Assignment 2

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1 (vi) [ISC 2018] : Find the approximate change in the volume 'V' of a cube of side x meters caused by decreasing the side by 1%.

Solution: Let V be the volume of a cube of side length x meters.
Let the change in side-length be Δx and the change in volume be ΔV

$$V = x^3 \tag{1}$$

Since Δx is small relative to x,

$$\frac{\Delta V}{\Delta x} \approx \frac{dV}{dx} \tag{2}$$

$$\frac{dV}{dx} = 3x^2 \tag{3}$$

Since,

$$\frac{\Delta x}{x} = -0.01 \tag{4}$$

$$\Rightarrow \Delta x = -0.01x \tag{5}$$

$$\Rightarrow \Delta V = 3x^2 \times (-0.01x) \tag{6}$$

$$\Rightarrow \Delta V = -0.03x^3 \tag{7}$$

$$\frac{\Delta V}{V} = -\frac{0.03x^3}{x^3} \tag{8}$$

$$\frac{\Delta V}{V} = -0.03 \tag{9}$$

Let the new volume of the cube be V'

$$V' = V + \Delta V \tag{10}$$

$$\Rightarrow V' = x^3 - 0.03x^3 \tag{11}$$

$$\Rightarrow V' = 0.97x^3 \tag{12}$$

The volume of the cube approximately decreases by 3% to $0.97x^3$.

For a more accurate result,

$$x' = x + \Delta x \tag{13}$$

$$\Rightarrow x' = x - 0.01x \tag{14}$$

$$\Rightarrow x' = 0.99x \tag{15}$$

$$V' = x'^3 \tag{16}$$

$$\Rightarrow V' = (0.99)^3 x^3 \tag{17}$$

$$\Rightarrow V' = 0.970299x^3 \tag{18}$$

$$\Rightarrow \Delta V = V' - V \tag{19}$$

$$\Rightarrow \Delta V = -0.029701x^3 \tag{20}$$

$$\Rightarrow \frac{\Delta V}{V} = -0.029701 \tag{21}$$

The volume of the cube decreases by 2.9701%.