Plasticity

- at or near room temperature, many materials (esp. metals) have a well defined yield strength
- at $\sigma < \sigma_y$ - elastic deformation - recoverable

  $\sigma \geq \sigma_y$ - plastic deformation - irrecoverable
- plastic behavior important for:
  - material design for strengthening (alloying, hardening)
  - metal forming operations
  - hardness of solids (friction, wear)
  - fracture (plastic zone at cracktip)

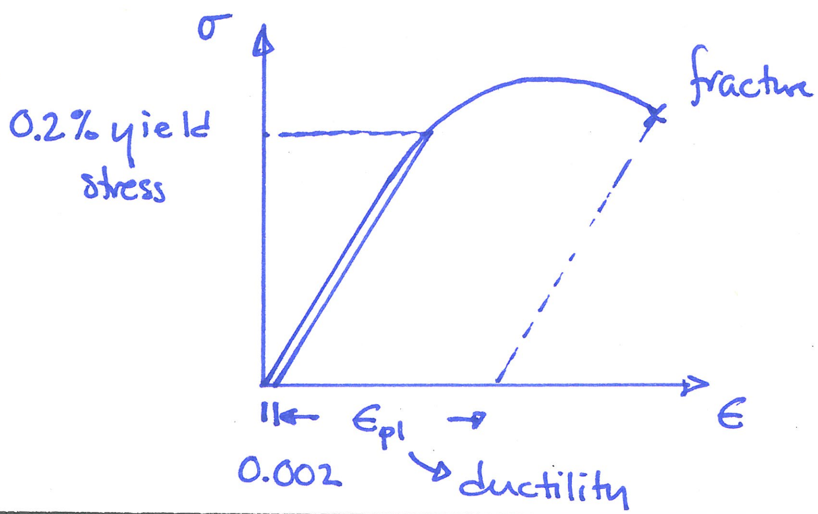
- some materials fracture before yield (eg. ceramics: SiC, WC)

  <u>but</u> fracture can be suppressed by imposing a hydrostatic stress field

  - then, even ceramics will yield
- this section of subject: low temperature plasticity
- at high temp metals + ceramics creep (metals @ $T/T_M > 0.3$, ceramics @ $T/T_M > 0.5$)
- examine creep later in subject

Measurement of yield strength

(1) uniaxial tension test



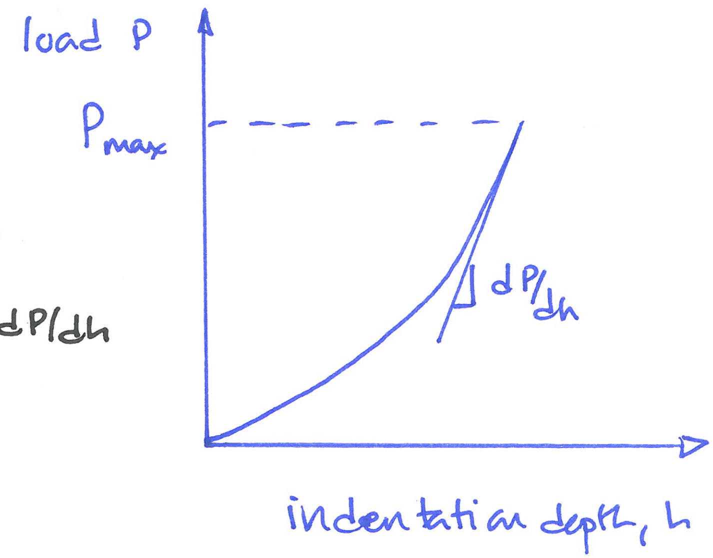
(2) hardness test (Vickers, Brinell, Rockwell)

- indenter pushed into specimen polished surface with known force
- size of imprint depends on σ_y
- various geometries of indenters used: sphere, pyramid, cone
- measure projected area or indent surface area
- advantages:
  - easy, don't need machined specimen
  - non-destructive
  - good for ceramics - constraint of surrounding material prevents fracture.

(3) nanoindentation

(insects) wings

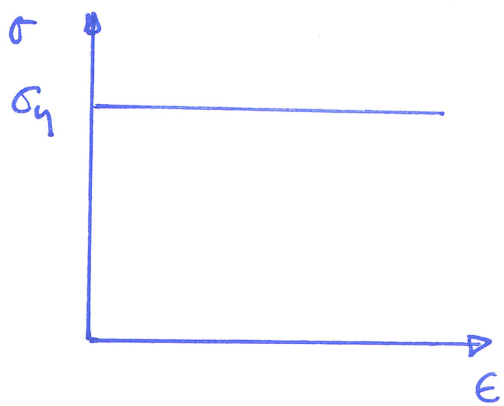
- good for thin films, small specimens
- local properties
- measure load P vs indentation depth, h
- can obtain E from unloading slope, dP/dh
- can obtain hardness, H, from P_{max}



