Stress field around a dislocation

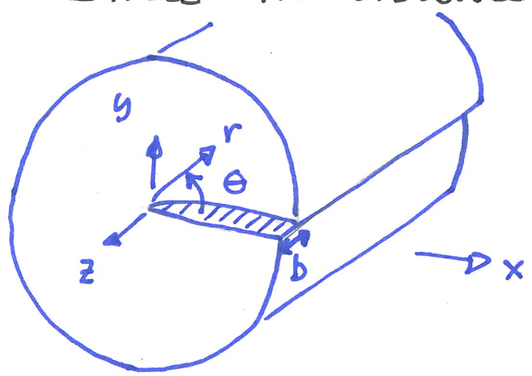
- dislocations produce stress fields around them
  - eg. figure: bottom in tension; top in compression
- calculate elastic properties of dislocations
- based on continuum elasticity
- works down to 5-10x atomic scale
- provides explanation of macroscopic plasticity

Dislocation theory:

- elastic stress field around a dislocation
- strain energy of a dislocation
- line tension
- glide force

Stress field around a dislocation

- screw dislocation has a simple elastic stress field
- stress arises from distortion of atoms
- stress in absence of external load: "self-stress" of dislocation



take circuit of 2π

$u_x = 0$

$u_y = 0$

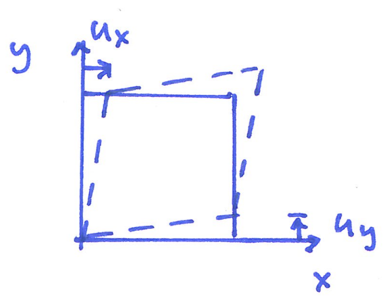
$u_z = b$ Burgers vector

displacement → strains → stresses

- displacement $u_z = b\,\dfrac{\theta}{2\pi} = \dfrac{b}{2\pi}\tan^{-1}\left(\dfrac{y}{x}\right)$ (assume linear interpolation & u_z independent of radius taken)

- normal strain: $\epsilon_{xx} = \dfrac{\partial u_x}{\partial x} = 0 \qquad \epsilon_{yy} = \dfrac{\partial u_y}{\partial y} = 0 \qquad \epsilon_{zz} = \dfrac{\partial u_z}{\partial z} = 0$

- shear strain $\gamma_{xy} = \dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x} = 0$



$$\gamma_{yz} = \cancel{\frac{\partial u_y}{\partial z}}^{\,0} + \frac{\partial u_z}{\partial y} = \frac{bx}{2\pi(x^2+y^2)}$$

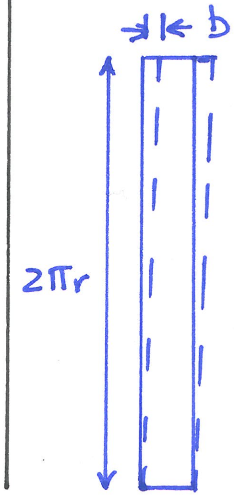
$$\gamma_{xz} = \frac{\partial u_z}{\partial x} + \cancel{\frac{\partial u_x}{\partial z}}^{\,0} = \frac{-by}{2\pi(x^2+y^2)}$$

$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = 0$

$$\sigma_{yz} = \frac{Gbx}{2\pi(x^2+y^2)} = \frac{Gb}{2\pi r}\cos\theta$$

$$\sigma_{xz} = \frac{-Gby}{2\pi(x^2+y^2)} = \frac{-Gb}{2\pi r}\sin\theta$$

Alternatively, in cylindrical coordinates unroll one piece of cylinder



$$\gamma_{\theta z} = \frac{b}{2\pi r}$$

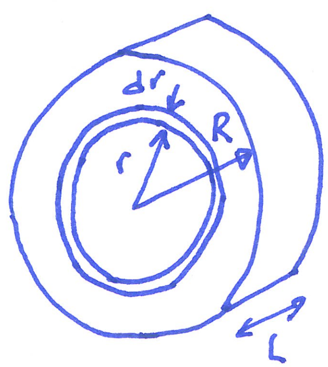
$$\sigma_{\theta z} = \frac{Gb}{2\pi r}$$

- stress varies as $1/r$
- at dislocation core, continuum assumption not realistic; ok down to ~ 4b
- r_0 = dislocation core radius ~ b to 4b
- edge dislocation - more complex stress state, but all non-zero components have factor $\boxed{\sigma \propto \frac{Gb}{2\pi r}}$ ⇒ this is often used as "order of magnitude" to represent dislocation stress field

Elastic strain energy of a dislocation

- stress field around a dislocation produces strain energy
- screw dislocation: $\sigma_{\theta z} = \dfrac{Gb}{2\pi r}$ (varies with radius, r)
- strain energy /vol $= \frac{1}{2}\sigma_{\theta z}\,\epsilon_{\theta z} = \dfrac{\sigma_{\theta z}^2}{2G}$
- $dU_{el} = \dfrac{\sigma_{\theta z}^2}{2G}\,dV$ ← volume, V

