Cairo University
Faculty of Engineering

Department of Computer Engineering

ELC 325B – Spring 2023

Digital Communications

Assignment #2

Submitted to

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Submitted by

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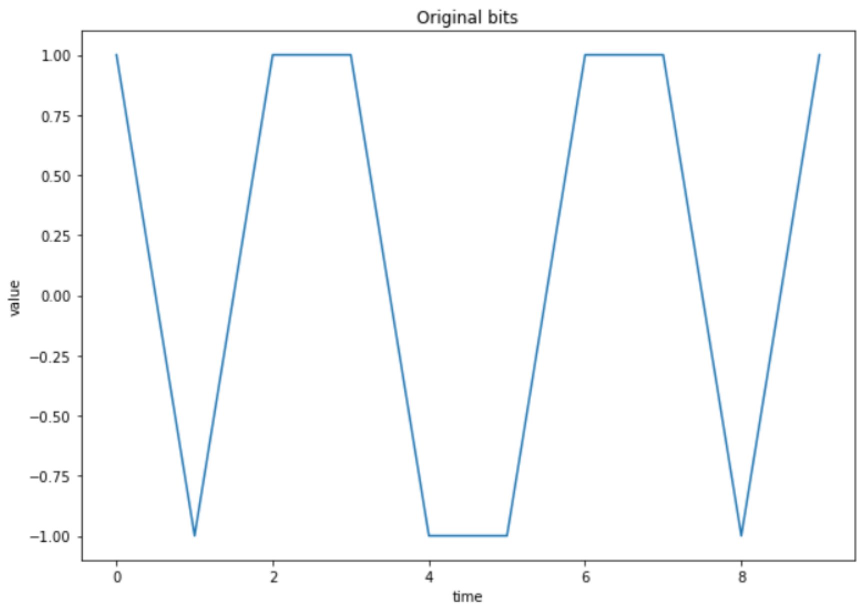
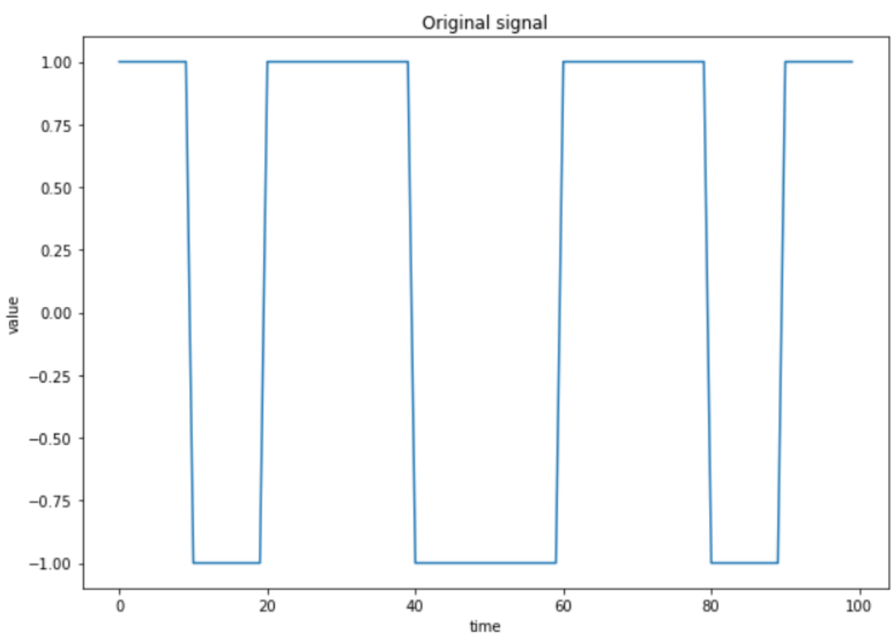
Part 2:

Figures:

Test for no of bits = 10, each bit from no of samples = 10

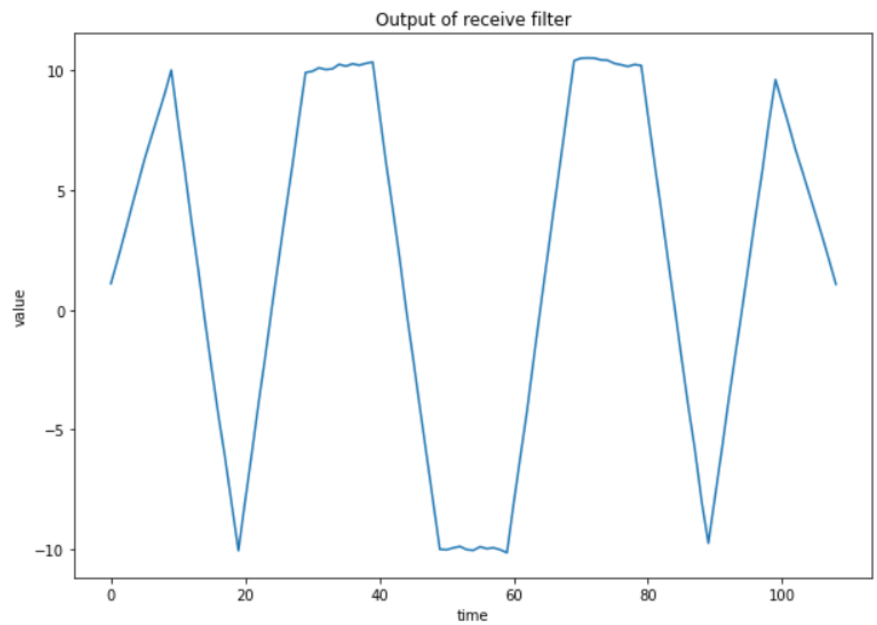
Original signal:

