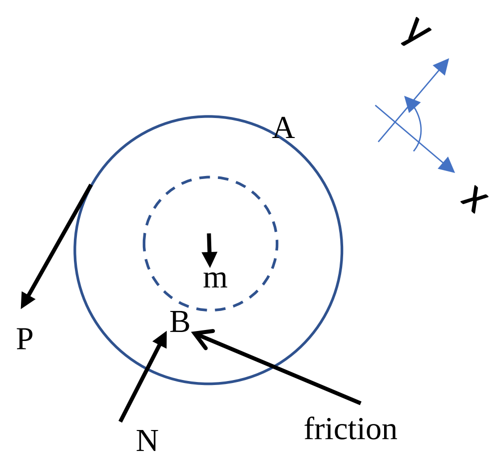
$$\sum F_y:\ 0 = N - P - mg\ cos\ (30^o)$$

$$\rightarrow N = 65 + 159.810.8660 = 192.4\ Newtons$$

Why friction in this direction? I am assuming the wheel will want to rotate CCW

$$\sum F_{x:}\quad - friction + mg\ sin30^o = m(a_G)_x$$

$$\sum M_G = \ I_G\ \alpha\ \rightarrow (65N)(0.9m)\text{-}friction(0.4m)\text{=}7.35\ \alpha$$



Solve the two equations:

$$\alpha = 2.98\frac{rad}{s^2}\ \ ;\ \ friction = \ 91.49\ Newtons$$

$$I_B \quad = \quad (15\,kg)(0.7\,meters)^2 + (15\,kg)(0.4)^2$$

$$I_B = 9.75\,kg \cdot m^2$$

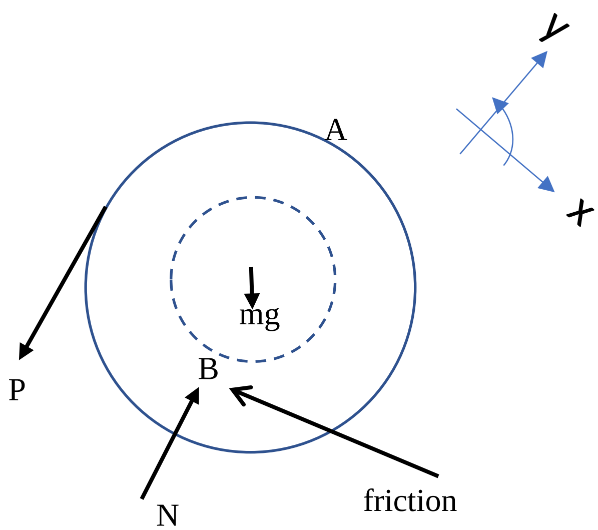
$$\alpha = \sum M_B / I_B$$

$$= \frac{[(-mg)(0.40m)(\sin 30)+(65N)(0.90m)]}{9.75\text{kg m}^2}$$

$$= \frac{\left[(-15\,kg)(9.81\frac{m}{s^2})(0.40m)(sin\,30)+(65N)(0.90m)\right]}{9.75kg\,m^2}$$

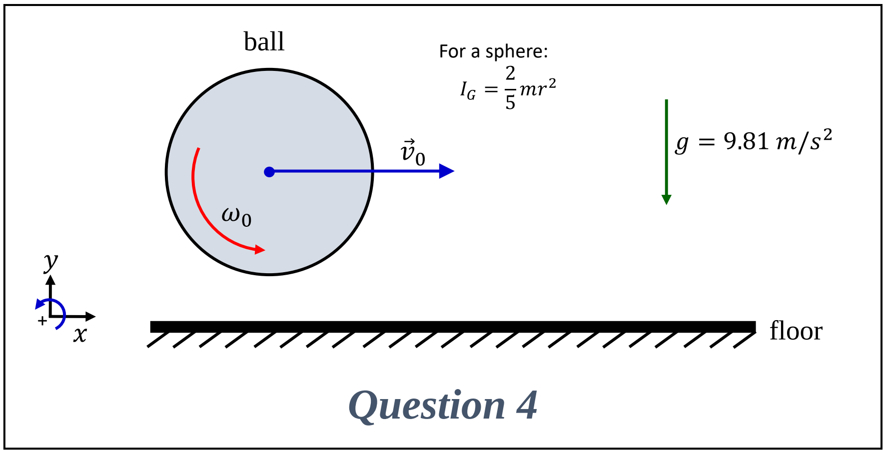
$$\alpha = 2.98\vec{k}\,\frac{rad}{s^2}$$