- consider a cylinder of material



$$dV = 2\pi r L\, dr$$

$$U_{el} = \int_{r_0}^{R} \frac{1}{2G}\left(\frac{Gb}{2\pi r}\right)^2 2\pi r L\, dr$$

r_0 = dislocation core radius

$$\frac{U_{el}}{L} = \int_{r_0}^{R} \frac{Gb^2}{4\pi r}\, dr$$

(neglect strain energy of dislocation core)

$$\boxed{\frac{U_{el}}{L} = \frac{Gb^2}{4\pi} \ln\left(\frac{R}{r_0}\right)}$$

elastic strain energy of dislocation (screw)

$R \sim$ distance between dislocations $\sim \sqrt{1/\rho}$ ← disloc. density

Strain energy of a dislocation

edge dislocation $\frac{U_{el}}{L} = \frac{Gb^2}{4\pi(1-\nu)} \ln\left(\frac{R}{r_0}\right)$

in general we can write $\boxed{\frac{U_{el}}{L} = \alpha G b^2}$ $\alpha \approx 0.5$ to 1.0

units: [J/m]

Line tension

- line tension is the change in strain energy when the dislocation is extended by a unit length
- equal to the energy/length

$\boxed{T = \frac{Gb^2}{2}}$ units [J/m = N]

- analogous to surface tension in a fluid

Glide force f on a dislocation

- force exerted on a dislocation normal to the dislocation line, when a stress is applied
- define per unit length of dislocation line
- calculate by equating external + internal work done when dislocation moves completely through a crystal

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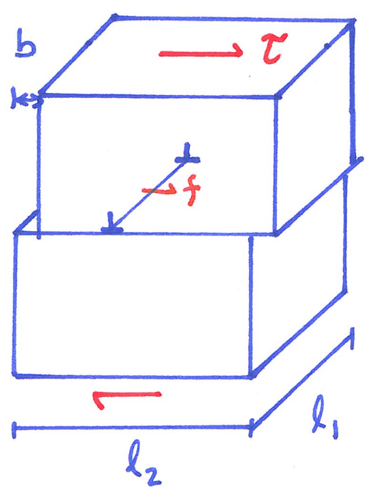

external work
$= \tau l_1 l_2 b$

internal work
$= f l_1 l_2$
(disl. moves l_2)

$\boxed{f = \tau b}$

- true for all types of dislocations
- τ = shear stress in glide plane, resolved in direction of b
- glide - dislocation moves in slip plane - contains both Burgers vector b & dislocation line, t

Lattice resistance

- stress required to move a dislocation through an otherwise perfect crystal
- covalently bonded solids: $\sigma_y \sim 10^{-2}$ to 10^{-1} E (intrinsically hard)
- pure metals: $\sigma_y \sim 10^{-6}$ to 10^{-2} E (intrinsically soft)
- ionic materials: $\sigma_y \sim 10^{-2}$ E (intermediate)
- metals - need to be strengthened - introduce obstacles to dislocation motion
  - alloying
  - solid solution hardening
  - precipitation hardening
  - dispersion hardening

Solid solution hardening

- individual solute atoms dissolved in primary metal

  e.g. brass: Zn in Cu
  bronze: Sn in Cu
  stainless steel: Ni, C in Fe

- elastic interaction between solute + primary metal
  - geometric misfit: dilute solution - individual atoms
    concentrated sol^{n} - clusters of atoms dissolve
    - short range order of solute
  - modulus misfit

Solid solution hardening

- shear stress required to move dislocation
- interaction energy between dislocation + solute is U_s
- glide force / length $= \tau^\circ_{ss} b$
- spacing of solute atoms $= L$
- concentration of solute atoms in the slip plane, $C = 1/L^2$
- work U_s must be done on a dislocation segment of length L to move it past atom of size b

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Work eq'n:

$$\tau^\circ_{ss} b L b = U_s$$

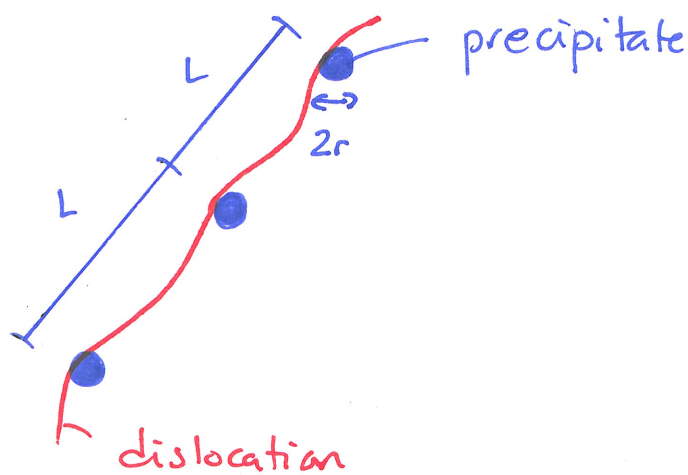
$$\tau^\circ_{ss} = \frac{U_s}{L b^2} = \frac{U_s \sqrt{C}}{b^2}$$

