5.5 Binary Search Trees
二叉搜索树

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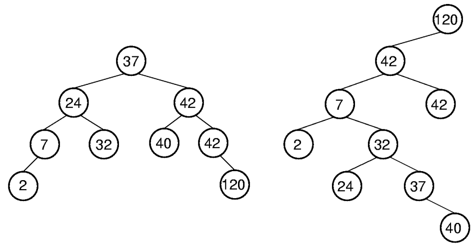
Defination of Binary Search Trees

- ➢ All items in the left subtree < the root.
- ➢ All items in the right subtree >= the root.
- ➢ Each subtree is itself a binary search tree.



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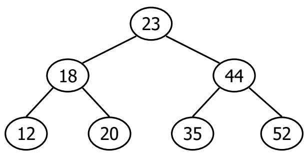
Binary Search Tree Traversals



preorder

23 18 12 20 44 35 52

postorder

12 20 18 35 52 44 23

inorder

12 18 20 23 35 44 52

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Binary Search Tree

- 搜索二叉树所涉及的基本操作
  - Search
  - Insert
  - remove
  - Deletemin
  - Traversal---print
    - inorder
- 1个指针 +1个整型变量就可描述一棵搜索二叉树
  - 1个根指针root指向BST的根结点
  - 1个整型变量nodecount存放BST中的结点数

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BST Class(1)

```
template <class E>
class BST {
private:
  BSTNode<E>* root;   // Root of the BST
  int nodecount;       // Number of nodes
  BSTNode<E>* findhelp( BSTNode<E>*, const E&) const;
  BSTNode<E>* inserthelp(BSTNode<E>*, const E&);
  BSTNode<E>* removehelp(BSTNode<E>*,
                     const E&,  BSTNode<E>*&);
  BSTNode<E>* deletemin(BSTNode<E>* , BSTNode<E>*&);
  void clearhelp(BSTNode<E>*);
  void printhelp(BSTNode<E>*, int) const;
```

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BST Class (2)

```
public:
   BST() { root = NULL; nodecount = 0; }

  ~BST() { clearhelp(root); }

  void clear() { clearhelp(root); root = NULL;
                 nodecount = 0; }

  bool remove(E& e) {
    BSTNode<E>* t = NULL;
    root = removehelp(root, e, t);
    if (t == NULL) return false;
    nodecount--;
    delete t;
    return true; }
```

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BST Class (3)

```
BSTNode<E>* find(const E& e) const
   { return findhelp(root, e); }

bool insert(const E& e) {
    root = inserthelp(root, e);
    nodecount++;
    return true; }

int size() { return nodecount; }

void print() const {      //相当于中序遍历
   if (root == NULL)
     cout << "The BST is empty.\n";
   else printhelp(root, 0);
}
};
```

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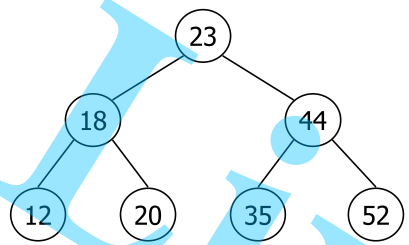
BST Search/find

步骤
1. 与根相比
2. 等于返回
3. 小于往左子树寻找
4. 大于往右子树寻找



1. Search 20
2. Search 45

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BST Class (4)-- findhelp

```
template <class E>  //返回指向找到结点的指针
BSTNode<E>* BST<E>:: findhelp( BSTNode<E>*
  subroot, const E& e) const {
  if (subroot == NULL) return NULL;
  if (e < subroot->element())
    return findhelp(subroot->left(), e);
  else if (e > subroot->element())
    return findhelp(subroot->right(), e);
  else { return subroot; }
}
```

O(d)

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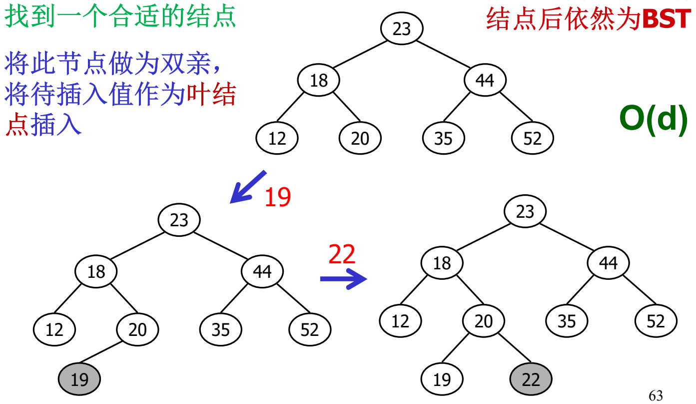
BST Insertion