data for σ_y

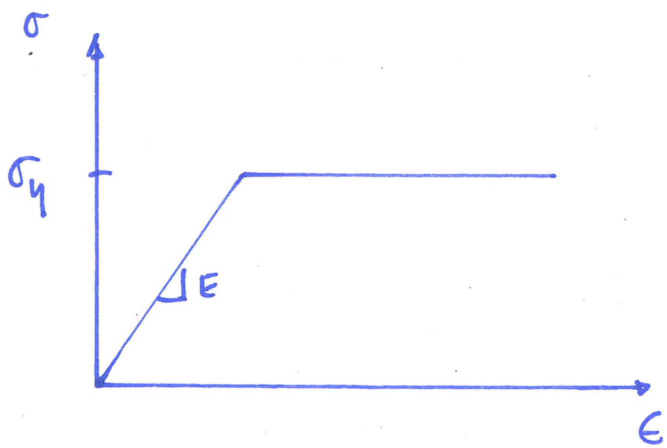
Idealizations of plastic behaviour

(1) rigid / perfectly plastic



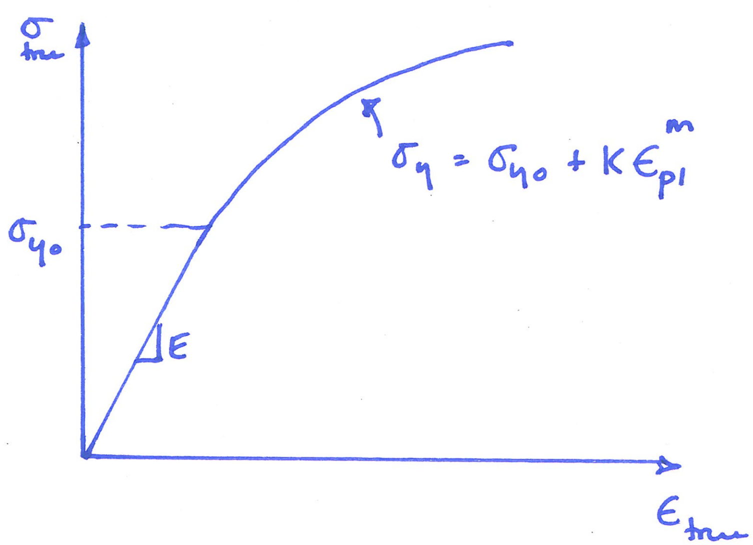
- rigid $\Rightarrow E \to \infty$
- elastic strain << plastic strain
- perfectly plastic $\Rightarrow \sigma_y$ = constant (neglects strain hardening)

(2) elastic/perfectly plastic



- use if plastic strain comparable to elastic strain
- but recall that limit of elasticity is about ~0.002
- only used if total strains are small

(3) elastic/strain hardening



- for true stress - true strain:

$$\sigma_y = \sigma_{y0} + K\epsilon^m$$

m = strain hardening exponent

$0.1 < m < 0.5$ typically.

Plasticity: assumptions

(1) plastic flow occurs at constant volume

$$\Delta V / V_0 = \epsilon_{11}^{pl} + \epsilon_{22}^{pl} + \epsilon_{33}^{pl} = 0$$

block demo

- plasticity associated with dislocation motion
- one block of material slips over another
- true for fully dense solids
- not true for cellular materials, granular materials (porous materials)

---

(2) modest hydrostatic pressures do not cause yielding (modest ~ $p < E/100$)

(3) neglect strain hardening - perfect plasticity

(4) material is isotropic

Yield criteria

- for multiaxial stress states, what combination of stress causes yield?
- various theories developed

  Rankine (1857)

  Tresca (1864, 1867)

  von Mises (1913)

Rankine: maximum stress criterion

- yield when maximum principal stress = σ_y. yield strength

  yield when $\boxed{\sigma_1 = \sigma_y}$

- but this predicts yield under hydrostatic stress - does not happen

Tresca: maximum shear stress criterion

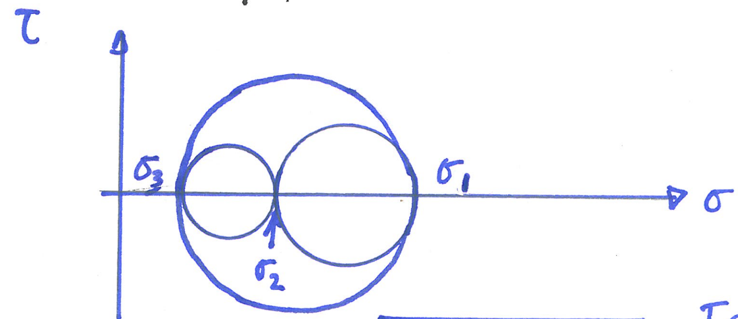
- yield when maximum shear stress in component equals maximum shear stress in uniaxial test at yield
- principal stresses: $\sigma_1 > \sigma_2 > \sigma_3$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Tresca
Eiffel tower