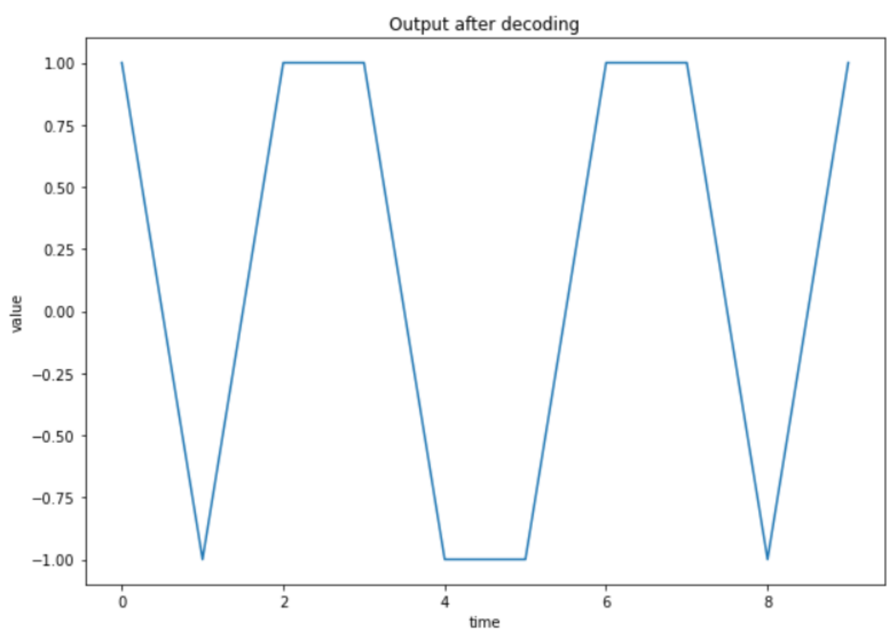




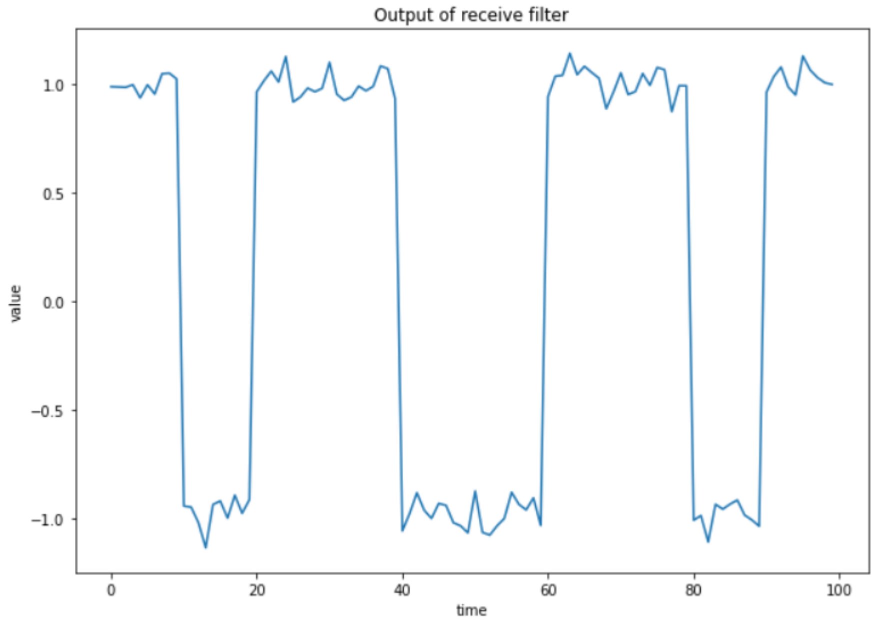
Output of the receive filter:

Case 1:

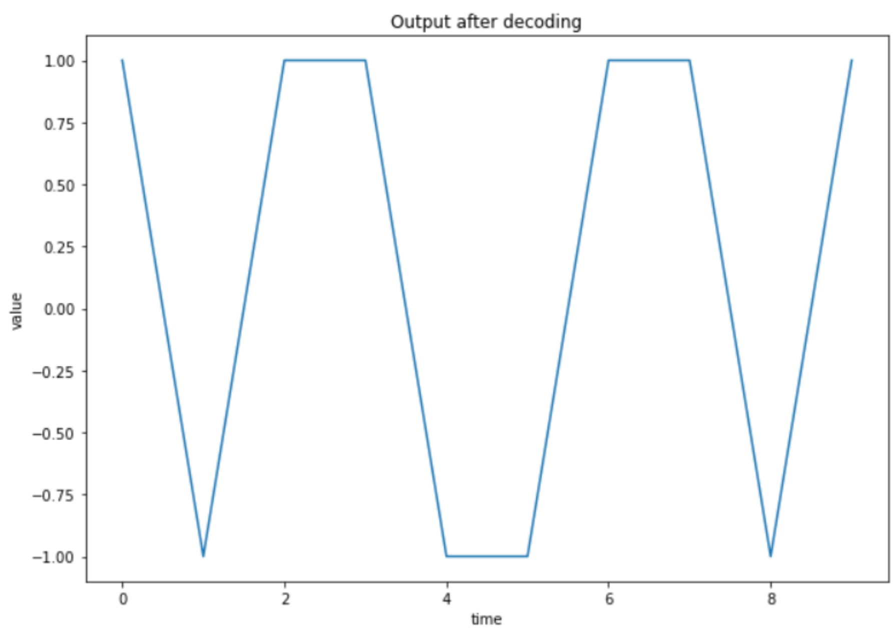




Case 2:

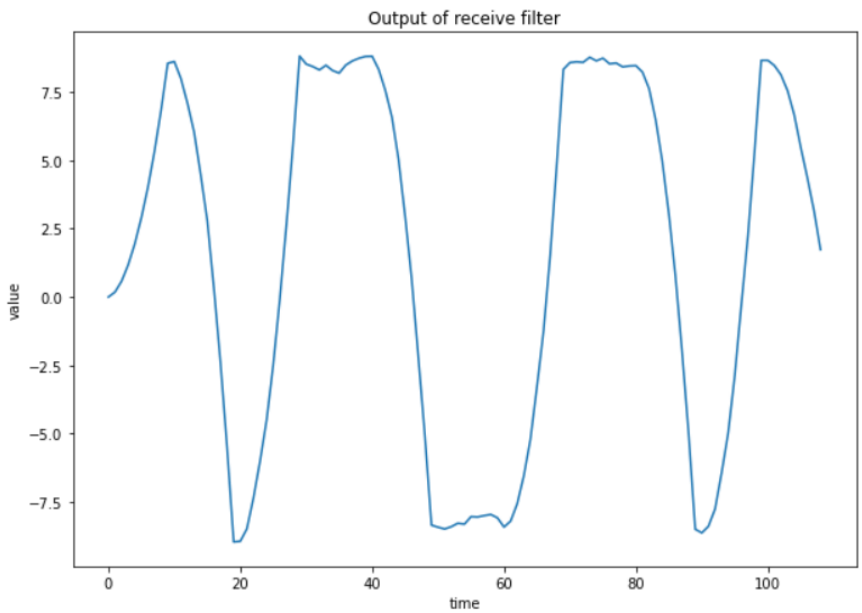


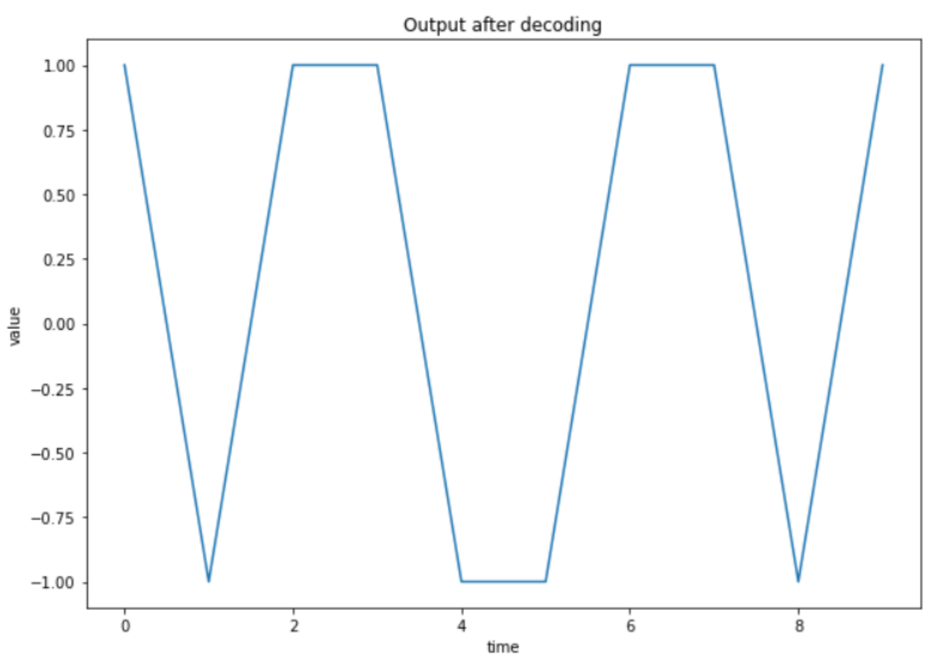
Output of receive filter
value
time
1.0
0.5
0.0
−0.5
−1.0
0
20
40
60
80
100



