Assignment 1

Anton Lindbro

9 november 2024

1 Introduction

In this assignment we have four differential equations that we are asked to examine and if possible find solutions for. We currently have 3 types of equations we can solve so we need to find out if any of the four given equations can fit into any of these three boxes of equations we can solve.

2 Classification

So we begin by classifying the given equations.

2.1 Equation 1

$$4xy - x\cos x + (2x^2y + 3\cos^2 y)\frac{dy}{dx} = 0 \tag{1}$$

By manipulating this we can find what type of equation it is

$$4xy - x\cos x + (2x^2 + 3\cos^2 y)\frac{dy}{dx} = 0 \tag{2}$$

$$4xy - x\cos x = -(2x^2 + 3\cos^2 y)\frac{dy}{dx} \tag{3}$$

$$(4xy - x\cos x)dx = -(2x^2 + 3\cos^2 y)dy \tag{4}$$

$$(4xy - x\cos x)dx + (2x^2 + 3\cos^2 y)dy = 0 \tag{5}$$

Calculating the partials we get that

$$\frac{d(4xy - x\cos x)}{dy} = 4x = \frac{d(2x^2 + 3\cos^2 y)}{dx} \tag{6}$$

So this is on exact form

2.2 Equation 2

$$(1 + y^3)x = e^x y^2 y' \tag{7}$$

Now we try to manipulate this equation aswell

$$(1 + y^3)x = e^x y^2 y' \tag{8}$$

$$y' = \frac{(1 + y^3)x}{e^x y^2} \tag{9}$$

$$y' = \frac{(1 + y^3)}{y^2} \frac{x}{e^x} \tag{10}$$

So this is a separable equation

2.3 Equation 3

$$yy' = \cos(xy^2) \tag{11}$$

This equation dosent fit into any of our boxes. This is not linear since we have a y^2 term that is also inside e cosine, it is not separable since we cant separate x and y because of the cosine. Although we can write it on the exact form

$$ydy = \cos(xy^2)dx \tag{12}$$

It is clear that the partial derivatives of these are not the same. So with the tools we have this cannot be solved.

2.4 Equation 4

$$\frac{y' - e^{2x+3\sin x}}{\cos x} = 3y \tag{13}$$

By manipulating this we can find what type of equation

$$y' - e^{2x+3\sin x} = 3y\cos x \tag{14}$$
$$y' - 3y\cos x = e^{2x+3\sin x} \tag{15}$$

This is linear

3 Solutions

3.1 Equation 1 exact

Since we have this is an exact form we have it as

$$M(x,y)dx + N(x,y)dy = 0 \tag{16}$$

In our case we have

$$M(x,y) = 4xy - x\cos x N(x,y) = 2x^2y + 3\cos^2 y \tag{17}$$

We now now that if we integrate these with respect to dx and dy respectively they should both equal the functions that solves this equation

$$\int M(x,y)dx = \int 4xy - x\cos x dx = 2x^2y - x\sin x - \cos x + h(y) \tag{18}$$

If we then take the y partial of this it should equal $N(x,y)$ and we can then find $h'(y)$

$$\frac{d}{dy}(2x^2y - x\sin x - \cos x + h(y)) = 2x^2 + h'(y) \tag{19}$$
$$2x^2 + h'(y) = 2x^2 + 3\cos^2(y) \tag{20}$$
$$h'(y) = 3\cos^2(y) \tag{21}$$
$$h(y) = \int 3\cos^2(y)dy = 3(\frac{1}{2} + \sin(2y)) \tag{22}$$
$$\tag{23}$$

This then gives us an implicit function for y

$$2x^2y + 3(\frac{1}{2} + \sin(2y)) = x \sin x + \cos x \tag{24}$$

3.2 Equation 2 separable

$$y' = (\frac{1 + y^3}{y^2})(\frac{x}{e^x}) \tag{25}$$

This can be solved with the following identity

$$\int \frac{y^2}{1 + y^3} dy = \int xe^{-x} dx \tag{26}$$

Calculating these integrals using substitution for the y integral and per partes for the x integral we get

$$\frac{1}{3} \ln 1 + y^3 = -xe^{-x} - e^{-x} \tag{27}$$

Solving this for y we get the answer

$$y = \sqrt[3]{e^{-3e^{-x}(x+1)} - 1} \tag{28}$$

3.3 Equation 4 linear

$$y' - 3y \cos x = e^{2x + 3 \sin x} \tag{29}$$

with the equation on the linear form we need to find a primitive to the function $3 \cos x$ which is the coefficient in front of y

$$\int 3 \cos x = -3 \sin x \tag{30}$$

Then using the general solution for linear first order ode we get the expression

$$y(x) = e^{2\sin x}\left(\int e^{-3\sin x} e^{2x+3\sin x} dx\right) \tag{31}$$

If we simplify and solve the integral we get

$$y(x) = e^{3\sin x}\left(\frac{e^{2x}}{2} + c\right) \tag{32}$$