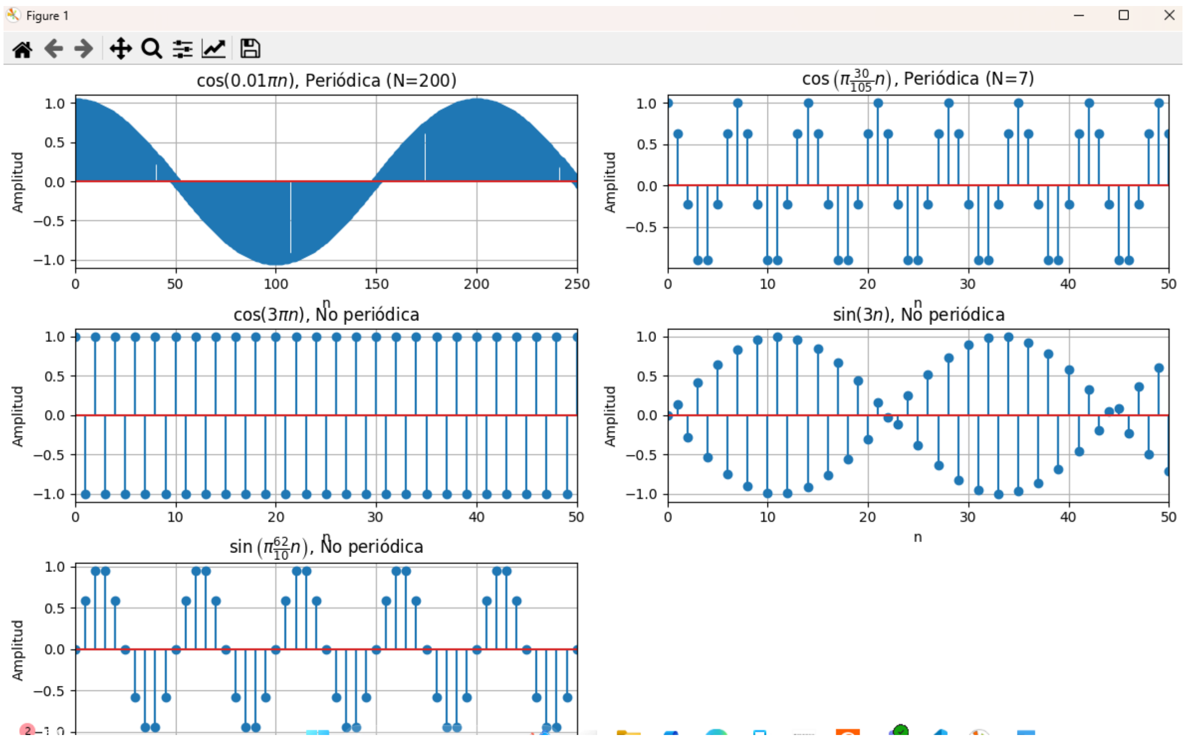
1.2 Determine which of the following sinusoids are periodic and compute their fundamental period.

(a) $\cos 0.01\pi n$ **(b)** $\cos\left(\pi \frac{30n}{105}\right)$ **(c)** $\cos 3\pi n$ **(d)** $\sin 3n$ **(e)** $\sin\left(\pi \frac{62n}{10}\right)$



