Assignment 6

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Question

Example 2.15 [Papoulis]: A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and the results are positive. Suppose that the person comes from a population of 100,000 where 2,000 people suffer from that disease. What can we conclude about the probability that the person under test has that particular cancer?

Let X_1 and X_2 be two Bernoulli Random Variables such that,

X_1	**Outcome**
0	Healthy
1	Has Cancer

Table: Bernoulli Distribution

X_2	**Outcome**
0	Negative
1	Positive

Table: Bernoulli Distribution

$$\Pr(X_1 = 1|X_2 = 1) = \frac{\Pr(X_2 = 1|X_1 = 1)\Pr(X_1 = 1)}{\Pr(X_2 = 1)} \tag{1}$$

$$\Pr(X_2 = 1) = \sum_{i=0}^{1} \Pr(X_2 = 1|X_1 = i)\Pr(X_1 = i) \tag{2}$$

$$\Pr(X_2 = 1) = 0.05 \times 0.98 + 0.95 \times 0.02 \tag{3}$$

$$\Pr(X_2 = 1) = 0.068 \tag{4}$$

$$\Pr(X_1 = 1|X_2 = 1) = \frac{0.95 \times 0.02}{0.068} \tag{5}$$

$$\Pr(X_1 = 1|X_2 = 1) = \frac{0.019}{0.068} \tag{6}$$

$$\Pr(X_1 = 1|X_2 = 1) = 0.2794 \tag{7}$$

Answer

Baye's theorem suggests that there is a 27.94% chance that this person has cancer even given he/she has been tested positive.