- in uniaxial test $\sigma_2 = \sigma_3 = 0$

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{\sigma_y}{2} \text{ @ yield}$$



yield when $\boxed{\sigma_1 - \sigma_3 = \sigma_y}$ Tresca criterion

---

von Mises: Maximum distortional energy criterion

- yield when equivalent stress equals yield stress in a uniaxial test

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + 3\sigma_{12}^2 + 3\sigma_{13}^2 + 3\sigma_{23}^2}$$

- yield when $\boxed{\sigma_{eq} = \sigma_y}$ von Mises criterion
- if use principal stresses, last 3 terms are zero
- can show this is equivalent to yield when distortional energy/vol (ie. shear) for general stress state equals distortional energy/vol in uniaxial test at yield

Example

- thin walled cylindrical pressure vessel $r = 0.5\,m$ $t = 0.01\,m$
- Al alloy $\sigma_y = 200$ MPa $p_{max} = ?$ to avoid yield (a) Rankine (b) Tresca (c) von Mises

$\sigma_1 = pr/t$ $\sigma_2 = pr/2t$ $\sigma_3 = 0$

(a) Rankine: yield when $\sigma_1 = pr/t = \sigma_y$; $p = \sigma_y\, t/r = 200\text{ MPa}\left(\frac{0.01}{0.5}\right) = 4\text{ MPa}$

(b) Tresca: $\sigma_1 - \sigma_3 = \sigma_y$ same as Rankine $p = 4$ MPa

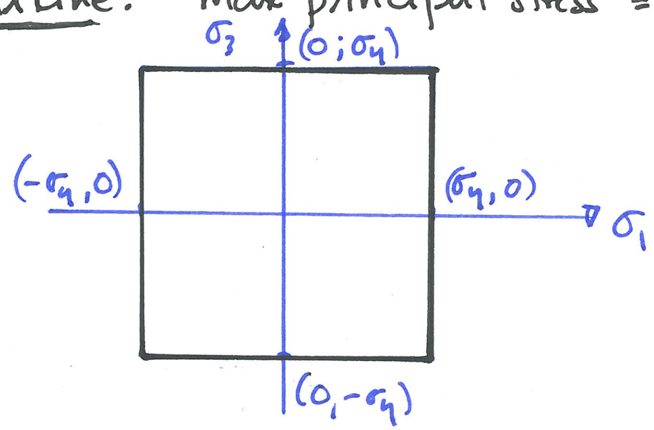
(c) von Mises: $\sigma_{eq} = \sqrt{\frac{1}{2}\left[\left(\frac{pr}{t} - \frac{pr}{2t}\right)^2 + \left(\frac{pr}{2t}\right)^2 + \left(\frac{pr}{t}\right)^2\right]} = \frac{\sqrt{3}}{2}\frac{pr}{t} = \sigma_y$ $p = \frac{200\,(.01)}{.5}\frac{2}{\sqrt{3}} = 4.61$ MPa

---

Comparison of Rankine, Tresca + von Mises criteria for plane stress

- consider principal stresses σ_1, σ_3 $\sigma_2 = 0$
- but here, σ_1 not necessarily greater than σ_3

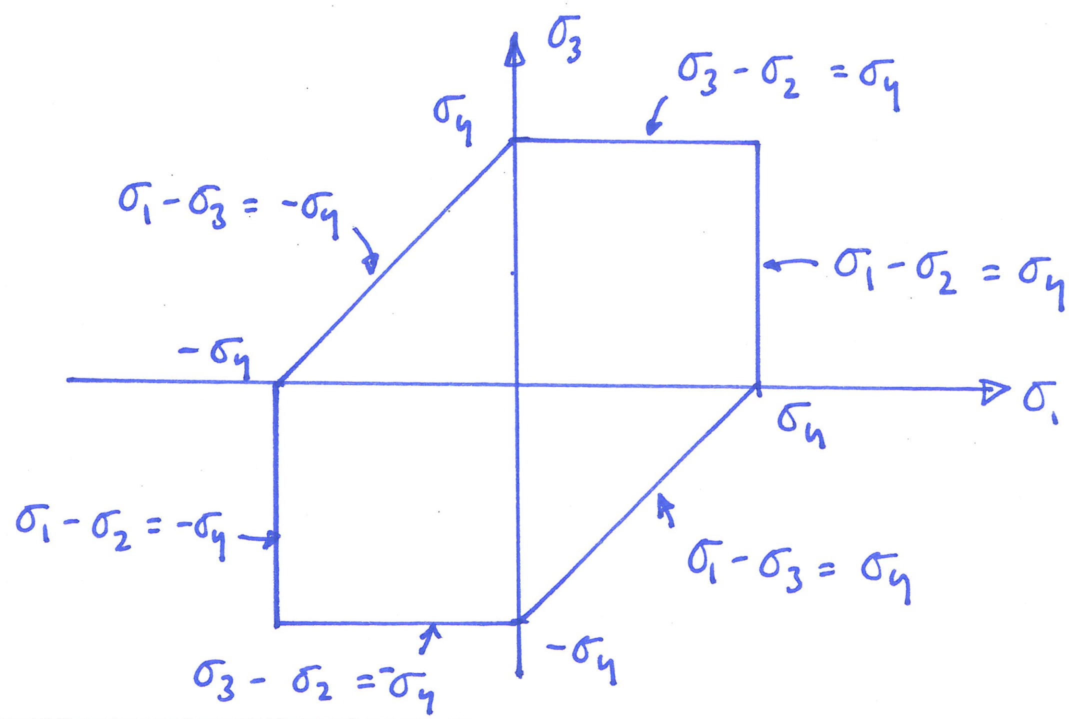
Rankine: max principal stress $= \sigma_y$