1.3 Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

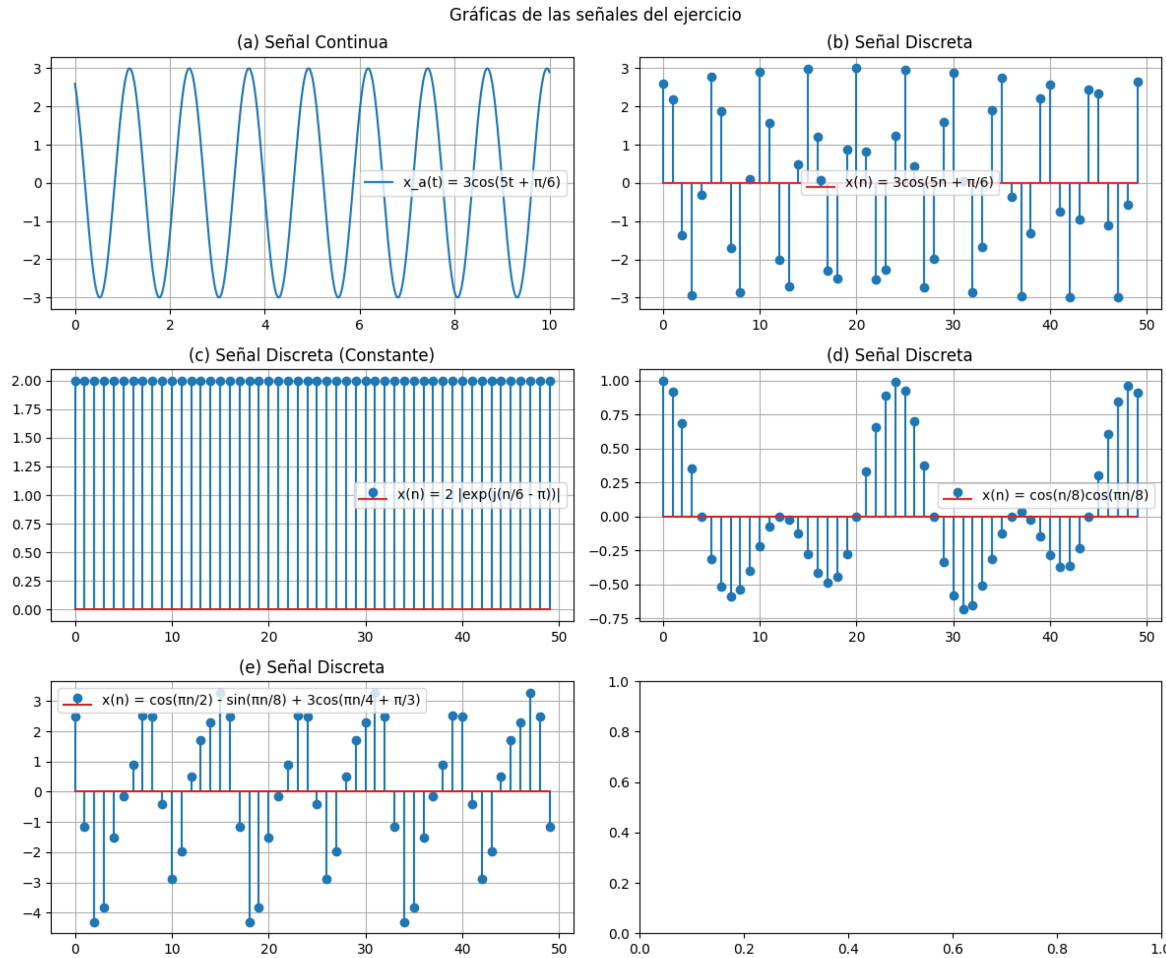
(a) $x_a(t) = 3\cos(5t + \pi/6)$

(b) $x(n) = 3\cos(5n + \pi/6)$

(c) $x(n) = 2\exp[j(n/6 - \pi)]$

(d) $x(n) = \cos(n/8)\cos(\pi n/8)$

(e) $x(n) = \cos(\pi n/2) - \sin(\pi n/8) + 3\cos(\pi n/4 + \pi/3)$



1.5 Consider the following analog sinusoidal signal:

$$x_a(t) = 3\sin(100\pi t)$$

(a) Sketch the signal $x_a(t)$ for $0 \leq t \leq 30$ ms.

(b) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = 1/F_s$, and show that it is periodic.

(c) Compute the sample values in one period of $x(n)$. Sketch $x(n)$ on the same diagram with $x_a(t)$. What is the period of the discrete-time signal in milliseconds?

(d) Can you find a sampling rate F_s such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?

1.5 $Xa(t) = 3Sin(100\pi t)$

Señal $Xa(t)$ Para $0 \leq t \leq 30$ ms

Amplitud = 3

Freq Angular $w = 100$

Frecuencia $= \frac{100}{2} = 50$

Periodo $= \frac{1}{50} = 0.02 = 20$ ms

Frecuencia de la señal muestreada

$Fs = 300$

$Ts = \frac{1}{Fs} = \frac{1}{300} = 3.33$ ms

$Xa(t) = 3Sen\left(\frac{100}{300}\pi t\right) = 3Sen\left(\frac{\pi}{3}t\right)$

