《习题册》参考答案及解答

第1-2页《样本空间与事件》

$$\equiv$$
. 1. $ig(1ig)\Omega=ig\{ig(x,yig):0\leq x\leq 1,0\leq y\leq 1ig\}$,

$$A=\left\{ \left(x,y
ight) \colon x^{2}+y^{2}<rac{1}{4},x\geq0,y\geq0
ight\}$$

$$B = \left\{ \left(x,y
ight) \colon 0 \leq x < y, rac{1}{3} < y \leq 1
ight\}$$

$$ig(2ig)\Omega = ig\{ig(i,jig): i,j=1,2,3,4,5,6ig\}$$

$$A = ig\{ig(i,jig): i = 1,3,5, j = 1,2,3,4,5,6ig\} \qquad \quad B = ig\{ig(1,3ig),ig(3,1ig),ig(2,2ig)ig\}$$

2.
$$(1)A\bar{B}\bar{C} = \left\{\omega: 0 \le \omega < \frac{1}{4}\right\}$$
 $(2)\bar{A}\bar{B}\bar{C} = \left\{\omega: \omega = 1\right\} = \left\{1\right\}$

$$ig(3ig)ar{A}\cupar{B}\cupar{C}=\overline{ABC}=ig\{\omega:0\leq\omega\leq1ig\}=\Omega$$

$$\left(4
ight)ar{A}\left(B\cup C
ight)=\left\{\omega:rac{1}{3}\leq\omega<1
ight\} \qquad \left(5
ight)A\cup B\cup C=\left\{\omega:0\leq\omega<1
ight\}$$

$$(2)$$
至少两个发生

$$(3)$$
A 发生,且 B , C 至少一个不发生 (4) 至多一个发生

第 3-4 页《概率的性质与古典概率》

$$P(\overline{A}) = 0.3 \Rightarrow P(A) = 0.7 \Rightarrow P(AB) = P(A) - P(A\overline{B}) = 0.3$$
$$\Rightarrow P(\overline{A} \cup \overline{B}) = P(\overline{AB}) = 1 - P(AB) = 1 - 0.3 = 0.7$$

$$\equiv$$
. 1. $\frac{2}{9}$

$$P(AC) = 0 \Rightarrow P(ABC) = 0$$

 $\Rightarrow P(\bar{A}\bar{B}\bar{C}) = P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$

$$= 1 - \mathbf{P}(A) - \mathbf{P}(B) - \mathbf{P}(C) + \mathbf{P}(AB) + \mathbf{P}(AC) + \mathbf{P}(BC) - \mathbf{P}(ABC)$$

$$= 1 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3} + \frac{1}{9} + \frac{1}{9} + 0 - 0 = \frac{2}{9}$$

2.
$$\frac{1}{12}$$
 3. $1-q$

三. 1. 教材习题一(A)三第 5 题

解答:
$$因 \mathbf{P}(A - B) = \mathbf{P}(A) - \mathbf{P}(AB)$$
, $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(AB)$, 则 $\mathbf{P}(A) - \mathbf{P}(B) \le \mathbf{P}(A) - \mathbf{P}(AB) \le \mathbf{P}(A) \le \mathbf{P}(A \cup B) \le \mathbf{P}(A) + \mathbf{P}(B)$, 即 $\mathbf{P}(A) - \mathbf{P}(B) \le \mathbf{P}(A - B) \le \mathbf{P}(A) \le \mathbf{P}(A \cup B) \le \mathbf{P}(A) + \mathbf{P}(B)$.

2. 证明:

$$\begin{split} \mathbf{P}\big(A\big) &\geq \mathbf{P}\big(A\big(B \cup C\big)\big) = \mathbf{P}\big(AB \cup A\,C\big) = \mathbf{P}\big(AB\big) + \mathbf{P}\big(A\,C\big) - \mathbf{P}\big(ABC\big) \\ &\geq \mathbf{P}\big(AB\big) + \mathbf{P}\big(A\,C\big) - \mathbf{P}\big(B\,C\big)\,, \end{split}$$

故

$$P(AB) + P(AC) - P(BC) \le P(A)$$

3.
$$(1) \frac{A_8^5}{8^5} = 0.205$$
 $(2) 1 - \frac{A_8^5}{8^5} = 0.795$ $(3) \frac{C_5^2 C_8^1 (A_7^3 + A_7^1)}{8^5} = 0.5298$

4. 教材习题一(**A**)三第 10 题

(1) 有放回抽取时:
$$\mathbf{P}(A) = \frac{2^2}{6^2} = \frac{1}{9}$$
; $\mathbf{P}(B) = \frac{4 \times 2 + 2 \times 4}{6^2} = \frac{4}{9}$; 因 $C = A \cup B$ 且 $AB = \varnothing$,所以 $\mathbf{P}(C) = \mathbf{P}(A) + \mathbf{P}(B) = \frac{5}{9}$

(2) 无放回抽取时:

用排列计算:
$$\mathbf{P}ig(Aig) = rac{\mathbf{A}_2^2}{\mathbf{A}_6^2} = rac{1}{15}$$
; $\mathbf{P}ig(Big) = rac{\mathbf{A}_4^1\mathbf{A}_2^1 + \mathbf{A}_2^1\mathbf{A}_4^1}{\mathbf{A}_6^2} = rac{8}{15}$; 因 $C = A \cup B$ 且 $AB = \varnothing$,所以 $\mathbf{P}ig(Cig) = \mathbf{P}ig(Aig) + \mathbf{P}ig(Big) = rac{9}{15} = rac{3}{5}$

用组合计算:
$$P(A) = \frac{C_2^2}{C_6^2} = \frac{1}{15}$$
; $P(B) = \frac{C_4^1 C_2^1}{C_6^2} = \frac{8}{15}$; 因 $C = A \cup B$ 且

$$AB=arnothing$$
,所以 $extbf{P}ig(Cig) = extbf{P}ig(Aig) + extbf{P}ig(Big) = rac{9}{15} = rac{3}{5}$

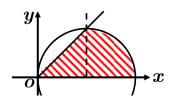
5.
$$p = \frac{\text{C}_{4}^{1}\text{C}_{13}^{7}\text{C}_{13}^{3}\text{C}_{13}^{1}\text{C}_{13}^{2}}{\text{C}_{52}^{13}}$$

第5-6页《几何概率、条件概率及乘法公式》

—. 1. **В** 2. **D** 3. **D**

$$\equiv$$
. 1. $\frac{2+\pi}{2\pi}$

这是一个二维几何概型问题:如图



样本空间为中心在 $\left(a,0\right)$ 处,半径为a的上半圆,其面积为 $m\left(\Omega\right)=rac{1}{2}\pi a^2$,设A表事

件"原点与该点的连线与x轴的夹角小于 $\frac{\pi}{4}$ ",则 $m(A) = \frac{1}{4}\pi a^2 + \frac{1}{2}a^2$,所以所求概

率为
$$\mathbf{P}ig(Aig) = rac{mig(Aig)}{mig(\Omegaig)} = rac{rac{1}{4}\pi a^2 + rac{1}{2}a^2}{rac{1}{2}\pi a^2} = rac{2+\pi}{2\pi}.$$

2. $\frac{3}{8}$

因事件A发生导致事件B发生,则 $A\subset B$ 或AB=A,事件B与事件C互斥,则 $BC=\emptyset$ 或 $B\subset ar{C}$,从而有 $ABar{C}=A$, $Bar{C}=B$,于是

$$\mathbf{P}\!\left(A\middle|B\bar{C}\right) = \frac{\mathbf{P}\!\left(AB\bar{C}\right)}{\mathbf{P}\!\left(B\bar{C}\right)} = \frac{\mathbf{P}\!\left(A\right)}{\mathbf{P}\!\left(B\right)} = \frac{\mathbf{0.3}}{\mathbf{0.8}} = \frac{\mathbf{3}}{8}$$

3. $\frac{6}{7}$

设 $oldsymbol{A}$ = "至少有一个女孩", $oldsymbol{B}$ = "至少有一个男孩",则 $oldsymbol{ar{A}}$ = "三个孩子全是男孩"

 \overline{AB} = "三个孩子全是男孩或全是女孩",从而有 $\mathbf{P}(\overline{A}) = \frac{1}{8}$, $\mathbf{P}(\overline{AB}) = \frac{2}{8}$,故所求

概率为
$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{1 - P(\overline{AB})}{1 - P(\overline{A})} = \frac{1 - \frac{2}{8}}{1 - \frac{1}{8}} = \frac{6}{7}$$

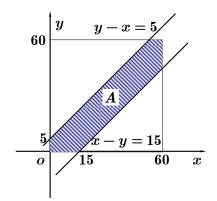
三. 1. **0.2986**

设王同学于 9 点 X 分到达,张同学于 9 点 Y 分到达,如图,则

$$\Omega = \left\{ \left(X, Y
ight) : 0 \leq X \leq 60, 0 \leq Y \leq 60
ight\}$$

设A = "两同学能见面",则

$$A = ig\{ig(X,Yig) : 0 \leq Y - X \leq 5ig\} \cup igg\{ig(X,Yig) : 0 \leq X - Y \leq 15ig\}$$



则所求概率为

$$p = rac{m\left(A
ight)}{m\left(\Omega
ight)} = 1 - rac{1}{2}rac{55^2 + 45^2}{60^2} = 0.2986$$

2. 0.7283

设A = "该种动物活到 10 岁",B = "该种动物活到 15 岁",由已知条件得所求

概率为
$$p = P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.67}{0.92} = 0.7283$$

3. **0.2333**, **0.4651**

因 $A\bar{B}=A-B$,所以

$$P(A|\bar{B}) = \frac{P(A\bar{B})}{P(\bar{B})} = \frac{P(A-B)}{1-P(B)} = \frac{0.14}{1-0.4} = \frac{0.2333}{1-0.4}$$

$$\overline{A} \cup B = \overline{\overline{A}}\overline{B} = A\overline{B} = A - B$$

$$\Rightarrow \mathbf{P}\left(\overline{A} \cup B\right) = 1 - \mathbf{P}\left(\overline{\overline{A} \cup B}\right) = 1 - \mathbf{P}(A - B) = 1 - 0.14 = 0.86$$

$$\Rightarrow P(B|\overline{A} \cup B) = \frac{P(B(\overline{A} \cup B))}{P(\overline{A} \cup B)} = \frac{P(B)}{P(\overline{A} \cup B)} = \frac{0.4}{0.86} = \frac{0.4651}{0.86}$$

4.
$$\mathbf{P}(B|A) = \frac{\mathbf{P}(AB)}{\mathbf{P}(A)} = \frac{\mathbf{P}(A) - \mathbf{P}(A\overline{B})}{\mathbf{P}(A)} \ge \frac{\mathbf{P}(A) - \mathbf{P}(\overline{B})}{\mathbf{P}(A)} = 1 - \frac{1 - p_2}{p_1}$$

设 $oldsymbol{A}=$ "甲机第一次攻击并击落乙机", $oldsymbol{B}=$ "乙机第一次攻击并击落甲机", $oldsymbol{C}=$ "甲机第二次攻击并击落乙机",则

(1)
$$P(B) = P(AB) + P(\overline{AB}) = 0 + P(\overline{A})P(B|\overline{A}) = 0.8 \times 0.3 = 0.24$$
;

(2)
$$P(A \cup C) = P(A) + P(C) - P(AC) = 0.2 + P(\overline{ABC}) - 0$$

 $= 0.2 + P(\overline{A})P(\overline{B}|\overline{A})P(C|\overline{AB})$
 $= 0.2 + 0.8 \times 0.7 \times 0.4 = 0.424$.

第7-8页《全概率与贝叶斯公式、事件的独立性与贝努利概型》

一. 1. **C**

$$P(\bar{B}|\bar{A} \cup B) = \frac{P(\bar{B}(\bar{A} \cup B))}{P(\bar{A} \cup B)} = \frac{P(\bar{A}\bar{B})}{P(\bar{A} \cup B)} = \frac{P(\bar{A})P(\bar{B})}{P(\bar{A}) + P(B) - P(\bar{A}B)}$$
$$= \frac{P(\bar{A})P(\bar{B})}{P(\bar{A}) + P(B) - P(\bar{A})P(B)} = \frac{0.2 \times 0.6}{0.2 + 0.4 - 0.2 \times 0.4} = 0.2308$$

2. **D**

3. **C**

$$P(A|B) + P(\bar{A}|\bar{B}) = 1 \Rightarrow P(A|B) = 1 - P(\bar{A}|\bar{B}) = P(A|\bar{B}) = \frac{P(A\bar{B})}{P(\bar{B})}$$
 $\Rightarrow P(A|B) = \frac{P(A) - P(AB)}{1 - P(B)} \Rightarrow P(A|B)[1 - P(B)] = P(A) - P(AB)$
 $\Rightarrow P(A|B) = P(A) \Rightarrow A = B$ 相互独立.

$$\equiv$$
. 1. $p^3(2-p)$

2.
$$\frac{48}{60}$$

因A与B互斥,故 $A\bar{C}$ 与 $B\bar{C}$ 互斥,从而有

$$\begin{split} \mathbf{P}\Big(A \cup B \middle| \bar{C}\Big) &= \frac{\mathbf{P}\Big(\!\big(A \cup B\big)\bar{C}\big)}{\mathbf{P}\Big(\bar{C}\Big)} = \frac{\mathbf{P}\Big(A\,\bar{C} \cup B\,\bar{C}\big)}{\mathbf{P}\Big(\bar{C}\Big)} = \frac{\mathbf{P}\Big(A\,\bar{C}\Big) + \mathbf{P}\Big(B\,\bar{C}\Big)}{\mathbf{P}\Big(\bar{C}\Big)} \\ &= \frac{\mathbf{P}\Big(A\Big)\mathbf{P}\Big(\bar{C}\Big) + \mathbf{P}\Big(B\Big)\mathbf{P}\Big(\bar{C}\Big)}{\mathbf{P}\Big(\bar{C}\Big)} = \mathbf{P}\Big(A\Big) + \mathbf{P}\Big(B\Big) = 0.8 \end{split}$$

思考题:一般情况下,A与C独立,B与C独立,则 $A \cup B$ 与C也独立吗?

设同学数为n,则由题意有 $1-\left(1-p
ight)^n=1-0.94^n\geq 0.95\Rightarrow n\geq 49$

三. 1. 0.15, 最可能乘火车

设 A_1,A_2,A_3,A_4 分别表他乘火车,轮船,汽车,飞机去上海参加会议,则 A_1,A_2,A_3,A_4 构成一个完备事件组, B 表他开会迟到,由题目已知条件可得

$$\left(1\right) \mathbf{P}\!\left(B\right) = \sum_{i=1}^{4} \mathbf{P}\!\left(A_{i}\right) \mathbf{P}\!\left(B\middle|A_{i}\right) = 0.3 \times \frac{1}{4} + 0.2 \times \frac{1}{3} + 0.1 \times \frac{1}{12} + 0.4 \times 0 = 0.15$$

$$(2) \mathbf{P} (A_{_{1}} | B) = \frac{\mathbf{P} (B | A_{_{1}}) \mathbf{P} (A_{_{1}})}{\sum_{_{i=1}^{4}}^{4} \mathbf{P} (A_{_{i}}) \mathbf{P} (B | A_{_{i}})} = \frac{0.3 \times \frac{1}{4}}{0.15} = 0.5$$

$$\mathbf{P}ig(A_2ig|Big) = rac{\mathbf{P}ig(Big|A_2ig)\mathbf{P}ig(A_2ig)}{\sum\limits_{i=1}^4\mathbf{P}ig(A_iig)\mathbf{P}ig(Big|A_iig)} = rac{\mathbf{0.2} imesrac{1}{3}}{\mathbf{0.15}} = \mathbf{0.4444}$$

$$\mathbf{P}ig(A_{_3}ig|Big) = rac{\mathbf{P}ig(Big|A_{_3}ig)\mathbf{P}ig(A_{_3}ig)}{\sum\limits_{i=1}^4\mathbf{P}ig(A_{_i}ig)\mathbf{P}ig(Big|A_{_i}ig)} = rac{0.1 imesrac{1}{12}}{0.15} = 0.0556$$

$$\mathbf{P}ig(A_{_4}ig|Big) = rac{\mathbf{P}ig(Big|A_{_4}ig)\mathbf{P}ig(A_{_4}ig)}{\sum\limits_{i=1}^4\mathbf{P}ig(A_{_i}ig)\mathbf{P}ig(Big|A_{_i}ig)} = rac{0.4 imes0}{0.15} = 0$$

因在诸 $\mathbf{P}(A_i|B)(i=1,2,3,4)$ 中, $\mathbf{P}(A_1|B)$ 最大,所以,若他迟到了,他最可能是乘火车去的.

2. 0.0171

设 \boldsymbol{A} 表"被检验者经检验认为没有患关节炎", \boldsymbol{B} 表"被检验者患关节炎",由贝叶斯公式有

$$\mathbf{P}\left(B\middle|A\right) = \frac{\mathbf{P}\left(B\right)\mathbf{P}\left(A\middle|B\right)}{\mathbf{P}\left(B\right)\mathbf{P}\left(A\middle|B\right) + \mathbf{P}\left(\bar{B}\right)\mathbf{P}\left(A\middle|\bar{B}\right)} = \frac{0.15 \times 0.1}{0.15 \times 0.1 + 0.9 \times 0.96} = 0.0171$$

3. (1)0.2157, (2)0.4095, 0.7678

(1) 设 A_i 表"从甲箱中取出的两件产品中有i(i=0,1,2)件次品",B表"从乙箱中取得次品",由全概率公式有

$$\mathbf{P}ig(Big) = \sum_{i=0}^{2} \mathbf{P}ig(A_{_{i}}ig)\mathbf{P}ig(Big|A_{_{i}}ig) = \sum_{i=0}^{2} rac{\mathbf{C}_{_{10}}^{2-i}\mathbf{C}_{_{5}}^{i}}{\mathbf{C}_{_{15}}^{2}} rac{\mathbf{C}_{_{3+i}}^{1}}{\mathbf{C}_{_{17}}^{1}} = rac{11}{51} = \mathbf{0.2157}$$

ig(2ig) 设 $oldsymbol{C_1}$ 为选自甲箱, $oldsymbol{C_2}$ 为选自乙箱, $oldsymbol{B_i}$ 表第 $oldsymbol{i}ig(i=1,2ig)$ 次取出正品,由全概率公式

$$\begin{split} \mathbf{P} \Big(B_{_{1}} \overline{B}_{_{2}} \cup \overline{B}_{_{1}} B_{_{2}} \Big) &= \mathbf{P} \Big(C_{_{1}} \Big) \mathbf{P} \Big(B_{_{1}} \overline{B}_{_{2}} \cup \overline{B}_{_{1}} B_{_{2}} \Big| C_{_{1}} \Big) + \mathbf{P} \Big(C_{_{2}} \Big) \mathbf{P} \Big(B_{_{1}} \overline{B}_{_{2}} \cup \overline{B}_{_{1}} B_{_{2}} \Big| C_{_{2}} \Big) \\ &= \frac{1}{2} \frac{\mathbf{C}_{_{5}}^{^{1}} \mathbf{C}_{_{10}}^{^{1}}}{\mathbf{C}_{_{15}}^{^{2}}} + \frac{1}{2} \frac{\mathbf{C}_{_{3}}^{^{1}} \mathbf{C}_{_{12}}^{^{1}}}{\mathbf{C}_{_{15}}^{^{2}}} = \frac{43}{105} = 0.4095 \end{split}$$

