Assignment 2 Mathematical methods in physics

Anton Lindbro

24 februari 2025

1 Problem 1

1.1

We are supposed to write a series solution for the given PDE. We have a string with fixed ends that is being driven by a timedependent force. The PDE is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + f(x,t)$$

Since we have fixed ends a sine series is appropriate for the solution. We can write that series as

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin\left(\frac{n\pi x}{L}\right)$$

1.2

Now we need to compute the series coeffcients for a cosine series for the driving force. We can write the driving force as

$$f(t) = \sum_{n=1}^{\infty} f_n \sin\left(\frac{n\pi x}{L}\right)$$

$$f_n = \frac{2}{L} \int_0^L f(t) \sin\left(\frac{n\pi x}{L}\right) dx$$

Since $f(t)$ is constant in x we get

$$
f_n = \frac{2f(t)}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right)dx = \frac{2f(t)}{L}\left[-\frac{L}{n\pi}\cos\left(\frac{n\pi x}{L}\right)\right]_0^L = \frac{2f(t)}{n\pi}\left(1-(-1)^n\right)
$$
$$
f_n = \frac{4f(t)}{n\pi} \text{ for n odd}
$$

1.3

Now we substitute the series solutions into the PDE and solve for $u_n(t)$

$$
\sum_{n=1}^{\infty}\frac{\partial^2 u_n}{\partial t^2}\sin\left(\frac{n\pi x}{L}\right) = -\frac{c^2\pi^2}{L^2}\sum_{n=1}^{\infty}u_n(t)n^2\sin\left(\frac{n\pi x}{L}\right)+\sum_{n=1}^{\infty}f_n\sin\left(\frac{n\pi x}{L}\right)
$$

Now we cancel all sin terms and get

$$
\frac{\partial^2 u_n}{\partial t^2} = -\frac{c^2\pi^2}{L^2}u_n(t)n^2 + f_n
$$

1.4

If we set $f(t) = \sin\omega t$ we can solve the ODE for $u_n(t)$

$$
\frac{\partial^2 u_n}{\partial t^2} = -\frac{c^2\pi^2}{L^2}u_n(t)n^2 + \frac{4\sin\omega t}{n\pi}
$$

1.5

1.6

2 Problem 2

2.1

2.2

3 Problem 3

3.1

3.2

3.3

3.4

4 Problem 4

5 Problem 5

5.1

5.2