Valores de $X(n)$ en el tiempo

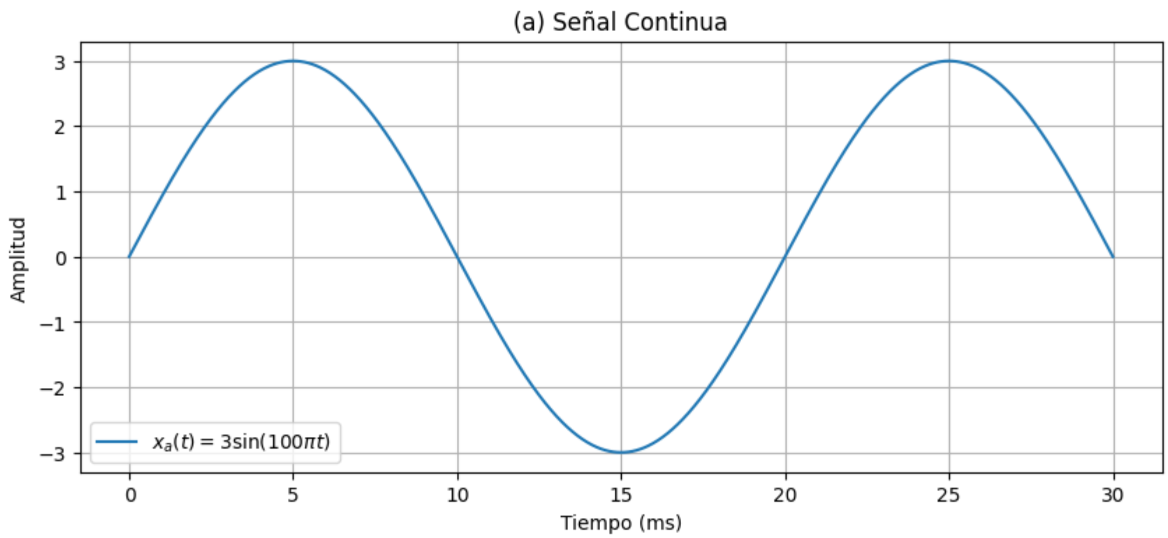
$0 = 3Sin(0) = 0$

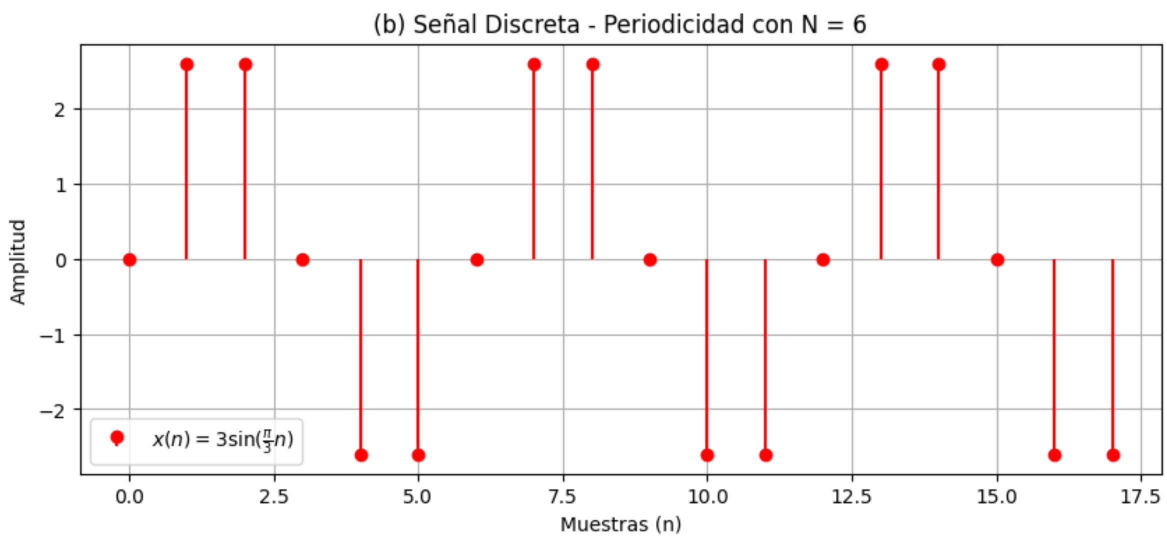
$1 = 3Sin(\pi/3) = 3(0.86) \approx 2.6$ ms

$2 = 3Sin(2\pi/3) = 3(0.86) \approx 2.66$ ms

$3 = 3Sen(3\pi/3) = 0$

$4 = 3Sen(4\pi/3) = 3(-0.86) = -2.66$





... number of periods of $x_a(t)$.

1.7 An analog signal contains frequencies up to 10 kHz.

(a) What range of sampling frequencies allows exact reconstruction of this signal from its samples?

(b) Suppose that we sample this signal with a sampling frequency $F_s = 8$ kHz. Examine what happens to the frequency $F_1 = 5$ kHz.

(c) Repeat part (b) for a frequency $F_2 = 9$ kHz.

1.7 frequencies up to 10kHz

a) Rango de frecuencias de muestreo

Nyquist
$F_s \geq 2 F_{s max}$

$10\,kHz = F_s \qquad 10(2) = 20\,kHz$

b) Muestreo con 8kHz

Limite de Nyq
$\frac{F}{2} = \frac{8k}{2} = 4\,kHz$

¿Qué pasa con 5kHz?
Sufre alising pues:
$Al = |F_i - kF_s| \quad k=1$
$5 - 1(8) = 5 - 8 = 3\,kHz$

Muestreo con 9k

Nyq =
$\frac{9}{2} = 4.5\,kHz$

$Al = 9 - (1)(9) = 0$

Con 9Hz se percibiría como 0 (como una DC)

1.8 An analog electrocardiogram (ECG) signal contains useful frequencies up to 100 Hz.

(a) What is the Nyquist rate for this signal?

(b) Suppose that we sample this signal at a rate of 250 samples/s. What is the highest frequency that can be represented uniquely at this sampling rate?