由条件概率公式及全概率公式有

$$\begin{split} \mathbf{P} \Big(B_{_{1}} \Big| \bar{B}_{_{2}} \Big) &= \frac{\mathbf{P} \Big(B_{_{1}} \bar{B}_{_{2}} \Big)}{\mathbf{P} \Big(\bar{B}_{_{2}} \Big)} = \frac{\mathbf{P} \Big(C_{_{1}} \Big) \mathbf{P} \Big(B_{_{1}} \bar{B}_{_{2}} \Big| C_{_{1}} \Big) + \mathbf{P} \Big(C_{_{2}} \Big) \mathbf{P} \Big(B_{_{1}} \bar{B}_{_{2}} \Big| C_{_{2}} \Big)}{\mathbf{P} \Big(C_{_{1}} \Big) \mathbf{P} \Big(\bar{B}_{_{2}} \Big| C_{_{1}} \Big) + \mathbf{P} \Big(C_{_{2}} \Big) \mathbf{P} \Big(\bar{B}_{_{2}} \Big| C_{_{2}} \Big)} \\ &= \frac{\frac{1}{2} \times \frac{10}{15} \times \frac{5}{14} + \frac{1}{2} \times \frac{12}{15} \times \frac{3}{14}}{\frac{1}{2} \times \frac{5}{15} + \frac{1}{2} \times \frac{3}{15}} = \frac{43}{56} = 0.7679 \end{split}$$

4. 0.2098, 0.0621

$$\left(1
ight)p=rac{ ext{C}_{40}^{3} ext{C}_{10}^{2}}{ ext{C}_{50}^{5}}=0.2098$$

$$\left(2\right)p = 1 - rac{ ext{C}_{40}^5}{ ext{C}_{50}^5} = 0.0621$$

第9-10页《第一章综合练习》

- 一. 1. **B** 2. **A**
- 二. 1. **0.1837**

设 A, B 分别表甲乙击中靶子,则所求概率为

$$\begin{split} \mathbf{P} \Big(A \, \overline{B} \Big| \, A \cup B \Big) &= \frac{\mathbf{P} \Big(A \, \overline{B} \Big)}{\mathbf{P} \Big(A \cup B \Big)} = \frac{\mathbf{P} \Big(A \Big) \mathbf{P} \Big(\overline{B} \Big)}{\mathbf{P} \Big(A \Big) + \mathbf{P} \Big(B \Big) - \mathbf{P} \Big(A \Big) \mathbf{P} \Big(B \Big)} \\ &= \frac{0.9 \times \Big(1 - 0.8 \Big)}{0.9 + 0.8 - 0.9 \times 0.8} = 0.1837 \end{split}$$

2.
$$\frac{1}{4}$$

三. 1. 教材习题一 (\mathbf{B}) 三第 1 题

证明:
$$\mathbf{P}(\overline{A}\overline{B}) = \mathbf{P}(\overline{A \cup B}) = \mathbf{1} - \mathbf{P}(A \cup B) = \mathbf{1} - \mathbf{P}(A) - \mathbf{P}(B) + \mathbf{P}(AB)$$

 $= \mathbf{1} - p - (\mathbf{1} - \sqrt{p}) + \mathbf{P}(AB) = \sqrt{p} - p + \mathbf{P}(AB)$
 $\geq \sqrt{p} - p(\because \mathbf{P}(AB) \geq 0) > 0(\because 0 p)$

2. 教材习题一 (\mathbf{B}) 三第 4 题

解答:设
$$A_1$$
, A_2 , A_3 分别表在 100, 150, 200 米处击中动物,由 $\mathbf{P} \Big(A_1 \Big) = \mathbf{0.6} = \frac{k}{100}$ 得 $k = 60$,从而得 $\mathbf{P} \Big(A_1 \cup \overline{A}_1 A_2 \cup \overline{A}_1 \overline{A}_2 A_3 \Big) = \mathbf{P} \Big(A_1 \Big) + \mathbf{P} \Big(\overline{A}_1 \Big) \mathbf{P} \Big(A_2 \Big| \overline{A}_1 \Big)$ $+ \mathbf{P} \Big(\overline{A}_1 \Big) \mathbf{P} \Big(\overline{A}_2 \Big| \overline{A}_1 \Big) \mathbf{P} \Big(A_3 \Big| \overline{A}_1 \overline{A}_2 \Big) = \mathbf{0.6} + \mathbf{0.4} \times \frac{60}{150} + \mathbf{0.4} \times \frac{90}{150} \times \frac{60}{200}$ $= \mathbf{0.832}$

3. 教材习题一**(B)**三第 5 题

解答: A表"选正确答案", B表"知道正确答案", 由贝叶斯公式得

$$egin{aligned} \mathbf{P}ig(m{B}ig|m{A}ig) &= rac{\mathbf{P}ig(m{A}ig|m{B}ig)\mathbf{P}ig(m{B}ig)}{\mathbf{P}ig(m{A}ig)\mathbf{P}ig(m{B}ig)+\mathbf{P}ig(m{A}ig|ar{m{B}}ig)\mathbf{P}ig(ar{m{B}}ig)} \end{aligned} = rac{p imes 1}{p imes 1+ig(1-pig) imes rac{1}{m}} = rac{mp}{mp+1-p}$$

4. 教材习题一**(B)**三第 3 题

解答:设A为"甲系统有效",B为"乙系统有效",则由题意有

$$P(A) = 0.92, \ P(B) = 0.93, \ P(B|\overline{A}) = 0.85,$$

从而有

$$P(AB) = P(B) - P(\overline{A}B) = P(B) - P(\overline{A})P(B|\overline{A}) = 0.962.$$

$$(1) P(A \cup B) = P(A) + P(B) - P(AB) = 0.988,$$

$$(2)P(A|\overline{B}) = \frac{P(A\overline{B})}{P(\overline{B})} = \frac{P(A) - P(AB)}{1 - P(B)} = 0.8286.$$

5. 教材习题一**(B)**三第8题

解答:每个能出厂的概率为 $p = 0.7 + 0.3 \times 0.8 = 0.94$,所以

- (1) 全部都能出厂的概率为 0.94^n ;
- (2) 恰有两个不能出厂的概率为 $C_n^2 0.06^2 0.94^{n-2}$.

第 11-12页《分布函数及离散型随机变量》

一. 1. $\bf B$ (利用分布函数的性质判断) 2. 1.

二. 1. 3 或7 (即
$$P(X = 2) = \frac{7}{30} = \frac{m}{10} \cdot \frac{10 - m}{9}$$
) 2. $\frac{11}{24}$

三.1. 教材习题二(A)三第1题

① 当
$$x < 0$$
时,显然有 $\mathbf{F}(x) = \mathbf{P}(X \le x) = \mathbf{P}(\varnothing) = 0$;

- ② 当 $0 \le x \le a$ 时,由题意有 $\mathbf{P}ig(X \in ig[0,xig]ig) = \delta x^3$ 且 $\mathbf{P}ig(X \in ig[0,aig]ig) = 1$,联立两式解得 $\delta = a^{-3}$,从而 $\mathbf{F}ig(xig) = \mathbf{P}ig(X \le xig) = \mathbf{P}ig(X \in ig[0,xig]ig) = a^{-3}x^3$;
- ③ 当x > a时,有 $\mathbf{F}(x) = \mathbf{P}(X \le x) = \mathbf{P}(\Omega) = 1$;

从而分布函数为
$$\mathbf{F}ig(xig) = egin{cases} 0, & x < 0 \ a^{-3}x^3, & 0 \leq x \leq a \ 1 & x > a \end{cases}$$

$$ext{P}iggl(rac{a}{3} < X \leq rac{2}{3}aiggr) = ext{F}iggl(rac{2}{3}aiggr) - ext{F}iggl(rac{a}{3}iggr) = a^{-3}iggl[iggl(rac{2}{3}aiggr)^3 - iggl(rac{a}{3}iggr)^3iggr] = rac{7}{27}\,.$$

2. 教材习题二(A)三第 2 题

由分布函数在x=1和x=2处的右连续性有0=a,1=a+b,解之得a=0,b=1

$$\mathbf{P}\!\left(X > \frac{3}{2}\right) = 1 - \mathbf{F}\!\left(\frac{3}{2}\right) = 1 - \left[a + b\!\left(\frac{3}{2} - 1\right)^{\!2}\right] = 1 - \left(\frac{3}{2} - 1\right)^{\!2} = \frac{3}{4}$$

3. 教材习题二(A)三第3题

显然,
$$X$$
的可能取值为 $1,2,3$ 且 $\mathbf{P}(X=1) = \frac{\mathbf{C}_3^1}{\mathbf{C}_4^2} = \frac{1}{2}; \mathbf{P}(X=2) = \frac{\mathbf{C}_2^1}{\mathbf{C}_4^2} = \frac{1}{3};$

$$\mathbf{P}ig(X=3ig) = rac{\mathbf{C}_1^1}{\mathbf{C}_4^2} = rac{1}{6}; \;\; egin{array}{c} igt X \sim egin{bmatrix} 2 & 3 & 4 \ rac{1}{2} & rac{1}{3} & rac{1}{6} \end{bmatrix}, \;\; eta$$
市函数为 $\mathbf{F}ig(xig) = egin{bmatrix} 0, & x < 1 \ rac{1}{2}, & 1 \le x < 2 \ rac{5}{6}, & 2 \le x < 3 \ 1, & x \ge 3 \end{bmatrix}$

4. 教材习题二(A)三第 4 题

显然 $m{X}$ 可能取 $m{0,1,2,3}$; 设 $m{A}_i$ 表 "在第 $m{i}ig(m{i=1,2,3}ig)$ 路口遇到红灯",则

$$\mathbf{P}ig(A_{i}ig) = rac{1}{2}, i = 1, 2, 3$$
 且 A_{1}, A_{2}, A_{3} 相互独立,所以有

$$\mathbf{P}\!\left(X=0\right)\!=\mathbf{P}\!\left(A_{_{\!\scriptscriptstyle 1}}\right)\!=\!\frac{1}{2};$$

$$\mathbf{P}(X=1) = \mathbf{P}(\overline{A}_{_{1}}A_{_{2}}) = \mathbf{P}(\overline{A}_{_{1}})\mathbf{P}(A_{_{2}}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4};$$

$$\mathbf{P}(X=2) = \mathbf{P}(\bar{A}_{_{1}}\bar{A}_{_{2}}A_{_{3}}) = \mathbf{P}(\bar{A}_{_{1}})\mathbf{P}(\bar{A}_{_{2}})\mathbf{P}(A_{_{3}}) = \frac{1}{8};$$

$$\mathbf{P}\big(X=3\big) = \mathbf{P}\big(\overline{A}_{_{1}}\overline{A}_{_{2}}\overline{A}_{_{3}}\big) = \mathbf{P}\big(\overline{A}_{_{1}}\big)\mathbf{P}\big(\overline{A}_{_{2}}\big)\mathbf{P}\big(\overline{A}_{_{3}}\big) = \frac{1}{8}.$$

故有
$$m{X} \sim egin{bmatrix} m{0} & m{1} & m{2} & m{3} \ m{1} & m{1} & m{1} & m{1} & m{1} \ m{2} & m{4} & m{8} & m{8} \end{bmatrix}$$

第 13-14页《常见离散型分布》

一. 1. **B** 2. **D** (用泊松分布近似计算)

二. 1.
$$e^{-1}$$
 (即为 $\mathbf{P}(\Delta > 0) = \mathbf{P}(4 - 4X > 0) = \mathbf{P}(X < 1) = \mathbf{P}(X = 0) = \frac{1^0}{0!}e^{-1} = e^{-1}$)

2. $\mathbf{C}_3^1 \times \mathbf{0.18} \times \mathbf{0.82}^2$ (设对 X 的 3 次取值中取到 1 的次数为 ξ ,而每次取到 1 的概率为

$$\mathbf{P}ig(X=1ig) = \mathbf{C}_2^1 \mathbf{0}.\mathbf{1}^1 \mathbf{0}.\mathbf{9}^1 = \mathbf{0}.\mathbf{18}$$
,从而有 $\boldsymbol{\xi} \sim \mathbf{B}ig(\mathbf{3}, \mathbf{0}.\mathbf{18}ig)$,所以所求概率为 $\mathbf{P}ig(\boldsymbol{\xi}=1ig) = \mathbf{C}_3^1 imes \mathbf{0}.\mathbf{18}^1 imes \mathbf{0}.\mathbf{82}^2 = \mathbf{C}_3^1 imes \mathbf{0}.\mathbf{18} imes \mathbf{0}.\mathbf{82}^2$)

三. 1. 教材习题二(A)三第 6 题

$$\left(1
ight)X\sim H\left(6,4,20
ight)$$
,所以 $ext{P}ig(X=kig)=rac{ ext{C}_{16}^{6-k} ext{C}_{4}^{k}}{ ext{C}_{20}^{6}}, k=0,1,2,3,4$,即

$$X \sim egin{bmatrix} 0 & 1 & 2 & 3 & 4 \ rac{ ext{C}_{16}^6 ext{C}_{4}^0}{ ext{C}_{20}^6} & rac{ ext{C}_{16}^5 ext{C}_{4}^1}{ ext{C}_{20}^6} & rac{ ext{C}_{16}^4 ext{C}_{2}^2}{ ext{C}_{20}^6} & rac{ ext{C}_{16}^3 ext{C}_{4}^3}{ ext{C}_{20}^6} & rac{ ext{C}_{16}^2 ext{C}_{4}^3}{ ext{C}_{20}^6} \ \end{pmatrix}$$

$$ig(2ig) Y \sim \mathrm{B}ig(6,0.2ig), \;$$
所以 $\mathrm{P}ig(Y=kig) = \mathrm{C}_6^k 0.2^k 0.8^{6-k}$, $k=0,1,2,3,4,5,6$

2. 教材习题二(A)三第7题

显然 $X \sim \mathbf{B} ig(15, 0.2 ig)$

$$(1) P(X = 3) = C_{15}^3 0.2^3 0.8^{12} = 0.2501$$

$$\big(2\big)\operatorname{P}\big(X\geq 2\big)=1-\operatorname{P}\big(X=0\big)-\operatorname{P}\big(X=1\big)$$

$$=1-0.8^{15}-15\times0.8^{14}\times0.2=0.8329$$

$$\begin{split} \left(3\right) \mathbf{P} \left(1 \leq X \leq 3\right) &= \mathbf{P} \left(X = 3\right) + \mathbf{P} \left(X = 2\right) + \mathbf{P} \left(X = 1\right) \\ &= \mathbf{C}_{15}^3 \mathbf{0.2}^3 \mathbf{0.8}^{12} + \mathbf{C}_{15}^2 \mathbf{0.2}^2 \mathbf{0.8}^{13} + \mathbf{C}_{15}^1 \mathbf{0.2}^1 \mathbf{0.8}^{14} = \mathbf{0.6130} \end{split}$$

$$\big(4\big)\, {\rm P}\big(X \le 1\big) = 1 - {\rm P}\big(X \ge 2\big) = 1 - 0.8329 = 0.1671$$

3. 教材习题二(A)三第 9 题

设 $oldsymbol{X}$ 为任意时刻同时出故障的车床台数,则 $oldsymbol{X} \sim \mathbf{B} ig(\mathbf{300}, \mathbf{0.01}ig)$

$$P(X = 4) = C_{300}^4 0.01^4 0.99^{296} = 0.1689$$

由泊松定理近似地有
$$X \sim \mathbf{P} \big(\mathbf{3} \big)$$
,所以 $\mathbf{P} \big(X = 4 \big) \doteq \frac{3^4}{4!} e^{-3} = \frac{27}{8} e^{-3} = 0.1680$

相对误差为
$$\frac{0.1689 - 0.1680}{0.1689} = 0.533\%$$

4. 教材习题二(A)三第 10 题

$$\left(1
ight)\mathrm{P}ig(X>rig)=\sum_{k=r+1}^{\infty}ig(1-pig)^{\!k-1}\,p=rac{pig(1-pig)^r}{1-ig(1-pig)}=ig(1-pig)^r$$

$$ig(2ig)$$
 由 $ig(1ig)$ 的结论知 $\mathbf{P}ig(X>t+rig)=ig(1-pig)^{r+t}$, $\mathbf{P}ig(X>tig)=ig(1-pig)^t$ 从而

$$egin{split} \mathbf{P}ig(X>r+tig|X>rig) &= rac{\mathbf{P}ig(X>r+t,X>rig)}{\mathbf{P}ig(X>rig)} = rac{\mathbf{P}ig(X>r+tig)}{\mathbf{P}ig(X>rig)} \ &= rac{ig(1-pig)^{r+t}}{ig(1-pig)^r} = ig(1-pig)^t = \mathbf{P}ig(X>tig). \end{split}$$

第 15-16页《连续型随机变量》

- 一. 1. C (用密度函数的特征(非负性和归一性)进行检验)
 - 2. **C**
 - 3. A 因密度函数为偶函数,则必有

$$1 = \int\limits_{-\infty}^{+\infty} fig(xig) dx = 2 \int\limits_{0}^{+\infty} fig(xig) dx = 2 ig[Fig(+\inftyig) - Fig(0ig) ig] = 2 - 2 Fig(0ig),$$

从而
$$\mathbf{F}(0) = \frac{1}{2}$$
; 所以

$$egin{split} \mathbf{P}ig(ig|Xig|>aig) &= 1 - \mathbf{P}ig(ig|Xig|\leq aig) = 1 - \int\limits_{-a}^{a} fig(xig) dx = 1 - 2\int\limits_{0}^{a} fig(xig) dx \ &= 1 - 2ig[\mathbf{F}ig(aig) - \mathbf{F}ig(0ig)ig] \ &= 1 - 2\mathbf{F}ig(aig) + 2\mathbf{F}ig(0ig) = 2ig[1 - \mathbf{F}ig(aig)ig] \end{split}$$

$$\equiv$$
. 1. $\frac{2}{\pi}$

曲归一性得
$$1=\int\limits_{-\infty}^{+\infty}fig(xig)dx=\int\limits_{0}^{1}rac{A}{\sqrt{1-x^2}}\,dx=Arcsin xig|_{0}^{1}=rac{\pi}{2}A\Rightarrow A=rac{2}{\pi}$$

2.
$$\frac{15}{8}\sqrt{\pi}$$

$$\int\limits_{-\infty}^{+\infty} x^{rac{3}{2}} fig(xig) dx = \int\limits_{0}^{+\infty} x^{rac{3}{2}} rac{1^2}{\Gammaig(2ig)} x^{2-1} e^{-x} dx = \int\limits_{0}^{+\infty} x^{rac{7}{2}-1} e^{-x} dx = \Gammaigg(rac{7}{2}igg) \ = rac{5}{2} imes rac{3}{2} imes rac{1}{2} \Gammaigg(rac{1}{2}igg) = rac{15}{8} \sqrt{\pi}$$

3.
$$\mathbf{B}(3, e^{-1})$$

$$oldsymbol{X} \sim eig(1ig)$$
,故 $oldsymbol{X}$ 的分布函数为 $oldsymbol{\mathrm{F}}ig(xig) = egin{cases} 1-e^{-x}, & x>0 \ 0, & x\leq 0 \end{cases}$

电子元件寿命大于 1 万小时的概率为
$$p=\mathbf{P}ig(X\geq 1ig)=1-\mathbf{F}ig(1ig)=e^{-1}$$

所以有 $Y \sim \mathbf{B}(3, e^{-1})$.