Tresca

$$\sigma_{max} - \sigma_{min} = \sigma_y$$

principal stresses



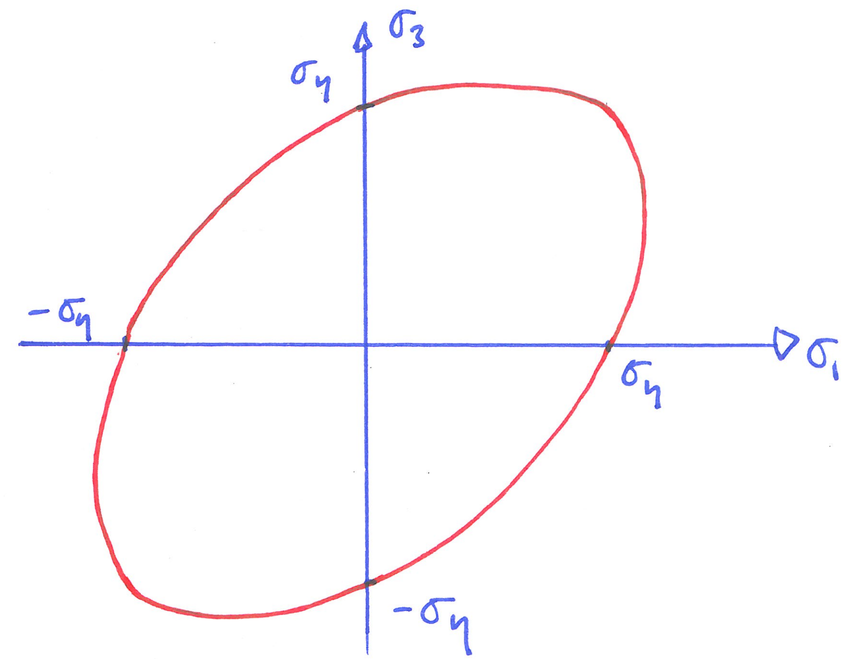
von Mises

$$\sigma_{eq}^2 = \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

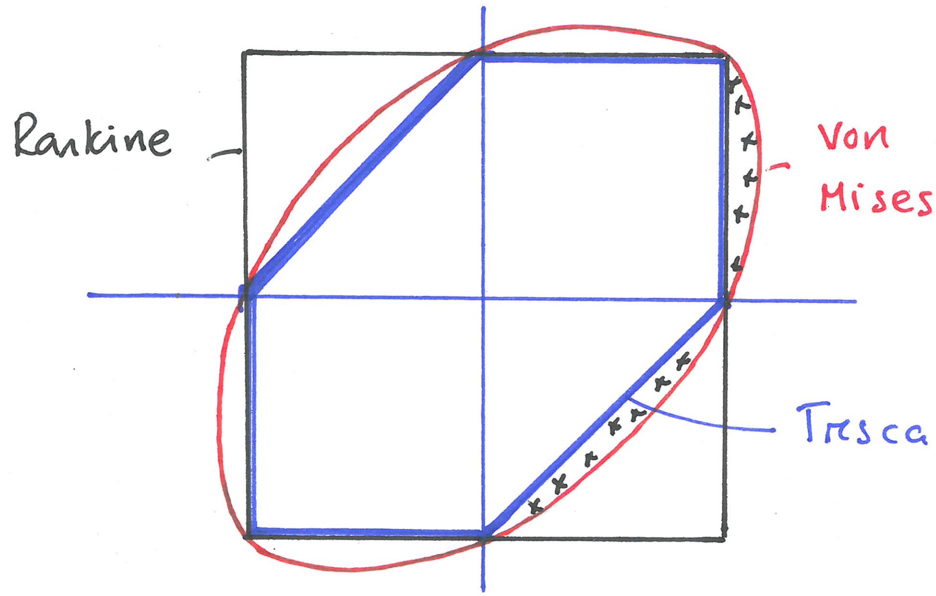
$$= \frac{1}{2}\left[\sigma_1^2 + \sigma_3^2 + \sigma_3^2 - 2\sigma_1\sigma_3 + \sigma_1^2\right]$$

$$\sigma_{eq}^2 = \sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 \Leftarrow \text{ellipse}$$

