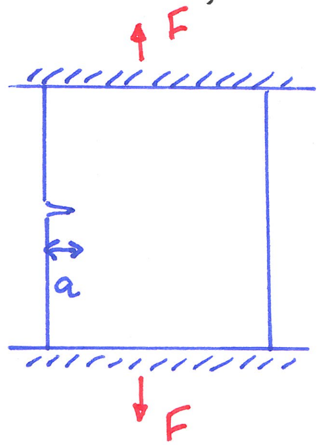
Fracture

- materials may fail by brittle fracture
- materials have cracks in them
- under critical conditions, cracks propagate → fracture
- examples: WW II Liberty ships: 2700 built, 1500 had signif. brittle fracture
  3 broke in two

  Boston molasses tank (Jan 1919) 2.3×10^6 gallons leaked
  50' tall (15m)
  90' diameter (27m)
  wall of molasses 35 mph
  21 people died

Griffith crack theory

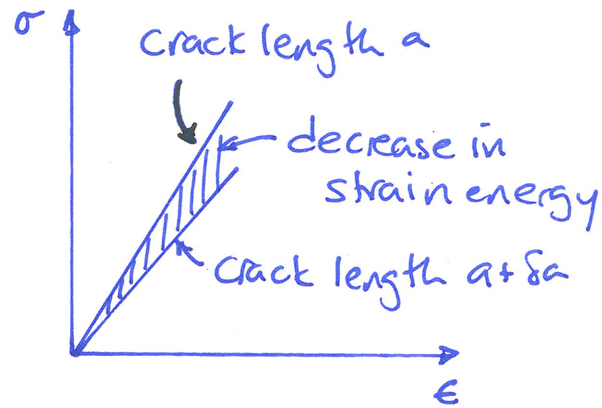
- consider an infinite plate, containing a crack, stressed in tension to some fixed displacement



- crack length, a
- plate thickness, t

- if the crack grows by δa, the stress relaxes & the strain energy decreases

roughly, the shaded area relaxes



decrease in strain energy

- roughly estimate the change in strain energy in going from

  no crack → crack of length a

$$U^{el} \sim \frac{\sigma^2}{2E} \frac{\pi a^2 t}{2}$$

- or, if crack grows by δa

$$\delta U^{el} = \frac{dU}{da}\delta a = \frac{\sigma^2}{2E} \frac{2\pi a t}{2} \delta a$$

- surface energy required for an increase in crack length by δa is:

$$2\Gamma t \delta a$$

- define G_c = energy absorbed per unit area of crack $= 2\Gamma$ [J/m^2]

  = critical strain energy release rate

$$2\Gamma t\,\delta a = G_c t\,\delta a$$

- equating work done in forming new surfaces with strain energy released:

$$\frac{\sigma^2}{2E} \pi a t\,\delta a = G_c t\,\delta a$$

$$\sigma^2 \pi a = 2EG_c$$

- this analysis is approximate
  - stress not constant at crack tip
  - relaxed area not semicircle

- exact analysis requires stress distribution ahead of crack tip (Inglis, 1913)

$$\sigma_{local} \propto \frac{\sigma^{\infty}\sqrt{\pi a}}{\sqrt{2\pi r}}$$

- exact analysis gives:

$$\sigma^2 \pi a = E G_c \qquad \text{(factor of 2 different from approximate analysis)}$$

or,

$$\underbrace{\sigma \sqrt{\pi a}}_{\text{loading conditions}} = \underbrace{\sqrt{E G_c}}_{\text{material properties}}$$

- also define $K_I = Y \sigma \sqrt{\pi a}$ = stress intensity factor [MPa $\sqrt{m}$]

  Y = geometrical factor, depends on loading configuration

- and $K_{Ic} = \sqrt{E G_c}$ = fracture toughness (material property)
  [MPa $\sqrt{m}$]