步骤：

1. 找到一个合适的结点
2. 将此节点做为双亲，将待插入值作为叶结点插入

插入原则：插入新结点后依然为**BST**



O(d)

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Iterative (non-Recursive非递归) BST Insertion Algorithm

```
Algorithm    insertBST (ref root <pointer>, val new <pointer>)
//       Pre    root is address of the root;  new is address of the new node
//       Post   new node inserted into the tree
    if (root = null)      root = new
    else
        pWalk = root
        loop (pWalk not null)       // Location found for the new node
            parent = pWalk
            if (new –> data < pWalk –> data)
                pWalk = pWalk –> left
            else
                pWalk = pWalk –> right
        if (new –> data < parent –> data)
            parent –> left = new
        else
            parent –> right = new
    return
End insertBST
```

Step1：找到一个合适的结点

Step2：将此节点做为双亲，将待插入值做为叶结点插入

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BST Class (5-1)--BST Inserthelp (non-Recursive非递归)

```
template <class E>  BSTNode<E>*  BST<E>::
inserthelp( BSTNode<Elem>* subroot,  const E& val) {
  if (subroot == NULL) // Empty: create node
        return  new BSTNode<E>(val, NULL,NULL);
  BSTNode<E>  * temp = subroot;
  BSTNode<E>  *parent;
  while (temp!=NULL) {
        parent=temp;
        if (val < temp->element())   temp = temp->left();
        else   temp=temp->>right();  }
  temp=new BSTNode<Elem>(val, NULL,NULL);
   if (val < parent->element())     parent->setLeft(temp);
  else   parent->setRight(temp);
  return subroot;   // Return tree with node inserted
}
```

O(d)

Step1

Step2

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Recursive(递归) BST Insertion Algorithm

```
Algorithm   insertBST (ref root <pointer>, val new<pointer> )
// Inserts a new node into BST using recursion
//      Pre    root is address of the root
//             new is address of the new node
//      Post   new node inserted into the tree
        if (root == null)
            root = new
      else
           if (new –> data < root –> data)
                insertBST (root –> left, new)
           else
                insertBST (root –> right, new)
      return
End   addBST
```

比较待插入值与根节点的大小

1）如果小于，将待插入值插入左子树

2）否则，将待插入值插入右子树

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BST Class (5-2)-- BST Insert (Recursive 递归)

比较待插入值与根节点的大小

1）如果小于，将待插入值插入左子树

2）否则，将待插入值插入右子树

```
template <class E,>  BSTNode<E>* BST<E>::
 inserthelp (BSTNode<E>* subroot, const E& val) {
   if (subroot == NULL) // Empty: create node
         return new BSTNode<Elem>(val,NULL,NULL);
   if (val < subroot->element()))
         subroot->setLeft( inserthelp(subroot->left(), val));
   else
         subroot->setRight( inserthelp(subroot->right(), val));

   return subroot;  // Return subtree with node inserted
}
```

O(d)

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BST Deletion

删除原则：删除结点后依然为BST

定义：从BST树中删除给定值

Step1：找到值等于待删除值的结点

Step2：根据该结点的特点采取不同的删除策略

- Leaf node: set the deleted node's parent link to null. Simplest
- Node having only left subtree: attach the left subtree to the deleted node's parent. simple
- Node having only right subtree: attach the right subtree to the deleted node's parent. simple
- Node having both subtrees   difficult

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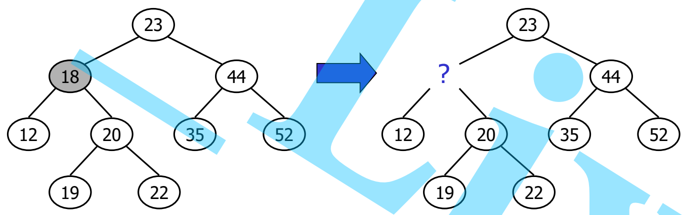
BST Deletion

---Node having both subtrees (1)

删除原则：删除结点后依然为BST

Example1：删除值为18的结点



从其子树中找一个值代替当前结点的值

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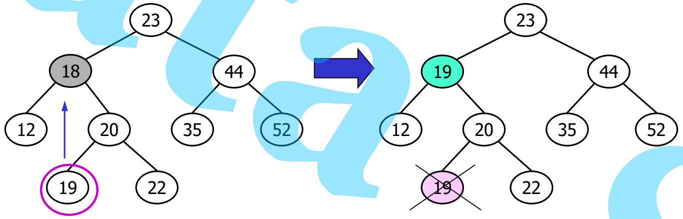
BST Deletion
---Node having both subtrees (2)

