Chapter 5 Binary Trees 二叉树

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Content

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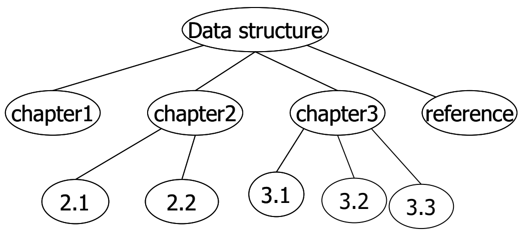
5.1 Basic Concepts of Tree and representation



Data structure
- chapter1
- Chapter2
  - 2.1
  - 2.2
- chapter3
  - 3.1
  - 3.2
  - 3.3
- reference

Compare to list, Tree structures permit both search and insert efficient to large collections of data

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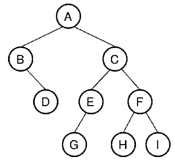
Terminology 术语(1)



- A tree consists of:
  - Nodes(结点): finite set of elements
  - Edges/branches(边): directed lines connecting the nodes
- For a node:
  - Degree(度): number of branches away from the node
- For a tree:
  - Root(根): node with indegree 0
  - nodes different from the root must have indegree 1

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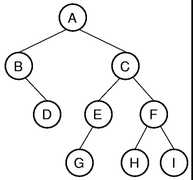
Terminology(2)

- Leaf(叶子) : node with degree 0
- Internal node (内部结点) : node that not a leaf
- Parent(双亲):
- Child(孩子):
- Siblings(兄弟): nodes with the same parent
- Path(路径): if $n_1, n_2, \ldots, n_k$ is a sequence of nodes in the tree such that n_i is the parent of n_{i+1} for $0 < i < k$, then this sequence is called a path from n_1 to n_k , the length of the path is $k-1$



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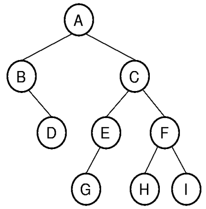
Terminology(3)

- Ancestor(前辈):
- Descendent(后代):
- Depth(深度) of a node M: the length of the path from root to M
- Level (层) : all nodes of depth d are at level d in the tree
- Height (高度) of a tree: the depth of deepest node in tree plus 1
- Sub-tree(子树): connected structure below the root



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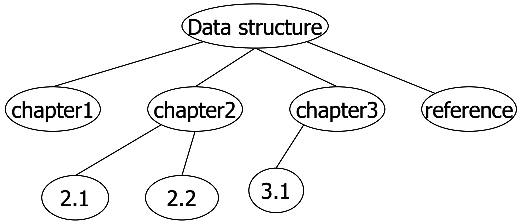
Tree Representation(1)

General tree

通用树



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Tree Representation(2)

Indented list

缩进式

Data structure
  chapter1
  Chapter2
    2.1
    2.2
  chapter3
    3.1
    3.2
    3.2
  reference

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Tree Representation(3)

Parenthetical list 括号式

Data Structure (chapter1 chapter2 (2.1 2.2)
chapter3(3.1 3.2 3.3) reference)

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5.2 Definition and properties of Binary tree

5.2.1 Definition of Binary tree 二叉树的定义

5.2.2 Properties of Binary tree 二叉树的性质

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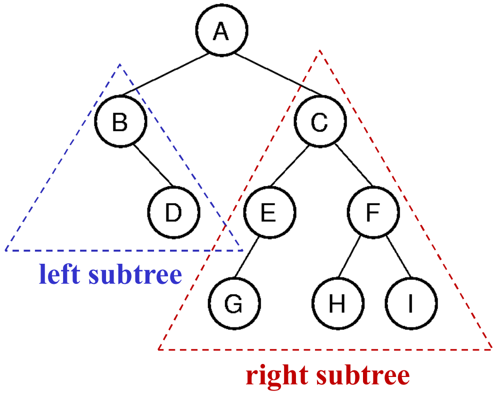
5.2.1 Definition of Binary tree

For binary tree(二叉树), any node cannot have more than two sub-trees(left and right)