三. 1. 教材习题二(A)三第 12 题

首先函数 $\varphi(x)$ 满足非负性;

其次证明存在c 使得函数 $\varphi(x)$ 满足归一性:由 $\int\limits_{-\infty}^{+\infty} \varphi(x) dx = \int\limits_{0}^{+\infty} \frac{x^2}{c^2} e^{-\frac{x^3}{c}} dx$ $= \frac{1}{3c} \int\limits_{0}^{+\infty} e^{-\left(\frac{x^3}{c}\right)} d\left(\frac{x^3}{c}\right) \text{ (此处应需} c > 0 \text{)} = \frac{1}{3c} \Gamma(1) = \frac{1}{3c} \text{ , 这说明当} c = \frac{1}{3} \text{ 时函数}$ $\varphi(x)$ 满足归一性;

所以, 当 $c = \frac{1}{3}$ 时函数 $\varphi(x)$ 为某连续型随机变量的密度函数. 此时

$$\mathbf{P}ig(X \leq 1ig) = \int\limits_0^1 9x^2 e^{-3x^3} dx = \int\limits_0^1 e^{-ig(3x^3ig)} dig(3x^3ig) = -e^{-3x^3}ig|_{x=0}^{x=1} = 1-e^{-3x^3}$$

2. 教材习题二(A)三第 14 题

$$egin{aligned} ig(1) & ext{ 由归一性有} \ 1 = \int\limits_{-\infty}^{+\infty} fig(xig) dx = \int\limits_{-A}^{A} rac{2}{\piig(1+x^2ig)} dx = rac{4}{\pi} \int\limits_{0}^{A} rac{1}{1+x^2} dx \ & = rac{4}{\pi} rctan x \Big|_{0}^{A} = rac{4}{\pi} rctan A \,, \,\,\, ext{所以} rctan A = rac{\pi}{4} \,, \,\,\, ext{从而} \, A = 1 \end{aligned}$$

$$\left(2
ight)\mathrm{F}\left(x
ight)=\int\limits_{-\infty}^{x}fig(tig)dt=\left\{egin{array}{c}0,&x\leq-1\ \int\limits_{-1}^{x}rac{2}{\piig(1+t^{2}ig)}dt=rac{2}{\pi}rctan\,x+rac{1}{2},&-1< x<1\ 1,&x\geq1\end{array}
ight.$$

3. 教材习题二(A)三第 15 题

$$\Big(1\Big)$$
 由归一性有 $1=\int\limits_{-\infty}^{+\infty}f\Big(x\Big)dx=\int\limits_{0}^{2}rac{1}{8}dx+\int\limits_{2}^{4}kxdx=rac{1}{4}+6k$,所以 $k=rac{1}{8}$;

$$egin{aligned} ig(2ig) \, \mathrm{F}ig(xig) &= \int\limits_{-\infty}^x fig(tig) dt = egin{cases} 0, & x \leq 0 \ \int\limits_0^x rac{1}{8} dt = rac{1}{8} x, & 0 < x < 2 \ \int\limits_0^z rac{1}{8} dt + \int\limits_2^x rac{1}{8} t dt = rac{1}{16} x^2, & 2 \leq x < 4 \ 1, & x \geq 4 \end{cases} \end{aligned}$$

4. 教材习题二(A)三第 16 题

显然 X 的分布函数为 $\mathbf{F}(x)=egin{cases} 1-e^{-\frac{x}{1000}}, & x>0 \\ 0, & x\leq 0 \end{cases}$, 每只元件寿命不超过 400 小时的概

率为
$$p = P(X \le 400) = F(400) = 1 - e^{-\frac{1}{1000} \times 400} = 1 - e^{-0.4}$$

设在仪器使用的最初 400 小时内元件的损坏数,则 $Y \sim \mathbf{B} \left(\mathbf{6}, \mathbf{1} - e^{-0.4} \right)$,从而有

$$(1) P(Y = 1) = C_6^1 \times (1 - e^{-0.4}) e^{-0.4 \times 5} = 6e^{-2} (1 - e^{-0.4});$$

$$(2) P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-0.4 \times 6} = 1 - e^{-2.4}$$

第 17-18页《随机变量函数的分布》

-. 1. D
$$(\mathbf{F}_{Y}(y) = \mathbf{P}(Y \le y) = \mathbf{P}(3X - 1 \le y) = \mathbf{P}(X \le \frac{y+1}{3}) = \mathbf{F}(\frac{y+1}{3})$$

2. C

$$\equiv$$
. 1. $f_{_Y} ig(y ig) = rac{1}{4} y^{-rac{1}{2}}, \ \ 0 < y < 4$

对任意
$$y \in \left(0,4
ight)$$
, $\mathbf{F}_{_{\!Y}}\left(y
ight) = \mathbf{P}\!\left(Y \leq y
ight) = \mathbf{P}\!\left(X^2 \leq y
ight) = \mathbf{P}\!\left(-\sqrt{y} \leq X \leq \sqrt{y}
ight)$
$$= \mathbf{P}\!\left(0 \leq X \leq \sqrt{y}\right) = \int\limits_{0}^{\sqrt{y}} \frac{1}{2} dx = \frac{1}{2}\sqrt{y} \;, \;\; \mathrm{所以} \, f_{_{\!Y}}\!\left(y\right) = \mathbf{F}_{_{\!Y}}'\left(y\right) = \frac{1}{4}\,y^{-\frac{1}{2}}$$

也可以直接利用平方变换的公式求解

2.
$$P(Y \ge 2) = P(1 - \sqrt[3]{X} \ge 2) = P(X \le -1) = F_X(-1) = 0$$

三. 1. 教材习题二(A)三第 18 题

$$egin{pmatrix} egin{pmatrix} 1\end{pmatrix} X \sim egin{bmatrix} -2 & -1 & 2 & 3 \ 0.2 & 0.3 & 0.2 & 0.3 \end{bmatrix} & egin{pmatrix} 2\end{pmatrix} Y \sim egin{bmatrix} 1 & 4 & 9 \ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

2. 教材习题二(**A**)三第 19 题

$$\begin{split} &(1) \ \mathbf{F}_{_{Y}} \left(y\right) = \mathbf{P} \left(Y \leq y\right) = \mathbf{P} \left(2X - 1 \leq y\right) = \mathbf{P} \left(X \leq \frac{y+1}{2}\right) = \mathbf{F}_{_{X}} \left(\frac{y+1}{2}\right) \\ & f_{_{Y}} \left(y\right) = \mathbf{F}_{_{Y}}' \left(y\right) = \frac{1}{2} f_{_{X}} \left(\frac{y+1}{2}\right) = \begin{cases} \frac{1}{2} e^{-\frac{y+1}{2}}, & y > -1, \\ 0, & y \leq -1 \end{cases} \\ &(2) \ \mathbf{F}_{_{Y}} \left(y\right) = \mathbf{P} \left(Y \leq y\right) = \mathbf{P} \left(e^{X} \leq y\right) = \begin{cases} 0, & y \leq 1 \\ \mathbf{P} \left(X \leq \ln y\right) = \mathbf{F}_{_{X}} \left(\ln y\right), & y > 1 \end{cases} \\ & f_{_{Y}} \left(y\right) = \mathbf{F}_{_{Y}}' \left(y\right) = \begin{cases} 0, & y \leq 1 \\ \mathbf{F}_{_{X}}' \left(\ln y\right) = \frac{1}{y} e^{-\ln y} = \frac{1}{y^{2}}, & y > 1 \end{cases} \\ &(3) \ \mathbf{F}_{_{Y}} \left(y\right) = \mathbf{P} \left(Y \leq y\right) = \mathbf{P} \left(X^{2} \leq y\right) = \begin{cases} \mathbf{P} \left(X \in \left[-\sqrt{y}, \sqrt{y}\right]\right), & y > 0 \\ 0, & y \leq 0 \end{cases} \\ & = \begin{cases} \mathbf{F}_{_{X}} \left(\sqrt{y}\right) - \mathbf{F}_{_{X}} \left(-\sqrt{y}\right), & y > 0 \\ 0, & y \leq 0 \end{cases} \\ & f_{_{Y}} \left(y\right) = \mathbf{F}_{_{Y}}' \left(y\right) = \begin{cases} \mathbf{F}_{_{X}}' \left(\sqrt{y}\right) - \mathbf{F}_{_{X}}' \left(-\sqrt{y}\right), & y > 0 \\ 0, & y \leq 0 \end{cases} \\ & = \begin{cases} \frac{1}{2\sqrt{y}} \left[f_{_{X}} \left(\sqrt{y}\right) + f_{_{X}} \left(-\sqrt{y}\right)\right] = \frac{1}{2\sqrt{y}} e^{-\sqrt{y}}, & y > 0 \\ 0, & y \leq 0 \end{cases} \\ & 0, & y \leq 0 \end{cases} \end{split}$$

3. 教材习题二(A)三第 22 题

$$egin{aligned} \mathbf{F}_{_{Y}}\left(y
ight) &= \mathbf{P}ig(Y \leq yig) = \mathbf{P}ig(X^2 \leq yig) = egin{cases} 0, & y \leq 0 \ \mathbf{P}ig(X \in ig[-\sqrt{y},\sqrt{y}ig]ig), & y \in ig(0,9ig) \ 1, & y \geq 9 \end{cases} \ &= egin{cases} 0, & y \leq 0 \ \mathbf{F}_{_{X}}ig(\sqrt{y}ig) - \mathbf{F}_{_{X}}ig(-\sqrt{y}ig), & y \in ig(0,9ig) \ 1, & y \geq 9 \end{cases} \end{aligned}$$

$$\begin{split} f_{_{Y}}\left(y\right) &= \mathbf{F}_{_{Y}}^{\prime}\left(y\right) = \begin{cases} \frac{1}{2\sqrt{y}} \Big[f_{_{X}}\left(\sqrt{y}\right) + f_{_{X}}\left(-\sqrt{y}\right)\Big], & y \in \left(0,9\right) \\ 0, & y \not\in \left(0,9\right) \end{cases} \\ &= \begin{cases} \frac{1}{2\sqrt{y}} \Big[\frac{1}{4} + \frac{1}{4}\Big] = \frac{1}{4\sqrt{y}}, & y \in \left(0,1\right] \\ \frac{1}{2\sqrt{y}} \Big[0 + \frac{1}{4}\Big] = \frac{1}{8\sqrt{y}}, & y \in \left(1,9\right) \\ 0, & y \not\in \left(0,9\right) \end{cases} \end{split}$$

第19-20页《第二章综合练习》

一. 1. \mathbf{C} (用分布函数的特征验证. 注意第二个答案, 若 $\mathbf{a} = \mathbf{2}, \mathbf{b} = -\mathbf{1}$ 不能保证

$$\mathbf{F}(x) = a\mathbf{F}_1(x) + b\mathbf{F}_2(x)$$
的非负性)

2. **B** 教材习题二(**B**)一第 4 题

$$X\sim eig(\lambdaig),Y=\minig(X,2ig)$$
,显然可见 Y 的有效值域为 $Rig(Yig)=ig(0,2ig)$,所以,当 $y\inig(0,2ig)$ 时," $Y\leq y$ " \Leftrightarrow " $\minig(X,2ig)\leq y$ " \Leftrightarrow " $X\leq y$ ",从而 $\mathbf{F}_{_{Y}}ig(yig)=\mathbf{P}ig(X\leq yig)=\mathbf{F}_{_{X}}ig(yig)=1-e^{-\lambda y}$,于是综上有

$$\mathbf{F}_{_{Y}}\left(y
ight) = egin{cases} 0, & y \leq 0 \ 1-e^{-\lambda y}, & y \in \left(0,2
ight) \ 1, & y \geq 2 \end{cases}$$

显然可见, $\mathbf{F}_{Y}(y)$ 在y=0处连续,在y=2处间断.

(本题中的随机变量Y是非离散非连续型随机变量)

$$\equiv$$
. 1. $\frac{9}{64}$

因
$$\mathbf{P}ig(X \leq 1/2ig) = \int\limits_0^{1/2} 2x dx = rac{1}{4}$$
,则 $Y \sim \mathbf{B}ig(3,1/4ig)$,故 $\mathbf{P}ig(Y = 2ig) = \mathbf{C}_3^2 ig(1/4ig)^2 rac{3}{4} = rac{9}{64}$

$$2. \ \frac{16\sqrt{2}}{3\sqrt{\pi}}$$

由归一性得

$$1=\int\limits_{-\infty}^{+\infty}fig(xig)dx=A\int\limits_{0}^{+\infty}x^{3/2}e^{-2x}dx=rac{A}{2^{5/2}}\int\limits_{0}^{+\infty}ig(2xig)^{3/2}\,e^{-2x}dig(2xig)=rac{A}{2^{5/2}}\Gammaig(5ig/2ig)$$

所以
$$A=rac{2^{rac{5}{2}}}{\Gammaiggl(rac{5}{2}iggr)}=rac{4\sqrt{2}}{rac{3}{4}\sqrt{\pi}}=rac{16\sqrt{2}}{3\sqrt{\pi}}$$

$$\equiv$$
. 1. $(1)\frac{9}{10}$ $(2)1-0.9^{10}-\mathrm{C}_{10}^{1}0.9^{9} imes0.1=1-1.9 imes0.9^{9}$ $(3)1-11e^{-10}$

(1)
$$P(|X| < 1.8) = \frac{3.6}{4} = 0.9$$

- $egin{aligned} ig(2ig)$ 设 Y 表 f10 次测量中误差绝对值大于 f1.8 的次数,则 $Y\sim Big(10,0.1ig)$,从而 $f Pig(Y\geq 2ig)=1-f Pig(Y=0ig)-f Pig(Y=1ig)=1-0.9^{10}-f C_{10}^10.9^9 imes0.1 \end{aligned}$
- $egin{aligned} ig(3ig)$ 设 Z 表 f100 次测量中误差绝对值大于 f1.8 的次数,则 $Z\sim Big(100,0.1ig)$,故近似地有 $Z\sim Pig(10ig)$,从而 f2D($Z\geq 2ig)=1-Pig(Z=0ig)-Pig(Z=1ig)=1-11e^{-10}$
- 2. 教材习题二(**B**)三第 4 题

显然密度函数是偶函数, 所以

$$\left(1\right)$$
 由归一性得 $1=\int\limits_{-\infty}^{+\infty}f\left(x
ight)dx=2A\int\limits_{0}^{+\infty}rac{1}{e^{x}+e^{-x}}dx$,作变量代换 $y=e^{x}$ 可得

$$1=2A\int\limits_{1}^{+\infty}rac{1}{1+y^{^{2}}}dx=2Arctan y\Big|_{1}^{+\infty}=rac{1}{2}\pi A\Rightarrow A=rac{2}{\pi}$$

$$ig(2ig)$$
 对任意 $x\in R$, $\mathbf{F}_{_{\! X}}ig(xig)=\int\limits_{-\infty}^x fig(tig)dt=rac{2}{\pi}\int\limits_{-\infty}^xrac{1}{e^t+e^{-t}}dt$,作变量代换 $y=e^t$ 可得

$$\mathbf{F}_{\!\scriptscriptstyle X}\left(x
ight)\!=\!rac{2}{\pi}\int\limits_{-\infty}^{x}rac{1}{e^{t}+e^{-t}}\,dt=rac{2}{\pi}rctan\,yigg|_{y=0}^{y=e^{x}}=rac{2}{\pi}rctan\,e^{x}\,,$$

于是有
$$extbf{P}ig(0 \leq X \leq 1ig) = extbf{F}_{_{X}}ig(1ig) - extbf{F}_{_{X}}ig(0ig) = rac{2}{\pi}rctan e - rac{1}{2}$$

$$\begin{array}{l} \left(3\right) \ \ \mathrm{F}_{_{Y}}\left(y\right) = \mathrm{P}\!\left(Y \leq y\right) = \mathrm{P}\!\left(e^{-|X|} \leq y\right) = \begin{cases} 0, & y \leq 0 \\ \mathrm{P}\!\left(\left|X\right| \geq -\ln y\right), & y \in \left(0,1\right) \\ 1, & y \geq 1 \end{cases} \\ = \begin{cases} 0, & y \leq 0 \\ \frac{4}{\pi} \int\limits_{-\ln y}^{+\infty} \frac{1}{e^{t} + e^{-t}} dt, & y \in \left(0,1\right) \\ 1, & y \geq 1 \end{cases} \\ \\ \mathbb{M}$$
从而有 $f_{_{Y}}\left(y\right) = \mathrm{F}'_{_{Y}}\left(y\right) = \begin{cases} \frac{4}{\pi \left(1 + y^{2}\right)}, & y \in \left(0,1\right) \\ 0, & y \not\in \left(0,1\right) \end{cases}$

3. 教材习题二(**B**)三第 8 题

显然,
$$X$$
的分布函数为 $\mathbf{F}_{X}(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-\frac{1}{60}x}, & x > 0 \end{cases}$

设 A_1,A_2,A_3 分别表年龄在15岁以下,15到50岁,50岁以上,则 A_1,A_2,A_3 构成一完备事件组且

$$\begin{split} \mathbf{P} \left(A_{_{1}} \right) &= \mathbf{F}_{_{X}} \left(\mathbf{15} \right) = 1 - e^{-\frac{1}{4}}, \\ \mathbf{P} \left(A_{_{2}} \right) &= \mathbf{F}_{_{X}} \left(\mathbf{50} \right) - \mathbf{F}_{_{X}} \left(\mathbf{15} \right) = \left(1 - e^{-\frac{50}{60}} \right) - \left(1 - e^{-\frac{15}{60}} \right) = e^{-\frac{1}{4}} - e^{-\frac{5}{60}} \\ \mathbf{P} \left(A_{_{3}} \right) &= 1 - \mathbf{F}_{_{X}} \left(\mathbf{50} \right) = 1 - \left(1 - e^{-\frac{50}{60}} \right) = e^{-\frac{5}{6}} \end{split}$$

再设B为某人得重病,则

$$\begin{aligned} & (1) \mathbf{P}(B) = \sum_{i=1}^{3} \mathbf{P}(A_i) \mathbf{P}(B|A_i) = \left[1 - e^{-\frac{1}{4}}\right] \times 0.1 + \left[e^{-\frac{1}{4}} - e^{-\frac{5}{6}}\right] \times 0.02 + e^{-\frac{5}{6}} \times 0.2 \\ & = 0.1 - 0.08e^{-\frac{1}{4}} + 0.18e^{-\frac{5}{6}} \approx 0.1159 \end{aligned}$$

$$(2) P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(B)} = \frac{\left|1 - e^{-\frac{1}{4}}\right| \times 0.1}{0.1159} = \frac{0.02212}{0.1159} = 0.1908$$

$$P(A_1)P(B|A_1) = \frac{\left|e^{-\frac{1}{4}} - e^{-\frac{5}{6}}\right| \times 0.02}{0.02212} = 0.1908$$

$$P(A_{2}|B) = \frac{P(A_{2})P(B|A_{2})}{P(B)} = \frac{\left|e^{-\frac{1}{4}} - e^{-\frac{5}{6}}\right| \times 0.02}{0.1159} = 0.0444$$

$$P(A_3|B) = 1 - 0.1908 - 0.0444 = 0.7648$$

所以若某人得病,他的年龄最可能的是50岁以上.