删除原则：删除结点后依然为BST

Using smallest value in the right subtree

Example1：删除值为**18**的结点



S1：找到右子树中最小值对应的结点
S2：用其值代替的当前结点的值
S3：删除该结点

叶子

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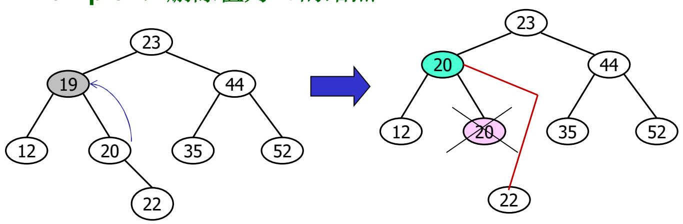
BST Deletion
---Node having both subtrees (3)

删除原则：删除结点后依然为BST

Using smallest node in the right subtree

Example2：删除值为**19**的结点



S1：找到右子树中最小值对应的结点
S2：用其值代替的当前结点的值
S3：删除该结点

只有右子树

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BST Deletion
---Node having both subtrees (4)

删除原则：删除结点后依然为BST

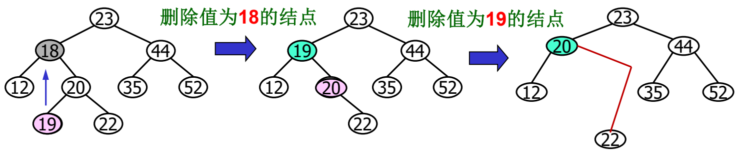
Using smallest node in the right subtree

① 找到右子树中的具有最小值的结点

✓ 要么为叶子结点，要么只有右子树

② 用该最小值替换待删结点的值

③ 删除具有最小值的结点（**simple**）



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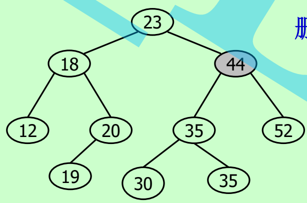
BST Delete

- Node having both subtrees
  - Using largest node in its left subtree

结合下图思考：*Why* 不用此策略？

删除值为**44**的结点



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BST Deletion

---算法步骤

Leaf Node or Node having only right subtree

Node having only left subtree

Step1： 找到根结点值等于待删除值的子树，

Step2： 如果该子树无左树，用该子树的右子树代替该子树。

否则如果该子树无右树，用该子树的左子树代替该子树

否则

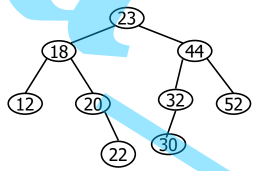
Node having both subtrees

① 找到右子树中的具有最小值的结点

✓ 要么为叶子结点，要么只有右子树

② 用该最小值替换待删结点的值

③ 删除具有最小值的结点



1 Delete 12

2 Delete 20

3 Delete 32

4 Delete 44

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BST Class (6)-- Removehelp

```
template <class E> BSTNode<E>*  BST<E>::
removehelp(BSTNode<E>* subroot, const E& e, BSTNode<E>* & t)  {
    if (subroot == NULL) return NULL;
    if (e < subroot->element( ) )
          subroot->setLeft(removehelp(subroot->left(), e, t));
    else if (e > subroot->element( )  )
          subroot->setRight(removehelp(subroot->right(), e, t));
    else {   // Found it
          BSTNode<E>* temp;    t=subroot;
           if (subroot->left() == NULL)     //没有左树
                 subroot = subroot->right();
          else if (subroot->right() == NULL)   //没有右树
                 subroot = subroot->left();
          else {    // Both children are non-empty
                 subroot->setRight(deletemin(subroot->right(), temp));
                 subroot->setElement(temp->element));  }  }
    return subroot;
}
```

O(d)

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BST Class (7)-- delete Minimum Value

带回两个指针

1. 指向已删除最小值的BST，返回值
2. 指向被删除的最小值结点，min

```
template <class E>
BSTNode<E>* BST<E>::
deletemin(BSTNode<E>* subroot,  BSTNode<E>* & min) {
  if (subroot->left() == NULL) {
    min = subroot;
    return subroot->right();
  }
  else {    // Continue left
     subroot->setLeft(deletemin(subroot->left(), min));
     return subroot;
  }
}
```

