Chapter 9 Searching

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9.1 Basic concepts

Search(查找)

Given: Distinct(无重复的) keys $k_1, k_2, \ldots, k_n$ and collection *Table* of *n* records of the form

$$(k_1, R_1), (k_2, R_2), \ldots, (k_n, R_n)$$

here R_j is the record information associated with key k_j for $1 <= j <= n$.

Search Problem: For key value K, locate the record (k_j, R_j) in *Table* such that $k_j = K$.

Search的评价指标： 平均查找长度ASL (Average Searching Length).

查找若干个记录需要的平均关键字比较次数

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Successful vs. Unsuccessful

A successful search is one in which a record with key $k_j = K$ is found.

An unsuccessful search is one in which no record with $k_j = K$ is found (and presumably no such record exists).

more expensive

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Extract-match query vs. range query
精确查找vs范围查找

An extract-match query is a search for the record whose key value matches a specified key value

A range query is a search for all records whose key value falls within a specified range of key values

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Approaches to Search

1. Sequential search (顺序查找)
   给定值依次和集合中各个记录的关键字进行比较，
   ASL取决于集合中记录的组织方式/顺序
   - Records Ordered by insert order
   - Records Ordered by search Frequency

   How to decide?
2. Binary search (二分/折半查找)
   - Records Ordered by key values
3. Direct access by key value (hashing 直接查找)
   本章重点
4. Tree indexing methods(索引查找).
   将在chapter 10 介绍

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9.2 Searching in the sorted Records

9.2.1 Records Ordered by key values

9.2.2 Records Ordered by search Frequency

9.2.3 Self-Organizing Lists

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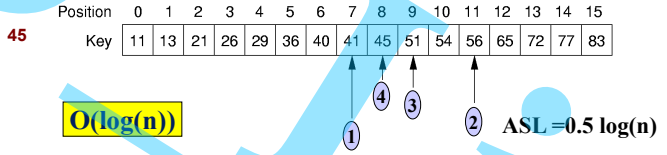
9.2.1 Records Ordered by key values
---Using binary search(折半查找)



O(log(n))

ASL =0.5 log(n)

思考：折半查找适合对存放在LList的有序集合进行查找吗？

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9.2.1 Records Ordered by key values
---Using binary search(折半查找)

```
int binary(int array[], int n, int K) {
    int l = -1;
    int r = n;     // l, r are beyond array bounds
    while ( (l+1) != r)  // Stop when l, r meet
    {
        int i = (l+r)/2;     // Check middle
        if (K < array[i])    r = i;   // Left half
        if (K == array[i]) return i; // Found it
        if (K > array[i])    l = i;   // Right half
    }
    return n; // Search value not in array
}
```

思考：折半查找适合对存放在LList的有序集合进行查找吗？

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9.2.2 Records Ordered by search Frequency

Order record by (expected) frequency of search occurrence.

- Perform sequential search

Expected search cost（ASL）：

1. 事先通过实验统计得到
2. 实时动态更新

$$ALS = 1p_1 + 2p_2 + \ldots + np_n.$$

$p_1, p_2, \ldots, p_n$ 为各记录被查找的归一化频度，
n 为记录集中记录条数

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Frequency Distributions example

(1) All records have equal frequency. $ASL = \sum_{i=1}^{n} i / n = (n+1)/2$

(2) Exponential Frequency

$$p_i = \begin{cases} 1/2^i & 1 \le i \le n-1 \\ 1/2^{n-1} & i = n \end{cases} \qquad ASL \approx \sum_{i=1}^{n} (i / 2^i) \approx 2.$$

(3) 80/20 rule（实际最常见）：

- 80% of search are to 20% of the records.
- For distributions following 80/20 rule,

实时动态更新 $ASL \approx 0.1n.$

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9.2.3 Self-Organizing Lists

- **Self-organizing(自组织) lists modify the order of records within the list based on the actual pattern of record search.**
- **Self-organizing lists use a heuristic(启发) for deciding how to reorder the records.**
  - **Count: Order by actual historical frequency of search.**
  - **Move-to-Front: When a record is found, move it to the front of the list.**
  - **Transpose: When a record is found, swap it with the record ahead of it**

original sequence: 0,1,2,3,4,5,6,7,8,9
Searching sequence 9, 9, 3, 0, 5, 5, 9

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Direct access by key value

不论是顺序查找还是折半查找，不论记录是按关键字还是按查找频度排序，平均查找长度ASL (Average Searching Length) 肯定大于1。

- 对于需频繁查找的记录集合(查找表)，希望ASL = 0
- 只有一个办法：预先知道所查关键字在表中的位置。
  即要求：在关键字与记录在表中的存储位置之间建立一个确定的关系---hashing

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9.3 Hashing

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9.3.1 What is hashing(散列)

Hashing: The process of **mapping(映射)** a **key** value to a **position** in a **table**.