第 21-22页《二维随机变量》

—. 1. **B** 2. **C**

$$\equiv$$
. 1. $\mathbf{F}(b,c) - \mathbf{F}(a,c)$ 2. 3 3. $p = \frac{10!}{1!5!3!1!} 0.07 \times 0.43^5 \times 0.35^3 \times 0.15$

三. 1. 教材习题三(A)三第1题

显然 X_1, X_2 都只能取 0,1,且

2. 教材习题三(A)三第3题

显然,X可取0,1,2,3,Y可取0,1,2,且

$$\begin{split} \mathbf{P}\big(X=0,Y=0\big) &= \mathbf{P}\big(\varnothing\big) = 0; & \mathbf{P}\big(X=0,Y=1\big) = \mathbf{P}\big(\varnothing\big) = 0; \\ \mathbf{P}\big(X=0,Y=2\big) &= \frac{\mathbf{C}_3^0\mathbf{C}_2^2\mathbf{C}_2^2}{\mathbf{C}_7^4} = \frac{1}{35}; & \mathbf{P}\big(X=1,Y=0\big) = \mathbf{P}\big(\varnothing\big) = 0; \\ \mathbf{P}\big(X=1,Y=1\big) &= \frac{\mathbf{C}_3^1\mathbf{C}_2^1\mathbf{C}_2^2}{\mathbf{C}_7^4} = \frac{6}{35}; & \mathbf{P}\big(X=1,Y=2\big) = \frac{\mathbf{C}_3^1\mathbf{C}_2^2\mathbf{C}_2^1}{\mathbf{C}_7^4} = \frac{6}{35}; \\ \mathbf{P}\big(X=2,Y=0\big) &= \frac{\mathbf{C}_3^2\mathbf{C}_2^0\mathbf{C}_2^2}{\mathbf{C}_7^4} = \frac{3}{35}; & \mathbf{P}\big(X=2,Y=1\big) = \frac{\mathbf{C}_3^2\mathbf{C}_2^1\mathbf{C}_2^1}{\mathbf{C}_7^4} = \frac{12}{35}; \end{split}$$

$$egin{aligned} \mathbf{P}ig(X=2,Y=2ig) &= rac{\mathbf{C}_3^2\mathbf{C}_2^2\mathbf{C}_2^0}{\mathbf{C}_7^4} = rac{3}{35}; & \mathbf{P}ig(X=3,Y=0ig) &= rac{\mathbf{C}_3^3\mathbf{C}_2^0\mathbf{C}_2^1}{\mathbf{C}_7^4} = rac{2}{35}; \\ \mathbf{P}ig(X=3,Y=1ig) &= rac{\mathbf{C}_3^3\mathbf{C}_2^1\mathbf{C}_2^0}{\mathbf{C}_7^4} = rac{2}{35}; & \mathbf{P}ig(X=3,Y=2ig) &= \mathbf{P}ig(oldsymbol{arnothing}ig) = \mathbf{0}. \end{aligned}$$

3. 教材习题三(A)三第 4 题

$$(1) \ \ 1 = \iint_{R^2} f \Big(x, y \Big) dx dy = \int\limits_{0}^{+\infty} dx \int\limits_{0}^{+\infty} c e^{-\left(2x + 4y \right)} dy = \frac{c}{8} \Rightarrow c = 8$$

$$(2) \ \mathbf{P}(X > 2) = \mathbf{P}(X > 2, Y < +\infty) = \int_{2}^{+\infty} dx \int_{-\infty}^{+\infty} f(x, y) dy$$

$$= 8 \int_{2}^{+\infty} dx \int_{0}^{+\infty} e^{-(2x+4y)} dy = \int_{2}^{+\infty} 2e^{-2x} dx \int_{0}^{+\infty} 4e^{-4y} dy = e^{-4};$$

$$\mathbf{P}(X > Y) = \int_{0}^{+\infty} 2e^{-2x} dx \int_{0}^{x} 4e^{-4y} dy = \int_{0}^{+\infty} (1 - e^{-4x}) 2e^{-2x} dx$$

$$= \int_{0}^{+\infty} 2e^{-2x} dx - \frac{1}{3} \int_{0}^{+\infty} e^{-6x} dx = 1 - \frac{1}{3} = \frac{2}{3};$$

$$\mathbf{P}(X + Y < 1) = \int_{0}^{1} 2e^{-2x} dx \int_{0}^{1-x} 4e^{-4y} dy = \int_{0}^{1} (1 - e^{-4(1-x)}) 2e^{-2x} dx$$

$$= \int_{0}^{1} e^{-2x} d(2x) - e^{-4} \int_{0}^{1} e^{2x} d(2x) = 1 - 2e^{-2} + e^{-4};$$

(3) 显然, 当
$$x \le 0$$
或 $y \le 0$ 时, 必有 $\mathbf{F}(x,y) = \mathbf{P}(X \le x, Y \le y) = 0$;

而当
$$x>0,y>0$$
时, $\mathbf{F}ig(x,yig)=\int\limits_{-\infty}^{x}dt\int\limits_{-\infty}^{y}fig(t,sig)ds$

$$=\int\limits_0^x 2e^{^{-2t}}dt\int\limits_0^y 4e^{^{-4s}}ds=ig(1-e^{-2x}ig)ig(1-e^{-4y}ig)$$
 综上得 $\mathbf{F}ig(x,yig)=egin{cases} ig(1-e^{-2x}ig)ig(1-e^{-4y}ig), & x>0, y>0 \ 0, & others \end{cases}.$

第 23-24 页《边缘分布、边缘密度及独立性》

- 一. 1. **D** (利用分布函数的性质判断)
- 2. **B** (利用密度函数的特征判断)
- 3. **C**

$$\equiv$$
 1. 0 2. $\mathbf{F}(x)(1-\mathbf{F}(y))$ 3. $\frac{1}{2}$

三. 1. 教材习题三(A)三第 6 题

$oxed{X \setminus Y}$	$y_{_1}$	$\boldsymbol{y}_{\scriptscriptstyle 2}$	$y_{_3}$	$\mathbf{P}ig(X=x_iig)=p_{i.}$
$oldsymbol{x}_{\!\scriptscriptstyle 1}$	'	,	1/12	1/4
$x_{_{2}}$	1/8	3/8	1/24	3/4
$\mathbf{P}ig(Y=y_{_j}ig)=p_{_{.j}}$	1/6	1/2	1/3	1

2. 教材习题三(A)三第8题

$$\begin{array}{l} \text{(1) } 1 = \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} f \Big(x,y \Big) dy = \int\limits_{0}^{\pi/2} dx \int\limits_{0}^{1} \frac{A \cos x}{\sqrt{1-y^2}} \, dy = A \int\limits_{0}^{\pi/2} \cos x dx \int\limits_{0}^{1} \frac{1}{\sqrt{1-y^2}} \, dy \\ = A \Big(\pi/2 \Big) \Rightarrow A = \frac{2}{\pi}; \end{array}$$

$$(2) \ \ f_{_{X}}\big(x\big) = \int\limits_{-\infty}^{+\infty} f\big(x,y\big) dy = \begin{cases} \dfrac{2}{\pi} \cos x \int\limits_{0}^{1} \dfrac{1}{\sqrt{1-y^{2}}} dy, \ \ 0 < x < \dfrac{\pi}{2} \\ 0, & others \end{cases} \\ = \begin{cases} \cos x, \ \ 0 < x < \pi/2 \\ 0, & others \end{cases} ; \\ f_{_{Y}}\big(y\big) = \int\limits_{-\infty}^{+\infty} f\big(x,y\big) dx = \begin{cases} \dfrac{2}{\pi\sqrt{1-y^{2}}} \int\limits_{0}^{\pi/2} \cos x dy, \ \ 0 < y < 1 \\ 0, & others \end{cases}$$

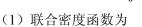
$$= egin{cases} rac{2}{\pi \sqrt{1 - y^2}}, \ 0 < y < 1 \ 0, & others \end{cases};$$

- (4) 由 3 个密度函数可知 $f\left(x,y\right)=f_{X}\left(x\right)f_{Y}\left(y\right), \forall x,y\in R$,所以 X 与 Y 相互独立!
- 3. 教材习题三(**A**)三第9题

如图,密度函数不为零的区域即图中阴影部分

$$\Omega = \left\{ \left(x,y
ight) \colon 0 < x < 1, 0 < y < x^2
ight\}$$
 ,

其面积为 $m(\Omega) = \int\limits_0^1 x^2 dx = \frac{1}{3}$,所以



$$fig(x,yig) = egin{cases} 3, & 0 < x < 1, 0 < y < x^2 \ 0, & others \end{cases}$$

(2) 边缘密度函数为

$$f_{_{X}}ig(xig) = \int\limits_{-\infty}^{+\infty} fig(x,yig) dy = egin{cases} \int\limits_{0}^{x^{2}} 3 dy = 3x^{2}, \ 0 < x < 1 \ 0, & others \end{cases}$$

$$f_{Y}\left(y
ight) = \int\limits_{-\infty}^{+\infty} fig(x,yig) dx = egin{cases} \int\limits_{\sqrt{y}}^{1} 3 dx = 3 \Big(1-\sqrt{y}\Big), & 0 < y < 1 \ 0, & others \end{cases}$$

(3) 在公共连续点 $\left(1/2,1/8\right)$ 处, $f_X\left(x\right)f_Y\left(y\right)\neq f\left(x,y\right)$,所以X与Y不相互独立!

第 25-26 页《条件分布》

1. 教材习题三(A)三第 10 题

(2)
$$P(X = 1|Y = 2) = \frac{P(X = 1, Y = 2)}{P(Y = 2)} = \frac{6/35}{10/35} = \frac{3}{5};$$

$$P(Y = 1|X = 2) = \frac{P(X = 2, Y = 1)}{P(X = 2)} = \frac{12/35}{18/35} = \frac{2}{3};$$

$$P(Y = 1|X \neq 2) = \frac{P(X \neq 2, Y = 1)}{P(X \neq 2)} = \frac{P(Y = 1) - P(X = 2, Y = 1)}{1 - P(X = 2)}$$

$$= \frac{20/35 - 12/35}{1 - 18/35} = \frac{8}{17};$$

(3)
$$X | Y = 2 \sim \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1/10 & 6/10 & 3/10 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1/10 & 6/10 & 3/10 \end{bmatrix}$$

(4) 由于
$$\mathbf{P}(X=2,Y=3) \neq \mathbf{P}(X=2)\mathbf{P}(Y=3)$$
, 所以 $X 与 Y$ 不独立!

2. 教材习题三(A)三第 11 题

由前面的计算结果知,

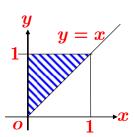
$$otin 0 < y < 1 ext{ ft}, \quad f_{X|Y}\left(x \middle| y
ight) = rac{f\left(x,y
ight)}{f_{Y}\left(y
ight)} = egin{dcases} rac{3}{3\left(1-\sqrt{y}
ight)} = rac{1}{1-\sqrt{y}}, & \sqrt{y} < x < 1 \ 0, & others \end{cases}$$

显然
$$f_{X|Y}\left(x\left|1/4
ight) = rac{f\left(x,1/4
ight)}{f_{Y}\left(1/4
ight)} = egin{cases} rac{3}{3\left(1-\sqrt{1/4}
ight)} = 2, & 1/2 < x < 1 \ 0, & others \end{cases}$$
 ,所以

$$ext{P}\Big(X < 2ig/3\Big|Y = 1ig/4\Big) = \int\limits_{-\infty}^{2/3} f_{X|Y}\Big(x\Big|1ig/4\Big) dx = \int\limits_{1/2}^{2/3} 2 dx = rac{1}{3};$$

$$\mathbf{P}ig(X < 2ig/3ig|Y > 1ig/4ig) = rac{\mathbf{P}ig(X < 2ig/3, Y > 1ig/4ig)}{\mathbf{P}ig(Y > 1ig/4ig)} = rac{\int\limits_{-\infty}^{2ig/3} dx \int\limits_{1/4}^{+\infty} fig(x,yig) dy}{\int\limits_{1/4}^{+\infty} f_Yig(yig) dy}$$

$$=rac{\int\limits_{1/2}^{2/3}dx\int\limits_{1/4}^{x^2}3dy}{\int\limits_{1/4}^{1}3\Big(1-\sqrt{y}\Big)dy}=rac{rac{5}{108}}{rac{1}{2}}=rac{5}{54}$$



3. 教材习题三(A)三第 14 题 如图,

$$f_{_{X}}ig(xig) = \int\limits_{-\infty}^{\infty} fig(x,yig) dy = egin{cases} \int\limits_{x}^{1} 3y dy = rac{3}{2}ig(1-x^2ig), & 0 < x < 1 \ 0, & others \end{cases}$$

$$f_{_{Y}}ig(yig) = \int\limits_{-\infty}^{\infty} fig(x,yig) dx = egin{cases} \int\limits_{0}^{y} 3y dx = 3y^2, \ 0 < y < 1 \ 0, & others \end{cases}$$

显然可见, $f_X(x)f_Y(y) \neq f(x,y)$, 所以 X = Y 不相互独立!

当
$$0 < x < 1$$
 时, $f_{Y|X}\left(y\Big|x
ight) = rac{f\left(x,y
ight)}{f_{X}\left(x
ight)} = egin{dcases} rac{3y}{3\left(1-x^2
ight)\!/2} = rac{2y}{1-x^2}, \ \ x < y < 1 \ \ 0, & others \end{cases}$

当
$$0 < y < 1$$
 时, $f_{X\mid Y}\left(x\middle|y
ight) = rac{f\left(x,y
ight)}{f_{Y}\left(y
ight)} = egin{cases} rac{3y}{3y^2} = rac{1}{y}, & 0 < x < y \ 0, & others \end{cases}$

4. 教材习题三(A)三第 12 题

由题意知,

当
$$0 < x < 1$$
 时, $f_{Y|X}ig(yig|xig) = egin{cases} 1/x, & 0 < y < x \ 0, & others \end{cases}$,

从而有

故有

$$f_{Y}ig(yig) = \int\limits_{-\infty}^{\infty} fig(x,yig) dx = egin{cases} \int\limits_{y}^{1} 3x dx = rac{3}{2}ig(1-y^2ig), & 0 < y < 1 \ 0, & others \end{cases}$$

于是有
$$\mathbf{P}ig(Y < 1/2ig) = \int\limits_{-\infty}^{1/2} f_Yig(yig) dy = \int\limits_{0}^{1/2} rac{3}{2}ig(1-y^2ig) dy = rac{11}{16}$$

第 27-28页《二维随机变量函数的分布》

-. 1. **0.64** 2.
$$p^2 + (1-p)^2$$

-	$oldsymbol{X}\setminus oldsymbol{Y}$	0	1	2		
前面已经得到	0		0			
	1	0	6/35	6/35	,	由此易得
			6/35 $12/35$			
_	3	2/35	2/35	0		

$$\Rightarrow Z \sim egin{bmatrix} 2 & 3 & 4 \ 10/35 & 20/35 & 5/35 \end{bmatrix} & U \sim egin{bmatrix} 1 & 2 & 3 \ 6/35 & 25/35 & 4/35 \end{bmatrix} \ V \sim egin{bmatrix} 0 & 1 & 2 \ 6/35 & 26/35 & 3/35 \end{bmatrix}$$

2. 教材习题三
$$(A)$$
三第 14 (3) 题

显然,
$$R(Z) = (0,1)$$
,对任意 $z \in (0,1)$,有

$$egin{aligned} \mathbf{F}_{\!Z}ig(zig) &= \mathbf{P}ig(Z \leq zig) = \mathbf{P}ig(Y - X \leq zig) = \iint\limits_{y - x \leq z} fig(x,yig) dx dy \ &= \int\limits_0^z dy \int\limits_0^y 3y dx + \int\limits_z^1 dy \int\limits_{y - z}^y 3y dx = \int\limits_0^z 3y^2 dy + \int\limits_z^1 3y z dy \ &= z^3 + rac{3}{2}zig(1 - z^2ig) = rac{3}{2}z - rac{1}{2}z^3 \end{aligned}$$

此时,Z的密度函数为 $f_Z\left(z
ight)=\mathbf{F}_Z'\left(z
ight)=rac{3}{2}ig(1-z^2ig)$,综上得Z的密度函数为

$$f_{_{\!Z}}\!\left(z
ight) = egin{cases} rac{3}{2}\!\left(1-z^{2}
ight), & z\in\left(0,1
ight), \ 0 & z
ot\in\left(0,1
ight). \end{cases}$$

3. 教材习题三(**A**)三第 15 题

显然,
$$R(Z) = (0,1)$$
,对任意 $z \in (0,1)$,有

$$\mathbf{F}_{\!\!\!Z}\!\left(z
ight) = \mathbf{P}\!\left(Z \leq z
ight) = \mathbf{P}\!\left(XY \leq z
ight) = \iint\limits_{xy \leq z} f\!\left(x,y
ight) dxdy$$

$$=\int\limits_{0}^{z}dx\int\limits_{0}^{1}dy+\int\limits_{z}^{1}dx\int\limits_{0}^{z/x}dy=\int\limits_{0}^{z}dx+\int\limits_{z}^{1}\Big(z\Big/x\Big)dx=z-z\ln z$$

此时, $oldsymbol{Z}$ 的密度函数为 $oldsymbol{f_Z}ig(oldsymbol{z}ig) = oldsymbol{\mathbf{F_Z'}}ig(oldsymbol{z}ig) = -\lnoldsymbol{z}$,综上得 $oldsymbol{Z}$ 的密度函数为

$$f_{\!\scriptscriptstyle Z}\!\left(z
ight) = egin{cases} -\ln z, & z \in \left(0,1
ight) \ 0 & z
ot\in \left(0,1
ight). \end{cases}$$

4. 教材习题三(**A**)三第 16 题

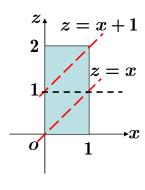
显然,
$$m{R}ig(m{Z}ig) = ig[0,2ig]$$
,对任意 $m{z} \in ig(0,2ig)$,有 $m{f}_{m{z}}ig(m{z}ig) = \int\limits_{-\infty}^{+\infty} m{f}_{m{X}}ig(m{x}ig)m{f}_{m{Y}}ig(m{z}-m{x}ig)m{d}m{x}$,要使

被积函数不为零,需
$$\begin{cases} 0 \leq x \leq 1 \\ 0 \leq z - x \leq 1 \end{cases}$$
,即 $\begin{cases} 0 \leq x \leq 1 \\ x \leq z \leq x + 1 \end{cases}$,

如图,从而有

(1) 当
$$z \in [0,1]$$
时, $f_z(z) = \int_0^z 1 dx = z$;

(2) 当
$$z\in\left[1,2\right]$$
时, $f_{z}\left(z\right)=\int\limits_{z-1}^{1}1dx=2-z$



5. 教材习题三(A)三第 17 题

显然,
$$R(Z) = (0, +\infty)$$
, 对任意 $z > 0$, 有

$$egin{aligned} \mathbf{F}_{\!Z}ig(zig) &= \mathbf{P}ig(Z \leq zig) = \mathbf{P}ig(Xig/Y \leq zig) = \int_{x/y \leq z} fig(x,yig) dx dy \ &= \int_0^{+\infty} dy \int_0^{yz} e^{-x-y} dx = \int_0^{+\infty} e^{-y} dy \int_0^{yz} e^{-x} dx = \int_0^{+\infty} e^{-y} ig(1-e^{-yz}ig) dy \ &= \int_0^{+\infty} e^{-y} dy - \int_0^{+\infty} e^{-y(z+1)} dy = 1 - rac{1}{z+1} \end{aligned}$$

此时,
$$m{Z}$$
的密度函数为 $f_Zig(zig) = \mathbf{F}_Z'ig(zig) = ig(1+zig)^{-2}$,综上得 $m{Z}$ 的密度函数为 $f_Zig(zig) = egin{cases} ig(1+zig)^{-2}, & z>0 \ 0, & z\leq 0 \end{cases}$

6. 教材习题三(A)三第 21 题

显然
$$T_i$$
的分布函数为 $\mathbf{F}_iig(tig) = \mathbf{F}ig(tig) = egin{cases} 1-e^{-0.2t}, & t>0 \ 0, & t\leq 0 \end{cases}$, $i=1,2,\cdots,5$.

