Bangladesh University of Engineering and Technology

Course No : EEE 6207

Course Name : Broadband Wireless Communication

Project Name: BER vs SNR analysis of MIMO systems for different diversity schemes and spatial multiplexing

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Abstract:

In Wireless communication, to improve data rate and reliability of the system multiple antennas are incorporated in the transmitter and receiver referred to as MIMO systems. MIMO systems can be of different configuration and different schemes can be employed to them depending on how much information is available at the transmitter. In this project different spatial diversity schemes with and without transmitter channel knowledge is simulated and their performance in compared. Also, spatial multiplexing systems with suboptimal linear detectors are simulated and comparison between different detection schemes are made.

Introduction:

Wireless channels are prone to fading due to multipath propagation, uncertainty of environment, movement of transmitter and receivers, scattering of particles etc. Fading significantly reduces the capacity and bit error rate performance of the channel. To mitigate fading we need diversity. Spatial diversity is the most popular of diversity employed in wireless systems due to its ability to provide diversity without causing penalty in bandwidth or speed of communication. Wireless systems where transmitter and receivers have multiple antenna are called MIMO systems.

In MIMO system, we can improve bit error rate performance by employing spatial diversity. In spatial diversity only one symbol in average is transmitted through all transmit antennas per symbol period. Depending on weather transmitter has channel knowledge or not the spatial diversity scheme may vary. If transmitter doesn't have channel knowledge we will need to apply space time block coding(STBC) in transmitter side achieving only diversity gain. If transmitter has channel knowledge then it can pre-code the symbols according to channel condition achieving both diversity and array gain. Spatial diversity systems in absence and presence of transmit CSI is studied and compared here. Also we make comparison between 2×1 and 2×2 MIMO systems.

We can use transmitter antennas to improve data rate by sending different independent symbols through each antenna. This is referred to as spatial multiplexing(SM). In SM, the maximum likelihood detector though optimal is complex to implement in hardware. Suboptimal linear detectors available in literature are zero-forcing and MMSE detector. Their performance and comparison is made in this project.

System Description:

Bit error rate(BER) vs Signal to noise ratio(SNR) comparison is studied for the following systems:

1) 2×1 MIMO (2 transmit antenna, 1 receive antenna) Spatial Diversity system with

i) No Transmit CSI, No Space time block coding in transmitter.

ii) No Transmit CSI, But Space time block coding is applied in transmitter.

iii) Transmit CSI available and Transmitter employing Transmit MRC i.e. matched beamforming.

2) 2×2 MIMO (2 transmit antenna, 2 receive antenna) Spatial Diversity system with

i) No transmit CSI, No Space time block coding in transmitter, Maximal Ratio Combining at receiver.

ii) No Transmit CSI, But Space time block coding is applied in transmitter.

iii) Transmit CSI available and Transmitter-Receiver pair employing precoding and postcoding i.e. Dominant eigenmode transmission.

3) 2×2 MIMO (2 transmit antenna, 2 receive antenna) Spatial Multiplexing system

i) employing Zero-Forcing(ZF) detector.

ii) employing Minimum Mean Squared Error(MMSE) detector.

Simulation Setup:

i) We assume Rayleigh flat fading channel i.e. in baseband each channel is drawn from a circularly symmetric zero mean complex gaussian(CSZMCG) distribution.

ii) We assume QPSK modulation scheme for all systems.

iii) We assume that total energy available in transmitter per symbol duration is E_s. Then if there is N_T number of transmit antennas then energy per antenna is $\frac{E_s}{M_T}$.

iv) The noise is also drawn from a circularly symmetric zero mean complex gaussian(CSZMCG) distribution. We assume noise variance is N_0 i.e. variance of each of real and imaginary component is $\frac{1}{2}$ and they are independent.