- states of stress inside envelope – elastic
- " " " on " – yield



Comparison of yield criteria



- data[(x)] lie between Tresca & von Mises
- closer to von Mises
- Rankine does not represent data
- max difference between Tresca + von Mises (for plane stress) is for pure shear ($\sigma_1 = -\sigma_3$)

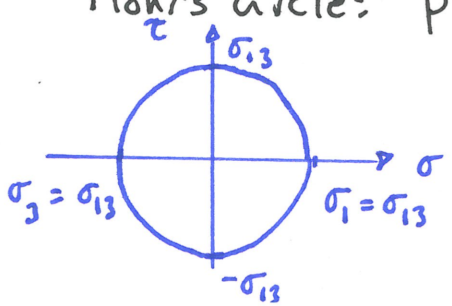
---

for pure shear:

Tresca $\sigma_{max} - \sigma_{min} = \sigma_{13} - (-\sigma_{13}) = \sigma_y \Rightarrow \sigma_{13} = \frac{\sigma_y}{2} = 0.5\,\sigma_y$.

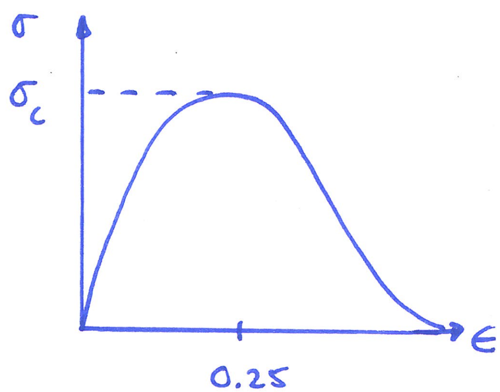
von Mises: $\sqrt{3}\,\sigma_{13} = \sigma_y \quad \sigma_{13} = \frac{\sigma_y}{\sqrt{3}} = 0.577\,\sigma_y$ (13% difference)

Mohr's circle: pure shear, σ_{13}



Plasticity: Microscopic aspects: Theoretical cohesive strength

- energy-separation curve can be differentiated to give force-separation curve (σ-ϵ)



σ_c = cohesive stress to pull atoms apart

occurs at $\epsilon \sim 0.25$

- idealize as $\sigma = \sigma_c \sin 2\pi\epsilon$
- at $\epsilon = 0$ $\left.\frac{d\sigma}{d\epsilon}\right|_{\epsilon=0} = 2\pi\sigma_c \cos 2\pi\epsilon = E$ or $\boxed{\sigma_c = E/2\pi}$

- similarly, in shear, $\tau_c = G/2\pi$
- more exact estimates give $\sigma_c \sim 0.05$ to $0.1\,E$ (using more realistic interatomic force curves)
- measured σ_y/E for typical metal alloys ~ 0.002 (high strength alloys ~ 0.007)
- much lower than theoretical cohesive strength
- plasticity results from dislocation motion
- yield stress = stress at which dislocations move through crystal lattice

1934
GI Taylor
E Orowan
M Polanyi

Dislocations: geometrical aspects

- dislocation is a line defect in the stacking of atoms in crystal lattice

Edge dislocation

- extra half plane of atoms in one part of crystal structure
- Burgers circuit
  - make clockwise circuit in lattice with dislocation
  - make same circuit (same # steps in each direction) in perfect lattice without dislocation - it does not close

---

  - vector from finish point to start = Burgers vector $\vec{b}$
  - also define vector parallel to dislocation line, t
  - edge dislocation: $\vec{b} \perp \vec{t}$

Screw dislocation

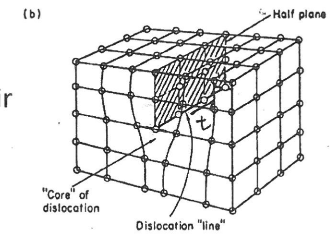
- two blocks of material sheared across partial cut through material
- make Burgers circuit same way
- $\vec{b} \parallel \vec{t}$

EDGE DISLOCATION

$t \parallel$ DISLOCATION LINE

$\vec{b} \perp \vec{t}$

Ashby MF and Jones DRH (1996)
Engineering Materials I: An Introduction to their Properties and Applications . Second Edition
Butterworth Heinemann.



Hull D and Bacon DJ (1984) An Introduction to Dislocations
3rd Edition Butterworth Heinemann

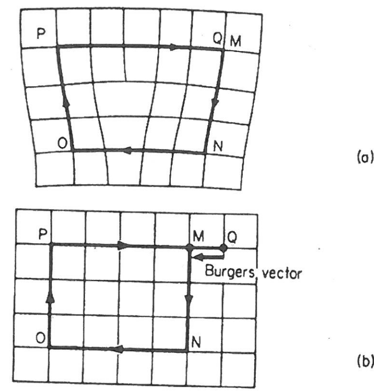


FIG. 1.19. (a) Burgers circuit round an edge dislocation, (b) the same circuit in a perfect crystal; the closure failure is the Burgers vector.