(1) 并联时,系统的寿命 $T_{\!\scriptscriptstyle eta} = \max_{1 \leq i \leq 5} \left\{ X_i
ight\}$,其分布函数为

$$\mathbf{F}_{\!\scriptscriptstyleeta}ig(tig) = ig(\mathbf{F}ig(tig)ig)^{\!\scriptscriptstyle 5} = egin{cases} ig(1-e^{-0.2t}ig)^{\!\scriptscriptstyle 5}\,, & t>0\ 0, & t\leq 0 \end{cases},$$

使用寿命大于1万小时的概率为

$$ext{P}ig(T_{_{\#}}>1ig)=1- ext{F}_{_{\#}}ig(1ig)=1-ig(1-e^{-0.2}ig)^{\!5}=0.9998;$$

(2) 串联时,系统的寿命 $T_{\scriptscriptstyle \parallel} = \min_{1 \leq i \leq 5} \left\{ X_i
ight\}$,其分布函数为

$$\mathbf{F}_{\!\scriptscriptstyle{f \#}}ig(tig) = \mathbf{1} - ig(\mathbf{1} - \mathbf{F}ig(tig)ig)^{\!\scriptscriptstyle{5}} = egin{cases} 1 - e^{-t}, & t > 0 \ 0, & t \leq 0 \end{cases},$$

从而其密度函数为
$$f_{\scriptscriptstyle \parallel}ig(tig) = egin{cases} e^{-t}, & t>0 \ 0, & t\leq 0 \end{cases}$$

使用寿命大于1万小时的概率为

$$P(T_{_{\oplus}} > 1) = 1 - F_{_{\oplus}}(1) = 1 - (1 - e^{-1}) = e^{-1} = 0.3679.$$

第29-30页《第三章综合练习》

- 一. 1. 教材习题三 (\mathbf{B}) 一第 2 题 (\mathbf{A})
 - 2. 教材习题三(B)一第3题 (D)

显然,
$$X$$
 的分布函数为 $\mathbf{F}_{X}\left(x\right)=egin{cases}1-e^{\lambda x}, & x>0\\0, & x\leq0\end{cases}$

$$oldsymbol{Y}$$
 的分布函数为 $oldsymbol{\mathrm{F}}_{\!Y}ig(yig) = egin{cases} 0, & x < 0 \ 1/2, & 0 \leq x < 1 \ 1 & x \geq 1 \end{cases}$

从而N的分布函数为

$$\mathbf{F}_{_{N}}\left(z
ight) = 1 - \left[1 - \mathbf{F}_{_{X}}\left(z
ight)
ight]\!\left[1 - \mathbf{F}_{_{Y}}\left(z
ight)
ight] = egin{dcases} 0, & z < 0 \ rac{1}{2}, & z = 0 \ 1 - rac{e^{-\lambda z}}{2}, & 0 < z < 1 \ 1, & z \geq 1 \end{cases}$$

二. 1. 教材习题三
$$\left(\mathbf{B}\right)$$
二第 3 题 $\frac{1}{2}$

2. 类似教材习题三
$$\left(\mathbf{B}\right)$$
三第 5 题 $f_{Z}\left(z\right)=rac{1}{2}\left[f_{Y}\left(z\right)+f_{Y}\left(z-1\right)
ight]$

三. 1.
$$ig(1ig)$$
 由题意有 $oldsymbol{Z}ig|oldsymbol{Y}=oldsymbol{n}\sim \mathrm{B}ig(oldsymbol{n},0.2ig)$,所以 $igPig(oldsymbol{Z}=oldsymbol{k}ig|oldsymbol{Y}=oldsymbol{n}ig)=\mathrm{C}_n^k0.2^k0.8^{n-k}$, $oldsymbol{k}=oldsymbol{0},oldsymbol{1},oldsymbol{2},\cdots,oldsymbol{n}$;

$$egin{split} ig(2)\ & ext{ if } k \leq n ext{ if }, \ & \mathbf{P}ig(Z=k,Y=nig) = \mathbf{P}ig(Y=nig)\mathbf{P}ig(Z=kig|Y=nig) \ & = rac{30^n}{n\,!}e^{-30}\mathrm{C}_n^k0.2^k0.8^{n-k}, \quad k=0,1,2,\cdots,n \end{split}$$

当
$$k>n$$
时, $\mathrm{P}ig(Z=k,Y=nig)=0$

2. 教材习题三 (\mathbf{B}) 三第2题

先计算X与Y的(边缘)密度函数为

$$egin{aligned} f_{_{X}}ig(xig) &= \int\limits_{-\infty}^{+\infty} fig(x,yig) dy = \left\{igg|_{-1}^{+1} rac{1}{4}ig(1+xyig) dy = rac{1}{2}, \ |x| < 1 \ 0, & |x| \geq 1
ight. \ \left(\int\limits_{-1}^{+\infty} rac{1}{4}ig(1+xyig) dx = rac{1}{2}, \ |y| < 1
ight. \end{aligned}$$

$$f_{Y}\left(y
ight)=\int\limits_{-\infty}^{+\infty}f\left(x,y
ight)dx=egin{cases} \int\limits_{-1}^{+1}rac{1}{4}ig(1+xyig)dx=rac{1}{2},\ \left|y
ight|<1 \ 0, &\left|y
ight|\geq 1 \end{cases}$$

易见,在三个密度函数的公共连续点 $\left(\frac{1}{2},\frac{1}{2}\right)$ 处, $f\left(x,y\right)=\frac{5}{16} \neq \frac{1}{4}=f_{_X}\left(x\right)f_{_Y}\left(y\right)$,

所以X与Y不独立!

令
$$U = X^2, V = Y^2$$
,显然 $R(U) = R(V) = [0,1]$,当 $0 < u < 1, 0 < v < 1$ 时,

$$egin{aligned} \mathbf{F}ig(u,vig) &= \mathbf{P}ig(U \leq u, V \leq vig) = \mathbf{P}ig(X^2 \leq u, Y^2 \leq vig) \ &= \mathbf{P}igg(-\sqrt{u} \leq X \leq \sqrt{u}, -\sqrt{v} \leq Y \leq \sqrt{v}igg) \ &= \int\limits_{-\sqrt{u}}^{\sqrt{u}}igg(\int\limits_{-\sqrt{v}}^{-\sqrt{v}}rac{1}{4}ig(1+xyig)dyigg)dx = \sqrt{uv} \end{aligned}$$

从而(U,V)的联合密度函数为

$$\psiig(u,vig) = rac{\partial^2 \mathrm{F}ig(u,vig)}{\partial u \partial v} = egin{cases} rac{1}{4\sqrt{uv}}, & 0 < u < 1, 0 < v < 1 \ 0, & others \end{cases}$$

而U与V的(边缘)密度函数为

$$\psi_{_U}ig(uig) = \int\limits_{-\infty}^{+\infty} \psiig(u,vig) dv = egin{cases} \int\limits_0^1 rac{dv}{4\sqrt{uv}} = rac{1}{2\sqrt{u}}, & 0 < u < 1 \ 0, & others \end{cases},$$

$$\psi_{_{V}}ig(vig) = \int\limits_{-\infty}^{+\infty} \psiig(u,vig) du = egin{cases} \int\limits_{_{0}}^{1} rac{du}{4\sqrt{uv}} = rac{1}{2\sqrt{v}}, & 0 < v < 1 \ 0, & others \end{cases},$$

显然可见,对任意的u,v,都有 $\psi(u,v)=\psi_U(u)\psi_V(v)$,所以U与V相互独立,

即 X^2 与 Y^2 相互独立!

3. 教材习题三(**B**)三第6题

$$ig(X,Yig)$$
的联合密度为 $fig(x,yig) = egin{cases} 1/4, & ig(x,yig) \in ig[1,3ig] imes ig[1,3ig] \ 0, & others \end{cases}$

显然 $U = \left| X - Y \right|$ 的值域为 $R\left(U \right) = \left[0, 2 \right]$,对任意的 $u \in \left(0, 2 \right)$,有

$$\mathbf{F}_{\!\scriptscriptstyle U}\left(u\right) = \mathbf{P}\!\left(U \leq u\right) = \mathbf{P}\!\left(\!\left|X - Y\right| \leq u\right) = \int\limits_{|x - y| \leq u} f\!\left(x, y\right) \! dx dy$$

$$=rac{4-ig(2-uig)^2}{4}=u-rac{1}{4}u^2$$
 ,

从而密度函数为
$$p_{_U}ig(uig) = egin{cases} 1 - rac{1}{2}u, & u \in ig(0,2ig) \ 0, & u
ot\in ig(0,2ig) \end{cases}$$

第 31-32 页

$$\Box$$
. 1. $a = 2, b = 8$

2. **780** 设抽得**3** 张奖券的总金额为**ξ**,则**ξ**
$$\sim \begin{bmatrix} 600 & 900 & 1200 \\ {C_8^3 \big/ C_{10}^3} & {C_8^2 C_2^1 \big/ C_{10}^3} & {C_8^1 C_2^2 \big/ C_{10}^3} \end{bmatrix}$$
,从而
$$f \mathbf{E}(\xi) = 600 \times \frac{\mathbf{C}_8^3}{\mathbf{C}_{10}^3} + 900 \times \frac{\mathbf{C}_8^2 \mathbf{C}_2^1}{\mathbf{C}_{10}^3} + 1200 \times \frac{\mathbf{C}_8^1 \mathbf{C}_2^2}{\mathbf{C}_{10}^3} = 780$$

3.
$$-\frac{67}{45}$$
, $9\frac{8}{15}$

三. 1. 教材习题四(A)三第 2 题

显然,X可取1,2,3,4,5,7,8,9,10,11,12,其分布律为

平均得分为
$$\mathbf{E}(X) = 1/6\sum_{i=1}^{5}i + 1/36\sum_{i=7}^{12}i = \frac{15}{6} + \frac{57}{36} = \frac{147}{36} = \frac{49}{12}$$

2. 教材习题四 (\mathbf{A}) 三第 4 题

由分布函数的右连续性得
$$A-rac{B}{25}=0$$
 $\left(1\right)$,再由 $\mathbf{F}\left(+\infty\right)=1$ 得 $A=1\left(2\right)$,联立 $\left(1\right)\left(2\right)$

两式即得
$$A=1,B=25$$
,从而密度函数为 $f(x)=\mathrm{F}'(x)=egin{cases} 0, & x\leq 5 \ 50x^{-3}, & x>5 \end{cases}$,于是动

物的平均寿命为
$$\mathrm{E}ig(Xig)=\int\limits_{-\infty}^{+\infty}xfig(xig)dx=\int\limits_{5}^{+\infty}x50x^{-3}dx=50\int\limits_{5}^{+\infty}x^{-2}dx=10$$

3. 教材习题四(A)三第 5 题

设工厂售出一台设备获利为
$$Y$$
,则 $Y=gig(Xig)=egin{cases} 600,&X<1\ 1000,&X\geq 1 \end{cases}$,从而有

$$egin{aligned} \mathrm{E}ig(Yig) &= \int\limits_{-\infty}^{+\infty} fig(xig) gig(xig) dx = \int\limits_{0}^{1} 0.25 e^{-0.25 x} imes 600 dx + \int\limits_{1}^{+\infty} 0.25 e^{-0.25 x} imes 1000 dx \ &= 600 \Big(1 - e^{-0.25}\Big) + 1000 e^{-0.25} = 600 + 400 e^{-0.25} \end{aligned}$$

4. 教材习题四(A)三第7题

显然
$$\mathbf{E}(X) = 1$$
, $\mathbf{D}(X) = 1$, $\mathbf{E}(Y) = 2$, $\mathbf{D}(Y) = 2$, 从而有

(1)
$$\mathbf{E}(X-Y) = \mathbf{E}(X) - \mathbf{E}(Y) = 1 - 2 = -1;$$

(2)
$$\mathbf{E}(2X^2 + 3Y^2) = 2\mathbf{E}(X^2) + 3\mathbf{E}(Y^2)$$

= $2\left[\mathbf{D}(X) + \left(\mathbf{E}(X)\right)^2\right] + 3\left[\mathbf{D}(Y) + \left(\mathbf{E}(Y)\right)^2\right] = 22;$

(3) 若
$$X$$
与 Y 相互独立,则 $\mathbf{E}(XY) = \mathbf{E}(X)\mathbf{E}(Y) = \mathbf{2}$.

5. Y 表每次检验的次品数,则 $Y \sim \mathbf{B} ig(\mathbf{10,0.1} ig)$

$$p = P(Y > 1) = 1 - 0.9^{10} - C_{10}^1 0.1^1 0.9^{10} = 0.2639$$

$$Z \sim \mathrm{B}ig(4,0.2639ig), \quad \mathrm{E}ig(Xig) = 4 imes 0.2639 = 1.06$$

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- —. 1. **A** 2. **C** 3. **B**

- \equiv 1. **0.9** 2. $2a^2$ 3. $k! \frac{1}{a^k}$

三. 1. 教材习题四(A)三第 10 题

$$\mathrm{E}ig(Xig)=\int\limits_{-\infty}^{+\infty}xfig(xig)dx=\int\limits_{-1}^{1}xrac{1}{\pi\sqrt{1-x^2}}dx=0;$$

$$\mathrm{E}ig(X^2ig) = \int\limits_{-\infty}^{+\infty} x^2 fig(xig) dx = \int\limits_{-1}^1 x^2 rac{1}{\pi\sqrt{1-x^2}} dx = rac{1}{2};$$

$$\left(\int rac{x^2 dx}{\sqrt{a^2-x^2}} = -rac{x}{2}\sqrt{a^2-x^2} + rac{a^2}{2}rcsinrac{x}{a} + \mathrm{C}
ight)$$

$$\mathrm{D}ig(Xig) = \mathrm{E}ig(X^2ig) - ig(\mathrm{E}ig(Xig)ig)^2 = rac{1}{2}.$$

2. 教材习题四(A)三第 13 题

$$\mathrm{E}ig(Yig)=\int\limits_{0}^{1/2}2x^{2} imes2dx=rac{1}{6};\quad \mathrm{E}ig(Y^{2}ig)=\int\limits_{0}^{1/2}ig(2x^{2}ig)^{2} imes2dx=rac{1}{20};$$

$$D(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{20} - \frac{1}{36} = \frac{1}{45}$$

3. 教材习题四(A)三第 12 题

由于
$$X \sim Uig(-1,3ig), Y \sim eig(2ig), Z \sim \Gammaig(2,2ig)$$
,所以

$$\mathrm{E}\left(X
ight)=1,\,\mathrm{E}\left(Y
ight)=rac{1}{2},\,\mathrm{E}\left(Z
ight)=1,\,\mathrm{D}\left(X
ight)=rac{4}{3},\,\mathrm{D}\left(Y
ight)=rac{1}{4},\,\mathrm{D}\left(Z
ight)=rac{1}{2}$$

且 X,Y,Z 相互独立,从而有

(1)
$$\mathbf{E}(U) = \mathbf{E}(3X - 2XY + 4YZ - 2)$$

 $= 3\mathbf{E}(X) - 2\mathbf{E}(X)\mathbf{E}(Y) + 4\mathbf{E}(Y)\mathbf{E}(Z) - 2$
 $= 3 \times 1 - 2 \times 1 \times \frac{1}{2} + 4 \times \frac{1}{2} \times 1 - 2 = 2$

(2)
$$\mathbf{D}(V) = \mathbf{D}(X - 2Y + 3Z - 2)$$

= $\mathbf{D}(X) + 4\mathbf{D}(Y) + 9\mathbf{D}(Z) = \frac{4}{3} + 4 \times \frac{1}{4} + 9 \times \frac{1}{2} = \frac{41}{6}$

4.
$$\mathbb{E}(X-c)^2 = \mathbb{D}(X-c) + \left[\mathbb{E}(X-c)\right]^2 = \mathbb{D}(X) + \left[\mathbb{E}(X-c)\right]^2 \ge \mathbb{D}(X)$$

5. **1.5**

互独立且
$$X = \sum_{i=1}^5 X_i$$
 , $X_i \sim \begin{bmatrix} 1 & 0 \\ i/10 & 1-i/10 \end{bmatrix}$, $i=1,2,3,4,5$, 从而有

$$\mathrm{E}\left(X_{_{i}}
ight) = rac{i}{10}, \ \mathrm{E}\left(X_{_{i}}^{^{2}}
ight) = rac{i}{10}, \ \mathrm{D}\left(X_{_{i}}
ight) = rac{i\left(10-i
ight)}{100}, \ i=1,2,3,4,5 \ ,$$

所以有

$$\mathbf{E}(X) = \mathbf{E}\left(\sum_{i=1}^{5} X_{i}\right) = \sum_{i=1}^{5} \mathbf{E}(X_{i}) = \frac{1+2+3+4+5}{10} = 1.5,$$

$$\mathrm{D}ig(Xig) = \mathrm{D}igg(\sum_{i=1}^5 X_iigg) = \sum_{i=1}^5 \mathrm{D}ig(X_iig) = \sum_{i=1}^5 rac{iig(10-iig)}{100} = rac{9+16+21+24+25}{100} = 0.95$$

第 35-38页《协方差与祖吴系数》

- 一. 1. **B**
- 2. **A**
- 3. **C**

$$\equiv$$
 1. $\frac{4}{3} + \frac{1}{2\sqrt{3}}$ 2. $\frac{1}{4}$ 3. $\frac{2}{3}\sigma^2$

2.
$$\frac{1}{4}$$

$$3. \quad \frac{2}{3}\sigma^2$$

三. 1. 教材习题四(A)三第 15 题

$$\mathrm{E}ig(XYig)=\int\limits_{-\infty}^{+\infty}dx\int\limits_{-\infty}^{+\infty}xyfig(x,yig)dy=\int\limits_{0}^{1}xdx\int\limits_{0}^{1}yig(2-x-yig)dy=rac{1}{6}$$

边缘密度函数为

$$f_{_{X}}ig(xig)=\int\limits_{-\infty}^{+\infty}fig(x,yig)dy=egin{cases} \int\limits_{0}^{1}ig(2-x-yig)dy=rac{3}{2}-x,\ 0\leq x\leq 1\ 0, & x
ot\in[0,1] \end{cases}$$

$$f_{Y}\left(y
ight)=\int\limits_{-\infty}^{+\infty}fig(x,yig)dx=egin{cases} \int\limits_{0}^{1}ig(2-x-yig)dx=rac{3}{2}-y,\ 0\leq y\leq 1\ 0, & y
ot\in[0,1] \end{cases}$$

从而有

$$\mathrm{E}ig(Xig) = \int\limits_{-\infty}^{+\infty} x f_Xig(xig) dx = \int\limits_{0}^{1} x \Big(3ig/2 - x\Big) dx = rac{5}{12}$$

$$\mathrm{E}ig(X^2ig) = \int\limits_{-\infty}^{+\infty} x^2 f_Xig(xig) dx = \int\limits_{0}^{1} x^2 \Big(3ig/2 - x\Big) dx = rac{1}{4}$$

$$\mathbf{D}(X) = \mathbf{E}(X^2) - (\mathbf{E}(X))^2 = \frac{1}{4} - \frac{5^2}{12^2} = \frac{36}{144}$$

$$\mathrm{E}ig(Yig) = \int\limits_{-\infty}^{+\infty} y f_{_{Y}}ig(yig) dy = \int\limits_{_{0}}^{_{1}} y \Big(3ig/2 - y\Big) dy = rac{5}{12}.$$

$$\mathrm{E}ig(Y^2ig)=\int\limits_{-\infty}^{+\infty}y^2f_{_Y}ig(yig)dy=\int\limits_{-\infty}^{1}y^2ig(3ig/2-yig)dy=rac{1}{4}$$

$$\mathbf{D}\big(Y\big) = \mathbf{E}\big(Y^2\big) - \big(\mathbf{E}\big(Y\big)\big)^2 = \frac{1}{4} - \frac{5^2}{12^2} = \frac{36}{144}$$

$$\mathrm{Cov}\big(X,Y\big) = \mathrm{E}\big(XY\big) - \mathrm{E}\big(X\big)\mathrm{E}\big(Y\big) = \frac{1}{6} - \frac{5^2}{12^2} = -\frac{1}{144}$$

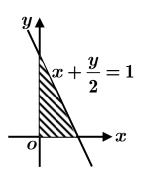
协方差阵为
$$V = egin{bmatrix} \mathbf{D}ig(Xig) & \mathbf{Cov}ig(X,Yig) \ \mathbf{Cov}ig(X,Yig) & \mathbf{D}ig(Yig) \end{bmatrix} = egin{bmatrix} rac{36}{144} & -rac{1}{144} \ -rac{1}{144} & rac{36}{144} \end{bmatrix}$$

相关系数为
$$R\left(X,Y\right) = \frac{\operatorname{Cov}\left(X,Y\right)}{\sqrt{\operatorname{D}\left(X\right)}\sqrt{\operatorname{D}\left(Y\right)}} = \frac{-\frac{1}{144}}{\sqrt{\frac{36}{144}}\sqrt{\frac{36}{144}}} = -\frac{1}{36}$$

2. 教材习题四(A)三第 16 题

如图,联合密度为

$$egin{align} fig(x,yig) &= egin{cases} 1, & 0 < x < 1, 0 < y < 2 - 2x \ 0, & others \end{cases} \ & ext{E}ig(XYig) &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} xy fig(x,yig) dy \ &= \int\limits_{0}^{1} x dx \int\limits_{0}^{2-2x} y dy = rac{7}{6} \end{cases}$$