iv) We assume each channel in MIMO system has unity power gain i.e. $E\{|h|^2\} = 1$. Then variance of each of real and imaginary part of channel is $\frac{1}{2}$ and they are independent. This ensured that SNR per receiver is $\frac{E_s}{N_0} = \rho$.

v) We assume that an error in symbol detection means a single bit error. In this assumption Symbol error rate(SER) and bit error rate(BER) is connected by

$$BER = \frac{SER}{\log_2 M}, M\ is\ the\ modulation\ order$$

vi) We assume the channel is block fading i.e. while employing STBC the channel remains fixed for full block duration.

vii) We assume ergodic channel i.e. channel moves from one realization to other independently.

viii) We assume the receiver employs maximum likelihood detection is spatial diversity systems.

Results and Discussion:

2×1 MIMO Spatial Diversity system

We first investigate the case when there is no transmit CSI available and transmitter does not apply STBC coding. Then each transmitter naively sends the same symbol through the channel.

The receiver effectively sees

$$
\begin{aligned}
y &= \frac{\sqrt{E_s}}{\sqrt{N_T}} h_1 s + \frac{\sqrt{E_s}}{\sqrt{N_T}} h_2 s + \cdots . + \frac{\sqrt{E_s}}{\sqrt{N_T}} h_{N_T} s \\
&= \sqrt{E_s} \frac{1}{\sqrt{N_T}} \left(h_1 + \cdots . + h_{N_T} \right) s \\
&= \sqrt{E_s} h_{eff}\, s
\end{aligned}
$$

We can show if each of $h_1, \dots, h_{N_T}$ is CSZMCG with variance 1 when $h_{eff} = \frac{1}{\sqrt{N_T}} \left(h_1 + \cdots . + h_{N_T} \right)$ is also a CSZMCG with same variance. Then without any coding the channel is same as SISO performance. We see in $Fig. 1$ that in the below that without any transmit precoding both 2×1 and 4×1 system's BER vs SNR performance is same.

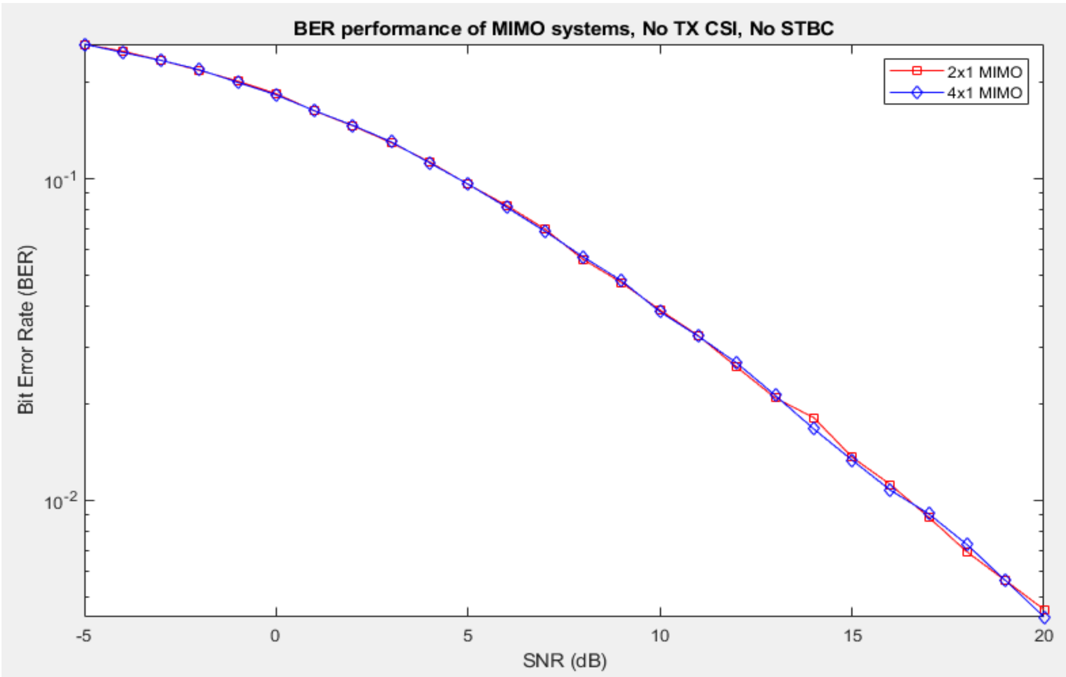

Fig.1 : Without STBC all MISO systems perform same equivalent to a SISO link.