<u>Griffith criterion</u>

if $\boxed{K_I = K_{Ic}}$ ⇒ fracture

$\boxed{Y \sigma \sqrt{\pi a} = K_{Ic}}$ ⇒ fracture

if $Y \sigma \sqrt{\pi a} < K_{Ic}$ ⇒ no fracture

crack propagation modes: I, II, III : see figure

Stress analysis of cracks

- stress field around crack tip : $\sigma_{local} \propto \frac{\sigma^{\infty}\sqrt{\pi a}}{\sqrt{2\pi r}}$
- singularity at $r=0$: $\sigma \propto 1/\sqrt{r}$ at $r=0$ $\sigma \to \infty$
- in practice, material near crack tip yields when local equivalent stress, $\sigma_e = \sigma_{yield}$
- get plastic zone at crack tip
- r_p = plastic zone size

- Griffith criterion is valid only if plastic zone small

  "small scale yielding" (SSY) : $\frac{r_p}{a} < 0.02$
- if have $r_p/a < 0.02 \Rightarrow$ linear elastic fracture mechanics (LEFM)
- if plastic zone large, then energy required for plastic deformation is significant & has to be accounted for in energy balance

  (we won't cover this)

Last time: Fracture

Stress field at a crack tip: $\sigma_{local} \propto \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \propto \frac{K_I}{\sqrt{2\pi r}}$

Griffith criterion: fracture when $\sigma\sqrt{\pi a} = \sqrt{EG_c}$ (infinite plate)

$Y\sigma\sqrt{\pi a} = \sqrt{EG_c}$ (other geometries)

$$\boxed{K_I = K_{Ic}}$$

$\sigma \propto \frac{1}{\sqrt{r}}$ ⇒ singularity @ crack tip

⇒ plasticity; plastic zone size, r_p

Griffith valid if small scale yielding: $\frac{r_p}{a} < 0.02$

(linear elastic fracture mechanics)

Plastic Zone Size

- to get first order estimate of r_p, plastic zone size:
  - assume · plane stress ($\sigma_z = 0$)
    · material elastic-perfectly plastic
  - set $\sigma_{yy} = \sigma_{yield}$ along line $\theta = 0$

$$\sigma_{yy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} = \frac{K_I}{\sqrt{2\pi r_p}} = \sigma_{yield}$$

$$\therefore\ r_p \approx \frac{K_I^2}{2\pi\,\sigma_{yield}^2} \qquad \text{(only for } \theta = 0\text{)}$$

- more exact estimate of plastic zone size found by converting (σ_x, σ_y, σ_{xy}) to principal stresses & using von Mises criterion: $\sigma_{eq} = \sigma_{yield}$
- can find shape of plastic zone for plane stress ($\sigma_{zz} = \sigma_{zy} = \sigma_{zx} = 0$)
  or plane strain ($\epsilon_{zz} = \epsilon_{zy} = \epsilon_{zx} = 0$)

- engineering formulae:

plane stress ($\sigma_z = 0$) $\quad r_p = \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_{yield}}\right)^2$

plane strain ($\epsilon_z = 0$) $\quad r_p = \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_{yield}}\right)^2$

Measurement of fracture toughness

- plastic zone size larger for plane stress than plane strain
- K_c larger for plane stress than plane strain
- want to measure lowest K_c (to be conservative)

  "plane strain fracture toughness"
- plane stress: thin specimen ($\sigma_z = 0$ at crack tip)
- plane strain: thick specimen ($\epsilon_z = 0$ at crack tip)

