

Assingment1

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Question

Find the max-product and max-min composition of relations R1 and

$$\text{R2 given as follows: } R1 = \begin{vmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{vmatrix} \text{ and } R2 = \begin{vmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{vmatrix}$$

Solution

$$\text{Given } R1 = \begin{vmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{vmatrix} \text{ and } R2 = \begin{vmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{vmatrix}$$

1 Max-Product

$$\mu T(x1, z1) = \max(\min(1.0 * 1.0), \min(0.3 * 1.0), \min(0.9 * 0.3), \min(0.0 * 0.2))$$

$$\mu T(x1, z1) = \max(1.0, 0.3, 0.27, 0.0)$$

$$\mu T(x1, z1) = 1.0$$

$$\mu T(x1, z2) = \max(\min(1.0 * 1.0), \min(0.3 * 0.0), \min(0.9 * 0.1), \min(0.0 * 0.3))$$

$$\mu T(x1, z2) = \max(1.0, 0.0, 0.09, 0.0)$$

$$\mu T(x1, z2) = 1.0$$

$$\begin{aligned}\mu T(x1, z3) &= \max(\min(1.0 * 0.9), \min(0.3 * 0.5), \min(0.9 * 0.0), \min(0.0 * 0.1)) \\ \mu T(x1, z3) &= \max(0.9, 0.15, 0.0, 0.0) \\ \mu T(x1, z3) &= 0.15\end{aligned}$$

$$\begin{aligned}\mu T(x2, z1) &= \max(\min(0.3 * 1.0), \min(1.0 * 1.0), \min(0.8 * 0.3), \min(1.0 * 0.2)) \\ \mu T(x2, z1) &= \max(0.3, 1.0, 0.24, 0.2) \\ \mu T(x2, z1) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x2, z2) &= \max(\min(0.3 * 1.0), \min(1.0 * 0.0), \min(0.8 * 0.1), \min(1.0 * 0.3)) \\ \mu T(x2, z2) &= \max(0.3, 0.0, 0.08, 0.3) \\ \mu T(x2, z2) &= 0.3\end{aligned}$$

$$\begin{aligned}\mu T(x2, z3) &= \max(\min(0.3 * 0.9), \min(1.0 * 0.5), \min(0.8 * 0.0), \min(1.0 * 0.1)) \\ \mu T(x2, z3) &= \max(0.27, 0.5, 0.0, 0.1) \\ \mu T(x2, z3) &= 0.27\end{aligned}$$

$$\begin{aligned}\mu T(x3, z1) &= \max(\min(0.9 * 1.0), \min(0.8 * 1.0), \min(1.0 * 0.3), \min(0.8 * 0.2)) \\ \mu T(x3, z1) &= \max(0.9, 0.8, 0.3, 0.16) \\ \mu T(x3, z1) &= 0.16\end{aligned}$$

$$\begin{aligned}\mu T(x3, z2) &= \max(\min(0.9 * 1.0), \min(0.8 * 0.0), \min(1.0 * 0.1), \min(0.8 * 0.3)) \\ \mu T(x3, z2) &= \max(0.9, 0.0, 0.1, 0.24) \\ \mu T(x3, z2) &= 0.24\end{aligned}$$

$$\begin{aligned}\mu T(x3, z3) &= \max(\min(0.9 * 0.9), \min(0.8 * 0.5), \min(1.0 * 0.0), \min(0.8 * 0.1)) \\ \mu T(x3, z3) &= \max(0.81, 0.40, 0.0, 0.08) \\ \mu T(x3, z3) &= 0.81\end{aligned}$$

$$\begin{aligned}\mu T(x4, z1) &= \max(\min(0.0 * 1.0), \min(1.0 * 1.0), \min(0.8 * 0.3), \min(1.0 * 0.2)) \\ \mu T(x4, z1) &= \max(0.0, 1.0, 0.24, 0.2) \\ \mu T(x4, z1) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x4, z2) &= \max(\min(0.0 * 1.0), \min(1.0 * 0.0), \min(0.8 * 0.1), \min(1.0 * 0.3)) \\ \mu T(x4, z2) &= \max(0.0, 0.0, 0.08, 0.3) \\ \mu T(x4, z2) &= 0.3\end{aligned}$$

$$\begin{aligned}\mu T(x4, z3) &= \max(\min(0.0 * 0.9), \min(1.0 * 0.5), \min(0.8 * 0.0), \min(1.0 * 0.1)) \\ \mu T(x4, z3) &= \max(0.0, 0.5, 0.0, 0.1) \\ \mu T(x4, z3) &= 0.5\end{aligned}$$