Next we apply Alamouti STBC scheme in the transmitter. Alamouti scheme can extract full diversity gain without any channel knowledge in transmitter. However, it doesn't have any array gain.

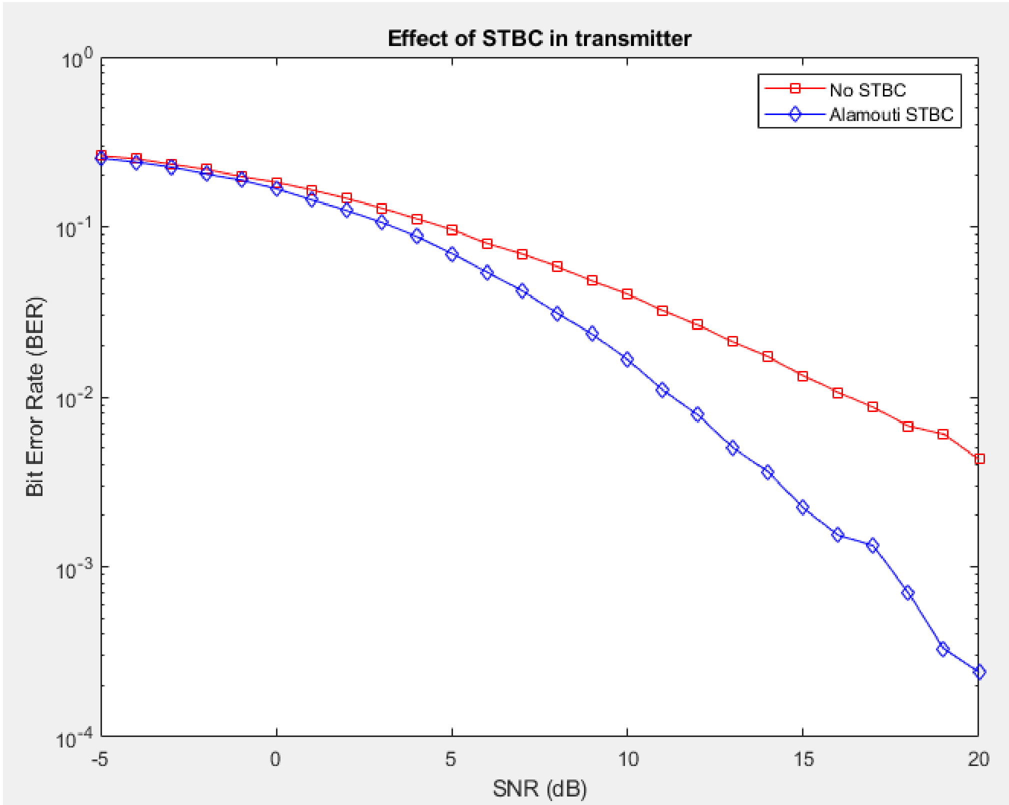

Fig.2 : STBC can achieve full diversity. The diversity decreases the slope of BER vs SNR curve.

The STBC scheme can gain significant performance improvement in moderate SNR regime due to its diversity gain as depicted in $Fig. 2$. Next, we assume that transmitter CSI is available. In that case transmitter can do precoding. Instead of sending naively same symbol, it can send symbol weighted according to channel condition to take benefit of good fading situations.