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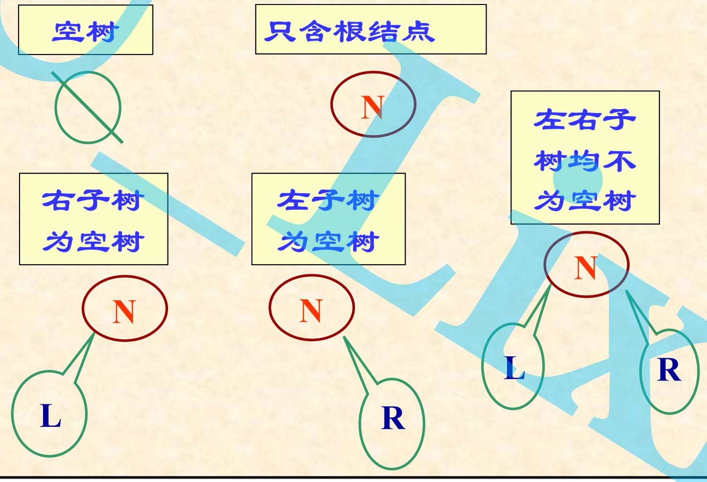
二叉树的五种基本形态



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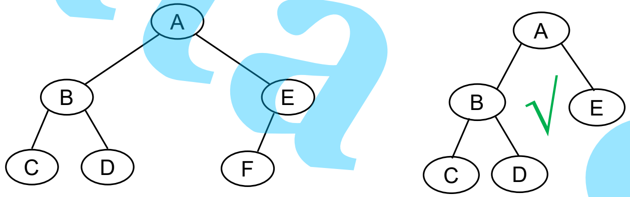
Full Binary Trees(满二叉树)

Each node in a full binary tree is either a leaf (degree is 0) or internal node with exactly two non-empty children(degree is 2).



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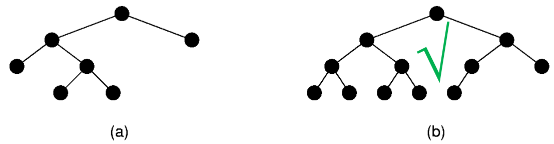
Complete Binary Trees(完全二叉树)

In the complete binary tree of height d,

1) all levels except level d-1 must be completely full

2) The d-1 level has all of its nodes filled from the left side.



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5.2.2 Properties(性质) of Binary tree

性质1： 在二叉树的第i (i≥0)层上至多有2^i个结点

证明（用归纳法）

1) $i = 0$ 时，只有1个根结点： $2^i = 2^0 = 1$

2) 假设i-1时命题成立，即i-1层最多有2^{i-1}个结点。

3）因为二叉树上每个结点至多有两棵子树，则第 i 层的结点数最多为 $2^{i-1} \times 2 = 2^i$ √

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性质2: 高度为k(k≥1)的二叉树上至多含2^k-1 个结点

证明:

根据性质1，高度为k的二叉树上的结点数至多为 $2^0+2^1+\cdots\cdots+2^{k-1} = 2^k-1$ √

高度为k(k≥1)的二叉树上至少含多少个结点呢？？

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性质3： 对任何一棵二叉树，若它含有n_0个叶子结点、n_2个度为2的内部结点，则必存在关系式：

$n_0 = n_2+1$

证明：

二叉树上结点总数 $n = n_0 + n_1 + n_2$

二叉树上分支总数 $b = n_1+2n_2$ （后继）

分支总数还可表示为 $b = n-1$ （前驱）

由此， $n0 = n2 + 1$ √

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性质4 具有n个结点的完全(complete)二叉树的高度为$\lfloor \log_2 n \rfloor+1$

证明：

1) 设完全二叉树的高度为 k
2) 则根据性质2 得 $2^{k-1} -1< n <= 2^k-1$

   即 $k-1 < \log_2 (n+1) <= k$
3) 因k只能是整数，因此，$k=\lfloor \log_2 n \rfloor+1$ √

推论：具有n个结点的二叉树的最大高度$H_{max} = n$，最小高度$H_{min} = \lfloor \log_2 n \rfloor + 1$

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性质5: The number of leaves in a non-empty full binary tree is one more than the number of internal nodes.