Alternate approach – noting this is rolling without slip



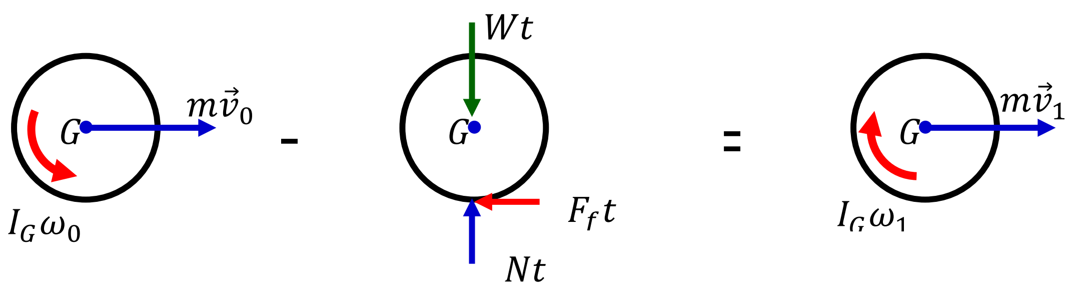
Question 4

A bowler throws a ball weighing 80 N and radius, $r_{ball} = 150mm$, along an alley (floor) with a forward velocity, $\vec{v}_0 = 15\,\vec{\imath}\,\frac{m}{s}$ and a backspin, angular velocity, $\vec{\omega}_0 = 12\,\vec{k}\,\frac{rad}{s}$.



Knowing that the **coefficient of kinetic friction between the ball and the alley is $\mu_k = 0.15$,**

a) determine the angular momentum of the ball at time, t_0 **[2 marks]**



$$r = \frac{d}{2} = 0.15\ m;\ m = \frac{W}{g} = 8.1549\ kg$$

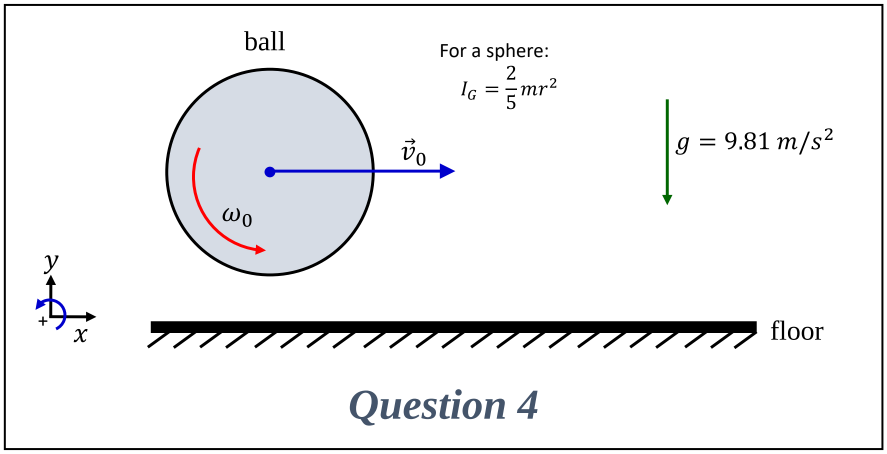
Moment of inertia of a sphere:

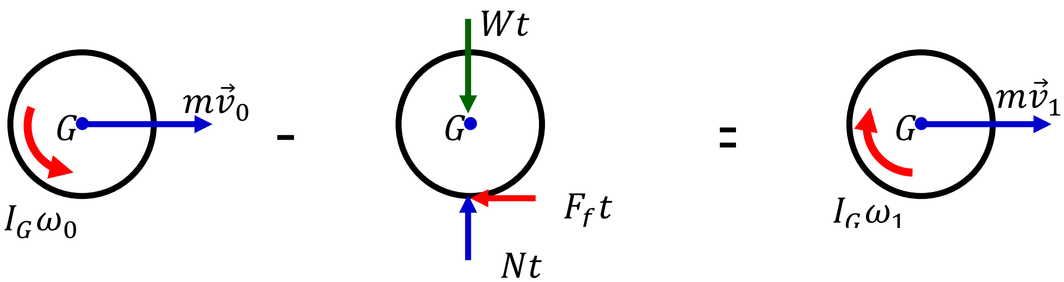
$$I_G = \frac{2}{5}mr^2 = \frac{2}{5}(8.1549)(0.15)^2 = 0.0734\ kg\ m^2$$

