Homework 2 GoA 3

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Flux

In order to compute the flux of the vectorfield $\boldsymbol{F}(x, y, z) = (x^4 z, x^3 y^2, 2x^3 yz)$ over the box defined by $0 \geq x \geq 1, 0 \geq y \geq 2, 0 \geq z \geq 1$ we use the divergens theorem since we have a vectorfield we can easily calculate the divergens of and a surface that encloses a simple volume the resulting tripple integral will be simple.

We begin by calculating the divergens of the field F

$$\frac{\partial F_x}{\partial x} = 4x^3 z \tag{1}$$
$$\frac{\partial F_y}{\partial y} = 2yx^3 \tag{2}$$
$$\frac{\partial F_z}{\partial z} = 2x^3 y \tag{3}$$
$$\nabla \cdot \boldsymbol{F} = 4x^3(y + z) \tag{4}$$

$$\iiint_D 4x^3(y + z)dxdydz \tag{5}$$

Where D is the box mentioned above. Using Fubinis therorem we can rewrite it as a iterated integral

$$\int_0^1 dz \int_0^2 dy \int_0^1 4x^3(y+z)dx \tag{6}$$

$$\int_0^1 dz \int_0^2 \left[x^4(y+z)\right]_0^1 dy = \int_0^1 dz \int_0^2 y + zdy \tag{7}$$

$$\int_0^1 \left[\frac{y^2}{2} + yz\right]_0^2 dz = \int_0^1 2 + 2zdz \tag{8}$$

$$\left[2z + z^2\right]_0^1 = 3 \tag{9}$$

Since the divergence theorem gives the wrong orientation we flip the sign and get that the flux over the box is -3

Series

In order to determine the convergence of the series below prove that the absolute value of the series converges.

$$\sum_{n=1}^{\infty} \frac{(-5)^{n+1}}{(2n)^n} \tag{10}$$

Using the root test we find that the series is absolute convergent as follows.

$$\lim_{n\to\infty} \sqrt[n]{\frac{5^{n+1}}{(2n)^n}} \tag{11}$$

$$\lim_{n\to\infty} \frac{5^{1+\frac{1}{n}}}{(2n)} \tag{12}$$

Since the exponent of the numerator goes to one when n grows 12 can be written as

$$\lim_{n\to\infty} \frac{5}{(2n)} = 0 \tag{13}$$

So by the root test the series in 11 is convergent and therfore 10 is absolutely convergent.