If channel vector is $\boldsymbol{h}$ then we transmit the vector $\boldsymbol{h}^{\boldsymbol{H}}\mathbf{s}$ through the channel. This nullifies the phase shift through the channel and ensures that each stream combines in phase in receiver. This is called transmit MRC or matched beam-forming. This can extract full diversity gain and array gain as well. Since array gain is missing in Alamouti scheme, the performance is transmit MRC is superior to Alamouti STBC scheme. The array gain available in transmit MRC doesn't affect slope, but adds a constant BER improvement factor. We show comparison of all three cases in $Fig. 3$.

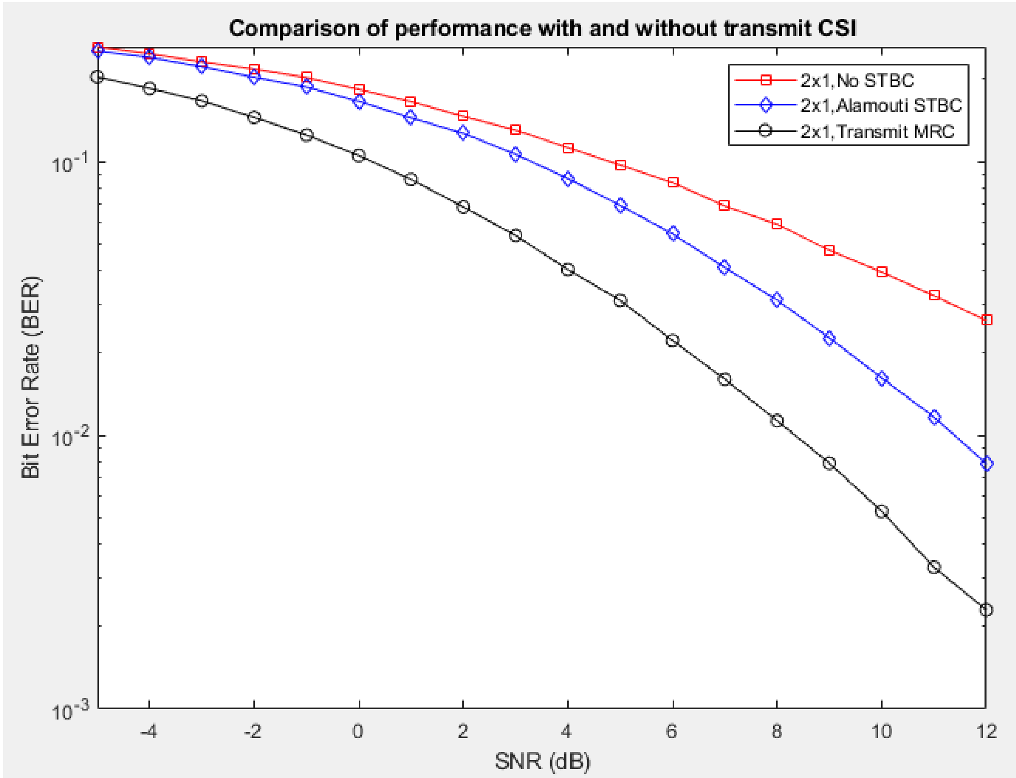

Fig.3 : If Transmitter CSI is available then array gain in addition to diversity gain is extractable as well. The addition of array gain doesn't affect slope but makes a shift in the bit error rate curve.

2×2 MIMO Spatial Diversity system

In 2×2 MIMO Systems, we have additional receive diversity and receiver array gain available. We first investigate the case when there is no transmit CSI available and transmitter does not apply STBC coding. Then each transmitter naively sends the same symbol through the channel.