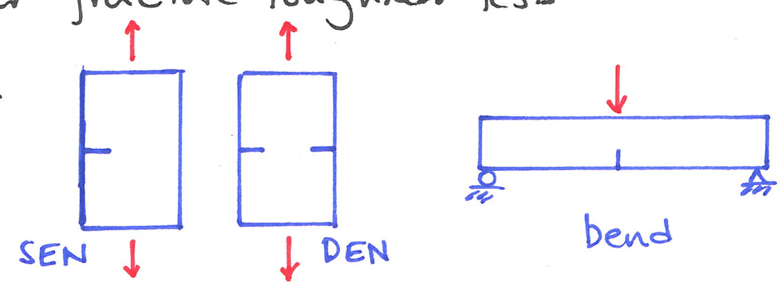
- ASTM requires specimen thickness, $B > 25\, r_p$ ← plane strain value

  for valid fracture toughness test
- various geometries used for fracture toughness tests

  e.g. single edge notch

  double edge notch

  bend test



- data for K_{Ic}, G_c ⇒ metals - high - large plastic zone

  ceramics - low - small plastic zone

Micromechanisms of fast fracture

- ceramics + glasses: brittle, cracks propagate easily
- metals typically ductile - resistant to crack propagation, but can become brittle at low temperatures
- polymers - intermediate, and can be made tougher by making them into composites.

Mechanism of crack propagation: cleavage

- fracture surface of ceramic or glass - smooth
- featureless flat surface, with little or no plastic deformation
- ceramics + glasses have high yield strengths, $\sigma_y \Rightarrow$ little plastic deformation
- local stress breaks apart interatomic bonds
- crack propagates by separating a pair of atomic planes, giving rise to atomically flat surface by cleavage

Mechanisms of crack propagation: ductile tearing

- fracture surface of cracked ductile metal that has fractured is rough
- a lot of plastic work taken place to form rough surface
- plastic zone at crack tip $r_p \sim \frac{1}{2\pi} \frac{K_{IC}^2}{\sigma_y^2}$
- soft metals have low σ_y, large r_p
- hard ceramics have high σ_y, small r_p

Ductile tearing

- even when nominally pure, most metals contain small inclusions of chemical compounds formed by reaction of metal + impurity atoms
- within plastic zone, plastic flow takes place around these inclusions, leading to elongated cavities
- as plastic flow progresses, cavities link up, turning a sharp crack into a blunt one, reducing the stress concentration
- ductile tearing consumes a lot of energy by plastic flow, increasing G_c & K_{IC}
- explains why ductile metals so tough

Ductile to brittle transition

- whether or not behavior is ductile or brittle depends on yield strength, σ_y + size of plastic zone
- σ_y depends on temperature & strain rate:

$$\sigma_y = \sigma_y^\circ \left[1 - \frac{kT}{Q_b} \ln \frac{\dot{\gamma}}{\dot{\gamma}_0}\right]$$

σ_y° = yield strength at 0°K

Q_b = energy/bond

k = Boltzmann's constant

T = absolute temp. (°K)

$\dot{\gamma}$ = shear strain rate

$\dot{\gamma}_0$ = material parameter

- also have $\quad r_p \propto \left(\frac{K_I}{\sigma_y}\right)^2$
- as temperature T decreases, yield strength σ_y increases, plastic zone size r_p decreases
- as $T \downarrow$, material becomes more brittle
- get ductile-brittle transition as temperature decreases (or as strain rate $\dot{\gamma}$ increases)

Constance Tipper

From Wikipedia, the free encyclopedia

Constance Fligg Elam Tipper	
Born	6 February 1894 New Barnet, Hertfordshire
Died	14 December 1995 (aged 101) Penrith, Cumbria
Education	Newnham College, Cambridge
Occupation	Metallurgist
Spouse(s)	George Tipper

Constance Fligg Elam Tipper (6 February 1894 – 14 December 1995) was an English metallurgist and crystallographer.

Constance Tipper specialized in the investigation of metal strength and its effect on engineering problems. During World War II she investigated the causes of brittle fracture in Liberty Ships. These ships were built in the US between 1941 and 1945, and were the first all-welded pre-fabricated cargo ships.

Tipper established that the fractures were not caused by welding, but rather by the steel itself. She demonstrated that there is a critical temperature below which the fracture mode in steel changes from ductile to brittle. Because ships in the North Atlantic were subjected to low temperatures, they were susceptible to brittle failure. These fatigue cracks were able to spread across the ship's welded joint plates, instead of stopping at plate edges of a riveted joint, as previously used.