Output after decoding
value
time
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
2
4
6
8

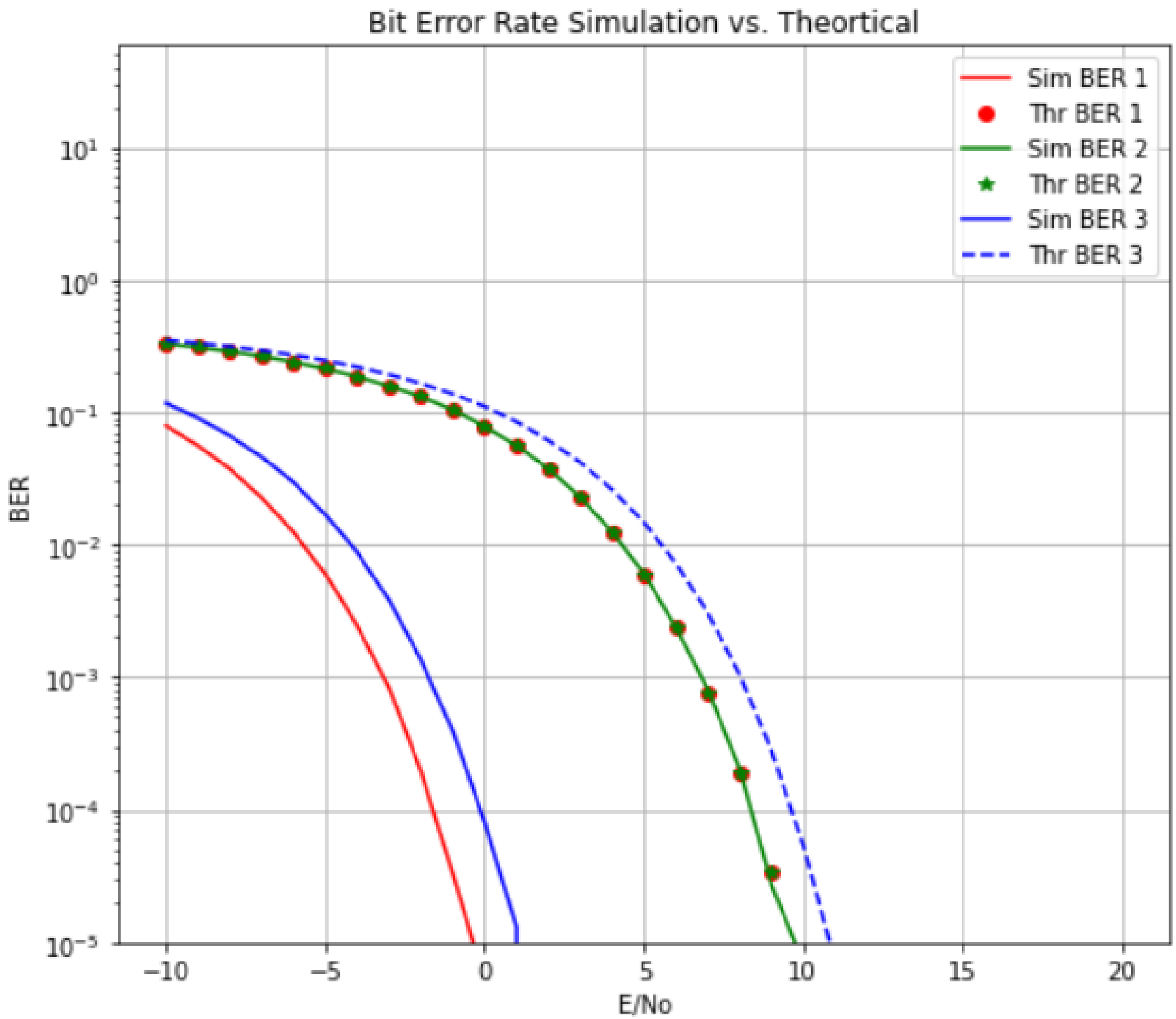
Case 3:


Output of receive filter
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
value
0
20
40
60
80
100
time


Output after decoding
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
value
0
2
4
6
8
time

Simulation & Theoretical BER vs. E/No:

Test for no of bits = 1000000, each bit from no of samples = 10



Note:

Thr BER 1 = Thr BER 2

avg sim BER 1 = 0.007015290322580645

avg sim BER 2 = 0.07850425806451612

avg sim BER 3 = 0.01229625806451613

min avg sim BER = 0.007015290322580645 -> case 1 of matched filter

avg sim BER 1 < avg sim BER 3 < avg sim BER 2

5. Is the BER an increasing or a decreasing function of E/No? Why?
increasing E/No -> decreasing BER

-> The sequence:
increase E/No
decrease No
decrease sigma of gaussian noise -> make gaussian noise distribution narrower
increase the transmitted energy to noise energy (increase SNR)
then noise didn't affect transmitted signal a lot
and make it easier to detect transmitted bits

-> The relationship between E/No ratio and BER is not linear, but logarithmic
BER decreases exponentially as the E/No ratio increases

6. Which case has the lowest BER? Why?
Case 1: The receive filter $h(t)$ is a matched filter with unit energy
it gives the lowest average simulation BER

because of that it uses a matched filter
which is designed to match the characteristics of the transmitted signal
and this makes the filter provides the best detection strategy
by correlating the received signal with a replica of the transmitted signal
so as a result the BER will be reduced and the SNR will be maximized