(a) angular momentum of the ball at time, t_0

$$I_G\omega_0 = 0.0734(12) = 0.8808\ kg\ m^2/s$$

b) find the time, t_1 at which the ball will start rolling without sliding **[4 marks]**





b) the time, t_1 at which the ball will start rolling without sliding

$$Nt - Wt = 0; N = W = 80\ N$$

$$F_f = \mu_k N = 0.15(80) = 12\ N$$

Linear impulse - momentum principle

$$m\vec{v}_0 - F_f t = m\vec{v}_1$$

$$\vec{v}_1 = \vec{v}_0 - \frac{F_f}{m}t = 15 - \frac{12}{8.1549}t = 15 - 1.4715t$$

Angular impulse - momentum principle

$$I_G\omega_0 - F_f tr = -I_G\omega_1$$

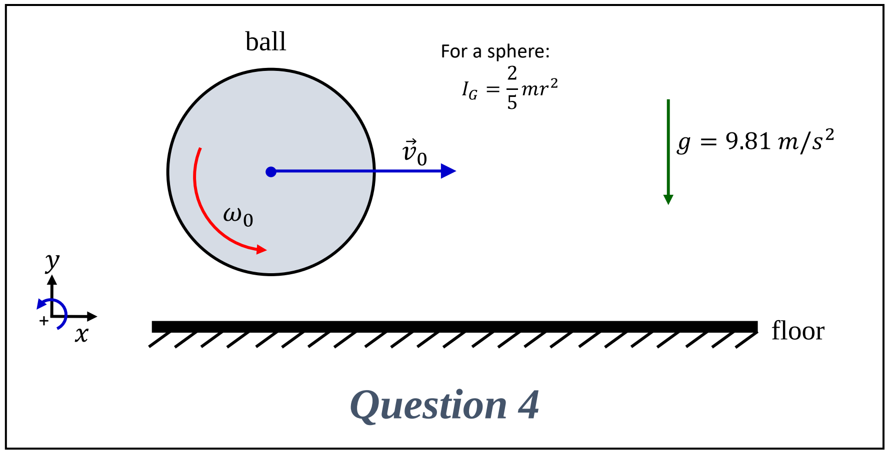
$$\omega_1 = \frac{F_f r}{I_G}t - \omega_0 = \frac{12(0.15)}{0.0734}t - 12 = 24.5232t - 12$$

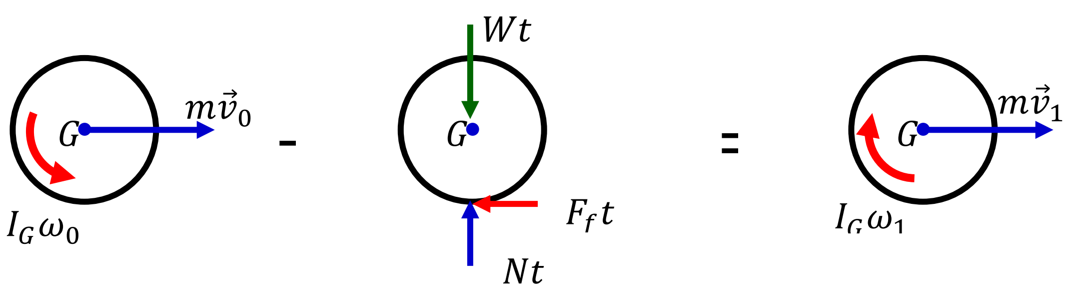
Slipping stops when: $\vec{v}_1 = \omega_1 r$