Proof: (性质3的特例)

for a non-empty full binary tree

因为 $N_{inte\_nod} = n_2$;

所以 $N_{leave} = n_0 = n_2+1 = N_{inte\_nod} +1$

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性质6: The number of empty subtrees in a non-empty Binary tree is one more than the number of nodes in the tree. $n_1+2n_0 = n+1$

Proof: left $=2*n_0 +n_1$

$=n_0 +n_0 + n_1$

$=n_2 +1 + n_0 + n_1$

$=n +1$

思考：非空二叉树中，非空子树的个数是多少呢？

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5.3 Binary Tree Node implementation

5.3.1 Pointer-based node implementation

5.3.2 Space Requirements and Overhead analysis

5.3.3 Array-based implementation for CBT

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二叉树的链式存储表示：链式二叉树

用2个类表达链式二叉树

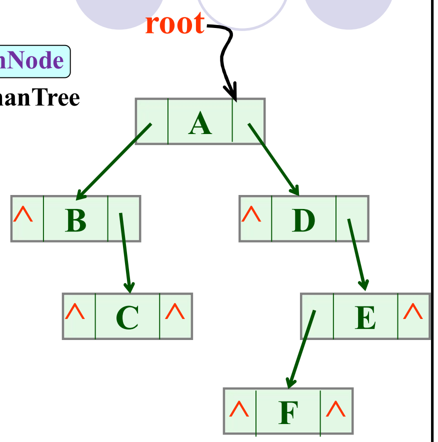
- 结点类：BSTNode/ VarBinNode
- 链式二叉树类：BST/HuffmanTree

二叉结点结构

left	data	right

1个根指针几乎就可描述一棵链式二叉树

- ✓ 根指针root指向根结点，
- ✓ 然后由根结点的左右孩子指针又分别指向其左右孩子，依此往下，就描述了整棵树



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二叉结点所涉及的基本操作有：

1.	返回结点值	element( )
2.	设置结点值	setElement(const E&)
3.	返回左孩子	left()
4.	设置左孩子	setLeft(BinNode*)
5.	返回右孩子	right()
6.	设置右孩子	setRight(BinNode*)
7.	是否叶子：	isLeaf()

二叉结点结构

left	data	right

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5.3.1 Pointer-based node implementation

二叉结点结构

left	data	right

```
// simple binary tree node imeplemetation
template <class E>
class BSTNode {
private:
  E it;          // The node's data value
  BSTNode* lc;   // Pointer to left child
  BSTNode* rc;   // Pointer to right child

public:
  BSTNode ( )   { lc = rc = NULL; }
  BSTNode (E  e, BSTNode*   l=NULL,  BSTNode*  r=NULL)
  {
     it = e;   lc = l;   rc = r;
  }
```

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Simple Binary tree node class(continue)

```
~BSTNode() { }
E& element() { return it; }  // 返回结点的元素值
void setElement (const E& e) { it = e; }  // 设置结点的元素值
BSTNode* left() const { return lc; } //返回结点的左孩子
void setLeft(BSTNode* b) { lc = b; }  // 设置结点的左孩子
BSTNode* right() const { return rc; } //返回结点的右孩子
void setRight(BSTNode* b) { rc = b; } //设置结点的右孩子
bool isLeaf()     //判断该结点是否为叶子
{ return (lc == NULL) && (rc == NULL); }
};
```

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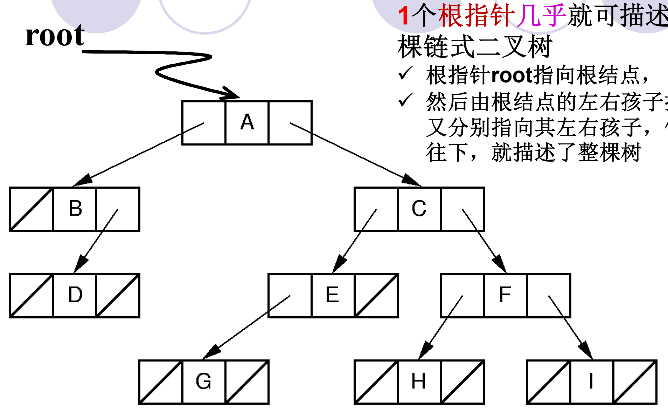
A typical Pointer-based binary tree