In 1949 Tipper was appointed Reader and became the only woman to be a full-time member of the Faculty of Engineering of Cambridge University.

She was the first person to use a scanning electron microscope (SEM) to examine metallic fracture faces. She used a scanning electron microscope built by Charles Oatley and his team, the second SEM ever built.

She retired in 1960. Her 100th birthday in 1994 was celebrated by Newnham College with the planting of the Tipper Tree, a sweet chestnut.

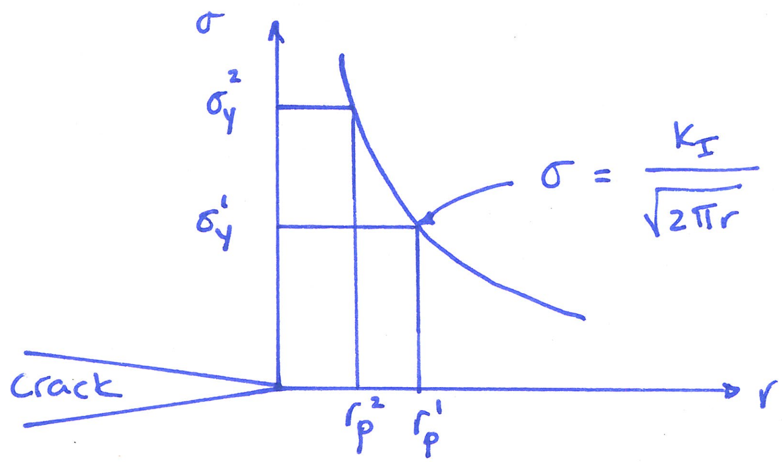
Works

- "The Production of Single Crystals of Aluminium and their Tensile Properties" (with H. C. H. Carpenter). *Proceedings of the Royal Society of London* (1921).
- *Deformation of Metal Crystals* (1935).
- *The Brittle Fracture Story* (1962).

References

- Charles, Jim and Gerry Smith. "Constance Tipper: her life and work", *Materials World* (1996).
- Hayes, Evelyn. "Dr. Constance Tipper: testing her mettle in a materials world", *Advanced Materials & Processes* (1998).
- Cathcart, Brian. " No dinner, but a nice box of chocs" (http://www.newstatesman.com/200402160008), *New Statesman* (2004). Retrieved on 27 May 2007.
- Hetzel, Phyllis. Obituary (http://findarticles.com/p/articles/mi_qn4158/is_19951220/ai_n14024925/print), *The Independent* (1995). Retrieved on 27 May 2007.

Ductile to brittle transition



- at temp T_1 $\sigma_y^1 \rightarrow r_p^1$
- at temp T_2 $\sigma_y^2 \rightarrow r_p^2$ $T_2 < T_1$

$T_2 \Rightarrow$ smaller plastic zone

$\Rightarrow$ more brittle behavior

Liberty ship failure - Constance Tipper identified ductile-brittle transition as cause

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Composites

- composites can be much more resistant to crack propagation (much tougher) than their constituents.
- eg. glass fibers in epoxy resin; glass, epoxy both brittle
- but composite is much tougher
- fibers act as crack stoppers
- as crack reaches fiber, it can debond, blunting the crack + arresting the crack

Composites

- rubber toughened epoxy
  - rubber particles act like little springs, clamping the crack closed again
  - increases load needed to propagate crack.
  - increases toughness.