Code:

```
import numpy as np
import matplotlib.pyplot as plt
import math
#############################################################

# generate random noise with gaussian distribution
def generate_gaussian_noise(mu, sigma, bits_no,bit_samples_no):
   return np.random.normal(mu, sigma, bits_no * bit_samples_no)
#############################################################

# generate random bits with values {-1,1} with probability = 0.5
def generate_random_bits(bits_no):
   return np.random.choice(a = [-1,1], size=bits_no, p=[1./2, 1./2])
#############################################################

def add_gaussian_noise(bits_no, bit_samples_no, signal_bits, noise):

   size = bits_no * bit_samples_no
   signal_samples=np.zeros(size)
   noisy_signal_samples=np.zeros(size)

    # repeat the bit with no of samples per bit
   for i in range(bits_no):
      # 0:9 10:19 20:39
      signal_samples[i * bit_samples_no : (i + 1) * bit_samples_no] = signal_bits[i]

   # add signal to the noise
   noisy_signal_samples = signal_samples + noise

   return signal_samples,noisy_signal_samples
#############################################################

def convolution(noisy_signal_samples,receive_filter,bits_no,bit_samples_no):

   size = bits_no * bit_samples_no
   result_samples = np.zeros(size)
   result_bits = np.zeros(bits_no)
   decoded_result_bits = np.zeros(bits_no)
```

```
    # convolution between signal and receive filter to get filter output
    if(receive_filter is None):
        result_samples = noisy_signal_samples
    else:
        result_samples = np.convolve(noisy_signal_samples, receive_filter)

    # sampling at T
    for i in range(bits_no):
        # 0:9->9 10:19->19 20:29->29
        result_bits[i]=result_samples[i * bit_samples_no + bit_samples_no - 1]

    # decoding to {-1,1}
    decoded_result_bits = np.sign(result_bits)

    return result_samples,result_bits,decoded_result_bits
##########################################################

def calc_simulation_error(signal_bits,decoded_result_bits,bits_no):
    error = np.sum(signal_bits != decoded_result_bits) / bits_no
    return error
##########################################################

def calc_theoretical_error(z):
     return math.erfc(z)
##########################################################

def test(receive_filter, thr_error_coeff, is_plot):
    sim_error, thr_error = [], []

    for E_No_db in range(-10, 21):
        E_No = 10 ** (E_No_db / 10) #get E/No from its db value
        E = 1
        sigma = np.sqrt(E/(2*E_No))

        noise = generate_gaussian_noise(0, sigma, bits_no, bit_samples_no)
        signal_samples, noisy_signal_samples = add_gaussian_noise(bits_no, bit_samples_no, signal_bits,
noise)
        result_samples,result_bits,decoded_result_bits = convolution(noisy_signal_samples, receive_filter,
bits_no, bit_samples_no)
```

```
        x = calc_simulation_error(signal_bits,decoded_result_bits,bits_no)
        y = 0.5 * calc_theoretical_error(thr_error_coeff * (E_No ** 0.5))
        sim_error.append(x)
        thr_error.append(y)

    if(is_plot):
        plt.figure(figsize=(10,7))
        plt.plot(range(0,len(signal_bits)), signal_bits)
        plt.title("Original bits")
        plt.xlabel('time')
        plt.ylabel('value')

        plt.figure(figsize=(10,7))
        plt.plot(range(0,len(signal_samples)), signal_samples)
        plt.title("Original signal")
        plt.xlabel('time')
        plt.ylabel('value')

        plt.figure(figsize=(10,7))
        plt.plot(range(0,len(result_samples)), result_samples)
        plt.title("Output of receive filter")
        plt.xlabel('time')
        plt.ylabel('value')

        plt.figure(figsize=(10,7))
        plt.plot(range(0,len(decoded_result_bits)), decoded_result_bits)
        plt.title("Output after decoding")
        plt.xlabel('time')
        plt.ylabel('value')

    return sim_error, thr_error
#########################################################

bits_no,bit_samples_no = 10 , 10
size = bits_no * bit_samples_no
signal_bits = generate_random_bits(bits_no)

receive_filter1 = np.ones(bit_samples_no)
receive_filter2 = None
receive_filter3 = np.linspace(0, 1, bit_samples_no) * np.sqrt(3)
```

```
sim_error1, thr_error1, sim_error2, thr_error2, sim_error3, thr_error3 = [], [], [], [], [], []

sim_error1, thr_error1 = test(receive_filter1, 1, True)
sim_error2, thr_error2 = test(receive_filter2, 1, True)
sim_error3, thr_error3 = test(receive_filter3, (3**0.5/2), True)
##########################################################

# test to simulate error
bits_no,bit_samples_no = 1000000 , 10
signal_bits = generate_random_bits(bits_no)

sim_error1, thr_error1 = test(receive_filter1, 1, False)
sim_error2, thr_error2 = test(receive_filter2, 1, False)
sim_error3, thr_error3 = test(receive_filter3, (3**0.5/2), False)

# plot
plt.figure(figsize=(8,7))
x_axis_range = range(-10, 21)

plt.semilogy(x_axis_range,sim_error1,'r-')
plt.semilogy(x_axis_range,thr_error1,'ro')
plt.semilogy(x_axis_range,sim_error2,'g-')
plt.semilogy(x_axis_range,thr_error2,'g*')
plt.semilogy(x_axis_range,sim_error3,'b-')
plt.semilogy(x_axis_range,thr_error3,'b--')

plt.title('Bit Error Rate Simulation vs. Theortical')
plt.legend(['Sim BER 1', 'Thr BER 1', 'Sim BER 2', 'Thr BER 2', 'Sim BER 3', 'Thr BER 3'])
plt.xlabel('E/No')
plt.ylabel('BER')
plt.yscale('log')
plt.ylim(10**(-4))
plt.grid()
plt.show()
```

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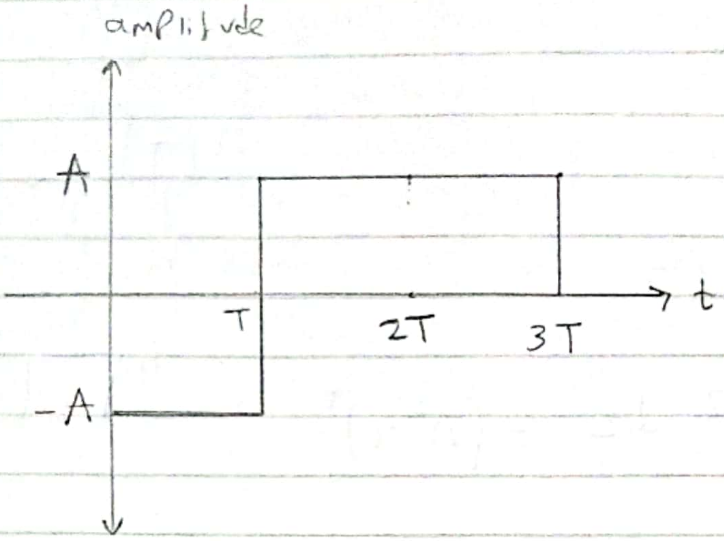
Part I

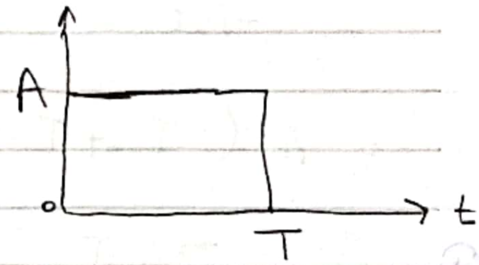
(a)