判断根节点是否有左子树

1）如果没有，根结点为最小值，返回其右子树

2）否则，从左子树中删除最小值（递归调用）

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BST Class (8)-- Printhelp (Inorder traversal)

```
template <class E>   void BST<E>::
printhelp(BSTNode<E>* subroot, int level) const
{
        if (subroot==NULL)  return;
         printhelp(subroot->left(),level+1);
         for(int i=0; i<level; i++)
                 cout << "*";
         cout<<subroot->element()<<endl;
         printhelp(subroot->right(),level+1);
 }   //中序遍历
```

O(n)

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BST Class (9)-- clearhelp ((postorder traversal)

```
template <class E>  void  BST<E>::
clearhelp(BSTNode<E>* subroot)
{
        if (subroot==NULL) return;
        clearhelp(subroot->left());
        clearhelp(subroot->right());
        delete subroot;
}
```

O(n)

后序遍历

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Cost of BST operations

search:

Insert:

Delete:

O(d)

Worst case: O(n) 单边树

Best case: O(log(n)) CBT

希望**BST**尽可能左右平衡

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An application example of BST

- 写一个程序，输入下列序列 构建**BST**，并测试插入，删除，查找, 打印等功能
- **37,24,42,7,2,40,42,32,120**
- **120,42,42,7,2,32,37,24,40**

输入序列顺序不同，构建的BST可能不同；
但是，中序遍历的结果却是绝对相同的。

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searchBT.h

课件 **P25-26, 61-63,65, 68(0r 70), 78-81**

结点class

BST class

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```
#include <iostream>
#include <stdio.h>
#include <stdlib.h>
#include "searchBT.h"
using namespace std;
void main() {
  BST<int> b1;
  int temp, i;
  cout<<" BST construction function test......"<<endl;
  cout<<"please input 9 int:";
  for(i=0;i<9;i++) {
        cin>>temp;   b1.insert(temp);   }
   cout<<" BST (inorder)"<<endl;    b1.print();
   cout<<" delete function test......"<<endl;
   cout<<"please input the data you want remove:";
   cin>>temp;
   b1.remove(temp);
   cout<<"after remove "<<temp<<"BST(inorder) is "<<endl;
   b1.print();
}
```

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5.6 Heaps(堆)

Also called Priority Queue
（优先队列）

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Priority Queue/优先队列

优先级高的先出队

When a collection of objects is organized by importance or priority, we call this a **Priority Queue**

基本操作：
Insert（Enqueue），插入一个新任务后依然需保持优先队列的特点
removeFirst（Dequeue)，　完成（删除）优先级最高任务后依然需保持优先队列的特点

实现：
一些简单的实现： list，BST
Heap（堆）： 普遍应用，和优先队列几乎被认为是同一个概念

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Defination of Heaps

①
Complete binary tree whose node with property:

1) value of any node **less than or equal** to that of its children (Min-heap, 小堆) or ②

2) value of any node **larger than or equal to** that of its children (Max-heap 大堆)

逻辑定义

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Defination of Heap

因为堆是CBT，所以堆通常用基于数组的方式来实现，即将BT中的结点按层由低到高，层内由左到右进行编号并存放于1维数组中，其结点下标满足下列关系式：

- PARENT(i) = (i-1)/2; /*父结点*/
- LEFT(i) = 2i+1; /* 左子结点 */
- RIGHT(i) = 2i+2; /* 右子结点 */
- n_0 =(int) ((n+1)/2) ; /* 叶子结点的个数 */

物理定义

Defination：n个元素组成的序列$\{k_0,k_1,k_2,\ldots,k_{n-1}\}$,当且仅当满足下列关系之一时，称之为堆

1） $k_i \le k_{2i+1}$,且 $k_i \le k_{2i+2}$, i =0,1,…, n/2-1 小堆

2) $k_i \ge k_{2i+1}$,且 $k_i \ge k_{2i+2}$, i =0,1,…, n/2-1 大堆

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Heap 与BST的区别

- BST ：
  - 左与右的关系
  - 不一定是CBT
  - 一般用基于指针的方式存储/实现
- heap:
  - 前辈与后辈的关系
  - 一定是CBT
  - 一般用基于数组的方式存储/实现

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Heap

- 1个数组 +2个整型变量就可描述一个堆
  - 1个数组存放heap中各结点的值
  - 1个整型变量maxSize存放数组的尺寸
  - 1个整型变量size存放 堆中的结点数
- heap所涉及的基本操作
  - Insert
  - remove
  - removeFirst
  - buildHeap

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maxHeap class(1)---Array based implement