① A **hash function** maps **key** values to **positions**. It is denoted by ***h***.

② A **hash table**(哈希表/散列表) is an **array** that holds the **record**s. It is denoted by **HT**.

- ✓ HT has *M* slots/positions, indexed form 0 to *M*-1.

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9.3.2 Hash Functions

Hash 函数构造原则

① **MUST** return a value **within** the hash table **index range** (0 ~ M-1).

② **SHOULD** evenly distribute(均匀分布) the records stored among the hash table slots.

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Hash Functions Examples (1)

1) int h(int k) {
   return(k % M);
   }

除留余数法，H(k) = k%M
适合关键字的取值范围远大于哈希表长度 *M*.

2) Mid-square method: Square the key value, take the middle *r* bits from the result for a hash table of 2^r slots.

```
int h(int k) {
   return(k*k & 0x000FF000L) >>12);
   }
```

平方取中法，适合于关键字为数字, r的大小取决于关键字的取值范围

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Hash Functions Examples (2)

3) For strings: Sum the ASCII values of the letters and take results modulo *M*.

折叠法，适合于关键字为字符串或数字位数多，sum 远大于*M*

```
int h(char* k) {
   int i, sum;
   for (sum=0, i=0; k[i] != '\0'; i++)
      sum += (int) k[i];
   return(sum % M);
}
```

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Hash Functions Examples (3)

4) ELF Hash: From Executable and Linking Format (ELF), UNIX System V Release 4.

```
int ELFhash(char* key) {
   unsigned long h = 0;
   while(*key) {
      h = (h << 4) + *key++;
      unsigned long  g = h & 0xF0000000L;
      if (g)  h ^= g >> 24;
      h &= ~g;  }
   return h % M;
}
```

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构造哈希函数的其他方法

1) 直接定址法： h(key) = (a × key + b)%M

2) 随机数法： h(key) = Random(key)

3) 数字分析法：提取分布均匀的若干位或它们的组合作为地址

实际应用时采用何种构造哈希函数的方法取决于关键字集合的情况(包括关键字的取值范围和分布)，总的原则是使产生冲突的可能性降到尽可能地小。

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Collisions（冲突）

- 哈希函数是一个压缩映射，在一般情况下，很容易产生“冲突(Collisions)”现象，
  - 即： key1≠key2， 而 h(key1) = h(key2)
- Collisions are inevitable in most applications.
  - h(k) = k %M，具有相同余数的关键字在HT中有冲突

很难找到一个不产生冲突的哈希函数。一般情况下，只能构建/选择恰当的哈希函数，使冲突尽可能少地产生；之外，还需要找到一种“处理冲突”的方法。

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9.3.3 collision resolution/处理冲突的方法

Each record *i* has a home position $h(k_i)$. If another record occupies *i*'s home position

collisions occur

What to do? **How to do?**

“处理冲突”的实际含义是

为产生地址冲突的记录寻找下一个哈希地址

1. Closed Hashing 闭域法/开放定址法
2. Open Hashing 开域法/链地址法

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9.3.3.1 Closed Hashing(闭域法/开放定址法)

- stores all records directly in the hash table.

为每个记录求得一个地址序列(Probe sequence)

$H_0, H_1, H_2, \ldots, H_s$ $1 \le s \le m-1$， m：哈希表的长度

其中：$H_0 = h(key)$ home position

$H_i = (H_0 + d_i) \% m$, $i=1, 2, \ldots, s$

Probe Function 探测函数 $P(i)=d_i$

- 增量序列d_i的几种取法

1) 线性探测： $d_i = i$

2) 平方探测： $d_i = 1^2, -1^2, 2^2, -2^2, \ldots,$

3) 随机探测： d_i 是一组伪随机数列

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Pseudo-Random Probing

- Select a (random) permutation(序列) of the numbers from 1 to *m*-1 作为增量序列：

$$r_1, r_2, \ldots, r_{m-1}$$

- All insertions and searches use the same permutation.

Example: Hash table size of $m = 101$

- $r_1=2$, $r_2=5$, $r_3=32$.
- $h(k_1)=30$, $h(k_2)=28$.
- Probe sequence for k_1: 30, 32, 35, 62.
- Probe sequence for k_2: 28, 30, 33, 60.