If channel between i^{th} receive antenna and j^{th} transmit antenna is h_{ij} then received vector is

$$y_1 = \frac{\sqrt{E_s}}{\sqrt{2}}\,(h_{11}s + h_{12}s) = \sqrt{E_s}\frac{1}{\sqrt{2}}\,(h_{11} + h_{12})s = \sqrt{E_s}h_{eff,1}\, s$$

$$y_2 = \frac{\sqrt{E_s}}{\sqrt{2}}\,(h_{21}s + h_{22}s) = \sqrt{E_s}\frac{1}{\sqrt{2}}\,(h_{21} + h_{22})s = \sqrt{E_s}h_{eff,2}\, s$$

We see without block coding the effective system becomes equivalent to 1×2 system. The receiver does Maximal Ratio Combining which can extract full receive diversity and receiver array gain. We see in $Fig. 4$ that 2×2 no TX CSI,no STBC case performs better than 2×1 no TX CSI, no STBC due to receiver diversity and array gains.

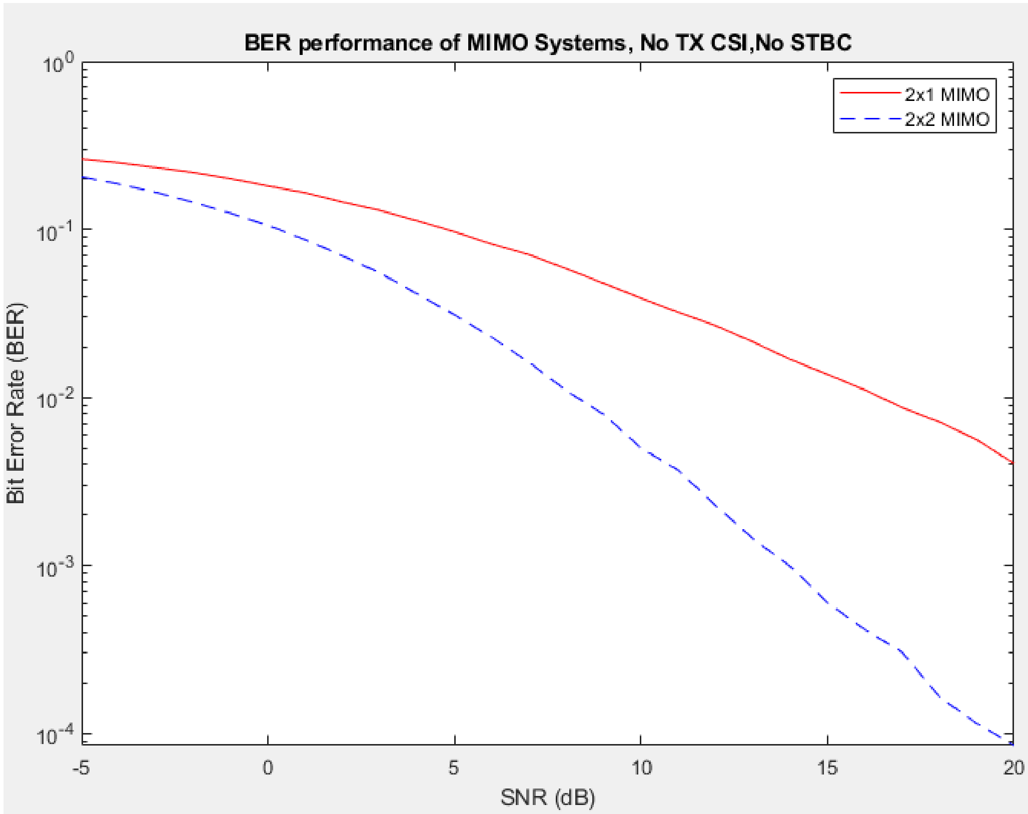

Fig.4 : 2×2 case has receive diversity and receive array gain available. The array gain shifts the curve while the diversity gain changes the slope.

In absence of transmit CSI, to avail array gain the transmitter can do Alamouti Coding. In this case alamouti coding can extract full $2 \times 2 = 4$ diversity but since alamouti can't extract transmitter array gain the array gain available is of order 2.