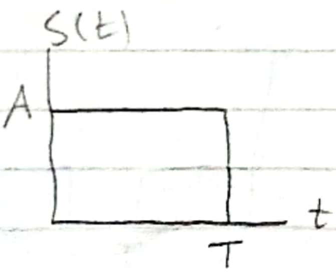


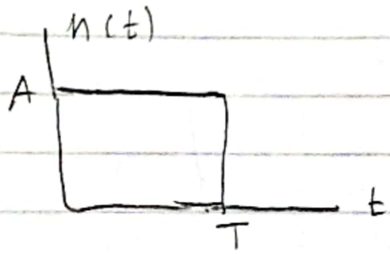
(b) $g(t) = A\,\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right)$

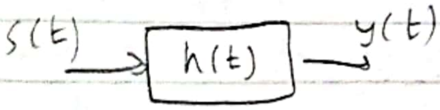
$h(t) = g(T-t) = A\,\mathrm{rect}\left(\frac{T-t-\frac{T}{2}}{T}\right) = h(t)$

$h(t) = A\,\mathrm{rect}\left(\frac{\frac{T}{2}-t}{T}\right)$



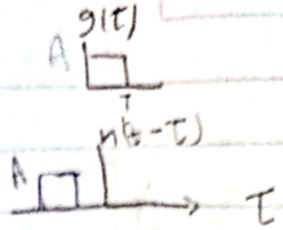






$y(t) = s(t) * h(t)$

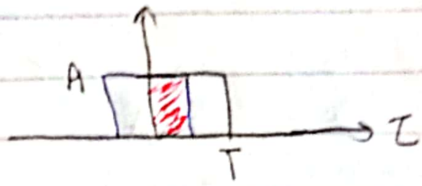
$$y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$



* for $t < 0$

$y(t) = 0$

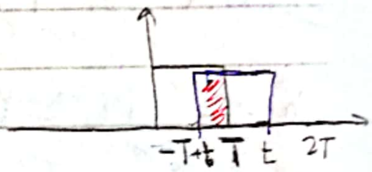
* for $0 < t < T$



$$y(t) = \int_0^t A^2 \, d\tau = \left(A^2 \tau\right)_0^t$$

$$= A^2 t$$

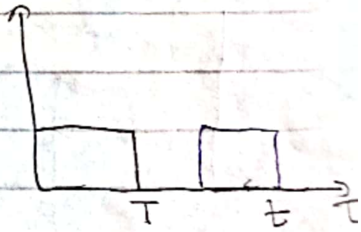
* for $T < t < 2T$



$$y(t) = \int_{-T+t}^{T} A^2 \, d\tau = \left[A^2 \tau\right]_{-T+t}^{T}$$

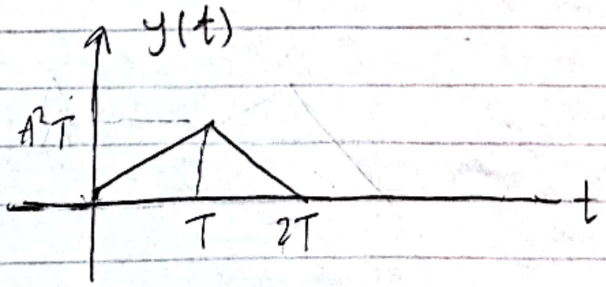
$$= A^2(T + T - t) = A^2(2T - t)$$

* for $t > 2T$



$y(t) = 0$

$$y(t) = \begin{cases} 0 & t < 0 \\ A^2 t & 0 < t < T \\ A^2(2T - t) & T < t < 2T \\ 0 & t > 2T \end{cases}$$



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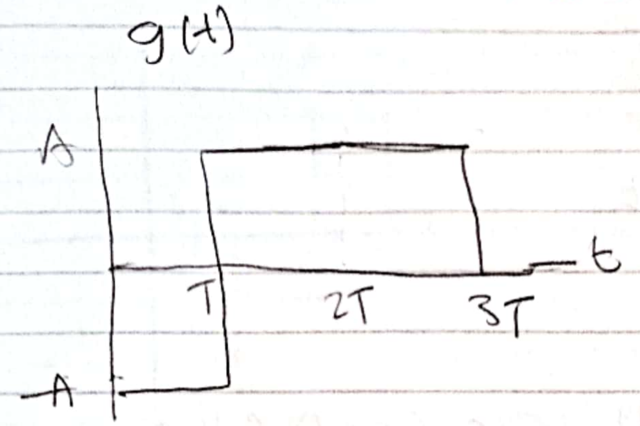
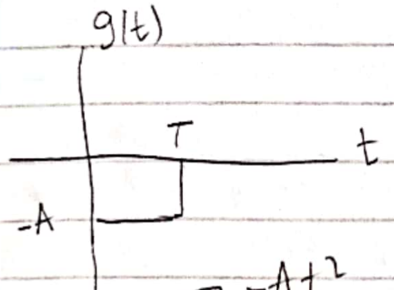
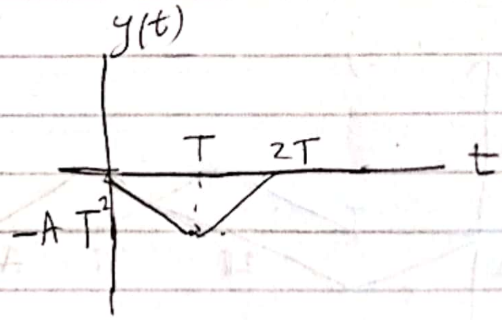


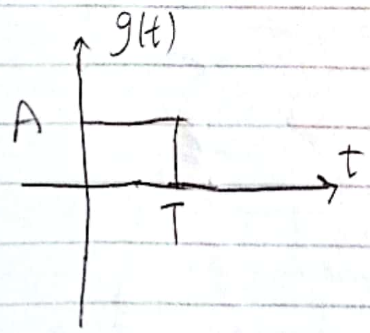
from the last result

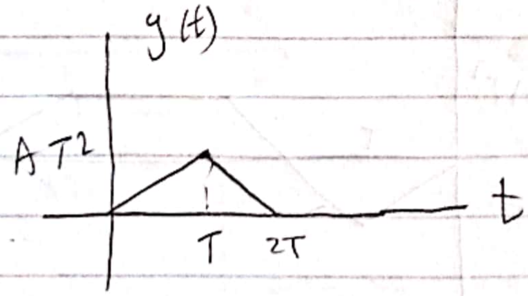


$\Longrightarrow$



$$y(t) = \begin{cases} -At^2 & 0 < t < T \\ -A^2(2T-t) & T < t < 2T \\ 0 & \text{otherwise} \end{cases}$$





$$y(t) = \begin{cases} At^2 & 0 < t < T \\ A^2(2T-t) & T < t < 2T \\ 0 & \text{otherwise} \end{cases}$$

