Introduction.

- this class covers both mechanics + mechanical behaviour of materials
- mechanics - response of components to loading

  deformation ↙ ↘ failure load

- response depends on:

  (a) internal forces + moments generated by external forces + moments

  (b) material properties e.g. stiffness, strength

  (c) component geometry

- mechanical behaviour of materials
  - on atomic level, what controls mechanical properties?

    e.g. stiffness - binding energy

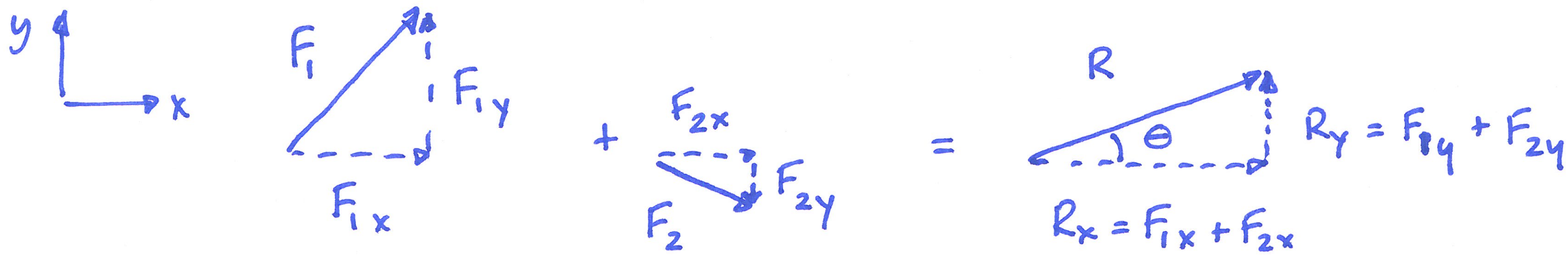
    strength - defects - ductile metals - dislocations

    brittle glass - cracks

Review: Forces

Force - vector - magnitude + direction

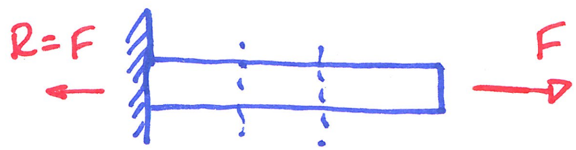
- add by vector addition



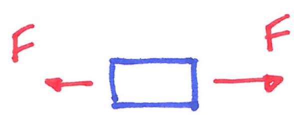
R : magnitude $= \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}$ direction $\theta = \arctan \dfrac{F_{1y} + F_{2y}}{F_{1x} + F_{2x}}$

$R_x = \sum_i F_{ix}$ $R_y = \sum_i F_{iy}$

External + internal forces



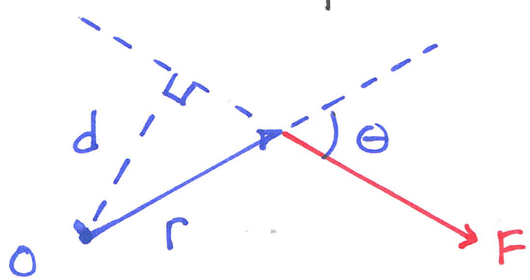
external force: action of other bodies on member



internal force: within a member

Review: Moments

- tendency of a force to cause rotation about a point or line
- Moment = cross product of vector, r, from the point (or line) to the point of application of the force, F, with the force



$\bar{M} = \bar{r} \times \bar{F}$ CROSS-PRODUCT USEFUL FOR 3D

Moment is a vector.

