Consider the following initial-boundary value problem modeling heat flow in a wire.

(PDE) $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ for $0 < x < \pi,\ t > 0$

(BC) $u(0,t) = 0, u(\pi,t) = 0,\ t > 0$

(IC) $u(x,0) = \sin^2(x)$, for $0 < x < \pi$

The general solution is given by, $u(x,t) = \sum_{n=1}^{\infty} b_n e^{-2n^2 t}\sin(nx)$,

with coefficients $b_n = \frac{2}{\pi}\int_0^{\pi} \sin^2(x)\sin(nx)dx = \frac{1}{\pi}\int_0^{\pi}(1 - \cos(2x))\sin(nx)$.

For the even coefficients, $\{b_2, b_4, b_6, \dots\}$, the integrand will have ODD symmetry about $x = \pi/2$ implying $b_n = 0$.

Determine b_n when n is odd. [1] Evaluate the first three coefficients, $\{b_1, b_3, b_5\}$, and write out the final solution, $u(x,t)$, through the first three non-zero terms in its series representation.