边缘密度函数为

$$f_{_{X}}ig(xig)=\int\limits_{-\infty}^{+\infty}fig(x,yig)dy=egin{cases} \int\limits_{0}^{2-2x}1dy=2-2x,\ 0\leq x\leq 1\ 0, & x
ot\in[0,1] \end{cases}$$

$$f_{_{Y}}\left(y
ight)=\int\limits_{-\infty}^{+\infty}f\left(x,y
ight)dx=egin{cases} \int\limits_{0}^{1-y/2}1dx=1-rac{y}{2},\ 0\leq y\leq 2\ 0, \qquad y
ot\in\left[0,2
ight] \end{cases}$$

从而有

$$(1) \quad \mathbf{E}\left(X\right) = \int_{-\infty}^{+\infty} x f_X\left(x\right) dx = \int_{0}^{1} x \left(2 - 2x\right) dx = \frac{1}{3}$$

$$\mathbf{E}\left(Y\right) = \int_{-\infty}^{+\infty} y f_Y\left(y\right) dy = \int_{0}^{2} y \left(1 - y/2\right) dy = \frac{2}{3}$$

$$(2) \quad \mathbf{E}\left(X^2\right) = \int_{-\infty}^{+\infty} x^2 f_X\left(x\right) dx = \int_{0}^{1} x^2 \left(2 - 2x\right) dx = \frac{1}{6}$$

$$egin{aligned} \mathbf{D}ig(m{X}ig) &= \mathbf{E}ig(m{X}^2ig) - ig(\mathbf{E}ig(m{X}ig)ig)^2 = rac{1}{6} - rac{1}{9} = rac{1}{18} \ &= ig(m{Y}^2ig) = \int\limits_{-\infty}^{+\infty} y^2 f_Yig(yig) dy = \int\limits_0^2 y^2 ig(1 - yig/2ig) dy = rac{2}{3} \ &= ig(m{Y}ig) = \mathbf{E}ig(m{Y}^2ig) - ig(\mathbf{E}ig(m{Y}ig)ig)^2 = rac{2}{3} - rac{4}{9} = rac{2}{9} \end{aligned}$$

(3)
$$Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{1}{6} - \frac{1}{3} \times \frac{2}{3} = -\frac{1}{18}$$

$$R\left(X,Y\right) = \frac{\operatorname{Cov}\left(X,Y\right)}{\sqrt{\operatorname{D}\left(X\right)}\sqrt{\operatorname{D}\left(Y\right)}} = \frac{-\frac{1}{18}}{\sqrt{\frac{1}{18}\sqrt{\frac{2}{9}}}} = -\frac{1}{2}$$

- (4) 因 $R(X,Y)(=-1/2) \neq 0$, 即X 与 Y相关, 当然不独立.
- 3. 教材习题四(A)三第 17 题

$$\operatorname{Cov}ig(X+Y,X-Yig) = \operatorname{Cov}ig(X,Xig) + \operatorname{Cov}ig(Y,Xig) - \operatorname{Cov}ig(X,Yig) - \operatorname{Cov}ig(Y,Yig)$$

$$= \operatorname{D}ig(Xig) - \operatorname{D}ig(Yig)$$

4. 教材习题四(A)三第 18 题

直接由上题的结论有

$$egin{aligned} \operatorname{Cov}ig(U,Vig) &= \operatorname{Cov}ig(lpha X + eta Y, lpha X - eta Yig) = \operatorname{D}ig(lpha Xig) - \operatorname{D}ig(eta Yig) \\ &= lpha^2\operatorname{D}ig(Xig) - eta^2\operatorname{D}ig(Yig) = lpha^2\sigma^2 - eta^2\sigma^2 = ig(lpha^2 - eta^2ig)\sigma^2 \end{aligned}$$

因X与Y相互独立,故

$$\mathbf{D}ig(Uig) = \mathbf{D}ig(lpha X + eta Yig) = lpha^2 \mathbf{D}ig(Xig) + eta^2 \mathbf{D}ig(Yig) = ig(lpha^2 + eta^2ig)\sigma^2$$

$$\mathbf{D}\big(V\big) = \mathbf{D}\big(\alpha X - \beta Y\big) = \alpha^2 \mathbf{D}\big(X\big) + \beta^2 \mathbf{D}\big(Y\big) = \big(\alpha^2 + \beta^2\big)\sigma^2$$

从而有

$$R\!\left(X,Y\right) = \frac{\mathrm{Cov}\!\left(X,Y\right)}{\sqrt{\mathrm{D}\!\left(X\right)}\sqrt{\mathrm{D}\!\left(Y\right)}} = \frac{\left(\alpha^2 - \beta^2\right)\sigma^2}{\sqrt{\left(\alpha^2 + \beta^2\right)\sigma^2}\sqrt{\left(\alpha^2 + \beta^2\right)\sigma^2}} = \frac{\alpha^2 - \beta^2}{\alpha^2 + \beta^2}$$

5. 教材习题四(A)三第 14 题

由联合分布律易得

$$m{X} \sim egin{bmatrix} m{1} & m{2} & m{3} \ 4/9 & 1/3 & 2/9 \end{bmatrix}\!\!, \;\; m{Y} \sim egin{bmatrix} m{0} & m{1} & m{2} \ 5/9 & 1/3 & 1/9 \end{bmatrix}\!\!, \;\; m{X}m{Y} \sim egin{bmatrix} m{0} & m{1} & m{2} \ 5/9 & 2/9 & 2/9 \end{bmatrix}$$

所以

(1)
$$E(X) = 1 \times 4/9 + 2 \times 3/9 + 3 \times 2/9 = 16/9$$

 $E(X^2) = 1^2 \times 4/9 + 2^2 \times 3/9 + 3^2 \times 2/9 = 34/9$
 $D(X) = E(X^2) - [E(X)]^2 = 34/9 - 16^2/9^2 = 50/81$
 $E(Y) = 0 \times 5/9 + 1 \times 3/9 + 2 \times 1/9 = 5/9$
 $E(Y^2) = 0^2 \times 5/9 + 1^2 \times 3/9 + 2^2 \times 1/9 = 7/9$
 $D(Y) = E(Y^2) - [E(Y)]^2 = 7/9 - 5^2/9^2 = 38/81$
(2) $E(XY) = 0 \times 5/9 + 1 \times 2/9 + 2 \times 2/9 = 2/3$
 $Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{2}{3} - \frac{16}{9} \times \frac{5}{9} = -\frac{26}{81}$
 $R(X,Y) = \frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{-26/81}{\sqrt{50/81}\sqrt{38/81}} = -\frac{13}{5\sqrt{19}}$
(3) $D(X - 3Y) = D(X) + 9D(Y) - 6Cov(X,Y)$
 $= \frac{50}{91} + 9 \times \frac{38}{91} - 6 \times (-)\frac{26}{91} = \frac{548}{91}$

第 39-40 页《第四章综合练习》

$$-. 1. \frac{8}{9}$$
 2. $\sqrt{\frac{m}{m+n}}$

二.1. 教材习题四 (\mathbf{B}) 三第1题

设应组织y吨货源,才能使平均收益最大,则必有 $y \in (2000,4000)$.

设收益为Y,由题意有

$$Y=gig(Xig)=egin{cases} 3y, & X>y \ 3X-ig(y-Xig)=4X-y, & X\leq y \end{cases}$$

于是平均收益为

$$egin{align} ext{E}ig(Yig) &= ext{E}ig(gig(Xig)ig) = \int\limits_{-\infty}^{+\infty} gig(xig)f_Xig(xig)dx = \int\limits_{2000}^{y} rac{4x-y}{2000}dx + \int\limits_{y}^{4000} rac{3y}{2000}dx \ &= rac{1}{1000}ig[-y^2 + 7000y - 2000^2ig] \end{aligned}$$

显然可见,当组织的货源为y=3500吨时,平均收益 $\mathbf{E}ig(Yig)$ 最大.

2. 教材习题四(**B**)三第2题

设X为停车次数,并设

$$m{X}_i = egin{cases} m{1}, & \hat{m{x}} i$$
站有人下车 $m{0}, & \hat{m{x}} i$ 站无人下车 $m{i} = m{1}, m{2}, \cdots, m{10}$,则 $m{X} = \sum_{i=1}^{10} m{X}_i$

由题意有

$$egin{aligned} oldsymbol{X}_i \sim egin{bmatrix} 0 & 1 \ 0.9^{20} & 1-0.9^{20} \end{bmatrix}, & i=1,2,\cdots,10 \,, & oldsymbol{oldsymbol{eta}} oldsymbol{\mathrm{E}}ig(oldsymbol{X}_iig) = 1-0.9^{20}, \, i=1,2,\cdots,10 \end{aligned}$$

从而有平均停车次数为
$$\mathbf{E}(X) = \mathbf{E}\left(\sum_{i=1}^{10} X_i\right) = \sum_{i=1}^{10} \mathbf{E}(X_i) = \mathbf{10}(\mathbf{1} - \mathbf{0.9^{20}}).$$

3. 教材习题四**(B)**三第 6 题

$$\begin{array}{l} \text{(1)} \ \ \mathrm{E} \Big(X \Big) = \int\limits_{-\infty}^{+\infty} x f \Big(x \Big) dx = \int\limits_{-1}^{1} x \left| x \right| dx = 0 \ , \\ \\ \mathrm{E} \Big(X^2 \Big) = \int\limits_{-\infty}^{+\infty} x^2 f \Big(x \Big) dx = \int\limits_{-1}^{1} x^2 \left| x \right| dx = 2 \int\limits_{0}^{1} x^2 x dx = \frac{1}{2} \ , \\ \\ \mathrm{D} \Big(X \Big) = \mathrm{E} \Big(X^2 \Big) - \Big[\mathrm{E} \Big(X \Big) \Big]^2 = \frac{1}{2} \end{array}$$

(2) 因
$$\mathbf{E}\left(X\left|X\right|\right) = \int_{-\infty}^{+\infty} x\left|x\right| f\left(x\right) dx = \int_{-1}^{1} x\left|x\right|^{2} dx = 0$$
, $\mathbf{E}\left(X\right) = 0$, 所以 $\mathbf{Cov}\left(X,\left|X\right|\right) = \mathbf{E}\left(X\left|X\right|\right) - \mathbf{E}\left(X\right)\mathbf{E}\left(\left|X\right|\right) = 0$;

(3)
$$\operatorname{Cov} \left(X, \left| X \right| \right) = 0$$
 表明 $R \left(X, \left| X \right| \right) = 0$,从而 X 与 $\left| X \right|$ 不相关! 但 X 与 $\left| X \right|$ 不是相互独立的.这是因为:对任意 $x \in \left(0, 1/2 \right)$,有 $\operatorname{P} \left(X < x \right) < 1$,从而

$$\mathbf{P}(X < x, |X| \le x) = \mathbf{P}(|X| \le x) \neq \mathbf{P}(X < x)\mathbf{P}(|X| \le x)$$
所以 $X = |X|$ 不相互独立!

4. 教材习题四**(B)**三第 8 题

证明 由题意知(X,Y)的联合分布律为

$$\begin{split} \mathbf{P}\big(X=1,Y=1\big) &= \mathbf{P}\big(AB\big), \ \mathbf{P}\big(X=-1,Y=-1\big) = \mathbf{P}\big(\bar{A}\bar{B}\big) \\ \mathbf{P}\big(X=-1,Y=1\big) &= \mathbf{P}\big(\bar{A}B\big), \ \mathbf{P}\big(X=1,Y=-1\big) = \mathbf{P}\big(A\bar{B}\big) \end{split}$$

从而有

$$XY \sim egin{bmatrix} 1 & -1 \ Pig(ABig) + Pig(ar{A}ar{B}ig) & Pig(ar{A}Big) + Pig(Aar{B}ig) \end{pmatrix},$$

故

$$\begin{split} \mathbf{E}\big(XY\big) &= \mathbf{1} \times \left[\mathbf{P}\big(AB\big) + \mathbf{P}\big(\bar{A}\bar{B}\big)\right] + \left(-1\right) \times \left[\mathbf{P}\big(\bar{A}B\big) + \mathbf{P}\big(A\bar{B}\big)\right] \\ &= \mathbf{P}\big(AB\big) + \mathbf{P}\big(\bar{A}\bar{B}\big) - \mathbf{P}\big(\bar{A}B\big) - \mathbf{P}\big(A\bar{B}\big) \\ &= \mathbf{P}\big(AB\big) + \mathbf{P}\Big(\overline{A \cup B}\Big) - \left[\mathbf{P}\big(B\big) - \mathbf{P}\big(AB\big)\right] - \left[\mathbf{P}\big(A\big) - \mathbf{P}\big(AB\big)\right] \\ &= \mathbf{3}\mathbf{P}\big(AB\big) + \left[\mathbf{1} - \mathbf{P}\big(A \cup B\big)\right] - \mathbf{P}\big(B\big) - \mathbf{P}\big(A\big) \\ &= \mathbf{4}\mathbf{P}\big(AB\big) + \mathbf{1} - \mathbf{2}\mathbf{P}\big(B\big) - \mathbf{2}\mathbf{P}\big(A\big) \end{split}$$

边缘分布律为

$$X \sim egin{bmatrix} 1 & -1 \ \mathbf{Pig(A)} & \mathbf{Pig(ar{A}ig)} \end{bmatrix}\!, \quad Y \sim egin{bmatrix} 1 & -1 \ \mathbf{Pig(B)} & \mathbf{Pig(ar{B}ig)} \end{bmatrix}\!,$$

故

$$\mathbf{E}(X) = \mathbf{1} \times \mathbf{P}(A) + (-1) \times \mathbf{P}(\overline{A}) = \mathbf{P}(A) - \mathbf{P}(\overline{A}) = \mathbf{2P}(A) - \mathbf{1}$$

$$\mathbf{E}(Y) = \mathbf{1} \times \mathbf{P}(B) + (-1) \times \mathbf{P}(\overline{B}) = \mathbf{P}(B) - \mathbf{P}(\overline{B}) = \mathbf{2P}(B) - \mathbf{1}$$

于是
$$X 与 Y$$
 不相关 \Leftrightarrow $\mathbf{E}(XY) = \mathbf{E}(Y)\mathbf{E}(X)$

$$\Leftrightarrow 4\mathbf{P}\!\left(AB\right) + 1 - 2\mathbf{P}\!\left(B\right) - 2\mathbf{P}\!\left(A\right) = \left[2\mathbf{P}\!\left(A\right) - 1\right]\!\left[2\mathbf{P}\!\left(B\right) - 1\right]$$

$$\Leftrightarrow P(AB) = P(A)P(B) \Leftrightarrow A 与 B$$
 相互独立

第 41-42 页《正 交分布》

一. 1. **A**

$$\begin{split} p_{_1} &= \mathbf{P} \Big(X \leq \mu - 2 \Big) = \Phi \bigg(\frac{\mu - 2 - \mu}{2} \bigg) = \Phi \Big(-1 \Big) = 1 - \Phi \Big(1 \Big) \\ \\ p_{_2} &= \mathbf{P} \Big(Y \geq \mu + 3 \Big) = 1 - \mathbf{P} \Big(Y \leq \mu + 3 \Big) = 1 - \Phi \bigg(\frac{\mu + 3 - \mu}{3} \bigg) = 1 - \Phi \Big(1 \Big) \end{split}$$

2. **C**

$$\begin{split} \mathbf{P} \Big(\! \big| X - \mu \big| < q \Big) > \mathbf{P} \Big(\! \big| Y - \mu \big| < q \Big) \Rightarrow \mathbf{P} \Bigg(\! \Big| \frac{X - \mu}{\sigma_1} \! \Big| < \frac{q}{\sigma_1} \Big) > \mathbf{P} \Bigg(\! \Big| \frac{Y - \mu}{\sigma_2} \! \Big| < \frac{q}{\sigma_2} \Big) \\ \Rightarrow 2 \Phi \Bigg(\frac{q}{\sigma_1} \Bigg) - 1 > 2 \Phi \Bigg(\frac{q}{\sigma_2} \Bigg) - 1 \Rightarrow \Phi \Bigg(\frac{q}{\sigma_1} \Bigg) > \Phi \Bigg(\frac{q}{\sigma_2} \Bigg) \Rightarrow \sigma_1 < \sigma_1 \end{split}$$

3. **C**

曲
$$X \sim f\left(x
ight) = rac{1}{\sqrt{3\pi}} e^{rac{1}{3}\left(2x-x^2-1
ight)} = rac{1}{\sqrt{2\pi}\sqrt{3/2}} e^{-rac{\left(x-1
ight)^2}{2\left(\sqrt{3/2}
ight)^2}}$$
,即 $X \sim \mathbf{N}\left(1,3/2
ight)$,从而 $\mathbf{E}\left(X
ight) = 1, \mathbf{D}\left(X
ight) = 3/2$

二. 1. **0.2**

因
$$X\sim \mathrm{N}ig(\mu,\sigma^2ig)$$
且 $\mathrm{P}ig(X>2ig)=0.5$,则 $\mu=2$, $\mathrm{P}ig(X\leq 2ig)=0.5$,从而 $\mathrm{P}ig(0< X<2ig)=\mathrm{P}ig(2< X<4ig)=0.3$,所以 $\mathrm{P}ig(X<0ig)=\mathrm{P}ig(X\leq 2ig)-\mathrm{P}ig(0< X<2ig)=0.5-0.3=0.2$

2. **0.12**

因
$$X\sim \mathrm{N}ig(2,4ig)$$
 且 $\mathrm{P}ig(ig|X-2ig|>aig)\geq 0.95$,则 $\mathrm{P}igg(ig|igX-2igg|\leq rac{a}{2}ig)\leq 0.05$,即 $2\Phiig(a/2ig)-1\leq 0.05$,所以 $\Phiig(a/2ig)\leq 0.525$,查表得 $a/2\leq 0.06$,因而 $a\leq 0.12$.

