밑바닥 딥러닝 P.102 [식 5.13] 증명

$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \times W^T$ 증명

i) $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $W = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

$Y = X^T W + B = (x_1 w_{11} + x_2 w_{21} + b_1,\ x_1 w_{12} + x_2 w_{22} + b_2,\ x_1 w_{13} + x_2 w_{23} + b_3)$

$= (y_1, y_2, y_3)$

ii) $\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \times \frac{\partial Y}{\partial X}$.

$\frac{\partial Y}{\partial X} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} \end{pmatrix}$ ∵ vector-by-vector

$= \begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ w_{13} & w_{23} \end{pmatrix}$

$= W^T$

$\therefore \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y} \times W^T$

$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \times X^T$ 증명

i) $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $W = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

$Y = X^T W + B = (x_1 w_{11} + x_2 w_{21} + b_1,\ x_1 w_{12} + x_2 w_{22} + b_2,\ x_1 w_{13} + x_2 w_{23} + b_3)$

$= (y_1, y_2, y_3)$

ii) $\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \times \frac{\partial Y}{\partial W}$.

∵ vector - by - matrix

$$\frac{\partial Y}{\partial W} = \begin{pmatrix} \frac{\partial y_1}{\partial w_{11}} & \frac{\partial y_1}{\partial w_{12}} & \frac{\partial y_1}{\partial w_{13}} & \frac{\partial y_1}{\partial w_{21}} & \frac{\partial y_1}{\partial w_{22}} & \frac{\partial y_1}{\partial w_{23}} \\ \frac{\partial y_2}{\partial w_{11}} & \frac{\partial y_2}{\partial w_{12}} & \frac{\partial y_2}{\partial w_{13}} & \frac{\partial y_2}{\partial w_{21}} & \frac{\partial y_2}{\partial w_{22}} & \frac{\partial y_2}{\partial w_{23}} \\ \frac{\partial y_3}{\partial w_{11}} & \frac{\partial y_3}{\partial w_{12}} & \frac{\partial y_3}{\partial w_{13}} & \frac{\partial y_3}{\partial w_{21}} & \frac{\partial y_3}{\partial w_{22}} & \frac{\partial y_3}{\partial w_{23}} \end{pmatrix}$$

$$= \begin{pmatrix} x_1 & 0 & 0 & x_2 & 0 & 0 \\ 0 & x_1 & 0 & 0 & x_2 & 0 \\ 0 & 0 & x_1 & 0 & 0 & x_2 \end{pmatrix}$$

$\doteq X^T$ ∵ 각 행이 X^T의 요소로 구성되며 반복되기 때문에 전치행렬을 X^T로 일반화 가능하다.