
Case study I.1 – Scales in Nature

You may need the following formulae:

- Gravitational potential between two bodies $V_{\rm gr} = G\frac{M_1\,M_2}{r}$;
- Coulomb potential between two charges $V_e = k_e\frac{q_1\,q_2}{r}$;
- Non-relativistic kinetic energy of a particle $E_{\rm kin} = \frac{p^2}{2m}$;
- Energy of an object at rest $E = M\,c^2$; energy of a moving relativistic object is: $E = \sqrt{(Mc^2)^2 + (pc)^2}$
- Heisenberg uncertainty principle $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ which, for the limiting situations can be used to relate momentum and position of the particle $x \sim h/p$ (de Broglie wavelength).

All constants used above, as well as proton/neutron mass $m_p \simeq m_n$, electron mass $m_e \ll m_p$, and electron/proton charge, can be found online.

Q1. Measurement units of particle physics.

(a) Write down the definition of eV (electron-Volt). What is measured in these unites: voltage, speed, energy, current, charge, mass?

(b) Define by analogy pV (proton-Volt). Compute the ratio 1 pV/1 eV.

(c) Higgs boson observed at LHC has the estimated mass of 125 GeV. How many kilograms is that?

(d) If you are 100% efficient in using the energy provided by the battery below, how many batteries as on the figure do you need to create one Higgs boson?

[the figure shows a battery of size AA, with the following text visible:
1700mAh, Li-Polymer Battery, USB Rechargable Battery, 1.5V]

Q2. Google for sizes of different objects in the Universe, from very small to very large, and make a $\log_{10} - \log_{10}$ plot of mass versus size, similar to the one you saw in the introductory lecture (also available online).

Objects in Nature are formed as a result of an equilibrium between competing forces. In the next three questions, you will need to estimate the size (radius) or the mass of the objects below and compare your answers to the numbers observed experimentally.

Q3. Atom and its nucleus

(a) If to compare Compton wave length and de Broigle wave length of an object, which one is shorter? Is it possible that an at least rough equality between these two can be reached?

(b) Compton wave length is the lowest bound on the size of an object of given mass. It can be estimated in the following way: kinetic energy arising from Heisenberg uncertainty principle is compatible with Einstein energy of an object. Intuitive interpretation: if you try to compress an object, beyond some threshold the operation will require the energy sufficient for creation of another copy of the object. So you start multiplying an object instead of compressing it.

Proton has the size very close to its Compton wave length. Estimate the size of a proton given its mass.

(c) Atoms are formed as a balance between Coulomb energy and kinetic energy (of an electron). Estimate the size of a hydrogen atom.

(d) If a proton is scaled up to the size of a coin (10kr), in comparison how large the hydrogen atom shall be?

Q4. Stars

(a) In stars, gravity overcomes electric forces to ignite fusion. Consider a gas giant planet composed entirely from hydrogen whose atoms are, due high pressures, are densely packed. Estimate at what mass such a planet can potentially start fusion, express your answer in terms of Solar and Jupiter masses.

(b) When fusion ceases (end of star's main sequence), electron degeneration pressure prevents the ex-star from the gravitational collapse, a *white dwarf* is formed. Electron degneration pressure is a quantum effect: each electron occupies a segment of space of de Broglie wavelength size and, because electrons are fermions, two electrons won't occupy the same segment of space. Estimate the size (radius) of a white dwarf which weights 1 solar mass, express the answer in terms of Sun's radius and Earth's radius [1].

(c) If gravity is so powerful that it overcomes electron degeneration pressure, neutron degeneration pressure is the next in magnitude countering force. In case the latter is able to balance gravity, a neutron star is formed. Estimate size of a neutron star who has the mass of 1.5 Sun[2].

Q5. Black holes and Planck scale

(a) If gravity is still more powerful, the ex-star collapses to a black hole. Find a relation between the black hole size and its mass. How large is a black hole with the mass of the Eearth?

(b) Black hole mass is the upper bound on the object's mass/energy of given size. The Compton wave length formula offers the lower bound for the object's mass/energy. Intersection of two bounds is the Planck scale. Estimate the Planck mass, Planck distance, and Planck time.

(c) Demonstrate that if we use Planck measurement units, the main universal constants $G, \hbar, c$ shall be equal to 1.

[1] By googling online, you will likely find a more sophisticated reasoning giving more accurate estimates. I *will not accept this as the answer* because the goal is you try things yourself.

[2] Neutron starts with mass equal to that of the Sun cannot exist, they would be white dwarfs instead