Fatigue

- repeated stress cycles can cause failure at stresses less than the ultimate tensile strength, σ_{UTS}, or even the yield strength, σ_y

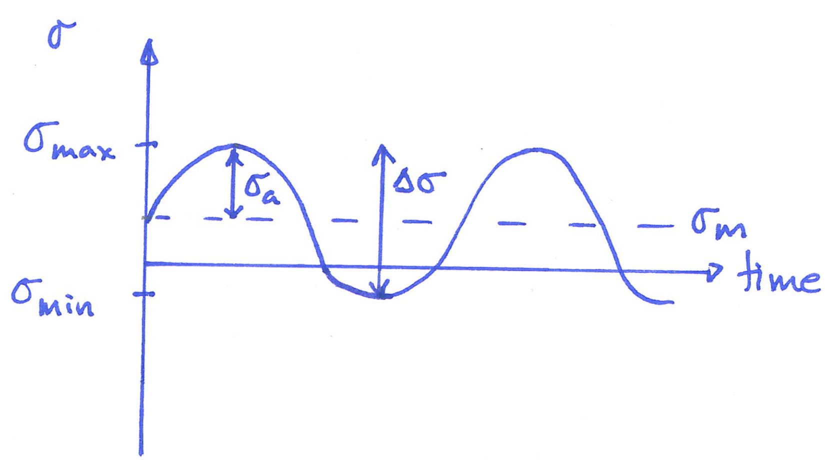
(a) Fatigue of uncracked components

- no pre-existing crack
- e.g. small components - gear teeth
- fracture controlled by initiation of crack

(b) Fatigue of cracked components

- pre-existing crack
- large structures e.g. bridges, ships, pressure vessels
- fracture controlled by propagation of crack

(a) Fatigue of uncracked components



$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

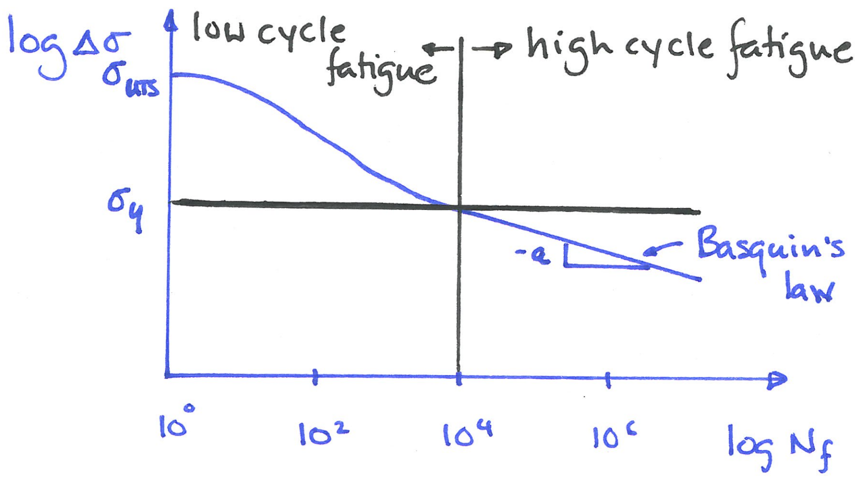
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

N = number of cycles of stress

N_f = number of cycles to failure

Fatigue data for uncracked specimens typically plotted as:



low cycle fatigue:

σ_{max} or $|\sigma_{min}| > \sigma_y$

$N_f < 10^4$ cycles (roughly)

high cycle fatigue:

σ_{max} or $|\sigma_{min}| < \sigma_y$

$N_f > 10^4$ cycles (roughly)

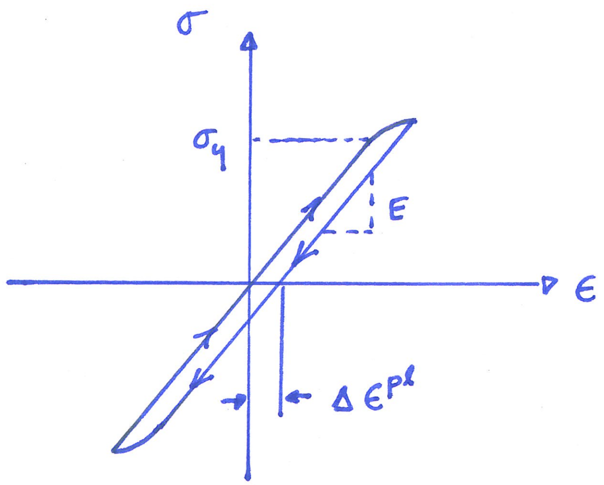
Empirical relations for fatigue of uncracked components