$$15 - 1.4715t = (24.5232t - 12)(0.15)$$

$$t = 3.262\ s$$

c) find the velocity of the ball at time, t_1 **[4 marks]**





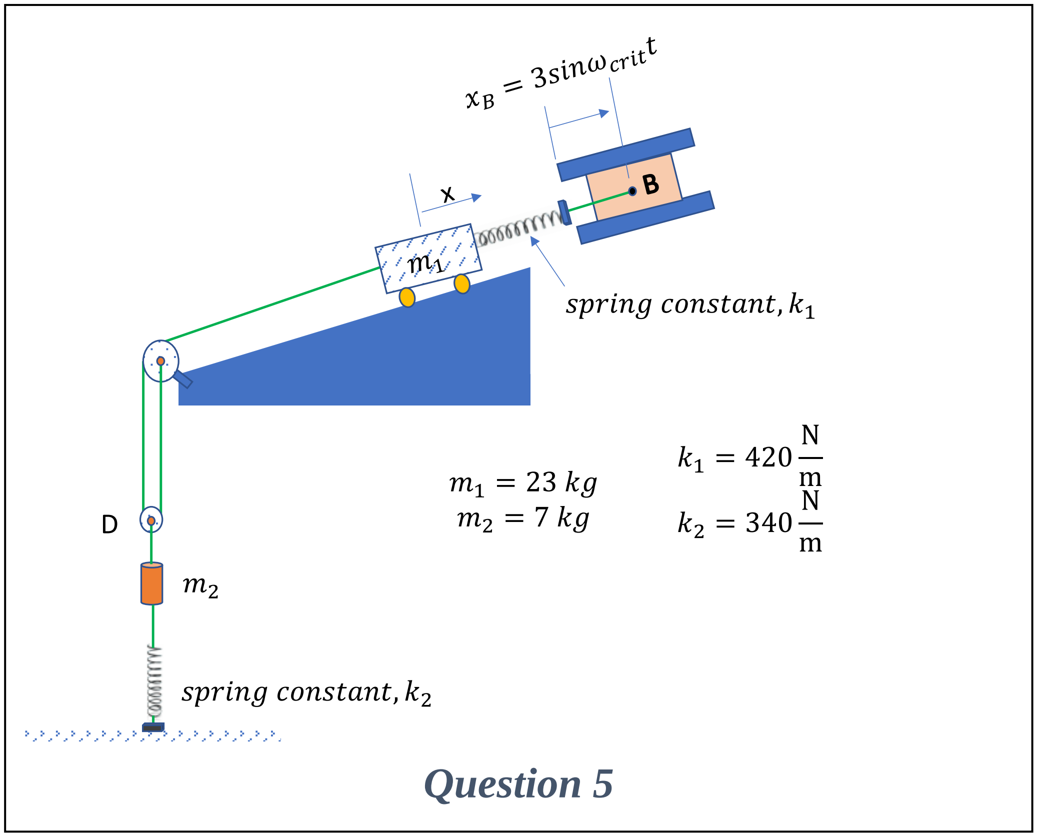
$$\vec{v}_1 = 15 - 1.4715t$$

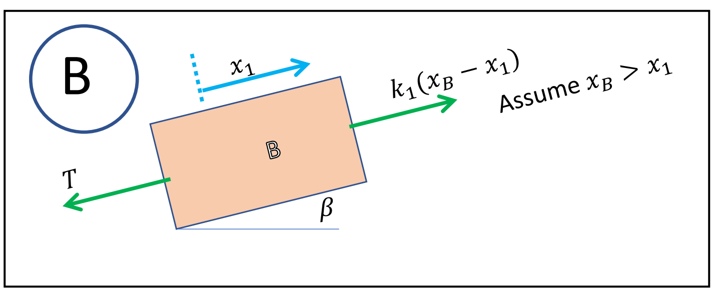
$$\vec{v}_1 = 15 - 1.4715(3.262) = 10.2\ m/s$$