- 2D: Magnitude of moment = (magnitude of force) × perpendicular distance between point O (about which moment is taken) + line of action of force F

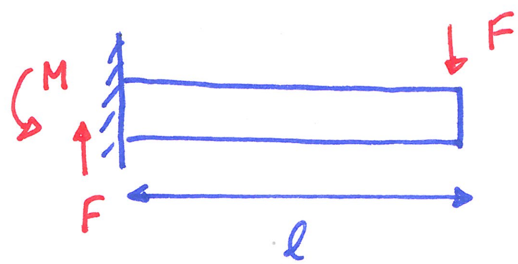
$$|M| = |Fd| = rF\sin\theta$$ in above diagram.

direction: right hand rule: thumb on point O, fingers curl in sense of rotation

CCW + (thumb out) CW − (thumb into board/page)

Review: Free Body Diagram

- diagram showing point or body with all external forces + moments, including reactions.
- include dimensions, angles



Equilibrium

Static equilibrium: $\Sigma F = 0 \quad \Sigma M = 0$

Components: $\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$

2D: $\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_o = 0$

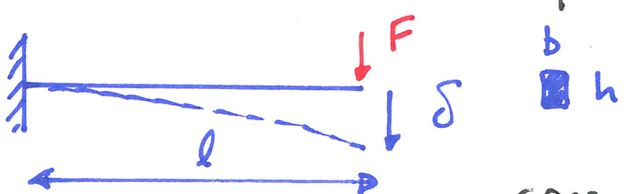
if $\Sigma F \neq 0 \Rightarrow F = ma \Rightarrow$ translation

if $\Sigma M \neq 0 \Rightarrow M = m \alpha r \Rightarrow$ rotation

Review: Units - SI

mass: kg   length: m   time: s   force: N.

- check units on equations!



beam material   cross-section

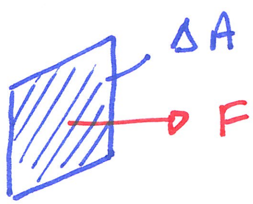
Young's modulus, $E\ [N/m^2]$

$$\delta = \frac{4 P l^3}{E b h^3}$$

$$[m] = \frac{[N]\,[m^3]}{\left[\frac{N}{m^2}\right][m]\,[m^3]} = [m] \checkmark$$

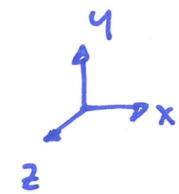
Stress

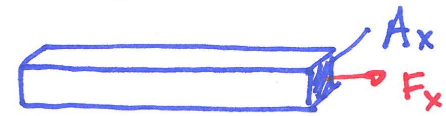
- Stress = force per unit area
- Normal stress, σ = force perpendicular to the area, per unit area
- Holds true at a point



$$\sigma = \lim_{\Delta A \to 0} \frac{F}{\Delta A}$$

- Can average stress over an area:





$$\sigma_{xx} = \frac{F_x}{A_x}$$ (Galileo, 1638)

A_x ← normal to area in x direction

- Sign convention: tensile stress positive
- units: $\frac{N}{m^2}$ = Pascal, Pa

Pascal slides

- usually MPa $= 10^6$ Pa $= 145$ psi

  GPa $= 10^9$ Pa

- engineering stress $\sigma = \frac{F}{A_0}$ ← initial area

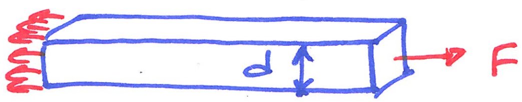
Normal stress

- to have uniform normal stress in a bar, the load F must act through the centroid of the bar's cross-section

$$\text{centroid } (\bar{x}, \bar{y}) = \left(\frac{\int x \, dA}{\int dA}, \frac{\int y \, dA}{\int dA} \right)$$

- at end of bar, near point of application of load, stress will not be uniform unless load applied evenly over area A