- first consider fatigue under zero mean stress ($\sigma_m = 0$)
- high cycle fatigue : $\boxed{\text{Basquin's law:} \quad \Delta\sigma N_f^a = C_1}$

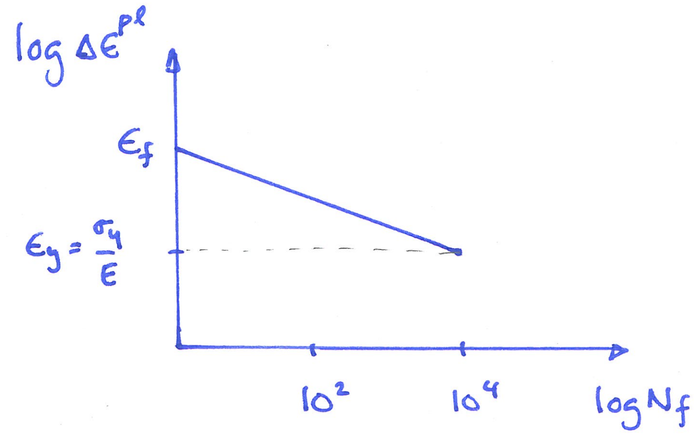
a, C_1 material properties

typically $1/15 < a < 1/8$

- <u>low cycle</u> fatigue: $\boxed{\text{Coffin-Manson law: } \Delta\epsilon^{pl} N_f^{\,b} = C_2}$



b, C_2 material properties, $b \approx 0.5$ to 0.6



- for a mean tensile stress ($\sigma_m > 0$), stress range must be decreased to preserve same N_f
- $\boxed{\text{Goodman's rule: } \Delta\sigma_{\sigma_m} = \Delta\sigma_0 \left(1 - \frac{\sigma_m}{\sigma_{urs}}\right)}$

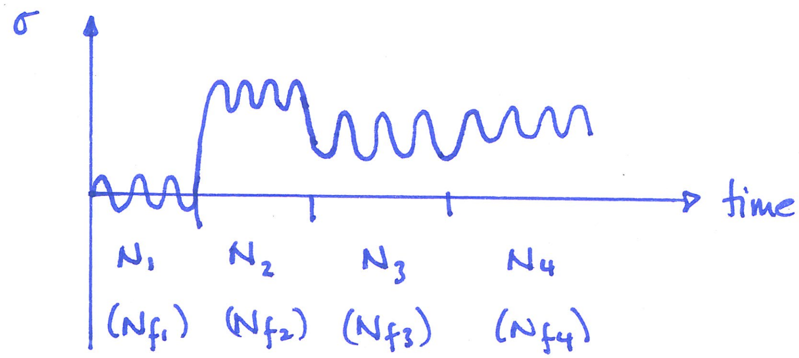
$\Delta\sigma_0$ = cyclic stress range for N_f cycles with $\sigma_m = 0$

$\Delta\sigma_{\sigma_m}$ = " " " " same N_f " " $\sigma_m > 0$

σ_{UTS} = ultimate tensile strength

Empirical relations for uncracked components

- If $\Delta\sigma$ varies over lifetime of component, then sum the damage for each $\Delta\sigma$



Miner's rule of cumulative damage $\boxed{\sum \frac{N_i}{N_{fi}} = 1}$

N_{fi} = no. cycles to fracture under stress in region i

$\frac{N_i}{N_{fi}}$ = fraction of lifetime used up after N_i cycles in that region