1个根指针几乎就可描述一棵链式二叉树

- ✓ 根指针root指向根结点，
- ✓ 然后由根结点的左右孩子指针又分别指向其左右孩子，依此往下，就描述了整棵树

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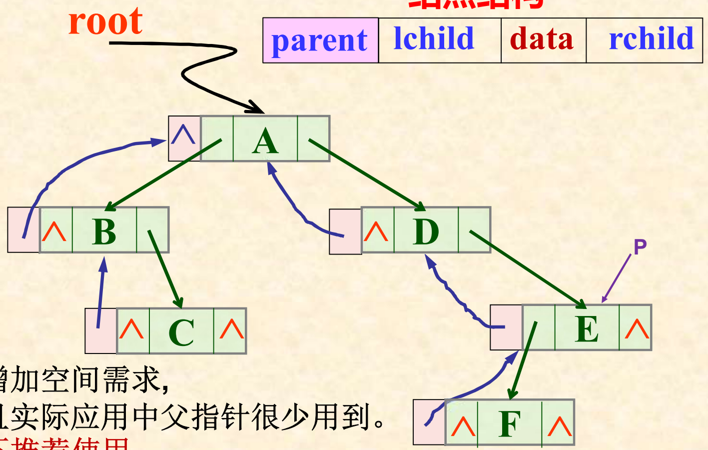
Add a upward pointer to the node's parent

结点结构

parent	lchild	data	rchild



增加空间需求，
且实际应用中父指针很少用到。
不推荐使用

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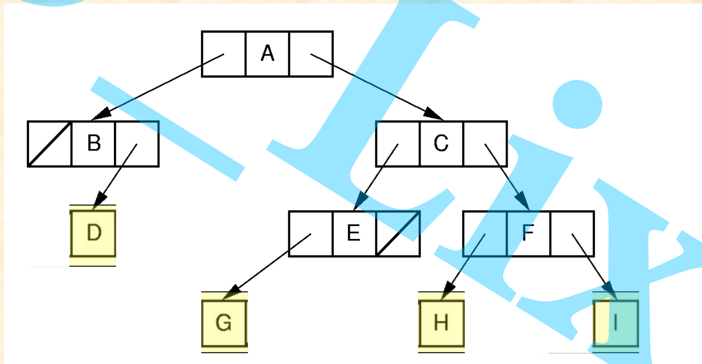
可变结构结点 VarBinNode

The structures of leaf and internal nodes are different:

leaf: data   internal nodes: lchild | data | rchild



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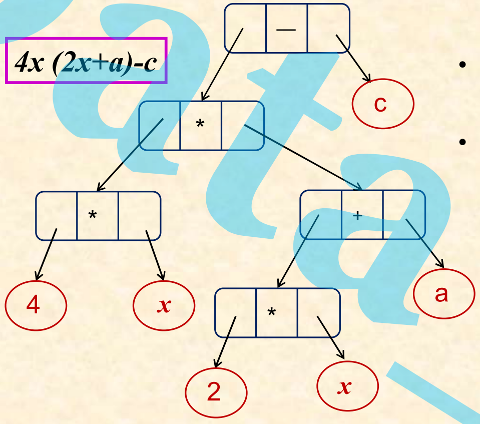
可变结构结点 VarBinNode的一个典型应用 ---expression tree/表达式树



- Each leaf is an operand 操作数
- The internal nodes are operators 操作符
- Sub-trees are sub-expressions 子表达式

1. 节省空间
2. 适合应用

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可变结构结点 VarBinNode

data

Leaf node的基本操作

1. 返回结点值 Val( )
2. 设置结点值 setVal(const E&)
3. 是否叶子: isLeaf()

lchild	data	rchild

Internal node 所涉及的基本操作

1. 返回结点值 Val( )
2. 设置结点值 setVal(const E&)
3. 是否叶子: isLeaf()
4. 返回左孩子 left()
5. 设置左孩子 setLeft(BinNode*)
6. 返回右孩子 right()
7. 设置右孩子 setRight(BinNode*)

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Binary Tree Node —VarBinNode(1)

```
class VarBinNode {   // Abstract base class
public:
   virtual  ~VarBinNode() {}
   virtual bool isLeaf() = 0;
};

class LeafNode : public VarBinNode { // Leaf
private:
  double var;                 // Operand value
public:
  LeafNode( const double& val)
    { var = val; }    // Constructor
  bool isLeaf() { return true; }
  double Val() { return var; }
  void setVal(const double& val)
    { var = val; }
};
```