```
template<class Elem> class maxHeap {
private:
  Elem* Heap;   // Pointer to the heap array
  int maxSize;    // Maximum size of the heap
  int size;       // Number of elems now in heap
  void siftDown(int); // Put element in place
public:
  maxHeap(Elem* h, int num, int max) {
    size=num;   maxSize=max;   Heap = new Elem[max];  }
  int heapSize() const  { return size  ; }
  bool isLeaf(int pos) const {
      return (pos >= size/2) && (pos < size);      }
  int leftChild(int pos) const  { return  2*pos+1; }
  int rightChild(int pos) const { return  2*pos+2;  }
  int parent(int pos) const   { return  (pos-1)/2; }
  void print( ) const   { …   }
  void clear( )   { …  }
  int find (const Elem&) { … }
```

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maxHeap class(2)---Array based implement

```
void buildHeap();
void insert(const Elem&);
Elem removeFirst( );
Elem remove(int);
};
```

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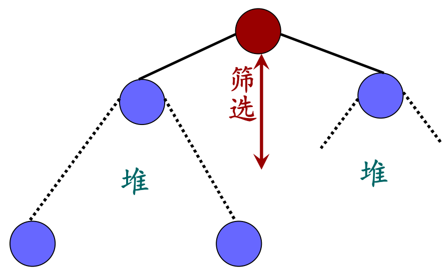
Siftdown /筛选

所谓“siftdown”指的是，对一棵左/右子树均为堆的完全二叉树，“调整”根结点使整个二叉树也成为一个堆。



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SiftDown

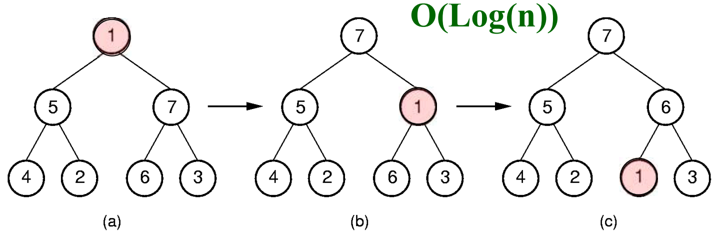
Siftdown/筛选的方法

S1： 将根结点作为当前结点

S2： 将当前结点值与其左、右子树的根结点值比较，并与三者中最大者进行交换；更新当前结点

S3：重复S2，直至叶子结点或无交换发生，所得结果即为堆。

O(Log(n))



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maxHeap class(3)--- SiftDown

```
template <class Elem>
void maxHeap<Elem>::siftDown(int pos) {
  while (!isLeaf(pos)) {
    int j = leftChild(pos);
    int rc = rightChild(pos);
    if ((rc<size) && (Heap[j]< Heap[rc]))
      j = rc;
    if ( Heap[pos] >= Heap[j] ) return;
    swap(Heap, pos, j); // 请自行写出该函数的代码
    pos = j;
  }
}
```

O(Log(n))

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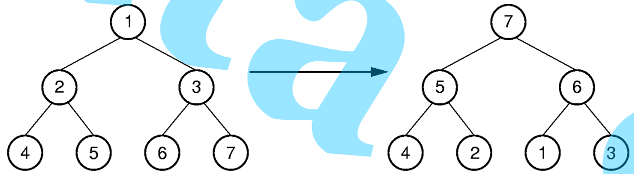
Building the MaxHeap

For fast heap construction:

•**Call siftdown for each item from high end(尾端) of array to low end（前端）** 从下往上/从后往前

•**Don't need to call siftDown on leaf nodes.(why?)**



(7-3), (5-2), (7-1), (6-1) **共交换4次**

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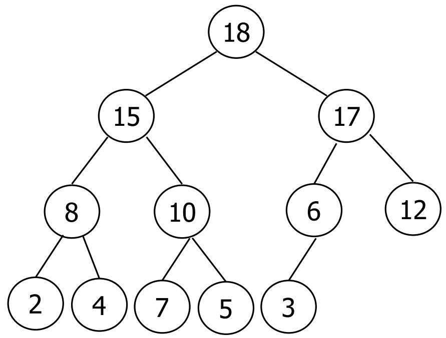
下列数组/**CBT**是大堆吗？若不是，请构建(要求写出具体过程)

5, 10, 12, 8, 15, 6, 17 , 2, 4, 7, 18, 3



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maxHeap class(4)--- BuildingHeap