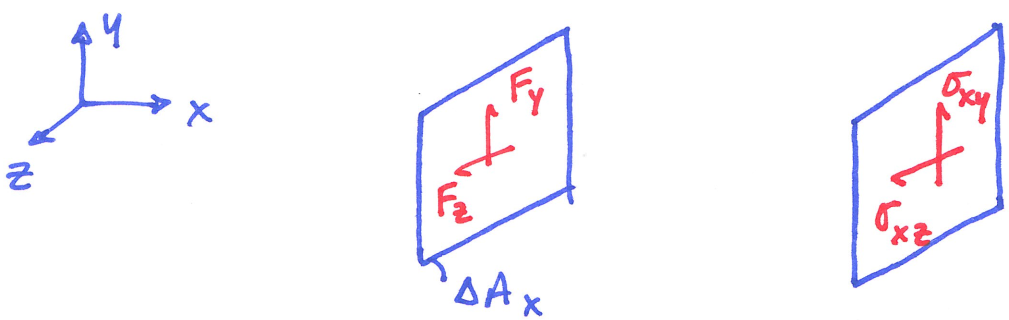
$$\sigma = \frac{F}{A}$$



- stress becomes uniform d from end (St. Venant's Principle)

Shear Stress

- shear stress = force parallel to area, per unit area, at a point



$$\sigma_{xy} = \lim_{\Delta A_x \to 0} \frac{F_y}{\Delta A_x}$$

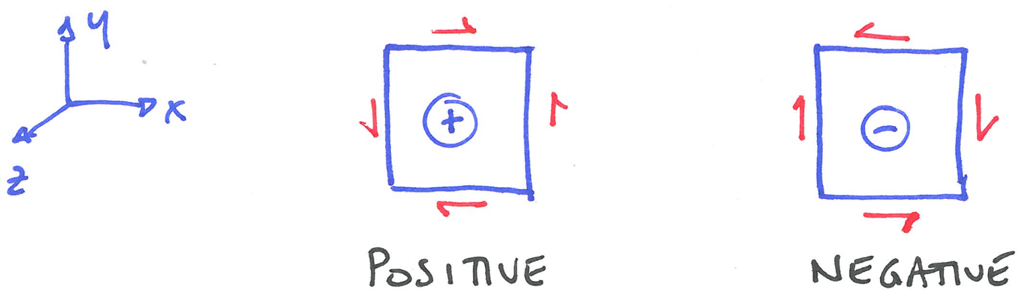
$$\sigma_{xz} = \lim_{\Delta A_x \to 0} \frac{F_z}{\Delta A_x}$$

OR, averaging over area, $\sigma_{xy} = \dfrac{F_y}{A_x}$, $\sigma_{xz} = \dfrac{F_z}{A_x}$ [Pa]

Shear - sign convention

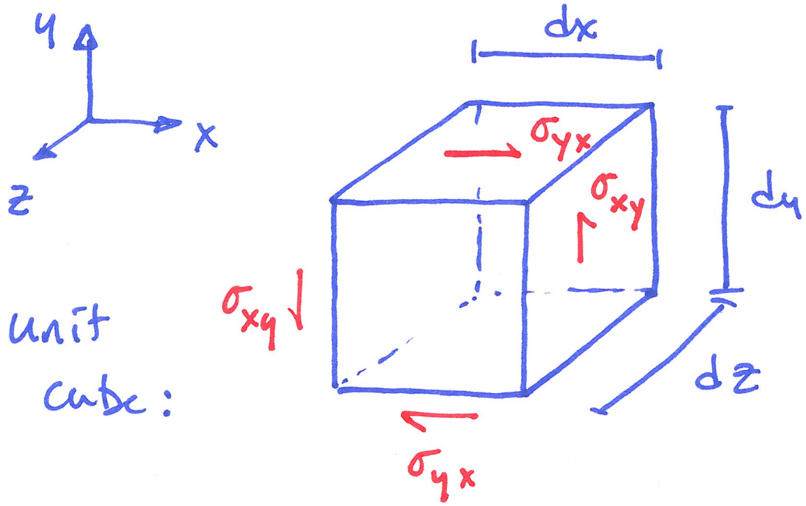
- define positive faces of element as those with normals in positive x y z directions

- positive shear acts in positive direction on positive face
  OR " " negative " " negative "



Shear Stress in a Plane

unit cube:



For equilibrium, shear forces appear in pairs, or couples ($\Sigma F = 0$)

also, for equilibrium $\Sigma M_o = 0$ ↻+

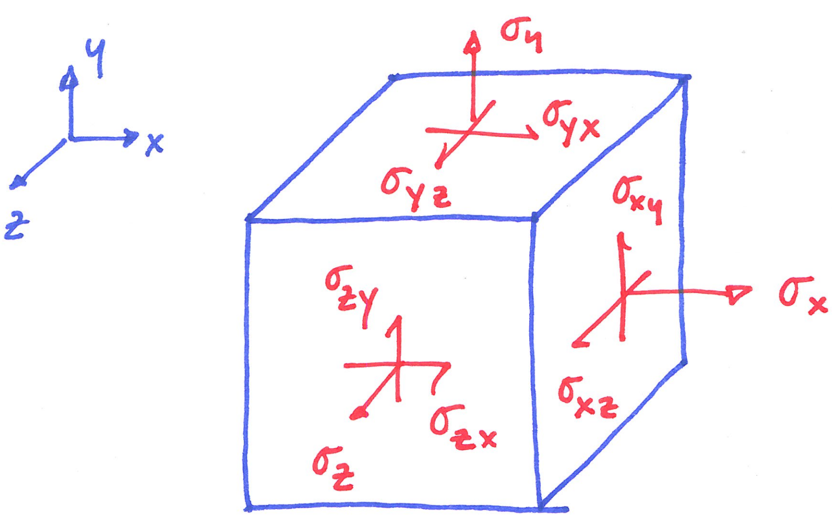
$$(\sigma_{xy})\, dy\, dz\, dx - \sigma_{yx}\, dx\, dz\, dy = 0$$

$$\Rightarrow \sigma_{xy} = \sigma_{yx}$$

similarly $\sigma_{xz} = \sigma_{zx}$

$\sigma_{yz} = \sigma_{zy}$

General stress state



(+ stresses on opposite faces)

9 components: 3 normal 6 shear.

write as matrix:

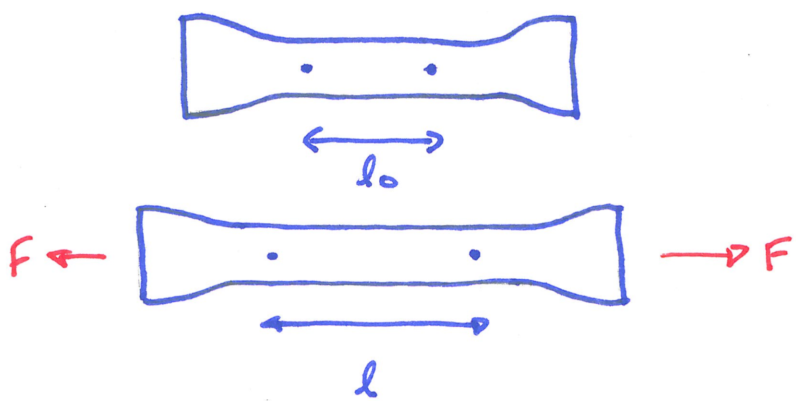
$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad \underline{\text{OR}} \quad \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

STRESS MATRIX SYMMETRIC

3 NORMAL, 3 SHEAR STRESSES

Normal strain, ϵ

- bar under axial load changes length
- normal strain is the amount of deformation, δ, per unit original length, ℓ_0



normal strain, $\epsilon = \dfrac{\ell - \ell_0}{\ell_0} = \dfrac{\delta}{\ell_0}$