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Binary Tree Node Class—VarBinNode(2)

```
// Internal node
class IntlNode : public VarBinNode {
private:
  VarBinNode* left;      // Left child
  VarBinNode* right;     // Right child
  char  opx;           // Operator value
public:
  IntlNode(const char& op,
             VarBinNode* l, VarBinNode* r)
    { opx = op; left = l; right = r; }
  bool isLeaf() { return false; }
  VarBinNode* left() { return left; }
  void setLeft(VarBinNode* l) {left = l;}
  varBinNode* right() { return right; }
  void setRight(VarBinNode* r) {right = r;}
  char Val() { return opx; }
  void setVal(char& op) {opx = op; }
};
```

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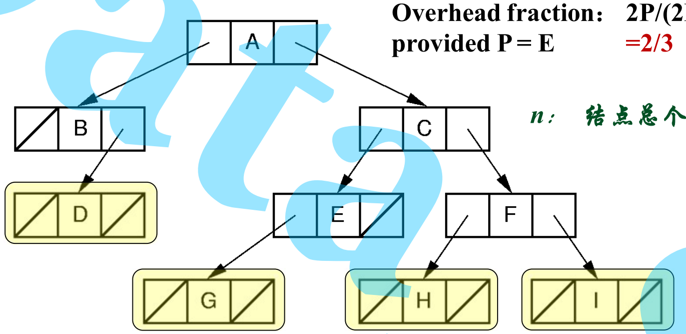
5.3.2 Space Requirements and Overhead analysis(1)

Overhead：存放数据所需空间之外的空间

所需空间：nE+2nP (Overhead)

Overhead fraction： 2P/(2P+E)

provided P = E =2/3



n： 结点总个数

二叉 Node

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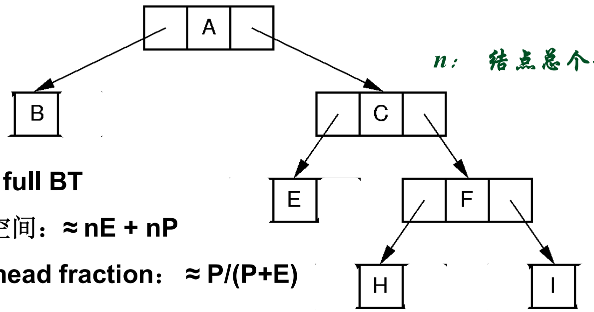
Space Requirements and Overhead analysis(2)



n： 结点总个数

For a full BT

所需空间：≈ nE + nP

Overhead fraction：≈ P/(P+E)

provided P = E = 1/2

`VarBinNode`

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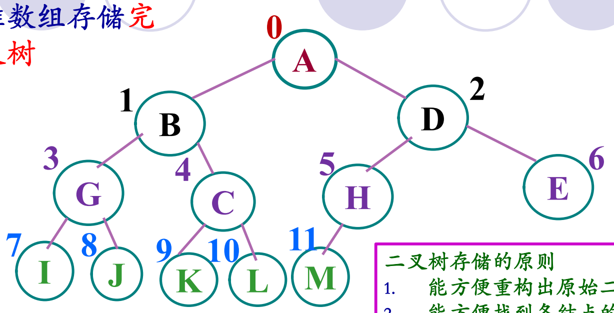
5.3.3 Array-based implementation for CBT

用一维数组存储完全二叉树



二叉树存储的原则
1. 能方便重构出原始二叉树
2. 能方便找到各结点的孩子

0	1	2	3	4	5	6	7	8	9	10	11
A	B	D	G	C	H	E	I	J	K	L	M

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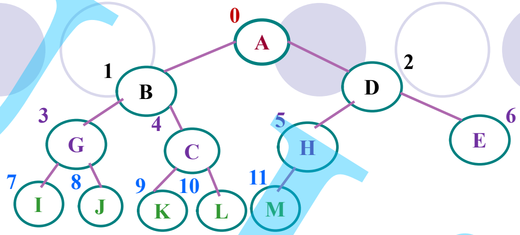

Position(数组下标)	0	1	2	3	4	5	6	7	8	9	10	11
BT元素	A	B	D	G	C	H	E	I	J	K	L	M
Parent	--	0	0	1	1	2	2	3	3	4	4	5
Left Child	1	3	5	7	9	11	--	--	--	--	--	--
Right Child	2	4	6	8	10	--	--	--	--	--	--	--
Left Sibling	--	--	1	--	3	--	5	--	7	--	9	--
Right Sibling	--	2	--	4	--	6	--	8	--	10	--	--