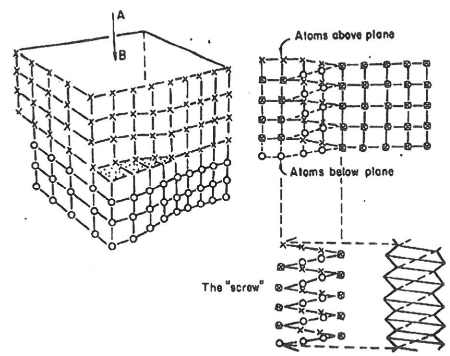
CW CIRCUIT

MNOPQ

POSITIVE EDGE
(HALF PLANE ATOMS ABOVE SLIP PLANE)

NEGATIVE EDGE
(HALF PLANE ATOMS BELOW SLIP PLANE)

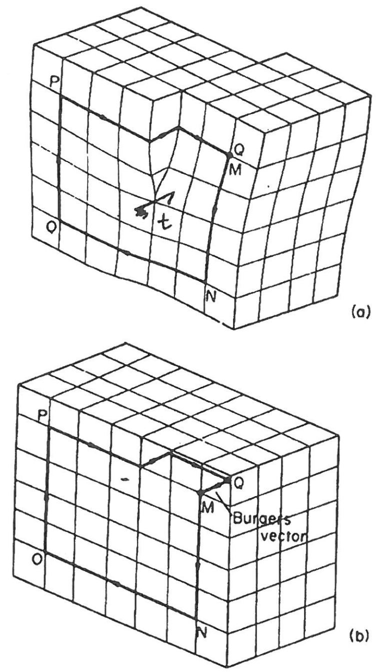
SCREW DISLOCATION



t ∥ DISLOC LINE

$\vec{b} \parallel \vec{t}$

Ashby MF and Jones DRH (1996)
Engineering Materials I: An Introduction to their Properties and Applications . Second Edition
Butterworth Heinemann.



Hull D and Bacon DJ (1984) An Introduction to Dislocations #rd Edition Butterworth Heinemann

FIG. 1.20. (a) Burgers circuit round a screw dislocation; (b) the same circuit in a perfect crystal; the closure failure is the Burgers vector.

Dislocations: geometrical aspects

- most dislocations mixed: part edge, part screw
- Burgers vector of dislocation can't change
- dislocation line cannot terminate within a crystal
- dislocation line can only terminate at a surface of a crystal (eg. free surface or grain boundary)
- dislocations can form closed loops

Dislocation motion

- two types of dislocation motion: glide + climb

Glide

- dislocation moves in the surface that contains both its line + Burgers vector
- results in slip - sliding of one plane of atoms over another on slip planes
- under shear stress, bonds sequentially break + reform so that dislocation moves through the lattice until it leaves at the surface
- net result is slip step of one atomic distance
- slip planes - most densely packed planes
- slip direction - direction in the slip plane in which atoms are most closely packed (i.e. in direction of closest packing)

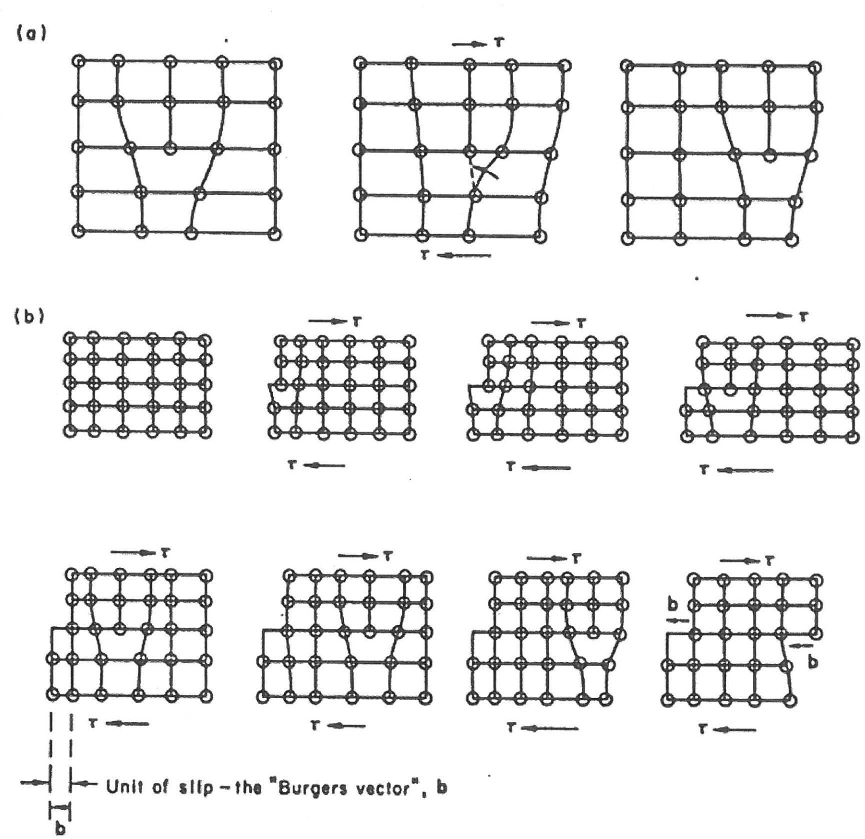
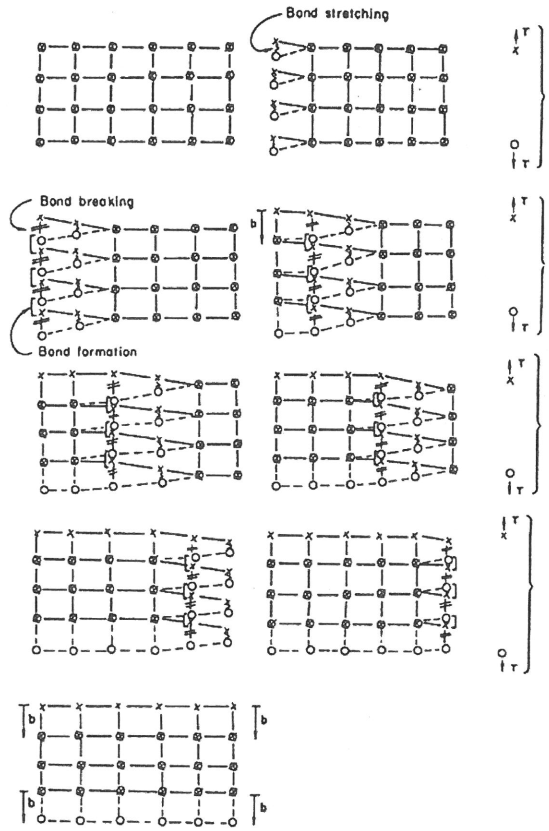