- strength increased with concentration of solute atoms & with interaction energy, U_s

$$\boxed{\tau^\circ_{ss} = \frac{U_s \sqrt{C}}{b^2}}$$

Precipitation hardening

- examples: dural - hard Al alloy   precipitate (ppt) of $CuAl_2$ in Al
  mild steel - iron carbide (Fe_3C) ppt in Fe
  nimonics - Ni_3Al in Ni
- precipitates are weak obstacles to dislocation motion
- dislocations can cut through their lattice
- work done in doing so raises σ_y



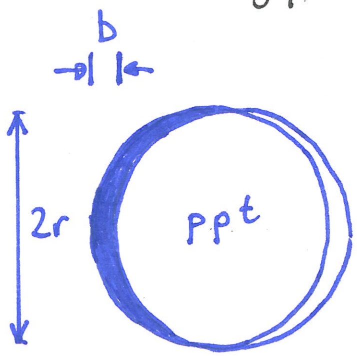
L = spacing between ppt.

r = radius of ppt.

- dislocation moves through entire diameter of ppt, 2r   ($r > b$)
- work done by dislocation = $\tau_p^o \, b L \, 2r$

Precipitation hardening (cont'd)

- energy required to cut through ppt. lattice per unit surface area $= \Gamma$



- energy to cut through ppt. lattice

$$\sim \Gamma \, 2rb \, 2 \qquad (2 \text{ surfaces})$$

- shear stress required to cut through ppt

$$\boxed{\tau_p^{\circ} = \frac{4rb\Gamma}{2rbL} = \frac{2\Gamma}{L}}$$

- more closely spaced ppt, higher σ_y
- high strength Al alloys, nimonics $L \sim 10^{-7}$ m

Dispersion hardening

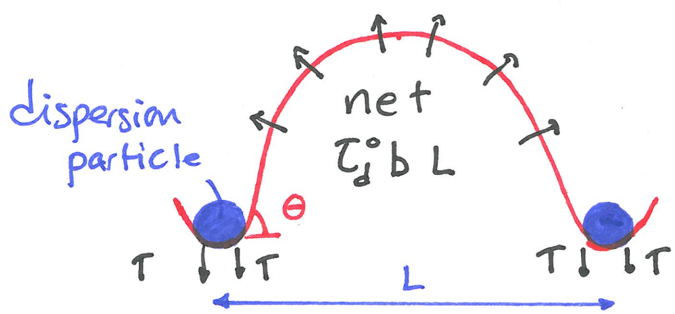
- dispersion - compound introduced into a crystal

  e.g. Al_2O_3 added to Al - sintered Al - powder metallurgy

- typically choose ceramics that are hard + strong
- dislocations cannot cut through dispersion

Dispersion hardening

- dislocations bow out between dispersion particles (pinned)



- maximum resistance to shear from line tension when $\theta = 90^\circ$
- force equilibrium

$$2T = \tau_d^\circ b L$$

$$T = Gb^2/2$$

$$\tau_d^\circ = \frac{2T}{bL} = \frac{Gb^2}{bL} = \frac{Gb}{L}$$

$$\boxed{\tau_d^\circ = \frac{Gb}{L}}$$

- note: to raise yield strength to $\tau^\circ/G = 10^{-3} \Rightarrow b/L = 10^{-3}$
- $b \sim 10^{-10}$ to 10^{-9} m $\Rightarrow L = 0.1\mu$ to 1μ
- dispersion particles have to be very fine to get small enough spacing

Frank-Read

Mill videos

Grain boundary strengthening

- empirical relationship: Hall-Petch

$$\tau^\circ_{gb} = \tau_0 + \frac{k}{\sqrt{d}}$$

τ_0 = lattice resistance

k = material property

d = grain size

- small grains - stronger

- arises from interaction of grain boundary + dislocation; not well understood

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Strengthening: summary

- yield strength is lattice resistence + sum of all strengthening mechanisms in alloy

$$\tau_y = \tau^\circ + \tau^\circ_s + \tau^\circ_p + \tau^\circ_d + \tau^\circ_{gb}$$

τ° ↓ lattice resistance of primary metal

τ°_s ↓ $\frac{u_s\sqrt{c}}{b^2}$

τ°_p ↓ $\frac{2\Gamma}{L}$

τ°_d ↓ $\frac{Gb}{L}$

τ°_{gb} ↓ $\frac{k}{\sqrt{d}}$