```
template <class Elem>
void maxHeap<Elem>:: buildHeap( ) {
   for(i = size/2-1; i >= 0; i--)
        siftDown(i);
}
```

O(n)

f(n)的具体计算公式见课本p184

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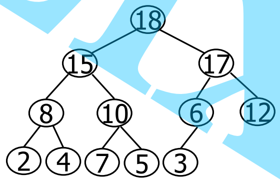
Insert a value in the MaxHeap

思路：

- 在堆末尾添加一取值为待插入值的叶子结点，作为当前结点，并size加1
- 将当前结点值与其双亲结点值比较，若大于则进行交换，并将其双亲作为当前节点；
- 重复上述操作，直至当前结点值小于等于其双亲结点值 或到达根结点。



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maxHeap class(5)--- Insert

```
template <class Elem>
void maxHeap<Elem>:: insert(const Elem& e)
{
    Assert( size < maxSize, "Heap is full");
    int curr = size;
    Heap[curr] = e;  size++;
    while(curr!=0 && Heap[curr]>Heap[parent(curr)])  {
          swap(Heap, curr,parent(curr)); curr=parent(curr);
    }
    return true;
}
```

O(Log(n))

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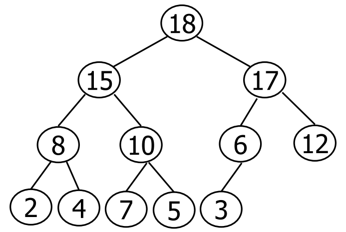
Remove First value in the Maxheap

思路：

作用？

- 将根结点值与最末叶子结点值进行交换，并size减1
- 对根结点 做 siftDown 操作



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maxHeap class (6)--Remove First Value

O(Log(n))

```
template <class Elem>
Elem maxHeap<Elem>::
removeFirst() {
  Assert ( size > 0, "Heap is empty");
  swap(Heap, 0, --size);   // Swap First with end
  if (size != 0) siftDown(0);
  return Heap[size];  // Return First value
}
```

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Remove 给定下标位置的值从maxHeap

思路：

作用？

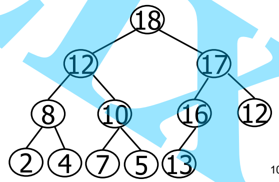
①将待删除结点作为当前结点

②将当前结点与最末叶子结点进行值交换，并size减1。

③将当前结点值与其双亲结点值比较，若大于则进行交换，同时将其双亲作为当前节点；

④重复步骤3，直至当前结点值小于其双亲结点值 或 到达根结点。

⑤对当前结点调用 siftDown



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maxHeap class(7) --Remove

```
template <class Elem>
Elem maxHeap<Elem>::remove(int pos) {
  Assert((pos>=0)&&(pos<size),"Bad position");
  if ( pos == size-1 ) size--;
  else {
    swap(Heap, pos, --size);
    while ((pos != 0) &&
           (Heap[pos] > Heap[parent(pos)])){
      swap(Heap, pos, parent(pos));
      pos = parent(pos);
    }
    if (size != 0) siftDown(pos);   }
  return Heap[size];
}
```

O(Log(n))

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An application example of heap

- 写一个程序，输入下列序列构建**maxHeap**，并测试插入，删除，查找，清空，打印等功能
- 1 2 3 4 5 6 7 8 9
- 5 2 9 3 7 6 8 4 1

输入序列顺序不同，构建的heap可能不同；但是，重复removeFirst直到堆为空得到的结果却是绝对相同的。

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```
……
#include "heap.h"   // maxHead class--课件中PP91,92,95,97,101,103
using namespace std;
void main() {
    int i;   double a[100], temp;
    cout<<"please input 7 data:"<<endl;
    for(i=0;i<7;i++)   cin>>a[i];
    maxHeap<double>  h1(a,7,100);
    cout<<"after buildHeap the heap is:"<<endl;
    h1.buildHeap();   h1.print();     cout<<endl;
    cout<<"insert function test........"<<endl;
    cout<<"please input the insert data:";
    cin>>temp;    h1.insert(temp);
    cout<<"after insert "<<temp<<" the heap is:"<<endl;
    h1.print();  cout<<endl;
    cout<<"removeFirst function test........"<<endl;
    while(h1.heapSize()) {
      temp=h1.removeFirst( );  cout<<temp<<"  ";    }
    cout<<endl;
    ......
}
```

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本章作业二

- 5.9
- 5.13 (b)
- 5.16

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