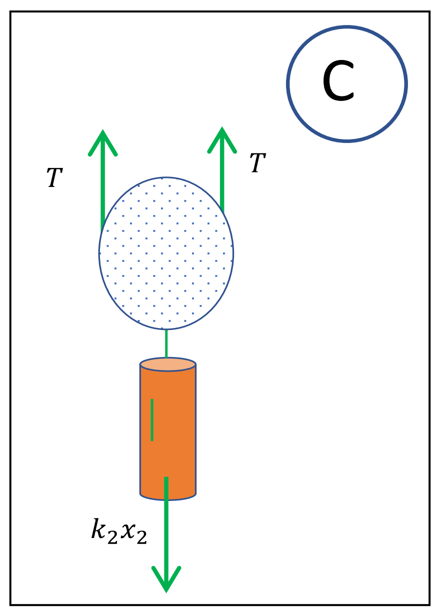
Question 5

Two masses are connected by an inextensible cable which passes over a massless pulley.

a) Draw the Free Body Diagrams including all forces on m_1 *and* m_2 of the system (**4 marks**)







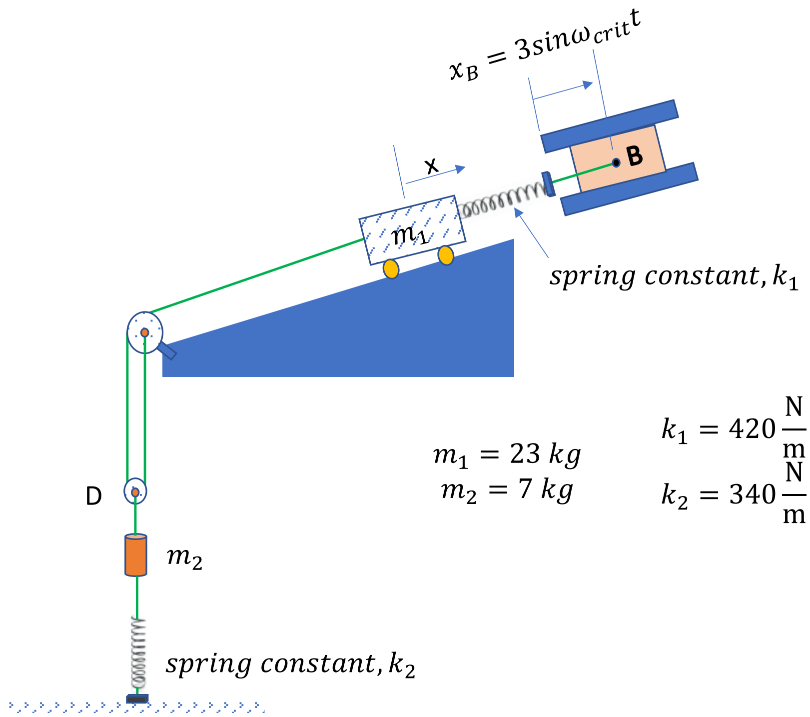
$\sum F_x = m_1\ddot{x}_1$ $\quad k_1(x_B - x_1) - T = m_1\ddot{x}_1$

$m_1\ddot{x}_1 + k_1x_1 + T = k_1 b\sin\omega_{crit}t$

$\sum F_x = m_2\ddot{x}_1$ $\quad 2T - k_2x_2 = m_1\ddot{x}_2$

$m_2\ddot{x}_2 + k_2x_2 = 2T$

b) What is the critical driving frequency, ω_{Crit}, of the Block B? (**6 marks**)



Question 5

Rearranging C yields D

$$T = \frac{m_2\ddot{x}_2 + k_2x_2}{2}$$

Putting D into B yields

$$m_1\ddot{x}_1 + k_1x_1 + \frac{m_2\frac{\ddot{x}_1}{2} + k_2\frac{x_1}{2}}{2} = k_1 b \sin\omega_{crit}t$$

Spare page, in case you ran out of room on one of the questions –
Indicate clearly which question you are writing.

5B continued

$$4(m_1\ddot{x}_1 + k_1x_1) + m_2\ddot{x}_1 + k_2x_1 = k_1b\sin\omega_{crit}t$$

$$99\ddot{x}_1 + 2020\,x_1 = 5040\,sin\,\omega_{crit}t$$

$$(4m_1 + m_2)\ddot{x}_1 + (4k_1 + k_2)x_1 = 4k_1b\sin\omega_{crit}t$$

$$\omega_n = \omega_{crit} = \sqrt{\frac{4k_1 + k_2}{4m_1 + m_2}} = 4.52\frac{rad}{s}$$

Spare page, in case you ran out of room on one of the questions –
Indicate clearly which question you are writing.