On contrary, if transmit CSI is available then transmitter can do dominant eigenmode transmission. If the channel matrix H has singular value decomposition of $H = UDV^H$ then the transmitter precodes the symbol as $\boldsymbol{v}_{\mathbf{max}}$s while the receiver postcodes as $\boldsymbol{u}_{max}^{\boldsymbol{H}}\boldsymbol{y}$ if y is the received vector where u_{max} and v_{max} are the left and right singular vector corresponding to maximum singular value of channel. This diagonalizes the channel and gives maximum receiver SNR.

In $Fig.5$ performance of all three cases for 2×2 MIMO system is shown. With diversity gain, Alamouti scheme has better slope than no STBC case. With full array gain, the dominant eigenmode transmission has a shift in BER curve from Alamouti scheme and is superior in performance.

In $Fig.6$ all the above mentioned 2×1 and 2×2 MIMO diversity schemes performance is plotted. To note few observation, notice that 2×2 no STBC case performs better than 2×1 Alamouti case since 2×2 no STBC has diversity and array gain order 2 while 2×1 Alamouti has only diversity order 2 and gain order 1.

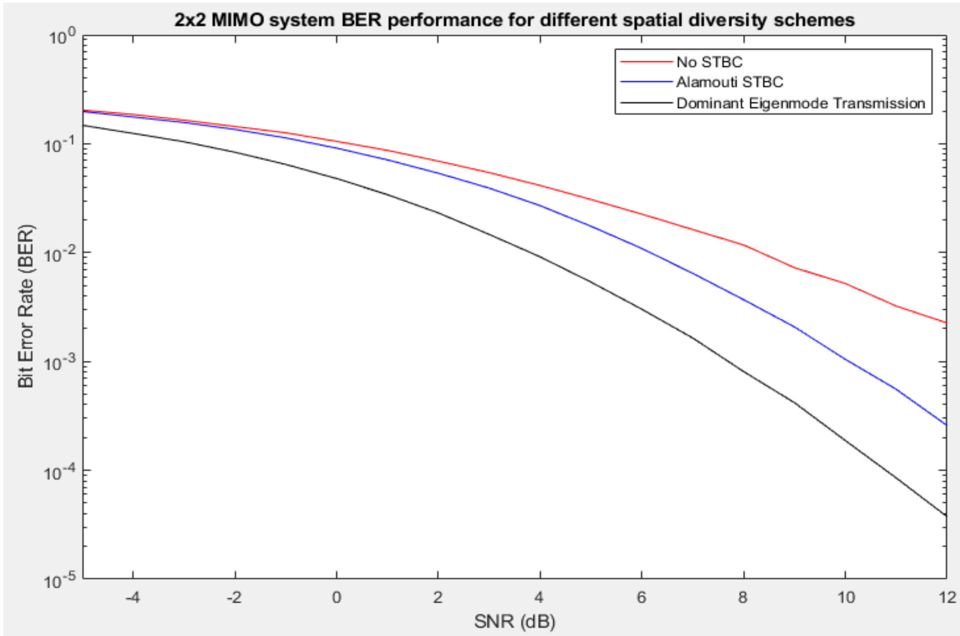

Fig.5 : 2×2 MIMO system. Without STBC we can't extract transmit diversity. Alamouti STBC extracts full $2 \times 2 = 4$ order diversity but can only extract receiver array gain. Dominant Eigenmode transmission can extract transmit array gain as well and thus has better bit error rate performance.

