Homework 2 GoA 3

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Flux

In order to compute the flux of the vectorfield $F(x, y, z) = (x^4z, x^3y^2, 2x^3yz)$ over the box defined by $0 \ge x \ge 1, 0 \ge y \ge 2, 0 \ge z \ge 1$ we use the divergens theorem since we have a vectorfield we can easily calculate the divergens of and a surface that encloses a simple volume the resulting tripple integral will be simple.

We begin by calculating the divergens of the field F

$$\frac{\partial F_x}{\partial x} = 4x^3 z \tag{1}$$

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$$\frac{\partial F_y}{\partial y} = 2yx^3 \tag{2}$$

$$\frac{\partial F_z}{\partial z} = 2x^3 y \tag{3}$$

$$\nabla \cdot \mathbf{F} = 4x^3 (y+z) \tag{4}$$

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$$\iiint_D 4x^3(y+z)dxdydz \tag{5}$$

Where D is the box mentioned above. Using Fubinis theorem we can rewrite it as a iterated integral

$$\int_0^1 dz \int_0^2 dy \int_0^1 4x^3 (y+z) dx \tag{6}$$

$$\int_0^1 dz \int_0^2 \left[x^4 (y+z) \right]_0^1 dy = \int_0^1 dz \int_0^2 y + z dy \tag{7}$$

$$\int_{0}^{1} \left[\frac{y^{2}}{2} + yz \right]_{0}^{2} dz = \int_{0}^{1} 2 + 2zdz$$

$$\left[2z + z^{2} \right]_{0}^{1} = 3$$
(8)

$$[2z + z^2]_0^1 = 3 (9)$$

Since the divergence theorem gives the wrong orientation we flip the sign and get that the flux over the box is -3

Series