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Close Hashing example

{ 19, 01, 23, 14, 55, 68, 11, 82, 36 } h(key) = key % 11

若采用线性探测再散列处理冲突 $d_i = i$

	0	1	2	3	4	5	6	7	8	9	10
	55	01	23	14	68	11	82	36	19		
i	0	0	1	0	2	5	1	4	0		

冲突次数：13

若采用二次探测再散列处理冲突 $d_i = 1^2, -1^2, 2^2, -2^2, \ldots,$

	0	1	2	3	4	5	6	7	8	9	10
	55	01	23	14	36	82	68		19		11
i	0	0	1	0	1	0	3				2

冲突次数：7

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Close Hashing example

若要再插入一个数，将其放置于剩余的2个空slot的概率分别为多大？

h(key) = key % 11

若采用线性探测再散列处理冲突 $d_i = i$

0	1	2	3	4	5	6	7	8	9	10
55	01	23	14	68	11	82	36	19		
								P：	10/11	1/11

若采用二次探测再散列处理冲突 $d_i = 1^2, -1^2, 2^2, -2^2, \ldots,$

0	1	2	3	4	5	6	7	8	9	10
55	01	23	14	36	82	68		19		11
						P：	5/11		6/11	

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Secondary Clustering

Old Probe Function 探测函数：P(K, i) = d_i

If two keys hash to the same slot, they follow the same probe sequence. This is called secondary clustering（二次聚集）.

For example
h(key) = key % 11 采用线性探测再散列处理冲突 $d_i = i$

Key1 =12, H_i: 1, 2, 3, 4, 5, 6,…..
key2=23, H_i: 1, 2, 3, 4, 5, 6,…..
key3= 34, H_i: 1, 2, 3, 4, 5, 6,…..

二次聚集容易导致较多的冲突次数

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Double hashing(双哈希）

Old Probe Function 探测函数：p(K, i) = d_i

To avoid secondary clustering, modify P(i) to be a function of the original key value K and count i:

New Probe Function P(K, i) = d_i* h_2(K)
哈希函数：h(K) √
增量序列：d_i √
二次哈希函数：h_2(K)

$H_i = (H_0 + P(K,i))\% m$
$= (h(k) + d_i * h_2(K))\% m, \quad i = 1, 2, \ldots, s$
① ②

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Double Hashing（双哈希）

example1 : Hash table of size M=11，$h(k)$= k % M,

$p(K, i) = i * h_2(K)$ ， $h_2(K)$ = k % 5 +1,

k_1 = 12, k_2 =23, k_3= 34,

- $h(k_1)$=1, $h(k_2)$=1, $h(k_3)$=1
- $h_2(k_1)$=3, $h_2(k_2)$=4, $h_2(k_3)$=5
- Probe sequence for k_1 is: 1, 4, 7, 10
- Probe sequence for k_2 is: 1, 5, 9, 13
- Probe sequence for k_3 is: 1, 6, 11, 16

OLD
1, 2, 3, 4
1, 2, 3, 4
1, 2, 3, 4

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double hashing example

{ 19, 01, 23, 12, 55, 68, 24, 86, 35 } h(key) = key % 11

若采用双哈希 $p(K, i) = i * h_2(K)$ 处理冲突，h2(K) = k % 5 +1

	0	1	2	3	4	5	6	7	8	9	10
	55	01	68	35	12	23		24	19	86	
i	0	0	0	1	1	1		1	0	0	

冲突总次数：4

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Insertion a record into hash table (close hashing)

```
// Insert e into HT
template <class Key, class Elem,  class KEComp,  classEEComp>
hashInsert(const Elem& e) {
  int home;       // Home position for e
  int pos = home = h(e.key); // Init
  for (int i=1;   !(EEComp::eq(EMPTY, HT[pos])); i++) {
    pos = (home + p(e.key, i)) % M;
    if (EEComp::eq(e, HT[pos]))     return false;  // Duplicate
  }
  HT[pos] = e;      // Insert e
  return true;
}
```

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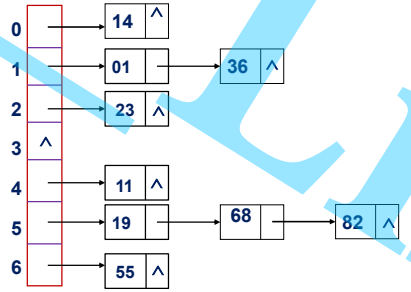
9.3.3.2 Open Hashing(开域法)/链地址法

将所有哈希地址相同的记录都链接在同一链表中

{ 19, 01, 23, 14, 55, 68, 11, 82, 36 } H(key) = key % 7



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9.3.4 Searching in HT

Search for the record with key *K* in HT :

1. Compute the home table location *h(K)*.
2. Starting with slot *h(K)*, locate the record containing key *K* using (if necessary) a collision resolution policy.