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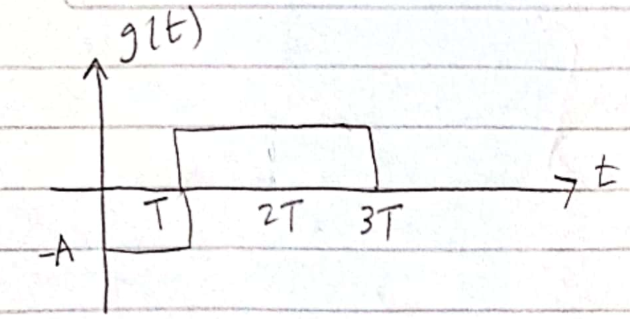
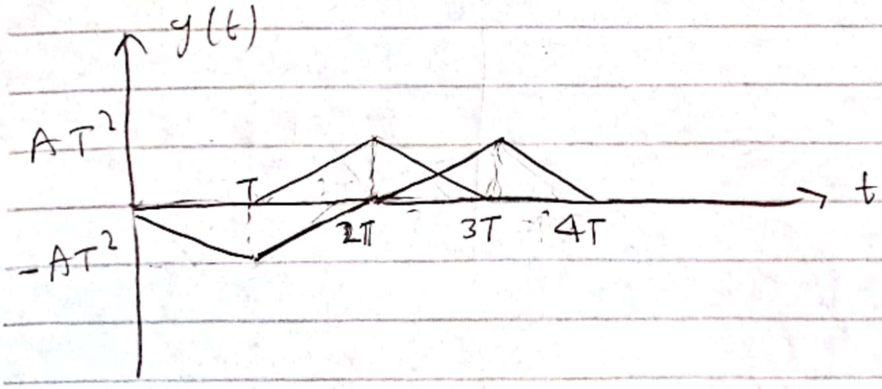
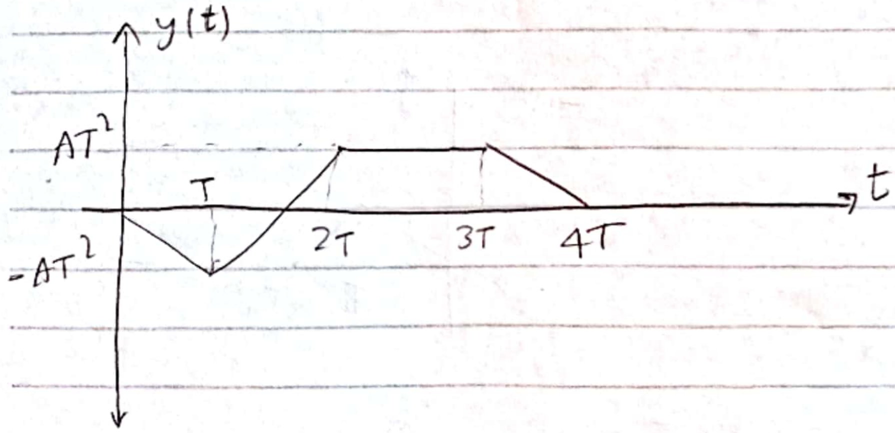


for this signal by applying super position to get y(t)



by applying summitions



this is the output of the matched filter

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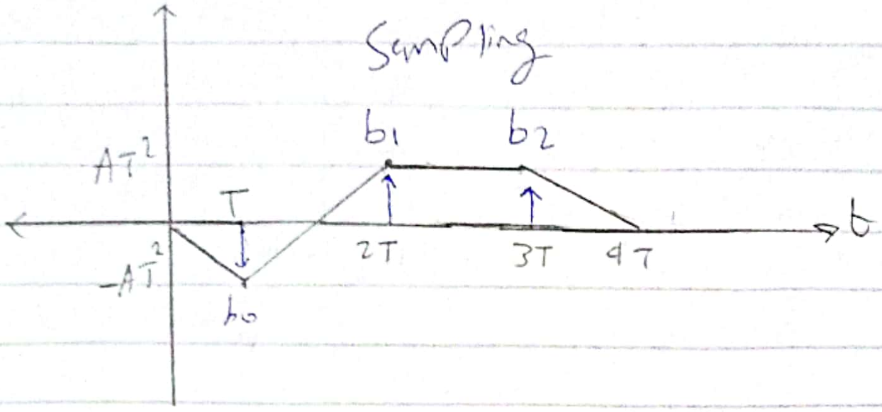
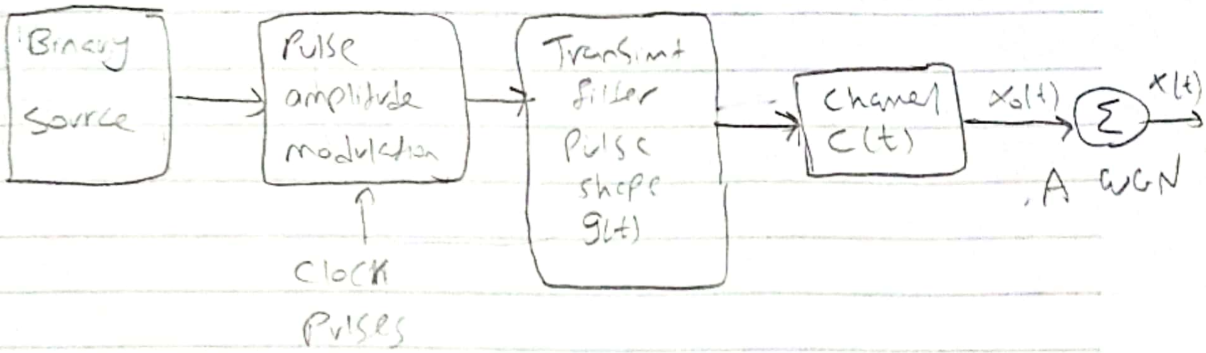
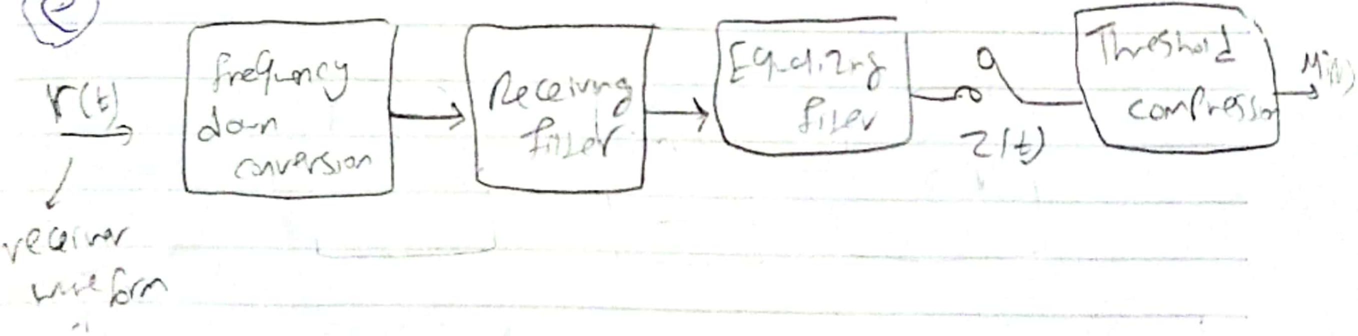
c



d



e

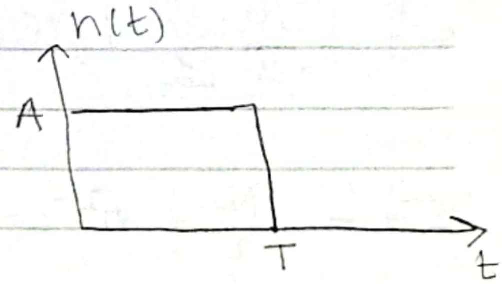


Part II $\quad g(t) = \begin{cases} -A \\ A, & 0 < t < T \end{cases}$

ⓐ the receiver filter $h(t)$ is a matched filter with unit energy

$$g(t) = A \,\text{rect}\left(\frac{t - \frac{T}{2}}{T}\right) \qquad r(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$h(t) = g(T-t) = A\,\text{rect}\left(\frac{T - t - \frac{T}{2}}{T}\right) = A\,\text{rect}\left(\frac{\frac{T}{2} - t}{T}\right)$$