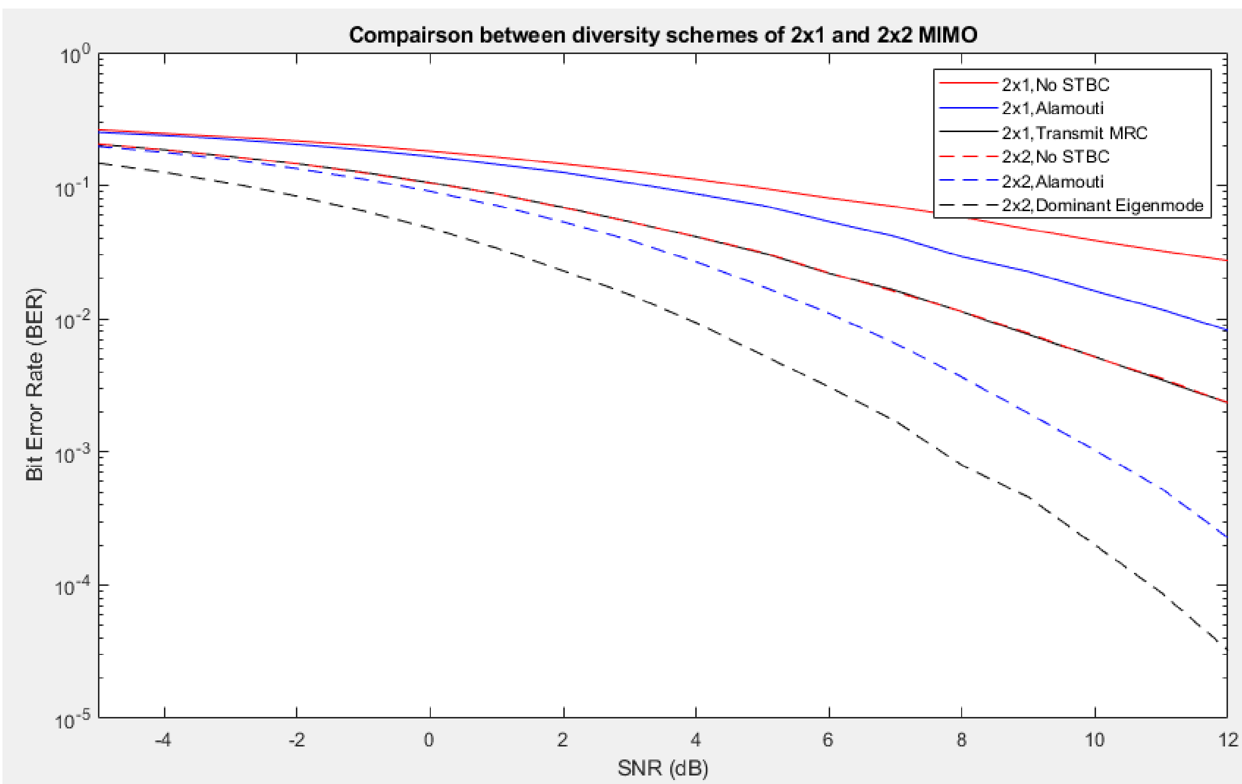

Fig.6 : Bit error rate performance of all $2 \times 1, 2 \times 2$ diversity schemes plotted together.

A table comprising of diversity and array gain order for all the schemes is mentioned below.

System Configuration	Diversity Configuration	Diversity Order	Array Gain Order
2×1	No STBC	1	1
2×1	Alamouti STBC	2	1
2×1	Transmit MRC	2	2
2×2	No STBC	2	2
2×2	Alamouti STBC	4	2
2×2	Dominant Eigenmode	4	4

From the table you can see 2×1, No STBC and 2×1,Transmit MRC has same diversity and array gain order and thus their curves overlap in $Fig. 6$.

2×2 MIMO Spatial Multiplexing system

In spatial multiplexing system, the transmitter sends independent symbols through the transmit antennas. In 2×2 MIMO SM the received vector is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\boldsymbol{y} = H\boldsymbol{s} + \boldsymbol{n}$$

The received symbol can be decoded by sub-optimal detector. If we restrict to linear detectors then detection is done by multiplying received vector with a matrix $\boldsymbol{W}$ i.e. $\boldsymbol{s_{detected}} = \boldsymbol{Wy}$.