1.8 ECG contiene frecuencias de hasta 100 Hz

a) Nyquist

$F_s \geq 2F_s$

$100(2) = 200\ Hz$

b) frecuencia máxima con 250 muestras

$F_{max} = \frac{250}{2} = 125\ Hz$

1.9 An analog signal $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled 600 times per second.

(a) Determine the Nyquist sampling rate for $x_a(t)$.

(b) Determine the folding frequency.

(c) What are the frequencies, in radians, in the resulting discrete time signal $x(n)$?

(d) If $x(n)$ is passed through an ideal D/A converter, what is the reconstructed signal $y_a(t)$?

1.9 $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled 600 times per second.

a) Nyquist Muestreo

$x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$

$F_1 = \frac{480\cancel{\pi}}{2\cancel{\pi}} = 240\ Hz$

$F_2 = \frac{720\cancel{\pi}}{2\cancel{\pi}} = 360\ Hz$

b) Determinar la frecuencia folding

$F_N = \frac{F_s}{2}$ $\quad F_N = \frac{600}{2} = 300\ Hz$

c) freq en radianes

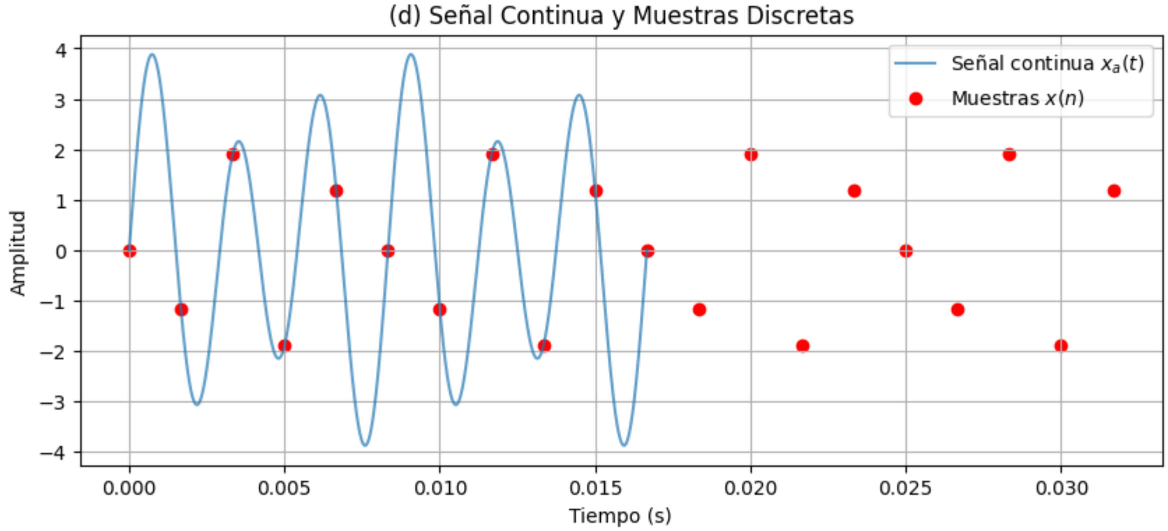
$\omega = 2\pi \frac{F}{F_s}$

$\omega_2 = 2\pi \frac{360}{600} = \underline{1.2\pi\ rad}$

$F_1 = 240\ Hz$

$\omega_1 = 2\pi \frac{240}{600} = 2\pi \times .4 = 0.8\pi\ rad$

(d) Señal Continua y Muestras Discretas



1.10 A digital communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

(a) What are the sampling frequency and the folding frequency?

(b) What is the Nyquist rate for the signal $x_a(t)$?

(c) What are the frequencies in the resulting discrete-time signal $x(n)$?

(d) What is the resolution Δ?

1.10 $Xa(t) = 3\cos 600\pi t + 2\cos 1800\pi t$

- El sistema opera a 10,000 bits
- Cada muestra en 1024 niveles (2^{10})

a) Determinar la frecuencia de muestreo y folding frecuency?

$$F_s = \frac{\text{Tasa bit}}{\text{Bits muestra}} = \frac{10{,}000}{10} = 1{,}000\ Hz$$

$$\text{Folding frec} = F_N = \frac{F_s}{2} = \frac{1000}{2} = 500\ Hz$$

b) Determinar la frecuencia de Nyquist para la señal $Xa(t)$

$$F_1 = \frac{600\pi}{2\pi} = 300\ Hz$$

$$F_2 = \frac{1800\pi}{2\pi} = 900\ Hz$$

c) frecuencia en señal discreta

$$w = 2\pi \frac{F}{F_s}$$

$$F_1 = 2\pi \frac{300}{1000} = 0.6\pi\ rad$$

$$F_2 = 2\pi \frac{900}{100} = 1.8\pi\ rad$$

d) Resolución

$$\Delta = \frac{2(5)}{1024} = \frac{10}{1024} = 0.0098\ \text{uni.}$$

Scribe

1.11 Consider the simple signal processing system shown in Fig. P1.11. The sampling periods of the A/D and D/A converters are $T = 5$ ms and $T' = 1$ ms, respectively. Determine the output $y_a(t)$ of the system, if the input is

$$x_a(t) = 3\cos 100\pi t + 2\sin 250\pi t \qquad (t \text{ in seconds})$$

The postfilter removes any frequency component above $F_s/2$.