Fig. 9.4. How an edge dislocation moves through a crystal. (a) Shows how the atomic bonds the dislocation break and reform to allow the dislocation to move. (b) Shows a complete s introduction of a dislocation into a crystal from the left-hand side, its migration through th expulsion on the right-hand side; this process causes the lower half of the crystal to slip under the upper half.

Ashby MF and Jones DRH (1996)
Engineering Materials I: An Introduction to their Properties and Applications . Second Edition Butterworth Heinemann.



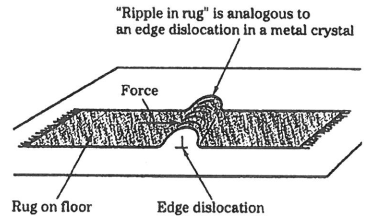
X = UPPER ATOMS

O = LOWER ATOMS

Fig. 9.8. Sequence showing how a screw dislocation moves through a crystal causing the lower half of the crystal (o) to slip by a distance b under the upper half (x).

Ashby MF and Jones DRH (1996)
Engineering Materials I: An Introduction to their Properties and Applications . Second Edition Butterworth Heinemann.

"CARPET RUCK"
ANALOGY.



Smith

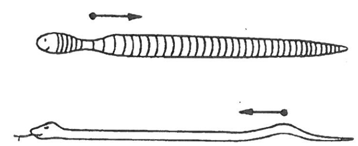


FIG. 4.6. Movement of an earthworm (above) and of a snake (below) by dislocations. (After Orowan, 1954.)

"WORM" ANALOGY

McClintock & Argon (1966)
Addison Wesley

McClintock F and Argon AS (1966) Mechanical Behavior of Materials. Addison Wesley

Dislocation motion

Climb

- dislocation moves out of glide surface
- requires diffusion - thermally activated process
- discuss in section on high temp creep in metals + ceramics

Kinks + Jogs

- If a section of dislocation glides on the slip plane, it acquires kinks
- if a section of dislocation climbs out of the slip plane & onto another, parallel slip plane, it acquires jogs

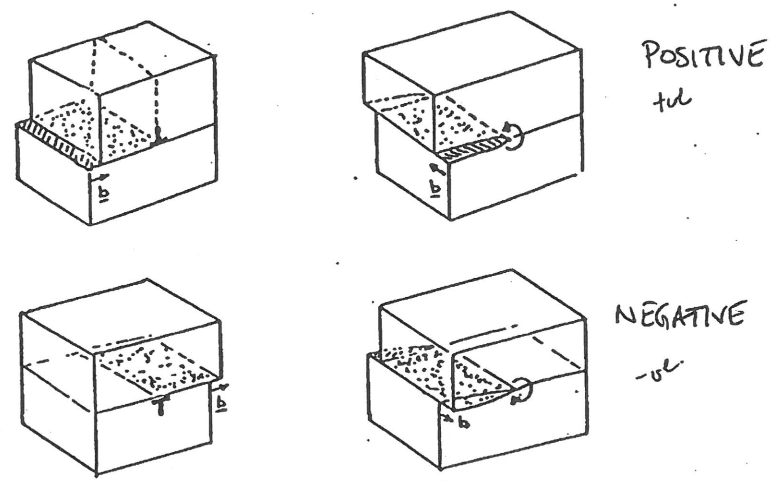
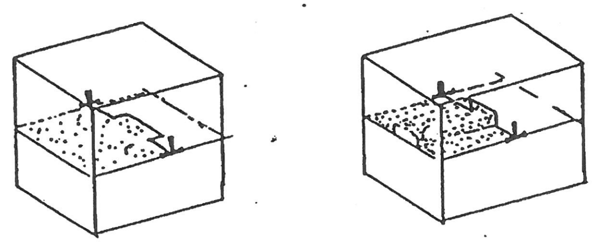