3. $\mathbf{0}$ 因 X 的密度函数是偶函数,所以 $\mathbf{E} ig(X^{2n+1} ig) = \mathbf{0} = \mathbf{E} ig(X ig)$,从而有 $\mathbf{Cov} ig(X, Y ig) =$

$$\mathbf{E}\big(XY\big) - \mathbf{E}\big(X\big)\mathbf{E}\big(Y\big) = \mathbf{E}\big(X^{2n+1}\big) - \mathbf{E}\big(X\big)\mathbf{E}\big(X^{2n}\big) = \mathbf{0} \,, \ \, \text{id} \, \mathbf{R}\big(X,Y\big) = \mathbf{0} \,.$$

$$\equiv$$
. 1. (1)0.9525 (2)0.3707 (3)0.3115 (4)0.7714

2. 教材习题五(**A**)三第2题

每个新生婴儿体重小于2719g的概率为

$$\mathbf{P}ig(X < 2719ig) = \Phiigg(rac{2719 - 3315}{575}igg) = \Phiig(-1.0365ig) = 1 - \Phiig(1.0365ig) = 0.15$$
 设所选的 $\mathbf{100}$ 个新生婴儿中体重小于 $\mathbf{2719}g$ 的个数为 Y ,则 $Y \sim \mathbf{B}ig(\mathbf{100}, \mathbf{0.15}ig)$,从而

所求概率为
$$P(Y \ge 2) = 1 - P(Y = 0) - P(Y = 1) = 1 - C_{100}^0 0.15^0 0.85^{100} -$$
 $-C_{100}^1 0.15^1 0.85^{99} = 1 - 0.85^{99} \times 15.85 = 1$

3. 教材习题五(A)三第3题

显然
$$Rig(Yig) = ig[0, +\inftyig)$$
. 对任意的 $y>0$, $\mathbf{F}_Yig(yig) = \mathbf{P}ig(Y\leq yig) = \mathbf{P}ig(|X|\leq yig) = 2\Phiig(yig) - 1$,此时 $f_Yig(yig) = \mathbf{F}_Y'ig(yig) = 2\varphiig(yig) = rac{2}{\sqrt{2\pi}}\,e^{-rac{y^2}{2}}$,于是综上有
$$f_Yig(yig) = \begin{cases} rac{2}{\sqrt{2\pi}}\,e^{-rac{y^2}{2}}, & y>0 \\ 0, & y\leq 0 \end{cases}$$

4. 教材习题五(A)三第 5 题

(1) 联合密度函数为
$$f(x,y) =$$

$$\begin{cases} \frac{1}{2}e^{-\frac{1}{2}y}, & x \in [0,1], y > 0 \\ 0, & others \end{cases}$$

(2)
$$a^2 + 2aX + Y = 0$$
有实根 $\Leftrightarrow X^2 \ge Y$,故所求概率为

$$egin{align} ext{P}ig(X^2 \geq Yig) &= \int\limits_0^1 dx \int\limits_0^{x^2} rac{1}{2} e^{-rac{1}{2}y} dy = 1 - \int\limits_0^1 e^{-rac{1}{2}x^2} dx \ &= 1 - \sqrt{2\pi} \int\limits_0^1 rac{1}{\sqrt{2\pi}} e^{-rac{1}{2}x^2} dx = 1 - \sqrt{2\pi} \left(\Phiig(1ig) - \Phiig(0ig)
ight) \ &= 1 - 0.3413 \sqrt{2\pi} = 0.1445 \ \end{split}$$

第 43-44 页 二维正态分布及自然指数分布族

一. 1. **B** 由二维正态密度定义,该密度函数可改写为

$$fig(x,yig) = rac{1}{\sqrt{3\pi}} \expigg[-rac{2}{3}ig(x^2+xy+y^2ig)igg]$$

$$=rac{1}{2\pi\sqrt{1-ig(-1\!\!\left/2
ight)^{\!2}}}\exp\!\left[\!-rac{1}{2ig[1-ig(\!-1\!\!\left/2
ight)^{\!2}ig]}\!\!\left(\!x^{2}-2ig(\!-1\!\!\left/2
ight)\!xy+y^{2}
ight)\!
ight]$$

- 2. \mathbf{C} 由上题显然可见 $\left(X,Y\right)\sim\mathbf{N}\left(0,0,1,1,-1/2\right)$,故 $X,Y\sim\mathbf{N}\left(0,1\right)$ 但不独立.
- 3. \mathbf{B} (U,V) 服从二维正态分布(因U,V的意一维线性组合服从一维正态),故: U,V独立 $\Leftrightarrow \mathbf{C}ov(U,V) = \mathbf{D}(X) \mathbf{D}(Y) = 0 \Leftrightarrow \sigma_1^2 = \sigma_2^2$

$$\equiv$$
. 1. $\frac{1}{3}$, 7, $\mathbf{N}\left(\frac{1}{3},7\right)$

2.
$$V(m) = 10p(1-p) = m - \frac{m^2}{10}$$
 (因为 $V(m) = D(X), m = E(X)$)

$$3. \ \frac{1}{2a} \sqrt{2\pi}$$

三. 1. 教材习题五(A)三第4题

显然, $R(Z) = [0, +\infty)$; 对任意的z > 0, $F_Z(z) = P(Z \le z) = P(\frac{1}{2}m(X^2 + Y^2) \le z)$

$$=P(X^2+Y^2 \leq rac{2z}{m}) = \int\!\!\!\int_{x^2+y^2 \leq rac{2z}{m}} rac{1}{2\pi\sigma^2} e^{-rac{x^2+y^2}{2\sigma^2}} dx dy$$

$$\stackrel{x=r\cos heta}{\longrightarrow}\int\limits_0^{2\pi}d heta\int\limits_0^{\sqrt{rac{2z}{m}}}rac{1}{2\pi\sigma^2}e^{-rac{r^2}{2\sigma^2}}rdr=1-e^{-rac{z}{m\sigma^2}};$$
 所以

$$F_{_{\!Z}}\!\left(z
ight)\!=\!egin{cases} 1-e^{-rac{z}{m\sigma^2}}, & z>0\ 0, & z\leq0 \end{cases}, \quad f_{_{\!Z}}\!\left(z
ight)\!=\!egin{cases} rac{1}{m\sigma^2}e^{-rac{z}{m\sigma^2}}, & z>0\ 0, & z\leq0 \end{cases}$$

2. 教材习题五(A)三第 6 题

$$C_{_{v}}=1\Rightarrow rac{\sqrt{D\left(X_{_{i}}
ight)}}{E\left(X_{_{i}}
ight)}=1\Rightarrow rac{\sigma_{_{i}}}{i}=1\Rightarrow \sigma_{_{i}}=i\,,\;\;i=1,2,3,4$$

于是,
$$X = \sum_{i=1}^4 X_i \sim N \left(\sum_{i=1}^4 \mu_i, \sum_{i=1}^4 \sigma_i^2 \right) = N \left(10, 30 \right)$$
,从而有 $P \left(2 < Z < 18 \right) = \Phi \left(\frac{18 - 10}{\sqrt{30}} \right) - \Phi \left(\frac{2 - 10}{\sqrt{30}} \right) = 2\Phi \left(\frac{2 - 10}{\sqrt{30}} \right) - 1 = \cdots$

3. 教材习题五(A)三第7题(略)

第 45-46 页 极阻定理

- —. 1. **С** 2. **С** 3. **А**
- \equiv . 1. $\frac{1}{n\lambda}$ 2. $\frac{1}{12}$
- 三. 1. 教材习题六(A)三第1题

$$egin{aligned} E\left(X
ight) &= \int\limits_{0}^{+\infty} x rac{x^m e^{-x}}{m!} dx = rac{\Gammaig(m+2ig)}{m!} = rac{ig(m+1ig)!}{m!} = m+1 \ E\left(X^2
ight) &= \int\limits_{0}^{+\infty} x^2 rac{x^m e^{-x}}{m!} dx = rac{\Gammaig(m+3ig)}{m!} = rac{ig(m+2ig)!}{m!} = ig(m+1ig)ig(m+2ig) \ Dig(Xig) &= Eig(X^2ig) - ig[Eig(Xig)ig]^2 = m+1 \ Pig(0 < X < 2ig(m+1ig)ig) = Pig(ig|X - Eig(Xig)ig| < ig(m+1ig) > 1 - rac{Dig(Xig)}{ig(m+1ig)^2} \ &= 1 - rac{m+1}{ig(m+1ig)^2} = rac{m}{m+1} \end{aligned}$$

2. 教材习题六(A)三第2题

$$\left(1
ight)rac{ heta}{2};\;\;\left(2
ight)E\left(X^{4}
ight)=\int\limits_{0}^{ heta}x^{4}rac{1}{ heta}dx=rac{1}{5} heta^{4};\;\;\left(3
ight) heta$$

3. 教材习题六**(A)**三第 4 题

设 $m{X}_i$ 为第 $m{i}$ 个灯泡的寿命,则 $m{E}ig(m{X}_iig)=m{0.2}, m{D}ig(m{X}_iig)=m{0.04}$, $m{i}=1,2,\cdots,200$.

由独立同分布中心极限定理,近似地有 $\sum_{i=1}^{200} X_i \sim N(40,8)$,则灯泡的平均寿命

$$ar{X}\sim Nig(0.2,0.0002ig)$$
,于是,所求概率为

$$Pig(ar{X}>0.21ig)=1-\Phiigg(rac{0.21-0.2}{\sqrt{0.0002}}igg)=1-\Phiig(0.71ig)=0.2389$$

4. 教材习题六**(A)**三第 6 题

设 $oldsymbol{X}$ 为正常工作的元件个数,由题意知, $oldsymbol{X} \sim oldsymbol{B} ig(oldsymbol{100,0.9} ig)$

ig(1ig)由二项分布以正态分布为极限的中心极限定理,近似地有 $oldsymbol{X} \sim oldsymbol{N}ig(90,9ig)$,

从而系统正常工作的概率为
$$P\left(X\geq 85
ight)=1-\Phi\left(rac{85-90}{3}
ight)=\Phi\left(rac{5}{3}
ight)=0.952$$

ig(2ig) 此时, $oldsymbol{X} \sim oldsymbol{B}ig(oldsymbol{n}, \mathbf{0.9}ig)$,由二项分布以正态分布为极限的中心极限定理,近似地

有
$$m{X} \sim m{N}ig(0.9n, 0.09nig)$$
,由题意应有 $m{P}ig(m{X} \geq 80\%nig) \geq 0.95$,即

$$\Phiigg(rac{0.9n-0.8n}{0.3\sqrt{n}}igg) \geq 0.95$$
,所以 $rac{0.9n-0.8n}{0.3\sqrt{n}} \geq 1.645$, $n \geq 25$

第 47-48 页 χ^2, t, F 分布

- —. 1. **D** 2. **C** 3. **D** 4. **C**
- \equiv . 1. $\frac{1}{3}$, 2 2. 0.7 3. t(9) 4. 0.4234
- 三. 1. 教材习题七(A)三第1题

$$\overline{x} = 50.56, s^2 = 4.28, b_2 = 3.80$$

2. 教材习题七(A)三第3题

$$rac{Y_{_{n}}}{6.4} = rac{\sum\limits_{i=1}^{n} X_{_{i}}^{^{2}}}{6.4} \sim \chi^{^{2}}ig(nig), \;\; Pig(Y_{_{n}} > 200ig) \leq 0.01 \Rightarrow Pigg(rac{Y_{_{n}}}{6.4} \leq rac{200}{6.4}igg) \geq 0.9$$

$$ightarrow rac{200}{6.4} \geq \chi_{_{0.9}}^{^{2}}ig(nig)$$
 $\Rightarrow 31.25 \geq \chi_{_{0.9}}^{^{2}}ig(nig)$ $\Rightarrow n \leq 22$

3. 教材习题七(**A**)三第5题

$$ar{X}\sim Niggl(52,rac{6.3^2}{36}iggr)$$

$$egin{aligned} ig(1ig) Pig(50.8 < ar{X} < 54.8ig) &= \Phiigg(rac{54.8 - 52}{6.3/6}igg) - \Phiigg(rac{50.8 - 52}{6.3/6}igg) \ &= \Phiig(2.67ig) - 1 + \Phiig(1.14ig) = 0.8691 \ ig(2ig) Eig|ar{X} - 52ig|^2 \le 0.05 \Rightarrow Dig(ar{X}ig) + ig(Eig(ar{X} - 52ig)ig)^2 \le 0.05 \Rightarrow Dig(ar{X}ig) \le 0.05 \ &\Rightarrow Dig(ar{X}ig) \le 0.05 \Rightarrow Dig(ar{X}ig) \le 0.05 \end{aligned}$$

4. 教材习题七(A)三第 6 题

$$\begin{split} \left(1\right)\bar{X} - \bar{Y} &\sim N \bigg(30 - 40, \frac{3^2}{9} + \frac{2^2}{12}\bigg) = N \bigg(-10, \frac{4}{3}\bigg) \\ P\left(\left|\bar{X} - \bar{Y}\right| < 12\right) &= \Phi \left(\frac{12 - (-10)}{\sqrt{4/3}}\right) - \Phi \left(\frac{-12 - (-10)}{\sqrt{4/3}}\right) \\ &= \Phi \left(11\sqrt{3}\right) - 1 + \Phi \left(\sqrt{3}\right) = \Phi \left(\sqrt{3}\right) = 0.9583 \\ \left(2\right) \frac{\bar{X} - 30}{S/3} &\sim t\left(8\right), P\left(\frac{\bar{X} - 30}{S/3} < C_1\right) = 0.975 \Rightarrow C_1 = t_{_{0.975}}\left(8\right) = 2.3060 \\ \frac{S_2^2/S_1^2}{\sigma_2^2/\sigma_1^2} &= \frac{S_2^2/S_1^2}{4/9} &\sim F\left(11, 8\right) \Rightarrow P\left(\frac{S_2^2}{S_1^2} < C_2\right) = P\left(\frac{S_2^2/S_1^2}{4/9} < \frac{C_2}{4/9}\right) = 0.05 \\ &\Rightarrow \frac{9}{4}C_2 = F_{_{0.05}}\left(11, 8\right) = \frac{1}{F_{_{0.05}}\left(8, 11\right)} \Rightarrow C_2 = \frac{4}{9}\frac{1}{F_{_{0.05}}\left(8, 11\right)} = \frac{4}{9} \cdot \frac{1}{2.95} = 0.151 \end{split}$$

—. 1. **С** 2. **С**

$$\equiv$$
 1. $\left(\overline{X} - 1/2\right)^3$ 2. $\overline{X}^2/3$

三. 1. 教材习题八 (\mathbf{A}) 三第 1 题

由于有两个未知参数, 所以考虑总体的一、二阶原点矩:

$$m_{_1}=Eig(Xig)=\mu, \ \ m_{_2}=Eig(X^2ig)=Dig(Xig)+ig[Eig(Xig)ig]^2=\mu^2+\sigma^2$$
反解得

$$\mu = m_{_1}, \sigma^{_2} = m_{_2} - m_{_1}^{^2};$$

用样本的一二阶原点矩替换总体的一二阶原点矩,即得 μ,σ^2 的矩估计为:

$$\hat{\boldsymbol{\mu}} = \overline{\boldsymbol{X}}, \quad \hat{\sigma}^2 = A_2 - \hat{X}^2 = B_2,$$

由样本算得 $\hat{x} = 6.20$, $b_2 = 0.38$, 所以 μ, σ^2 的矩估计值为:

$$\hat{\mu} = 6.2, \quad \hat{\sigma}^2 = 0.38.$$

因函数 $\sigma = \sqrt{\sigma^2}(\sigma > 0)$ 连续,所以 $\hat{\sigma} = \sqrt{\hat{\sigma}^2} = \sqrt{0.38} = 0.6164$.

干燥时间小于6.5小时的概率为 $P(X < 6.5) = \Phi(\frac{6.5 - \mu}{\sigma})$,它是关于 μ , σ 的连续函数,所以

其矩估计值为 $\hat{P}(X < 6.5) = \hat{\Phi}(\frac{6.5 - \mu}{\sigma}) = \Phi(\frac{6.5 - \hat{\mu}}{\hat{\sigma}}) = \Phi(\frac{6.5 - 6.2}{0.6164}) = \Phi(0.49) = 0.6879.$

2. 教材习题八(A)三第3题

由于仅一个未知参数, 故只需总体期望

- (1) $E(X) = \int_0^1 x(1-\theta)x^{-\theta}dx = \frac{1-\theta}{2-\theta}$,解之得 $\theta = \frac{1-2E(X)}{1-E(X)}$,用样本均值代替总体均值即得 θ 的矩估计量为 $\hat{\theta} = \frac{1-2\bar{X}}{1-X}$;
- (2) $E(X) = \int_0^{+\infty} x \frac{x}{\theta^2} e^{-\frac{x}{\theta}} dx = 2\theta$,解之得 $\theta = \frac{E(X)}{2}$,用样本均值代替总体均值即得 θ 的 矩估计量为 $\hat{\theta} = \frac{\bar{X}}{2}$.
- 3. 教材习题八(A)三第 4 题

$$X \sim f_{_{X}}ig(xig) = F_{_{X}}'ig(xig) = egin{cases} eta x^{-eta-1}, & x > 1 \ 0, & x \leq 1 \end{cases}$$

由于仅一个未知参数, 故只需总体期望

 $E(X) = \int_1^{+\infty} \beta x^{-\beta} dx = \frac{\beta}{1-\beta}$,解之得 $\beta = \frac{E(X)}{1+E(X)}$,用样本均值代替总体均值即得 β 的矩估计量为 $\hat{\beta} = \frac{\bar{X}}{1+\bar{X}}$.

4.
$$\hat{\boldsymbol{\beta}} = \overline{\boldsymbol{X}} / B_2$$
, $\hat{\boldsymbol{\alpha}} = \overline{\boldsymbol{X}}^2 / B_2$ (上课已讲)

5. 教材习题八**(A)**三第2题

由于有两个未知参数,所以考虑总体的一、二阶原点矩:

$$egin{aligned} m_{_1}&=Eig(Xig)=\int\limits_{ heta_1}^{+\infty}xrac{1}{ heta_2}e^{rac{x- heta_1}{- heta_2}}dx= heta_1+ heta_2,\ m_{_2}&=Eig(X^2ig)=\int\limits_{ heta}^{+\infty}x^2rac{1}{ heta_2}e^{rac{x- heta_1}{- heta_2}}dx=2 heta_2^2+ heta_1^2+2 heta_1 heta_2= heta_2^2+ig(heta_1+ heta_2ig)^2 \end{aligned}$$

反解得

$$\theta_{_{1}}=m_{_{1}}-\sqrt{m_{_{2}}-m_{_{1}}^{^{2}}},\ \theta_{_{2}}=\sqrt{m_{_{2}}-m_{_{1}}^{^{2}}};$$

用样本的一二阶原点矩替换总体的一二阶原点矩,即得 θ_1, θ_2 的矩估计量为:

$$\hat{ heta}_{_{1}} = ar{X} - \sqrt{B_{_{2}}} \; , \; \; \hat{ heta}_{_{2}} = \sqrt{A_{_{2}} - ar{X}^{^{2}}} = \sqrt{B_{_{2}}} \; .$$

第 51-52 页

—. 1. **D** 2. **A**

$$\equiv$$
 1. $\Phi\left(\frac{t-\overline{x}}{\sqrt{b_2}}\right)$ 2. $\min\left\{X_1, X_2, \dots, X_n\right\}$

3. 首先求的极大似然估计:

$$Lig(etaig) = \prod_{i=1}^n fig(x_i,etaig) = \prod_{i=1}^n rac{1}{2}eta^3 x_i^2 e^{-eta x_i} = rac{1}{2^n}eta^{3n}igg[\prod_{i=1}^n x_i^2igg] e^{-eta\sum_{i=1}^n x_i}$$

$$\ln Lig(etaig) = -n\ln 2 + 3n\ln eta + 2{\displaystyle\sum_{i=1}^n\ln x_i} - eta{\displaystyle\sum_{i=1}^n x_i}$$

$$rac{d \ln Lig(etaig)}{deta} = 0 \Rightarrow rac{3n}{eta} - \sum_{i=1}^n x_i^{} = 0$$

解之即得 $\boldsymbol{\beta}$ 的极大似然估计值为 $\hat{\boldsymbol{\beta}} = \frac{3}{\overline{x}}$; 所以 $\boldsymbol{\beta}$ 的极大似然估计量为 $\hat{\boldsymbol{\beta}} = \frac{3}{\overline{\overline{x}}}$;

因

$$m_{_3}=\mathrm{E}ig(X^3ig)=\int\limits_0^{_{+\infty}}x^3\,rac{eta^3}{\Gamma(3)}x^{_{3-1}}e^{-eta x}dx=rac{1}{2eta^3}\int\limits_0^{_{+\infty}}ig(eta xig)^{\!5}\,e^{-eta x}dig(eta xig)=rac{60}{eta^3}$$

是 $oldsymbol{eta}$ 的函数,该函数具有单值的反函数,所以其极大似然估计量为

$$\hat{m}_{_{3}}=rac{60}{\hat{eta}^{_{3}}}=rac{60}{\left(3ig/ar{X}
ight)^{^{3}}}=rac{20}{9}\,ar{X}^{_{3}}\,;$$

因 $V(m) = D(X) = \frac{3}{\beta^2} (\beta > 0)$ 具有单值反函数,所以V(m)的极大似然估计量为

$$\hat{V}\left(m
ight)\!=\!rac{3}{\left(ar{X}\!ig/3
ight)^2}=rac{ar{X}^2}{3}$$

三. 1. 教材习题八(A)三第3题

(1) 似然函数为
$$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) = \prod_{i=1}^{n} \left(1 - \sqrt{\theta}\right) x_i^{-\sqrt{\theta}} = \left(1 - \sqrt{\theta}\right)^n \left[\prod_{i=1}^{n} x_i\right]^{-\sqrt{\theta}}$$
 $\Rightarrow \ln L(\theta) = n \ln \left(1 - \sqrt{\theta}\right) - \sqrt{\theta} \sum_{i=1}^{n} \ln x_i$

从而有

$$rac{d \ln Lig(hetaig)}{d heta} = 0 \Rightarrow -rac{n}{2\Big(\sqrt{ heta}- heta\Big)} - rac{1}{2\sqrt{ heta}} \sum_{i=1}^n \ln x_{_i} = 0 \; .$$