Hence

$$\text{Max Product , T} = \text{R1} \cdot \text{R2} = \begin{vmatrix} 1.0 & 1.0 & 0.15 \\ 1.0 & 0.3 & 0.27 \\ 0.16 & 0.24 & 0.81 \\ 1.0 & 0.3 & 0.5 \end{vmatrix}$$

2 Max-Min Composition

Now,

$$\begin{aligned}\mu T(x1, z1) &= \max(\min(1.0, 1.0), \min(0.3, 1.0), \min(0.9, 0.3), \min(0.0, 0.2)) \\ \mu T(x1, z1) &= \max(1.0, 0.3, 0.3, 0.0) \\ \mu T(x1, z1) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x1, z2) &= \max(\min(1.0, 1.0), \min(0.3, 0.0), \min(0.9, 0.1), \min(0.0, 0.3)) \\ \mu T(x1, z2) &= \max(1.0, 0.0, 0.1, 0.0) \\ \mu T(x1, z2) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x1, z3) &= \max(\min(1.0, 0.9), \min(0.3, 0.5), \min(0.9, 0.0), \min(0.0, 0.1)) \\ \mu T(x1, z3) &= \max(0.9, 0.3, 0.0, 0.0) \\ \mu T(x1, z3) &= 0.9\end{aligned}$$

$$\begin{aligned}\mu T(x2, z1) &= \max(\min(0.3, 1.0), \min(1.0, 1.0), \min(0.8, 0.3), \min(1.0, 0.2)) \\ \mu T(x2, z1) &= \max(0.3, 1.0, 0.3, 0.2) \\ \mu T(x2, z1) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x2, z2) &= \max(\min(0.3, 1.0), \min(1.0, 0.0), \min(0.8, 0.1), \min(1.0, 0.3)) \\ \mu T(x2, z2) &= \max(0.3, 0.0, 0.1, 0.3) \\ \mu T(x2, z2) &= 0.3\end{aligned}$$

$$\begin{aligned}\mu T(x2, z3) &= \max(\min(0.3, 0.9), \min(1.0, 0.5), \min(0.8, 0.0), \min(1.0, 0.1)) \\ \mu T(x2, z3) &= \max(0.3, 0.5, 0.0, 0.1) \\ \mu T(x2, z3) &= 0.5\end{aligned}$$

$$\begin{aligned}\mu T(x3, z1) &= \max(\min(0.9, 1.0), \min(0.8, 1.0), \min(1.0, 0.3), \min(0.8, 0.2)) \\ \mu T(x3, z1) &= \max(0.9, 0.8, 0.3, 0.2) \\ \mu T(x3, z1) &= 0.9\end{aligned}$$

$$\begin{aligned}\mu T(x3, z2) &= \max(\min(0.9, 1.0), \min(0.8, 0.0), \min(1.0, 0.1), \min(0.8, 0.3)) \\ \mu T(x3, z2) &= \max(0.9, 0.0, 0.1, 0.3) \\ \mu T(x3, z2) &= 0.9\end{aligned}$$

$$\begin{aligned}\mu T(x3, z3) &= \max(\min(0.9, 0.9), \min(0.8, 0.5), \min(1.0, 0.0), \min(0.8, 0.1)) \\ \mu T(x3, z3) &= \max(0.9, 0.5, 0.0, 0.1) \\ \mu T(x3, z3) &= 0.9\end{aligned}$$

$$\begin{aligned}\mu T(x4, z1) &= \max(\min(0.0, 1.0), \min(1.0, 1.0), \min(0.8, 0.3), \min(1.0, 0.2)) \\ \mu T(x4, z1) &= \max(0.0, 1.0, 0.3, 0.2) \\ \mu T(x4, z1) &= 1.0\end{aligned}$$

$$\begin{aligned}\mu T(x4, z2) &= \max(\min(0.0, 1.0), \min(1.0, 0.0), \min(0.8, 0.1), \min(1.0, 0.3)) \\ \mu T(x4, z2) &= \max(0.0, 0.0, 0.1, 0.3) \\ \mu T(x4, z2) &= 0.3\end{aligned}$$

$$\begin{aligned}\mu T(x4, z3) &= \max(\min(0.0, 0.9), \min(1.0, 0.5), \min(0.8, 0.0), \min(1.0, 0.1)) \\ \mu T(x4, z3) &= \max(0.0, 0.5, 0.0, 0.1) \\ \mu T(x4, z3) &= 0.5\end{aligned}$$

Hence

$$\text{Max Min Composition , } T = R1 \cdot R2 = \begin{vmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.3 & 0.5 \\ 0.9 & 0.9 & 0.9 \\ 1.0 & 0.3 & 0.5 \end{vmatrix}$$