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For complete binary tree, the position relation can be calculated:

Parent(r) = (r-1)/2	if r > 0 and r < n.
Leftchild(r) = 2r + 1	if 2r + 1 < n.
Rightchild(r) = 2r + 2	if 2r + 2 < n.
Leftsibling(r) = r - 1	if r is even, r > 0, and r < n.
Rightsibling(r) = r + 1	if r is odd and r + 1 < n.

n： CBT中结点总个数

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二叉树存储的原则

1. 能方便重构出原始二叉树
   - ✓ 链式二叉树：
     本身就是树的形式
   - ✓ Array-based CBT
     从数组中第0个元素开始，按层构建，直到最后一个元素
2. 能方便找到各结点的孩子
   - ✓ 链式二叉树：
     从结点的lc，rc指针即可直接找到其左右孩子所在结点
   - ✓ Array-based CBT：设某个元素在数组中的下标为r
     Leftchild(r) = 2r + 1
     Rightchild(r) = 2r + 2

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链式非CBT与基于数组CBT总结

- 非CBT
  - 链式存储
  - 1个根指针—指向二叉树的根结点
  - 结点结构： 二叉结构，可变结构
- CBT
  - 基于数组存储
  - 1个数组和2个整形变量
    - 1个数组：按层存放结点元素值，即数组中每个元素对应一个结点
    - maxSize—数组大小，size—树中结点个数
  - 由任意数组元素下标可求出对应结点在CBT中的具体位置，及其孩子和双亲所对应的结点在数组中的位置

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5.4 Binary Tree Traversal(遍历)

5.4.1 Depth-First Traversal (深度优先遍历)

① Preorder traversal（前序遍历）

② Inorder traversal（中序遍历）

③ Postorder traversal（后序遍历）

5.4.2 Breadth-First Traversal（广度优先遍历）

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Traversals（遍历）

Any process for **visiting** each node **once and only once** in a **predetermined sequence**(预先确定的顺序） is called **traversal**.

“visting” 的含义可以很广，如：输出结点的信息,比较结点值与某一值的大小关系，修改节点数据等

- **Depth-First（深度优先）Traversal**
  - Preorder（前序） traversal: NLR
  - Postorder（后序） traversal: LRN
  - Inorder（中序） traversal: LNR
- **Breadth-First（广度优先） Traversal**

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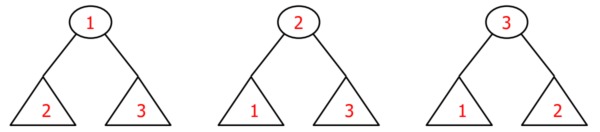
Depth-First Traversal



PreOrder(前序) NLR　InOrder(中序) LNR　PostOrder(后序) LRN

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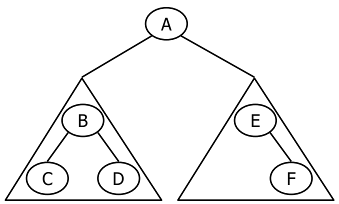
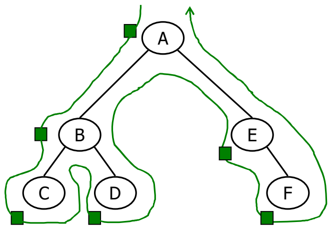
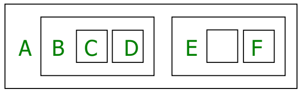
5.4.1 Preorder traversal（前序遍历）







Walking order

Processing order:
ABCDEF

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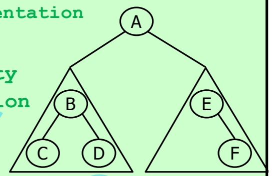
PreOrder Traversal c++ code

```
template <class E>      // Good implementation
void preOrder(BSTNode<E>* root) {
  if (root == NULL) return;  // Empty
  visit(root); // Perform some action
  preorder(root->left());
  preorder(root->right());
}
```




```
template <class Elem>  // Bad implementation
void preOrder2(BSTNode<Elem>* root) {
  visit(root); // Perform some action
  if (root->left() != NULL)
    preorder2(root->left());
  if (root->right() != NULL)
    preorder2(root->right());
}
```