$r(t) = g(t) + w(t)$

$y(t) = r(t) * h(t)$

$y(t) = g(t) * h(t) + w(t) * h(t)$

$\quad = g_o(t) + n(t)$

from part Ⓘ ⓑ

$$g_o(t) = \begin{cases} 0 & \text{otherwise} \\ A^2 t & 0 < t \leq T \\ A^2(2T - t) & T < t \leq 2T \end{cases}$$

$$\therefore y(T) = \begin{cases} -A^2 T + n(T) & \text{"0"} \\ A^2 T + n(T) & \text{"1"} \end{cases}$$

$N_y = E(y(T)) = E(g_o(T)) + E(n(T))$

$N_y = E(\pm A^2 T + n(T)) = \pm A^2 T + E(n(T))$

$E(n(T)) = E\left(\int w(\tau)\, h(T - \tau)\, d\tau\right)$

$\quad = E\left(\int w(\tau)\, g(\tau)\, d\tau\right) =$

$E\left(\int_0^T \pm A\, w(\tau)\, d\tau\right) = \int_0^T \pm A\, E(w(\tau))\, d\tau = 0$

$\therefore N_y = \pm A^2 T$

$$\therefore N_y = \begin{cases} -A^2 T & (0) \\ A^2 T & (1) \end{cases}$$

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$$\sigma_y^2 = Var(y(T)) = E(g_0(T) + n(T))$$

$$\sigma_y^2 = var(n(T)) = E(n^2(T)) - E(\underbrace{n(T)}_{0})^2$$

$$\sigma_y^2 = E(n^2(T)) = \int_{-\infty}^{\infty} S_n(f)\, df$$

$$\sigma_y^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0}{2} \int_0^T |h(t)|^2\, dt$$

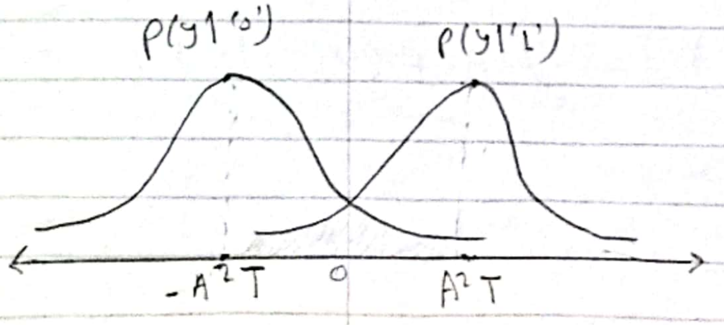
$$\sigma_y^2 = \frac{N_0}{2} * A^2 T = \frac{N_0 A^2 T}{2}$$

$$p(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu_y)^2}{2\sigma^2}}$$

$$p(y\,|\,'0') = \frac{1}{\sqrt{2\pi * \frac{N_0 A^2 T}{2}}} e^{-\frac{(y+A^2T)^2}{2 \times \frac{N_0 A^2 T}{2}}}$$

$$p(y\,|\,'0') = \frac{1}{\sqrt{\pi N_0 A^2 T}} e^{-\frac{(y+A^2T)^2}{N_0 A^2 T}}$$

$$p(y\,|\,'1') = \frac{1}{\sqrt{\pi N_0 A^2 T}} e^{-\frac{(y-A^2T)^2}{N_0 A^2 T}}$$



$$P(e|'0') = \int_0^\infty \frac{1}{\sqrt{\pi N_0 A^2 T}} e^{\frac{-(y+A^2T)^2}{N_0 A^2 T}} dy$$

let $z = \frac{y + A^2T}{\sqrt{N_0 A^2 T}}$

$$dz = \frac{dy}{\sqrt{N_0 A^2 T}}$$

$$P(e|'0') = \int_{\frac{A^2T}{\sqrt{N_0A^2T}}}^{\infty} \frac{1}{\sqrt{\pi}\cancel{\sqrt{N_0A^2T}}} \times e^{-z^2} \times \cancel{\sqrt{N_0A^2T}}\, dz$$

$$P(e|'0') = \int_{\frac{A^2T}{\sqrt{N_0A^2T}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = \frac{1}{2} \operatorname{erfc}\left(\frac{A^2T}{\sqrt{N_0 A^2 T}}\right)$$

$$P(e) = P(e|'1')\,P('1') + P(e|'0')\,P('0')$$

$$P('1') = P('0') = 0.5$$

$$\therefore P(e) = 2\,P(e|'0') \times 0.5 = P(e|'0')$$

$$\therefore P(e) = \frac{1}{2} \operatorname{errfc}\left(\frac{A^2T}{\sqrt{N_0 A^2 T}}\right)$$

$A = 1$, $T = 1$

$$\therefore P(e) = \frac{1}{2} \operatorname{errfc}\left(\frac{1}{\sqrt{N_0}}\right)$$

(b) the receive filter $h(t)$ is not existent $(h(t) = \delta(t))$

$y(t) = r(t) * h(t) = r(t) * \delta(t) = y(t)$

convolution with δ gives the same signal

$y(T) = g(T) + w(T)$

$y(T) = \pm A + w(T)$

$E(y(T)) = E(\pm A + w(T)) = \pm A + E(w(T))$ $\to 0$

$$N_y = \begin{cases} -A & ('0') \\ A & ('1') \end{cases}$$

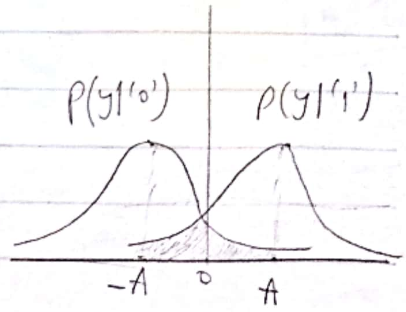
$\sigma_y^2 = Var(y(T)) = Var(\pm A + w(T)) = Var(w(T))$

$\sigma_y^2 = var(w(T)) = \frac{N_0}{2}$

$p(y|'0') = \frac{1}{\sqrt{N_0 \pi}} e^{-\frac{(y+A)^2}{N_0}} = N(A, \frac{N_0}{2})$