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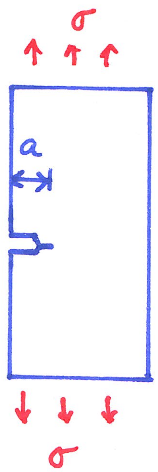
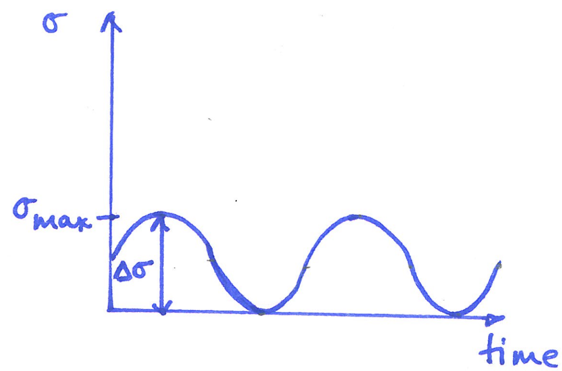
Fatigue of cracked components

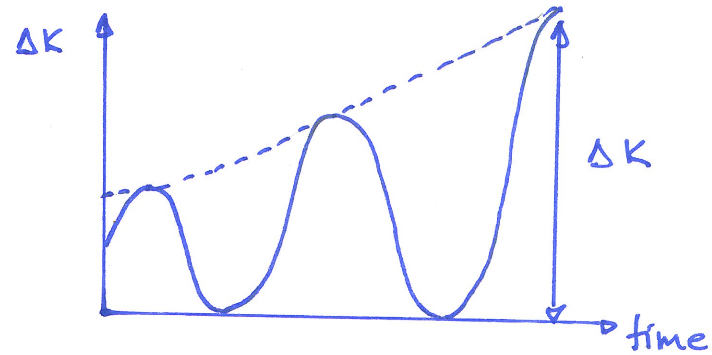
- detect cracks non destructively (eg. x-ray, ultrasound)
- initial crack length: measure, or take as less than resolution of measurement system.
- with cyclic load, cracks grow
- need to know number of cycles for crack to reach critical length for fracture

Fatigue of cracked components

- cyclically load specimen with sharp crack







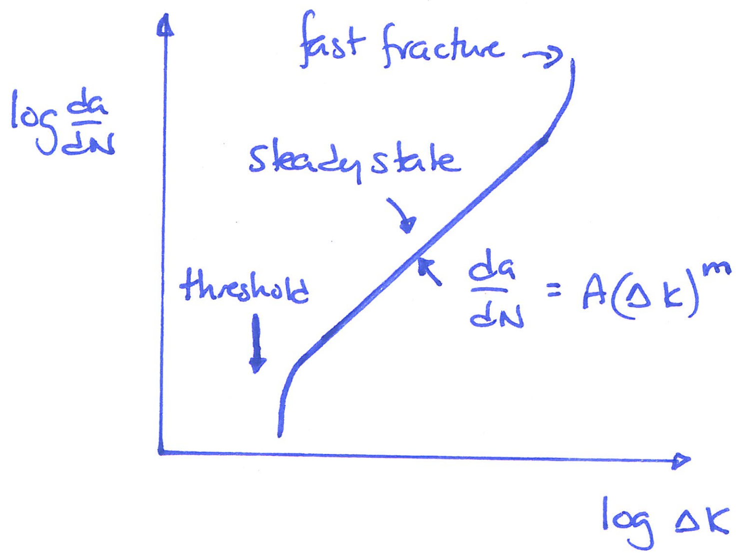
- as cycle stress at constant $\Delta\sigma$, crack length increases and since

$$K = \sigma\sqrt{\pi a} \Rightarrow K \text{ increases}$$

$$\Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi a}$$

- also find crack growth rate, da/dN, depends on ΔK

Fatigue of cracked components



$$\boxed{\text{Paris law: } \frac{da}{dN} = A(\Delta K)^m}$$

A, m material constants.

- If initial crack length, a_o, is measured

  & final crack length, a_f, for fracture can be calculated

then, safe number of cycles of loading can be estimated from:

$$N_f = \int_0^{N_f} dN = \int_{a_o}^{a_f} \frac{da}{A(\Delta K)^m}$$