For Zero-Forcing detection, $W_{ZF} = (H^H H)^{-1} H^H$ and For MMSE detection $W_{MMSE} = (H^H H + \sigma^2)^{-1} H^H$.

Where σ^2 is the variance of complex noise vector.

In $Fig. 7$ BER performance for both detector is plotted. Since ZF detector doesn't take noise into account it enhances noise. Noise enhancement of MMSE detector is lower. So, MMSE detector has superior performance.

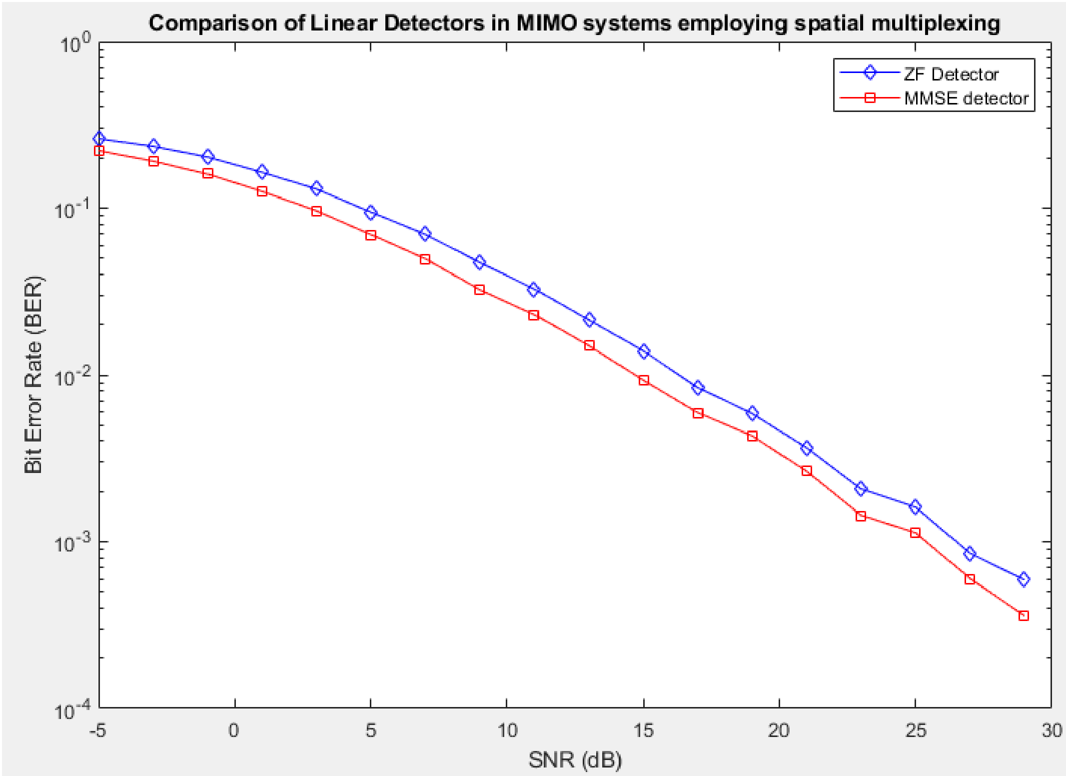

Fig.7 : In spatial multiplexing system, MMSE detector overperforms ZF detector.

Conclusion:

In conclusion, in this project the effect of different spatial diversity schemes with and without transmitter channel knowledge is studied. Also we have seen distinct the effects of diversity gains and array gain on bit error rate curve. More concretely, we have shown that diversity gain affects the slope whereas array gain shifts the curve. We have shown that in the absence of transmitter channel knowledge, STBC coding can be effective to achieve diversity gain. To achieve full array gain, channel knowledge at transmitter is mandatory.

MIMO systems employing spatial multiplexing is also studied. Their bit error rate performance under Rayleigh fading is observed. The effect and comparison of Zero Forcing and MMSE detector is shown. We have shown that MMSE detector has better detection performance.