$p(y|'1') = \frac{1}{\sqrt{N_0 \pi}} e^{\frac{-(y-A)^2}{N_0}} = N(-A, \frac{N_0}{2})$

Due to symmetry $\lambda = 0$, $P('0') = P('1')$



$P(e|'0') = \int_0^\infty p(y|'0')\, dy$

$= \int_0^\infty \frac{1}{\sqrt{N_0 \pi}} e^{\frac{-(y+A)^2}{N_0}} dy$

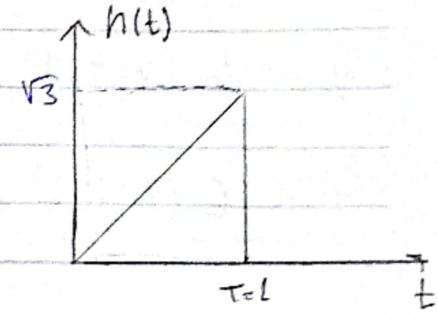
let $z = \frac{y+A}{\sqrt{N_0}}$, $dz = \frac{dy}{\sqrt{N_0}}$

$p(e|'0') = \int_{\frac{A}{\sqrt{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}\sqrt{N_0}} \times e^{-z^2} \times \sqrt{N_0}\, dz = \frac{1}{2} erfc\left(\frac{A}{\sqrt{N_0}}\right)$

$P(e) = p(e|'0') \times P('0') + P(e|'1')\, P('1') = 2P(e|'0') \times 0.5 =$

$P(e|'0') = \frac{1}{2} erfc\left(\frac{A}{\sqrt{N_0}}\right) = \frac{1}{2} erfc\left(\frac{1}{\sqrt{N_0}}\right)$

(c) The receive filter $h(t)$ has the following impulse response



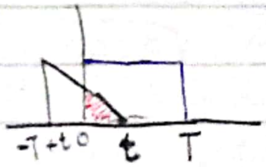
$$y(t) = r(t) * h(t) = g_o(t) + n(t)$$

$$g_o(t) = g(t) * h(t)$$

$$g_o(t) = \int_{-\infty}^{\infty} g(\tau)\, h(t-\tau)\, d\tau$$

* for $t < 0$

$$g_o(t) = 0$$



* for $0 < t \leq T$

$$g_o(t) = \int_0^t \sqrt{3} A \tau\, d\tau = \sqrt{3} A \left[\frac{\tau^2}{2}\right]_0^t = \frac{\sqrt{3}}{2} A t^2$$

$$= \sqrt{3} A t$$

$$y(T) = \begin{cases} -\frac{\sqrt{3}}{2} A T^2 + n(T) & \text{'0'} \\ \frac{\sqrt{3}}{2} A T^2 + n(T) & \text{'1'} \end{cases}$$

$$\mu_y = E(y(T)) = E(g_o(T)) + E(n(T))$$

$$\mu_y = E\left(\pm \frac{\sqrt{3}}{2} A T^2 + n(T)\right) = \pm \frac{\sqrt{3}}{2} A T^2 + E(n(T))$$

$$E(n(T)) = E\left(\int w(\tau) g(\tau)\, d\tau\right)$$

$$= E\left(\int_0^T \pm A\, w(\tau)\, d\tau\right) = \int_0^T \pm A\, E(w(\tau))\, d\tau = 0$$

$$\therefore \mu_y = \begin{cases} -\frac{\sqrt{3}}{2} A T^2 & \text{'0'} \\ \frac{\sqrt{3}}{2} A T^2 & \text{'1'} \end{cases}$$

$$\sigma_y^2 = Var(y(T)) = E(g_0(T) + n(T)) = E(\pm\frac{\sqrt{3}}{2}AT^2 + n(T))$$

$$\sigma_y^2 = Var(n(T)) = E(n^2(T)) + E(n(T))^2$$

$$\sigma_y^2 = E(n^2(T)) = \int_{-\infty}^{\infty} S_n(f)\,df$$

$$\sigma_y^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df = \frac{N_0}{2}\int_0^T |h(t)|^2 dt$$

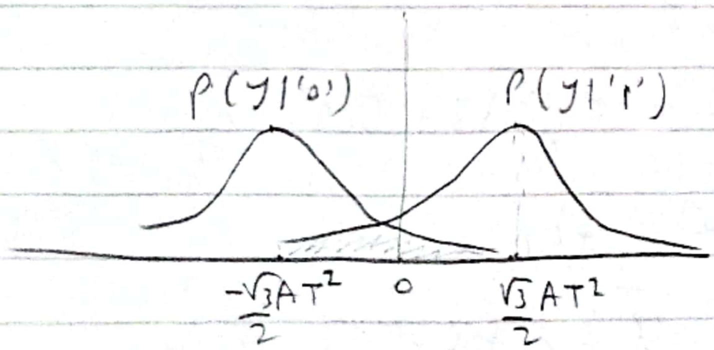
$$\sigma_y^2 = \frac{N_0}{2}\int_0^T 3t^2\,dt = \frac{N_0}{2}\left[3\frac{t^3}{3}\right]_0^T = \frac{N_0}{2}T^3$$

$$P(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-N)^2}{2\sigma^2}}$$

$$P(y|'0') = \frac{1}{\sqrt{2\pi \times \frac{N_0T^3}{2}}} e^{-\frac{(y+\frac{\sqrt{3}}{2}AT^2)}{N_0T^3}}$$

$$P(y|'0') = \frac{1}{\sqrt{\pi N_0 T^3}} e^{\frac{-(y+\frac{\sqrt{3}}{2}AT^2)}{N_0T^3}}$$

$$P(y|'1') = \frac{1}{\sqrt{\pi N_0 T^3}} e^{\frac{-(y-\frac{\sqrt{3}}{2}AT^2)}{N_0T^3}}$$



$$p(e|'0') = \int_0^\infty \frac{1}{\sqrt{\pi N_o T^3}} e^{-\frac{(y+\frac{\sqrt{3}}{2}AT^2)}{N_o T^3}}$$

$$\text{let } z = \frac{y+\frac{\sqrt{3}}{2}AT^2}{\sqrt{N_o T^3}}$$

$$dz = \frac{dy}{\sqrt{N_o T^3}}$$

$$p(e|'0') = \int_{\frac{\frac{\sqrt{3}}{2}AT^2}{\sqrt{N_o T^3}}}^{\infty} \frac{1}{\sqrt{\pi}\cdot\cancel{\sqrt{N_o T^3}}} e^{-z} \cancel{\sqrt{N_o T^3}}\, dz$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{\frac{\sqrt{3}}{2}AT^2}{\sqrt{N_o T^3}}\right)$$

assume $p('0') = p('1')$

$$p(e) = p(e|'0') = \frac{1}{2} \operatorname{erfc}\left(\frac{\frac{\sqrt{3}}{2}AT^2}{\sqrt{N_o T^3}}\right)$$

$$A = 1,\ T = 1$$

$$p(e) = p(e|'0') = \frac{1}{2} \operatorname{erfc}\left(\frac{\frac{\sqrt{3}}{2}}{\sqrt{N_o}}\right)$$