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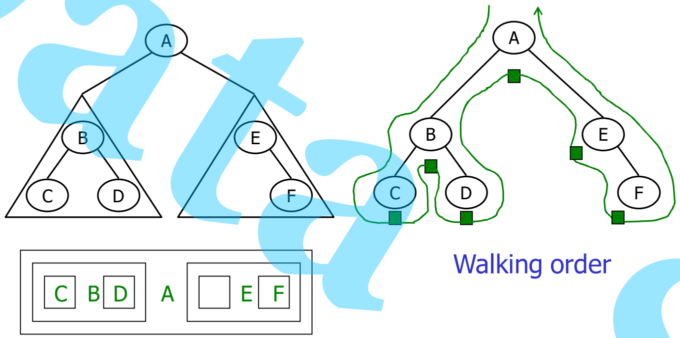
5.4.2 Inorder traversal（中序遍历）



Processing order

CBDAEF

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InOrder Traversal c++ code

```
template <class E>
void inOrder(BSTNode<E>* root) {
  if (root == NULL) return;  // Empty
  inorder(root->left());
  visit(root);  // Perform some action
  inorder(root->right());
}
```

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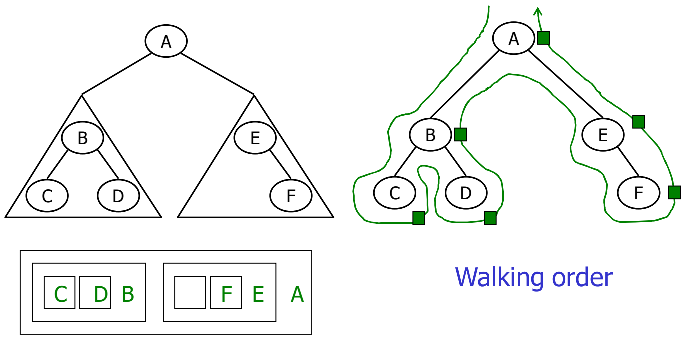
5.4.3 Postorder traversal（后序遍历）



Processing order

CDBFEA

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PostOrder Traversal c++ code

```
template <class E>
void postOrder(BSTNode<E>* root) {
  if (root == NULL) return;  // Empty
  postorder(root->left());
  postorder(root->right());
  visit(root);  // Perform some action
}
```

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由深度优先遍历序列恢复二叉树

- 已知前序序列和中序序列可唯一恢复二叉树

  example: 前序**ABCDEF**， 中序**CBDAEF**

- 已知后序序列和中序序列可唯一恢复二叉树

  example: 后序**CDBFEA**， 中序**CBDAEF**

- 已知前序序列和后序序列不可唯一恢复二叉树

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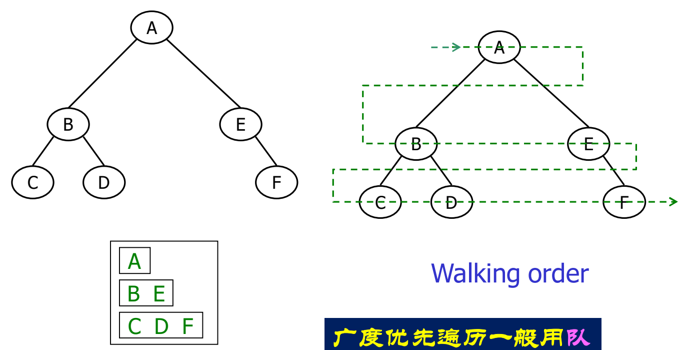
5.4.4 Breadth-First Traversal（广度优先遍历）



Walking order

广度优先遍历一般用队列的方式实现

Processing order

ABECDF

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Breadth-First Traversal pseudocode

Algorithm breadthFirst (BSTNode* root)
1 pointer = root
2 while (pointer not null)
  1 visit (pointer)
  2 if (pointer –> left not null)
    1 enQueue (pointer –> left)
  3 if (pointer –> right not null)
    1 enQueue (pointer –> right)
  4 if (not emptyQueue)
    1 deQueue (pointer)
  5 else
    1 pointer = null
End breadthFirst

有兴趣的同学课后自己编写对应的C++代码

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DFT & BFT实现总结

- 深度优先遍历（DFT）一般用递归(栈)的方式实现
- 广度优先遍历（BFT）一般用队列的方式实现

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本章作业一

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- 5.3
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