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Searching in HT using Closed hashing

1. 对于给定值 K，根据哈希函数及探测函数计算哈希地址
   H_0=h(K), H_i=(H_0+ P(K, i)) % m, i=1,2,…m-1;
2. 从i=0开始，比较K ? HT[H_i]， 若等于，查找成功，停止
3. 否则，查找不成功

Example : m=11 h(key) = key % 11

$H_i = (H_0 + d_i)$ % 11, *i=1, 2, …, 10*, d_i = *i*

0	1	2	3	4	5	6	7	8	9	10
55	01	23	14	68	11	82	36	19		

K=68 *i=2* 查找成功 SL=3

K=26 *i=5* SL=6 查找不成功

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哈希表查找(Closed hashing)的代价分析

Example : 依次查找 { 19, 01, 23, 14, 55, 68, 11, 82, 36 }

m=11, H(key) = key % m

$H_i = (H_0 + d_i)$ % m, *i=1, 2, …, m-1*, d_i = *i*

	0	1	2	3	4	5	6	7	8	9	10
	55	01	23	14	68	11	82	36	19		
SL	1	1	2	1	3	6	2	5	1		

平均查找长度ASL：(4*1+2*2+3+5+6)/9=2.44

通常ASL ≠ 0，这是因为有冲突存在

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Search in HT using closed hashing

```
// Search for the record with Key K
template <class Key, class Elem, class KEComp, class EEComp>
hashSearch(const Key &K, Elem &e) const {
  int home;          // Home position for K
  int  pos = home = H(K); // Initial posit
  for (int i = 1; !KEComp::eq(K, HT[pos]) &&
        !EEComp::eq(EMPTY, HT[pos]); i++)
      pos = (home + p(K, i)) % m; // Next
  if (KEComp::eq(K, HT[pos])) {    // Found it
       e = HT[pos];
       return true;
  }
  else return false;  // K not in hash table
}
```

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Searching in HT using Open hashing

1. 对于给定值K，根据哈希函数计算哈希地址 $H_0=H(K)$
2. 从H_0对应链表的第一个结点开始，比较结点记录关键字是否与K相等

   若相等，查找成功，返回
3. 返回 查找不成功

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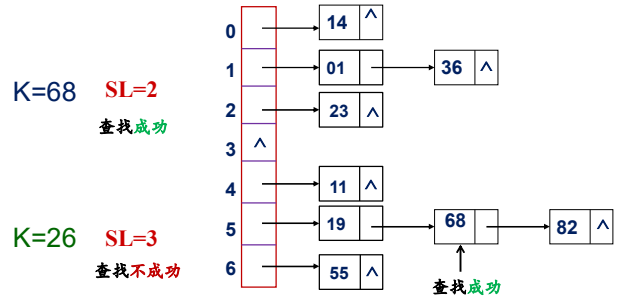
Searching in HT using Open hashing

Example :

H(key) = key % 7



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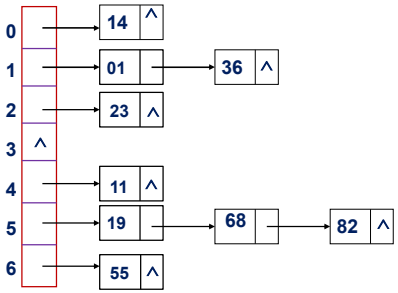
哈希表查找(Open hashing)的代价分析

Example ：依次查找 { 19, 01, 23, 14, 55, 68, 11, 82, 36 }

H(key) = key % 7

平均查找长度ASL:
(6*1+2*2+1*3)/9 =1.4



从查找过程得知，哈希表查找的ASL实际上并不等于零。Why?

因为有冲突存在

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决定哈希表查找的ASL的因素

1) 选用的哈希函数；
2) 选用的处理冲突的方法；
3) 哈希表饱和的程度：装载因子（load factor）α=n/m

   的大小（n—记录数， m—表的长度）

给定处理冲突方法，ASL是装载因子的函数。

线性探测 $ASL \approx \frac{1}{2}(1+\frac{1}{1-\alpha})$

随机探测 $ASL \approx -\frac{1}{\alpha}\ln(1-\alpha)$

链地址法 $ASL \approx 1+\frac{\alpha}{2}$

构造哈希表时，可选择一个适当的装载因子α，使得ASL限定在某个范围内。

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9.3.5 Deletion record from HT

- Deleting a record must not hinder(影响) later searches.
- We do not want to make positions in the hash table unusable because of deletion.

- these problems can be resolved by placing a special mark in place of the deleted record, called a tombstone.
- A tombstone will not stop a search, but that slot can be used for future insertions.

tombstones will add the ASL，try the following step to shorten the ASL

1. Local reorganizations.
2. Periodically rehash the table

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Chapter9 end

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