Assignment 8

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Question

Question:

The resistors r_1,r_2,r_3 and r_4 are independent random variables and each is uniform in the interval $(450, 550)$. Using the central limit theorem, find $\Pr(1900 \leq r_1 + r_2 + r_3 + r_4 \leq 2100)$.

CLT

Central limit Theorem:

If $X_1, X_2, \ldots, X_n \ldots$ is a sequence of random variables drawn from a population with an overall mean μ and variance σ^2, and if $\overline{X_n}$ is the sample mean of the first n samples, then the limiting form of the distribution, $Z = \lim_{n\to\infty} \left(\frac{\overline{X_n} - \mu}{\frac{\sigma}{\sqrt{n}}} \right)$, is a standard normal distribution.

Solution:

Since $r_i \, \forall i \in \{1, 2, 3, 4\}$ are i.i.d and are uniformly distributed in the interval $(450, 550)$,

$$E(r_i) = 500 \tag{1}$$

$$\sigma_i^2 = \int_{-\infty}^{\infty} (r - E(r_i))^2 \rho(r) dr \tag{2}$$

$$= \int_{450}^{550} (r - E(r_i))^2 \times \frac{1}{100} dr \tag{3}$$

$$= \int_{450}^{550} \frac{(r - 500)^2}{100} dr \tag{4}$$

(5)

Solution:

$$= \int_{-50}^{50} \frac{r^2}{100} dr \tag{6}$$

$$= \frac{50^2}{3} \tag{7}$$

Let,

$$Y = \sum_{1}^{4} r_i \tag{8}$$

Using Central Limit Theorem:

$$f_Y(y) \sim N\left(4 \times 500, \frac{2 \times 50}{\sqrt{3}}\right) \tag{9}$$

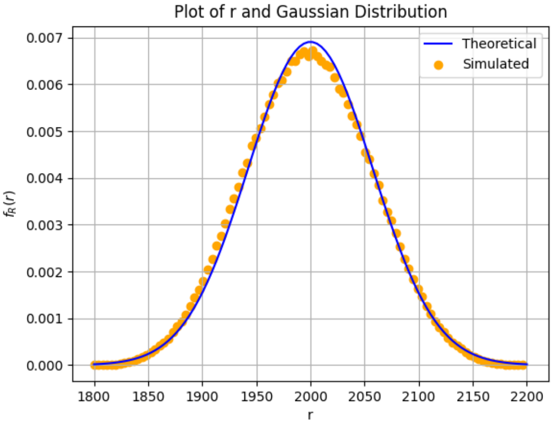

Figure: PDF of R

$$\Pr\left(1900 \le y \le 2100\right) = \int\limits_{1900}^{2100} f_Y(y)dy \tag{10}$$

$$= \int\limits_{1900}^{2100} \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{y-\mu}{\sqrt{2}\sigma}\right)^2} dy \tag{11}$$

$$= \int\limits_{-100}^{100} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} dy \tag{12}$$

$$\approx 0.91673 \tag{13}$$