解之即得 $m{ heta}$ 的极大似然估计值为 $\hat{m{ heta}} = \left[1 + rac{n}{\displaystyle\sum_{i=1}^n \ln x_i}
ight]^2$,其极大似然估计量为

$$\hat{ heta} = \left[1 + rac{n}{\displaystyle\sum_{i=1}^n \ln X_i}
ight]^2;$$

(2) 似然函数为
$$L(\theta) = \prod_{i=1}^n f(x_i, \theta) = \prod_{i=1}^n \frac{x_i}{\theta^2} e^{-\frac{x}{\theta}} = \theta^{-2n} \left[\prod_{i=1}^n x_i \right] e^{-\frac{1}{\theta} \sum_{i=1}^n x_i}$$
 $\Rightarrow \ln L(\theta) = -2n \ln \theta + \sum_{i=1}^n \ln x_i - \frac{1}{\theta} \sum_{i=1}^n x_i$

从而有

$$\frac{d\ln L\left(\theta\right)}{d\theta}=0 \Rightarrow -\frac{2n}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^n x_i=0\,,$$

解之即得 θ 的极大似然估计值为 $\hat{\theta} = \frac{1}{2n} \sum_{i=1}^n x_i = \frac{1}{2} \overline{x}$, 其极大似然估计量为

$$\hat{\theta} = \frac{1}{2}\bar{X}.$$

2. 教材习题八**(A)**三第4题

由分布函数得密度函数为 $f_{_{\! X}}ig(x,lpha,etaig) = egin{cases} lpha^etaeta x^{-eta-1}, & x \geq lpha \ 0, & x < lpha \end{cases}$

(1) 当
$$\alpha = 1$$
时,似然函数为 $L(\beta) = \prod_{i=1}^n f(x_i, \beta) = \prod_{i=1}^n \beta x_i^{-\beta - 1} = \beta^n \left[\prod_{i=1}^n x_i \right]^{-\beta - 1}$ $\Rightarrow \ln L(\beta) = n \ln \beta - (\beta + 1) \sum_{i=1}^n \ln x_i$

从而有

$$rac{d \ln Lig(etaig)}{deta} = 0 \Rightarrow rac{n}{eta} - \sum_{i=1}^n \ln x_i = 0 \; .$$

解之即得 $\boldsymbol{\beta}$ 的极大似然估计值为 $\hat{\boldsymbol{\beta}} = \frac{n}{\displaystyle\sum_{i=1}^{n} \ln x_i}$,其极大似然估计量为

$$\hat{eta} = rac{n}{\sum_{i=1}^{n} \ln X_i};$$

- (2) 当 $\beta=2$ 时,似然函数为 $L(\alpha)=\prod_{i=1}^n f(x_i,\alpha)=\prod_{i=1}^n 2\alpha^2x_i^{-3}=2^n\alpha^{2n}\left[\prod_{i=1}^n x_i\right]^{-3}$ 显然可见,似然函数 $L(\alpha)$ 关于 α 严格单增;与此同时,对任意 $i=1,2,\cdots,n$, $\alpha\leq x_i$,故 $\alpha\leq \min\left\{x_1,x_2,\cdots,x_n\right\}$,从而 $L(\alpha)$ 在 $\alpha=\min\left\{x_1,x_2,\cdots,x_n\right\}$ 处取得最大值,所以 α 的极大似然估计值为 $\hat{\alpha}=\min\left\{x_1,x_2,\cdots,x_n\right\}$,其极大似然估计量为 $\hat{\alpha}=\min\left\{X_1,X_2,\cdots,X_n\right\}$.
- 3. 教材习题八**(A)**三第 5 题

似然函数为
$$L\left(\sigma^2
ight) = \prod_{i=1}^n f\left(x_i,\sigma^2
ight) = \prod_{i=1}^n rac{1}{\sqrt{2\pi\sigma}} e^{-rac{x_i^2}{2\sigma^2}} = \left(2\pi
ight)^{-rac{n}{2}} \left(\sigma^2
ight)^{-rac{n}{2}} e^{-rac{1}{2\sigma^2}\sum_{i=1}^n x_i^2}$$
 $\Rightarrow \ln L\left(\sigma^2
ight) = -rac{n}{2}\ln\left(2\pi
ight) - rac{n}{2}\ln\left(\sigma^2
ight) - rac{1}{2\sigma^2}\sum_{i=1}^n x_i^2$

从而有

$$rac{d \ln Lig(\sigma^2ig)}{d\sigma^2} = 0 \Rightarrow -rac{n}{2\sigma^2} + rac{1}{2\sigma^4} \sum_{i=1}^n x_i^2 = 0 \; ,$$

解之即得 σ^2 的极大似然估计值为 $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n x_i^2$.

4. 教材习题八(A)三第 6 题

似然函数为
$$L\left(\mu,\sigma^2\right) = \prod_{i=1}^n f\left(x_i,\mu,\sigma^2\right) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma x_i} e^{-\frac{1}{2\sigma^2}(\ln x_i - \mu)^2}$$

$$= \left(2\pi\right)^{-\frac{n}{2}} \left(\sigma^2\right)^{-\frac{n}{2}} \left[\prod_{i=1}^n \frac{1}{x_i}\right] e^{-\frac{1}{2\sigma^2}\sum_{i=1}^n (\ln x_i - \mu)^2}$$

$$\Rightarrow \ln L\left(\mu,\sigma^2\right) = -\frac{n}{2} \ln\left(2\pi\right) - \frac{n}{2} \ln\left(\sigma^2\right) - \sum_{i=1}^n \ln x_i - \frac{1}{2\sigma^2} \sum_{i=1}^n \left(\ln x_i - \mu\right)^2$$

从而有

$$\begin{cases} \frac{\partial \ln L\left(\mu,\sigma^2\right)}{\partial \mu} = 0 \\ \frac{\partial \ln L\left(\mu,\sigma^2\right)}{\partial \sigma^2} = 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{\sigma^2} \sum_{i=1}^n \left(\ln x_i - \mu\right) = 0 \\ -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n \left(\ln x_i - \mu\right)^2 = 0 \end{cases},$$

解之即得
$$\mu, \sigma^2$$
的极大似然估计值为
$$\begin{cases} \hat{\mu} = \frac{1}{n} \sum_{i=1}^n \ln x_i \\ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n \left(\ln x_i - \frac{1}{n} \sum_{j=1}^n \ln x_j \right)^2, \text{ 极大似然估} \end{cases}$$

$$\hat{\mu} = rac{1}{n} \sum_{i=1}^n \ln X_i$$
 $\hat{\sigma}^2 = rac{1}{n} \sum_{i=1}^n \left(\ln X_i - rac{1}{n} \sum_{j=1}^n \ln X_j
ight)^2$.

第 53 - 54 页

$$\equiv$$
. 1. $\frac{1}{6}$ 2. $\frac{1}{2(n-1)}$

$$\equiv$$
. 1. $\mathbf{E}\left(\overline{X}^2\right) = \frac{\sigma^2}{n} + \mu^2
ightarrow \mu^2$

2. 教材习题八(A)三第 7 题

3. 教材习题八**(A)**三第 8 题

由《习题册》第 54 页第 3 题知 $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n X_i^2$.

(1)因
$$E(X) = 0$$
,所以 $E(X^2) = D(X) = \sigma^2$,从而有

$$Eig(\hat{\sigma}^2ig) = rac{1}{n}\sum_{i=1}^n Eig(X_i^2ig) = rac{1}{n}\sum_{i=1}^n Eig(X^2ig) = rac{1}{n}\sum_{i=1}^n \sigma^2 = \sigma^2,$$

所以 $\hat{\sigma}^2$ 为 σ^2 的无偏估计量.

(2) 因总体 $oldsymbol{X} \sim oldsymbol{N}ig(oldsymbol{0}, oldsymbol{\sigma^2}ig)$,所以

$$\chi_0^2 = \sum_{i=1}^n rac{X_i^2}{\sigma^2} \sim \chi^2ig(nig), \chi^2 = rac{ig(n-1ig)S^2}{\sigma^2} \sim \chi^2ig(n-1ig),$$

从而有

$$egin{split} D\Big(\hat{\sigma}^2\Big) &= Digg(rac{1}{n}\sum_{i=1}^nX_i^2igg) = Digg(rac{\sigma^2}{n}\chi_0^2igg) = rac{\sigma^4}{n^2}D\Big(\chi_0^2\Big) = rac{\sigma^4}{n^2}2n = rac{2\sigma^4}{n}; \ D\Big(S^2\Big) &= Digg(rac{\sigma^2}{n-1}\chi^2igg) = rac{\sigma^4}{ig(n-1ig)^2}D\Big(\chi^2\Big) = rac{\sigma^4}{ig(n-1ig)^2}2ig(n-1ig) = rac{2\sigma^4}{n-1}; \end{split}$$

显然可见
$$D(\hat{\sigma}^2) = rac{2\sigma^4}{n} < rac{2\sigma^4}{n-1} = D(S^2)$$
,所以 $\hat{\sigma}^2$ 比 S^2 更有效.

- 4. 教材习题八(A)三第 9 题 略
- 5. 教材习题八(A)三第 10 题 上课已讲!

第 55-56 页 《区间估计》

-. 1. B 2.
$$4u_{1-\frac{\alpha}{2}}^2 \frac{\sigma_0^2}{l^2}$$

二. 1. 教材习题八(A)三第 11 题

计算得 $\bar{x} = 575.2$; s = 8.7025

(1)因 $\mathbf{1}-\alpha=\mathbf{0.95}, \sigma^2=\sigma_0^2=\mathbf{25}$,故 $oldsymbol{\mu}$ 的 $\mathbf{95}\%$ 置信区间为

$$\left(\widehat{\widehat{\mu}}_{\!_{1}},\widehat{\widehat{\mu}}_{\!_{2}}\right) = \left(\overline{x} - u_{_{1 - \frac{\alpha}{2}}} \frac{\sigma_{_{0}}}{\sqrt{n}}, \overline{x} + u_{_{1 - \frac{\alpha}{2}}} \frac{\sigma_{_{0}}}{\sqrt{n}}\right) = \left(572.101, 578.299\right)$$

(2) 因 $\sigma=5$ 时, $C_v=rac{\sigma}{|\mu|}=rac{5}{\mu}\Big(\mu>0\Big)$ 关于 μ 严格单减,所以 C_v 的95%置信区间

$$onumber egin{aligned} etaigg(\widehat{m{C}}_v^1,\widehat{m{C}}_v^2igg) = \Big(m{C}_v\Big(\widehat{m{\mu}}_2\Big),m{C}_v\Big(\widehat{m{\mu}}_1\Big)\Big) = \left(rac{m{5}}{\widehat{m{\mu}}_2},rac{m{5}}{\widehat{m{\mu}}_1}
ight) = ig(m{0.00865},m{0.00874}ig); \end{aligned}$$

(3) 因 $\mathbf{1}-\alpha=0.95, t_{1-rac{lpha}{2}}ig(n-1ig)=t_{0.975}ig(9ig)=2.2622$,又 σ^2 未知,故 μ 的95%置信区间为

$$\left(\overline{x}-t_{\frac{1-\frac{\alpha}{2}}{2}}(n-1)\frac{s}{\sqrt{n}},\overline{x}+t_{\frac{1-\frac{\alpha}{2}}{2}}(n-1)\frac{s}{\sqrt{n}}\right)=\left(568.974,581.426\right).$$

2. 教材习题八(A)三第 12 题

经计算可得 $\bar{x}=10.08333,;\;s^2=0.06333,\;s=0.83467$,

查表可得
$$t_{1-rac{lpha}{2}}ig(n-1ig)=t_{0.95}ig(11ig)=1.7959$$
, $\chi_{rac{lpha}{2}}^2ig(n-1ig)=\chi_{0.05}^2ig(11ig)=4.575$,
$$\chi_{1-rac{lpha}{2}}^2ig(n-1ig)=\chi_{0.95}^2ig(11ig)=19.675$$
 .

因总体均值 μ 及方差 σ^2 未知,又n=12,故 μ,σ^2,σ 的置信度为90%的置信区间为

$$\left(\overline{x} - t_{1 - \frac{\alpha}{2}} (n - 1) \frac{s}{\sqrt{n}}, \overline{x} + t_{1 - \frac{\alpha}{2}} (n - 1) \frac{s}{\sqrt{n}}\right) = \left(9.6506, 10.5160\right)$$

$$egin{aligned} &\left(rac{\left(n-1
ight)s^2}{\chi^2_{1-rac{lpha}{2}}ig(n-1ig)}, rac{\left(n-1
ight)s^2}{\chi^2_{rac{lpha}{2}}ig(n-1ig)} = ig(0.0354, 0.1523ig) \ &\left(\sqrt{rac{\left(n-1
ight)s^2}{\chi^2_{lpha}ig(n-1ig)}}, \sqrt{rac{\left(n-1
ight)s^2}{\chi^2_{lpha}ig(n-1ig)}}
ight) = ig(0.1881, 0.3902ig) \end{aligned}$$

- 3. 教材习题八(A)三第 13 题 略
- 4. 教材习题八(A)三第 14 题 略

第 57-58 页

- 一. 1. **B** 2. **D** 教材习题七(**B**)一第 2 题 3. **C**
- \Box . 1. **16** 2. σ^2 3. $\frac{1}{X}$ $\frac{1}{X}$
- 三. 1. 教材习题五 (\mathbf{B}) 三第2题

$$egin{align*} E\left(M
ight) &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} \max\left(x,y
ight) f_{_{X}}\left(x
ight) f_{_{Y}}\left(y
ight) dy \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{x}^{+\infty} y \, rac{1}{2\pi} \, e^{-rac{x^{2}+y^{2}}{2}} dy + \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{x} x \, rac{1}{2\pi} \, e^{-rac{x^{2}+y^{2}}{2}} dy \ &= rac{1}{2\pi} iggl[\int\limits_{-\infty}^{+\infty} e^{-rac{x^{2}}{2}} dx \int\limits_{x}^{+\infty} e^{-rac{y^{2}}{2}} drac{y^{2}}{2} + \int\limits_{-\infty}^{+\infty} e^{-rac{y^{2}}{2}} dy \int\limits_{y}^{+\infty} e^{-rac{x^{2}}{2}} drac{x^{2}}{2} iggr] \ &= rac{1}{2\pi} iggl[\int\limits_{-\infty}^{+\infty} e^{-x^{2}} dx + \int\limits_{-\infty}^{+\infty} e^{-y^{2}} dy iggr] = rac{2}{\pi} \int\limits_{-\infty}^{+\infty} e^{-x^{2}} dx \ &= rac{1}{\pi} \int\limits_{0}^{+\infty} \left(x^{2}
ight)^{rac{1}{2}} e^{-x^{2}} dx^{2} = rac{1}{\pi} \Gamma\left(rac{1}{2}
ight) = rac{1}{\sqrt{\pi}} \ &E\left(N
ight) = \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} \min\left(x,y
ight) f_{_{X}}\left(x
ight) f_{_{Y}}\left(y
ight) dy \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x rac{1}{2\pi} e^{rac{x^{2}+y^{2}}{2}} dy + \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{x} y rac{1}{2\pi} e^{rac{x^{2}+y^{2}}{2}} dy \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x rac{1}{2\pi} e^{rac{x^{2}+y^{2}}{2}} dy + \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{x} y rac{1}{2\pi} e^{rac{x^{2}+y^{2}}{2}} dy \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x rac{1}{2\pi} e^{-rac{x^{2}+y^{2}}{2}} dy + \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{x} y rac{1}{2\pi} e^{-rac{x^{2}+y^{2}}{2}} dy \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x rac{1}{2\pi} e^{-rac{x^{2}+y^{2}}{2}} dy + \int\limits_{-\infty}^{+\infty} x \left(-\frac{x^{2}+y^{2}}{2} \right) \left(-\frac{x^{2}+y^{2}}{2} \right) \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x \left(-\frac{x^{2}+y^{2}}{2} \right) \left(-\frac{x^{2}+y^{2}}{2} \right) \left(-\frac{x^{2}+y^{2}}{2} \right) \left(-\frac{x^{2}+y^{2}}{2} \right) \ &= \int\limits_{-\infty}^{+\infty} dx \int\limits_{-\infty}^{+\infty} x \left(-\frac{x^{2}+y^{2}}{2} \right) \left(-\frac{x^{2}+y^{2}}{$$

$$egin{aligned} &=rac{1}{2\pi}iggl[\int\limits_{-\infty}^{+\infty}e^{-rac{y^2}{2}}dy\int\limits_{-\infty}^{y}xe^{-rac{x^2}{2}}dx+\int\limits_{-\infty}^{+\infty}e^{-rac{x^2}{2}}dx\int\limits_{-\infty}^{x}ye^{-rac{y^2}{2}}dyiggr] \ &=rac{1}{\pi}\int\limits_{-\infty}^{+\infty}e^{-rac{y^2}{2}}dy\int\limits_{-\infty}^{y}xe^{-rac{x^2}{2}}dx=rac{1}{\pi}\int\limits_{-\infty}^{+\infty}e^{-rac{y^2}{2}}dy\int\limits_{-\infty}^{y}e^{-rac{x^2}{2}}drac{x^2}{2} \ &=-rac{1}{\pi}\int\limits_{-\infty}^{+\infty}e^{-y^2}dy=-rac{2}{\pi}\int\limits_{0}^{+\infty}e^{-y^2}dy \ &=-rac{1}{\pi}\int\limits_{0}^{+\infty}\left(y^2
ight)^{-rac{1}{2}}e^{-y^2}dy^2=-rac{1}{\pi}\Gammaiggl(rac{1}{2}iggr)=-rac{1}{\sqrt{\pi}} \end{aligned}$$

2. 教材习题八**(B)**三第2题

矩估计法: 总体的一阶原点矩为

$$m_{_1}=Eig(Xig)=\int\limits_{_0}^1x heta dx+\int\limits_{_1}^2xig(1- hetaig)dx=rac{1}{2} heta+rac{3}{2}ig(1- hetaig)=rac{3}{2}- heta$$

反解得 $\theta = \frac{3}{2} - m_1$,用样本的一阶原点矩(即样本均值)代替总体的一阶原点矩得

未知参数 $m{ heta}$ 的矩估计量为 $\hat{m{ heta}}_{\scriptscriptstyle M}=rac{3}{2}-ar{m{X}}$,其矩估计值为 $\hat{m{ heta}}_{\scriptscriptstyle M}=rac{3}{2}-ar{m{x}}$;

极大似然估计法:
$$L(\theta) = \prod_{i=1}^n f(x_i; \theta) = \theta^N (1-\theta)^{n-N}$$

$$\Rightarrow \ln L(heta) = N \ln heta + (n-N) \ln (1- heta)$$
,由此得

$$rac{d \ln L(heta)}{d heta} = 0 \Leftrightarrow rac{N}{ heta} - rac{n-N}{1- heta} = 0$$
,解之即得 $heta$ 的矩估计值为 $\hat{ heta}_L = rac{N}{n}$.

3. 教材习题八**(B)**三第 3 题

此题即为在一个正态总体下,总体均值未知时对总体总体标准差的区间估计.

$$\left(\sqrt{\frac{\left(n-1\right)s^{2}}{\chi_{_{1-\alpha/2}}^{^{2}}\left(n-1\right)}},\sqrt{\frac{\left(n-1\right)s^{^{2}}}{\chi_{_{\alpha/2}}^{^{2}}\left(n-1\right)}}\right) = \left(\sqrt{\frac{0.17985}{\chi_{_{0.975}}^{^{2}}\left(9\right)}},\sqrt{\frac{0.17985}{\chi_{_{0.025}}^{^{2}}\left(9\right)}}\right)$$

$$=\left(\sqrt{rac{0.17985}{19.023}},\sqrt{rac{0.17985}{2.7}}
ight)=\left(0.0972,0.2581
ight)$$
,显然可见,在 95% 的置信度下,这批

零件长度的标准差符合设计要求.