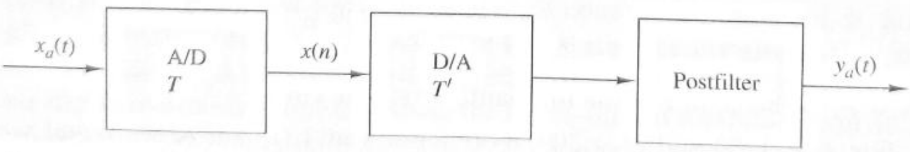


Figure P1.11

1.11 $Xa(t) = 3Cos\, 100\pi t + 2Sen\, 250\pi t$

$T = 5ms$

$FS = \frac{1}{T} = \frac{1}{5ms} = 200\ HZ$

$T = 1$

$FS = 1/T = 1/1ms = 1000\ HZ$

Alising

$F_1 = \frac{100\pi}{2\pi} = 500\ HZ \qquad F_2 = \frac{250}{2} = 125\ Hz$

1.15 *Sampling of sinusoidal signals: aliasing* Consider the following continuous-time sinusoidal signal

$$x_a(t) = \sin 2\pi F_0 t, \qquad -\infty < t < \infty$$

Since $x_a(t)$ is described mathematically, its sampled version can be described by values every T seconds. The sampled signal is described by the formula

$$x(n) = x_a(nT) = \sin 2\pi \frac{F_0}{F_s} n, \qquad -\infty < n < \infty$$

where $F_s = 1/T$ is the sampling frequency.

(a) Plot the signal $x(n)$, $0 \le n \le 99$ for $F_s = 5$ kHz and $F_0 = 0.5, 2, 3,$ and 4.5 kHz. Explain the similarities and differences among the various plots.

(b) Suppose that $F_0 = 2$ kHz and $F_s = 50$ kHz.

1. Plot the signal $x(n)$. What is the frequency f_0 of the signal $x(n)$?
2. Plot the signal $y(n)$ created by taking the even-numbered samples of $x(n)$. Is this a si ... 4 / 4 ... frequency?

1.15 $\quad x_a(t) = \sin 2\pi F_0 t$

$F_s = \frac{1}{T} \qquad x(n) = x_a(nT)\ \text{Sen}\left(2\pi \frac{F_0}{F_s} n\right)$

$F_s = 5\ KHZ$

$F_{señal} = 0.5, 2, 3, 4.5\ KHz$

$f_0 = \frac{F_0}{F_s} = \left\{\frac{0.5}{5}\ \frac{2}{5}\ \frac{3}{5}\ \frac{4.5}{5}\right\} = 0.1, 0.4, 0.6, 0.9$

Discreta

$F_0 = |F_s - F_0| = |5 - 4.5| = 0.5\ HZ$

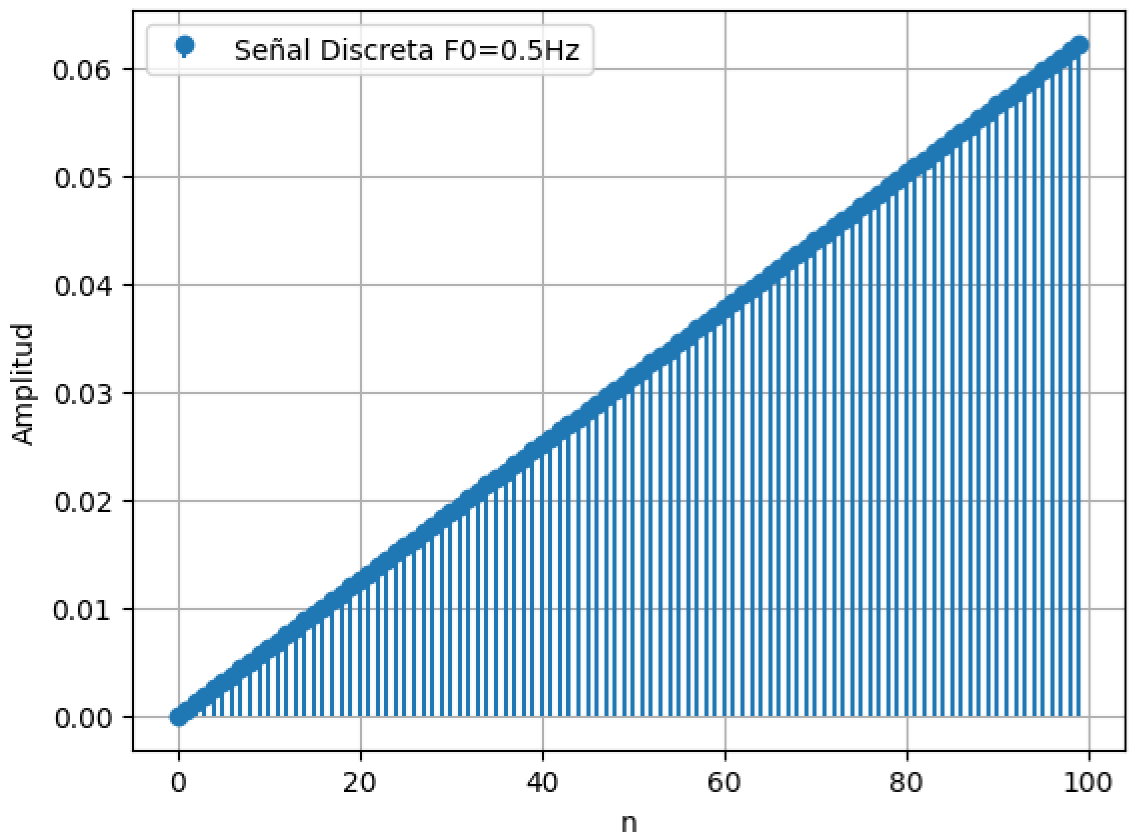
b) Análisis con $F_0 = 2\ kHz \quad F_s = 50\ kHz$