FIG 7.1 POSITIVE AND NEGATIVE EDGES AND SCREWS

Kink

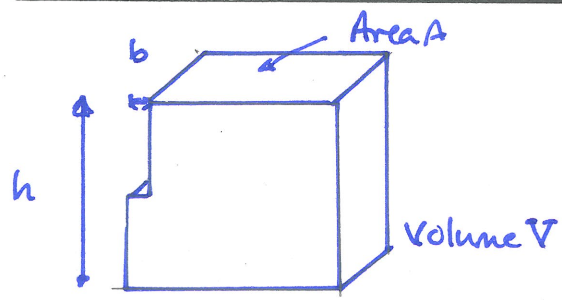


Jog

Ashby notes

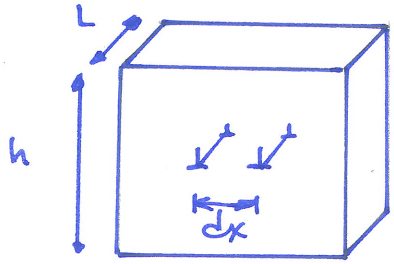
FIG 7.2 KINKS AND JOGS

Dislocation motion: Plastic shear strain



• If edge dislocation moves entirely through crystal on its slip plane

$$\gamma^P = \frac{b}{h} = \frac{bA}{V}$$



• if it moves incrementally by dx

$$d\gamma^P = b\,\frac{dA}{V} = \frac{bL\,dx}{V}$$

• if n dislocations $d\gamma^P = \dfrac{nbL\,dx}{V}$

• ρ = dislocation density = line length/volume = $\dfrac{nL}{V}$ $[L^{-2}]$

$$d\gamma^P = b\rho\,dx$$

$$\frac{d\gamma^P}{dt} = b\rho\,\frac{dx}{dt}$$

$$\boxed{\dot{\gamma}^P = b\rho v}$$

Orowan's equation

$\dot{\gamma}^P$ = plastic shear strain rate

b = Burgers vector

ρ = dislocation density → annealed $\rho \sim 10^5/cm^2$ → highly deformed metal $\rho \sim 10^{11}/cm^2$

v = velocity of dislocation

Egon Orowan

From Wikipedia, the free encyclopedia

Egon Orowan FRS[1] (Hungarian: ***Orován Egon***) (August 2, 1902 – August 3, 1989) was a Hungarian/British/U.S. physicist and metallurgist.

Contents

Life

Orowan was born in the Óbuda district of Budapest. His father, Berthold, was a mechanical engineer and factory manager, and his mother, Josze Spitzer Ságvári was the daughter of an impoverished land owner. In 1928, Orowan commenced his education at the Technical University of Berlin in mechanical and electrical engineering but soon transferred to physics, completing his doctorate on the fracture of mica in 1932. He seems to have experienced some difficulty in finding immediate employment and spent the next few years living with his mother and ruminating on his doctoral research.

In 1934, Orowan, roughly contemporarily with G. I. Taylor and Michael Polanyi, realized that the plastic deformation of ductile materials could be explained in terms of the theory of dislocations developed by Vito Volterra in 1905. Though the discovery was neglected until after World War II, it was critical in developing the modern science of solid mechanics.

After working for a short while on the extraction of krypton from the air for the manufacture of light bulbs, in 1937 Orowan moved to the University of Birmingham, UK where he worked on the theory of fatigue collaborating with Rudolf Peierls.

In 1939, he moved to the University of Cambridge where William Lawrence Bragg inspired his interest in x-ray diffraction. During World War II, he worked on problems of munitions production, particularly that of plastic flow during rolling. In 1944, he was central to the reappraisal of the causes of the tragic loss of many Liberty ships during the war, identifying the critical issues of the notch sensitivity of poor quality welds and the aggravating effects of the extreme low temperatures of the North Atlantic.

In 1950, he moved to the Massachusetts Institute of Technology where, in addition to continuing his metallurgical work, he developed his interests in geological and glacialogical fracture and in what he termed *socionomy*. In the latter study, Orowan developed the writings of the 14th century Tunisian historian Ibn Khaldun to forecast a supposed eventual failure of market demand similar to that claimed by Karl Marx. His ideas found little acceptance among the majority of economists.

Orowan - Ali Argon's PhD Thesis supervisor (1956)