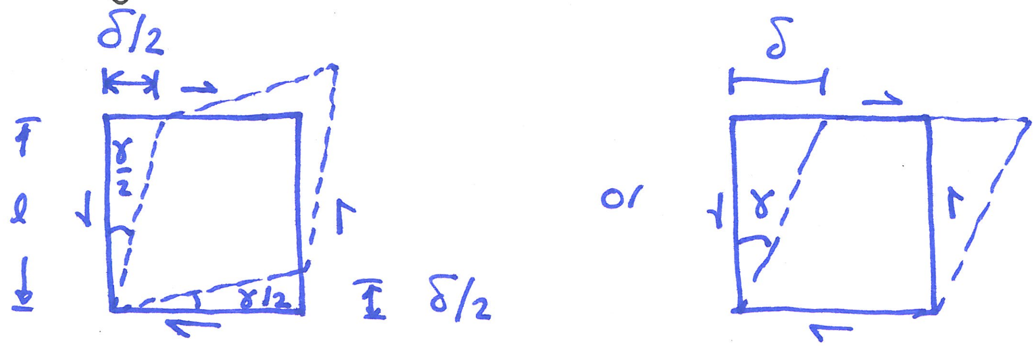
units: [-] or [mm/mm] or %

typical engineering strains of order 0.1%

eg. metals deform permanently at $\epsilon \sim 0.002$

Shear strain, γ

- shear stresses produce shear deformation + shear strain
- lengths of sides of element do not change, but simply rotate



angle of rotation gives shear strain

$\tan\gamma = \delta/\ell$

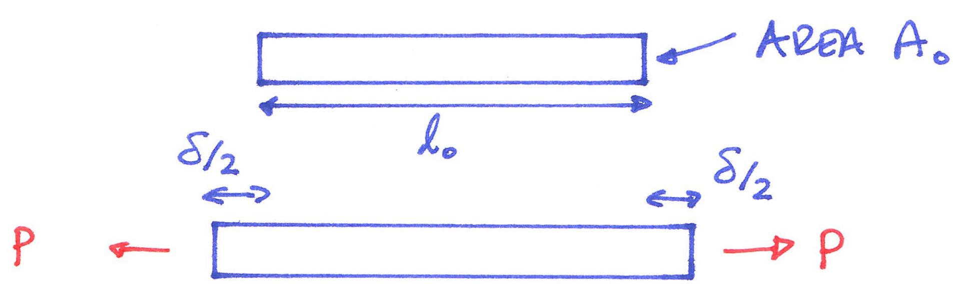
small γ, $\tan\gamma = \gamma$

$\gamma = \dfrac{\delta}{\ell}$ [-] $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$

3 INDEP. SHEAR STRAINS

Hooke's law for isotropic materials

- Hooke measured elongation, Δx, of springs under load, F
- Found: $$F = \underbrace{k}_{\text{spring constant}} \Delta x$$
- more generally, if a bar is made of a linear elastic, isotropic material + is loaded uniaxially:



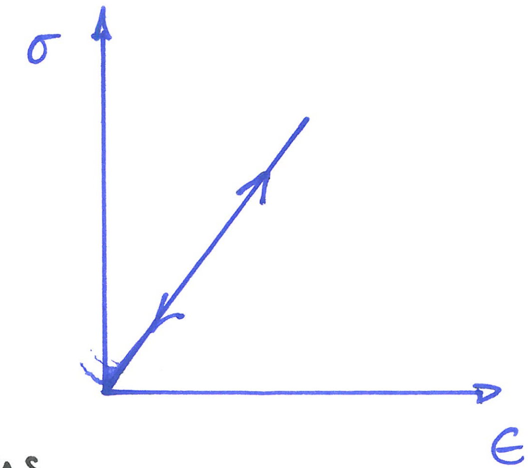
$$\frac{P}{A_0} = E \frac{\delta}{l_0}$$

E = Young's modulus [Pa]

$$\boxed{\sigma = E \epsilon}$$

Hooke's law (Hookean material)

- linear ⇒ $P \propto \delta$
- elastic ⇒ recover deformation on unloading
- many materials are linear elastic at small strains ($\epsilon < 0.002$)



- isotropic ⇒ properties are the same in all directions

  e.g. polycrystalline metal with randomly oriented grains
- anisotropic ⇒ properties different in different directions (e.g. wood - along/across grain)

Typical values of E (GPa)

diamond	1000 GPa
alumina	390
steel	200
aluminum	69
glass	69
polyethylene	0.2 - 0.7
rubbers	0.01 - 0.1

- Young's modulus data
  Ashby plot
- Robert Hooke - slide
  - video