$F_s = 50\ KHZ$

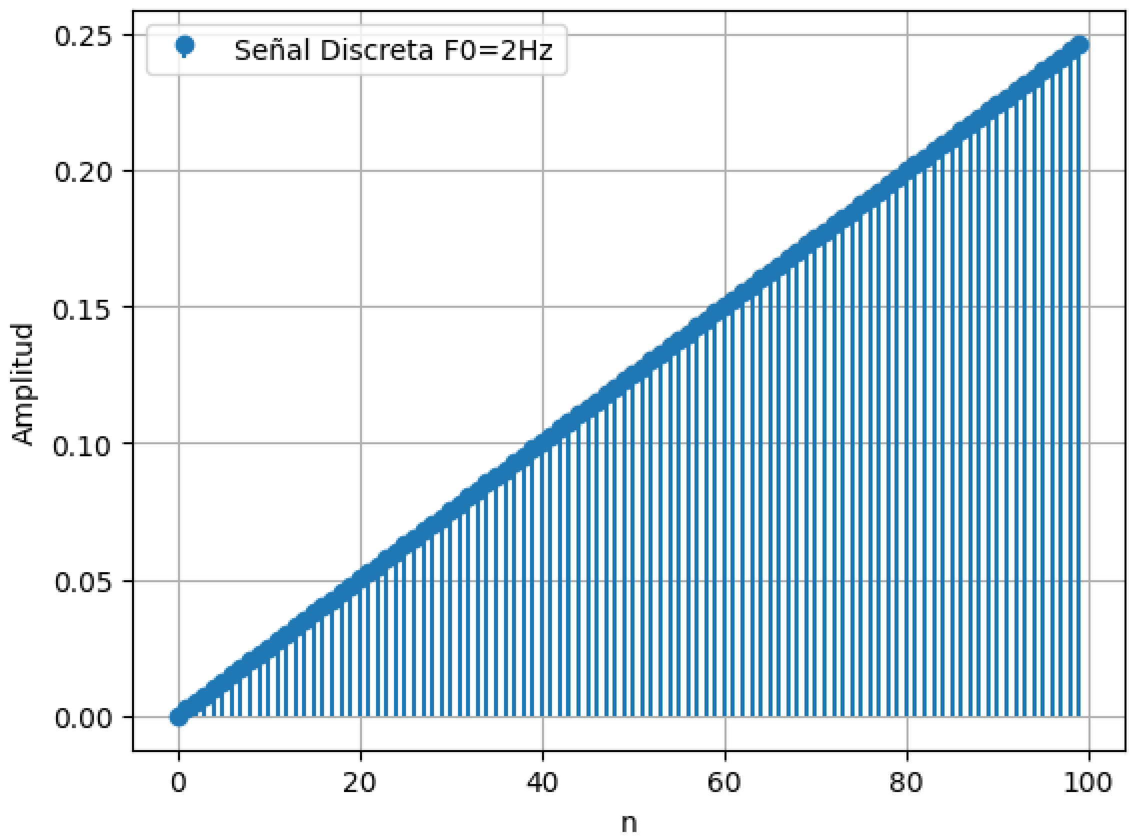
$f_0 = \frac{F_0}{F_s} = \frac{2}{50} = 0.04\ HZ$

