

Assignment 2

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1 (vi) [ISC 2018] : Find the approximate change in the volume 'V' of a cube of side x meters caused by decreasing the side by 1%.

Solution: Let V be the volume of a cube of side length x meters.
Let the change in side-length be Δx and the change in volume be ΔV

$$V = x^3 \quad (1)$$

Since Δx is small relative to x ,

$$\frac{\Delta V}{\Delta x} \approx \frac{dV}{dx} \quad (2)$$

$$(3)$$

Let the new volume of the cube be V' .

$$V' = V + \Delta V \quad (4)$$

Since,

$$\frac{\Delta x}{x} = -0.01 \quad (5)$$

$$\Rightarrow \Delta x = -0.01x \quad (6)$$

$$\frac{dV}{dx} = 3x^2 \quad (7)$$

$$\Rightarrow \Delta V = 3x^2 \times (-0.01x) \quad (8)$$

$$\Rightarrow \Delta V = -0.03x^3 \quad (9)$$

$$\frac{\Delta V}{V} = -\frac{0.03x^3}{x^3} \quad (10)$$

$$\frac{\Delta V}{V} = -0.03 \quad (11)$$

$$\Rightarrow V' = x^3 - 0.03x^3 \quad (12)$$

$$\Rightarrow V' = 0.97x^3 \quad (13)$$

The volume of the cube approximately decreases by 3% to $0.97x^3$.

For a more accurate result,

$$x' = x + \Delta x \quad (14)$$

$$V' = (x')^3 \quad (15)$$

$$\Delta V = V' - V \quad (16)$$

$$\Rightarrow x' = x - 0.01x \quad (17)$$

$$\Rightarrow x' = 0.99x \quad (18)$$

$$\Rightarrow V' = (0.99x)^3 \quad (19)$$

$$\Rightarrow V' = 0.970299x^3 \quad (20)$$

$$\Rightarrow \Delta V = -0.029701x^3 \quad (21)$$

$$\Rightarrow \frac{\Delta V}{V} = -0.029701 \quad (22)$$

The volume of the cube decreases by 2.9701%.