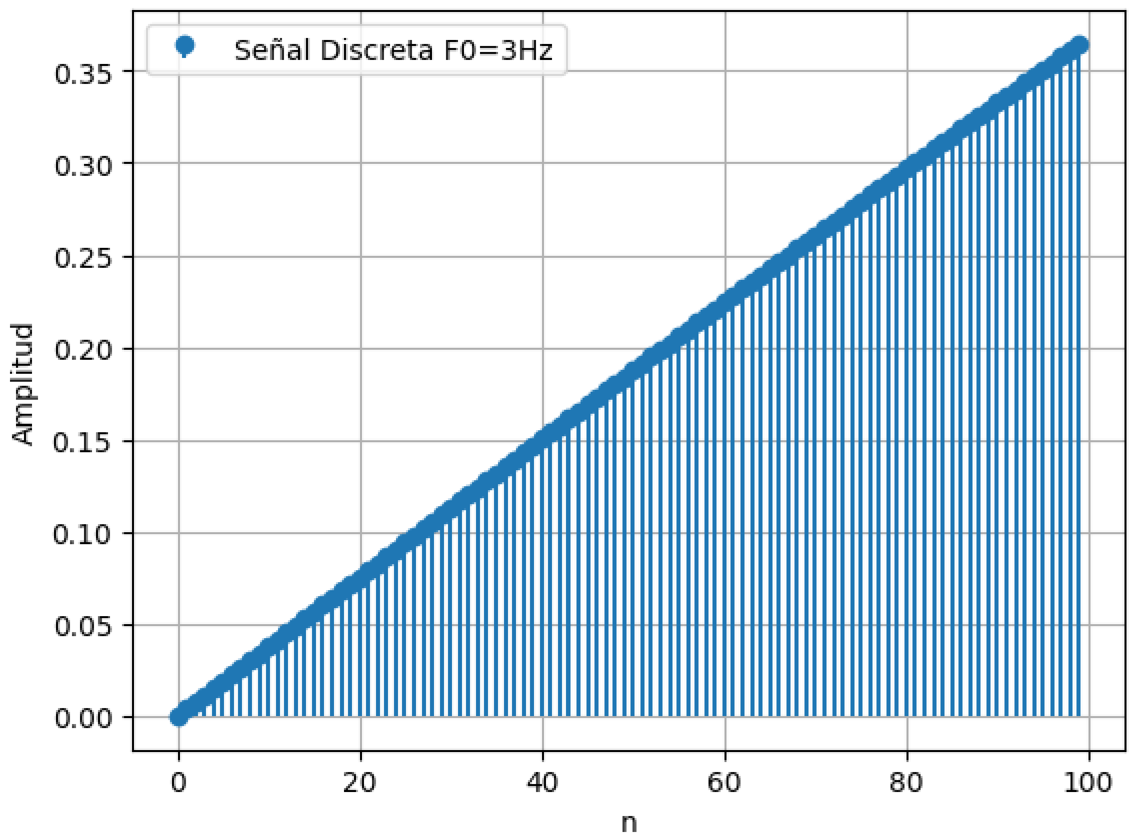
$x(n) = \text{Sen}(2\pi \cdot 0.04 n)$

Scribe

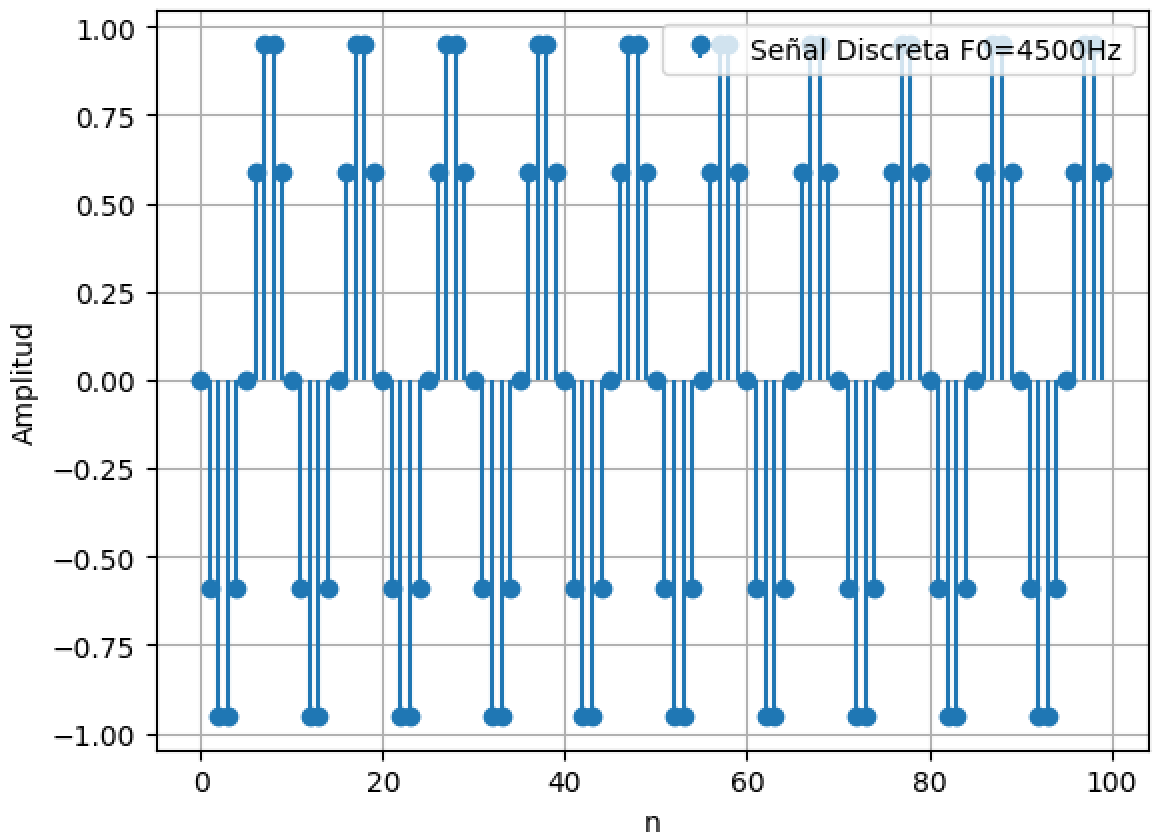


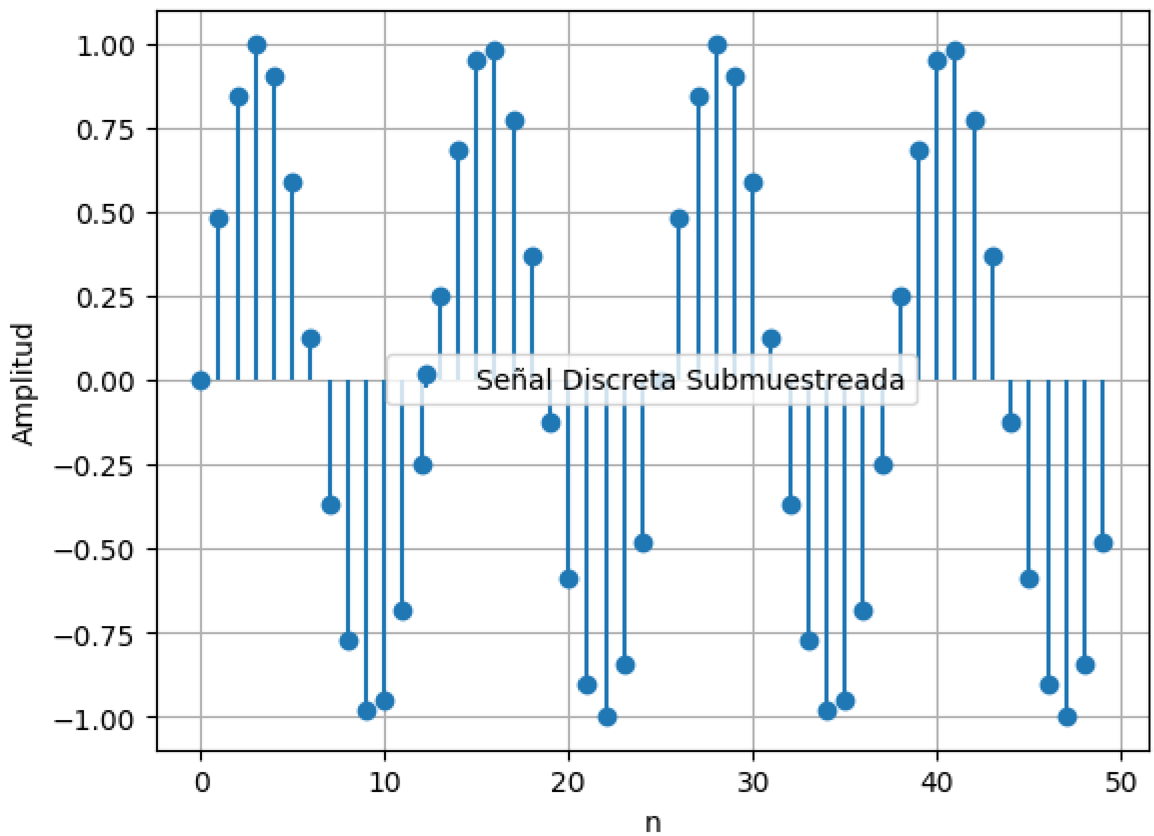
Señal Discreta F0=0.5Hz
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Amplitud
0
20
40
60
80
100
n


Señal Discreta F0=2Hz
0.25
0.20
0.15
0.10
0.05
0.00
Amplitud
0
20
40
60
80
100
n



Señal Discreta F0=3Hz
Amplitud
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
20
40
60
80
100
n


Señal Discreta F0=4500Hz
Amplitud
n
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
20
40
60
80
100


Señal Discreta Submuestreada
Amplitud
n
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
10
20
30
40
50