Optics in Matlab
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I. Problem: Diffraction of a slit

Use the waveamp.m matlab script (uploaded on the web site) to study the diffraction of a slit that has the width of w You can start with w = 10 µm and λ = 630 nm and you can let the light propagate to a d = 200 µm distance.

- Show that approximately, the image (diffraction pattern) of the slit is a Fourier transform, i.e. the U(x0) diffraction pattern is a Fourier transform of the U(x) aperture function with $fx = \frac{x_0}{\lambda d}$. Spatial frequency components (there is a 1 scale factor between the spatial and frequency domain). Show how the validity of the approximation changes as we go further from the slit or change the wavelength. See the Figure 1. for some hint.
- Show that you can actually generate Fourier transform of different U(x) aperture functions, check how accurate this hardware Fourier transform is.

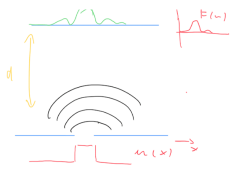

Fig. 1: Representation of task.

Solution:

My results of the MATLAB simulation for the mechanical Fourier transform and the calculated Fourier transform of the slit with varying width are displayed in Figure 2.-6. The mechanical Fourier transform was determined using the waveamp.m function to calculate the light intensity from a light source going through a slit. The result of that function can be seen in the upper image of each figure. The mechanical Fourier transform can be created by selecting a line (red line on the image) or circular section (black curve on the image) of these images at a certain distance. The mechanical Fourier transform is shown in the bottom left images. The calculated Fourier transform is presented by the bottom right image of each figure. One unit represents 400 nm in the simulation. If we set the slit width to 4, 6, 8, 10... unit diffraction occurs only because the width of the slit is comparable to the wavelength (630nm) of the light. The suggested scaling factor between the mechanical and calculated Fourier transforms does not align with the expected results in the simulation.

On Figure 2.-6. it can be seen that using smaller slit widths result in fewer harmonics appearing in the signal. The number of harmonics in the mechanical Fourier transform can vary depending on the selection plane's distance from the source. Further selection planes display fewer harmonics, while closer selection planes display more (comparing Figure 3. and 4.).

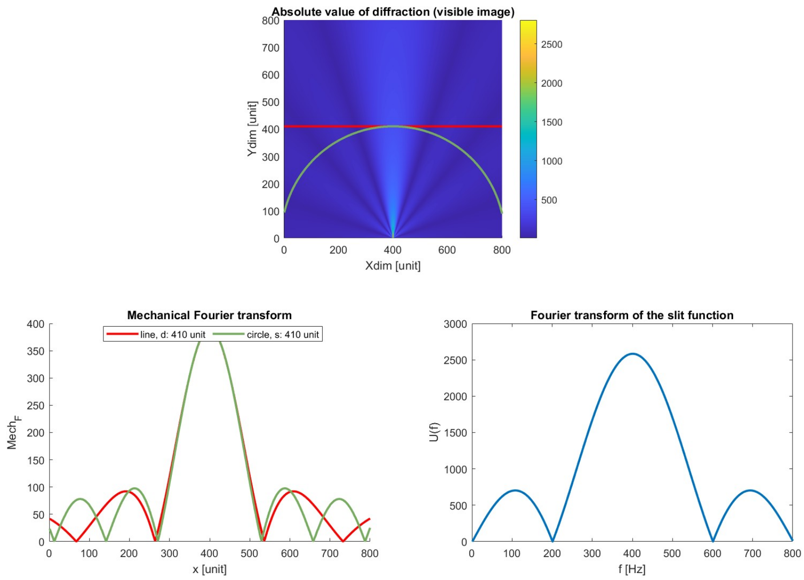

Fig. 2: Top: visible image of diffraction, Bottom left: Mechanical Fourier transform, Bottom right: Calculated Fourier Transform, using the slit width of 4 units and distance of 410 units.

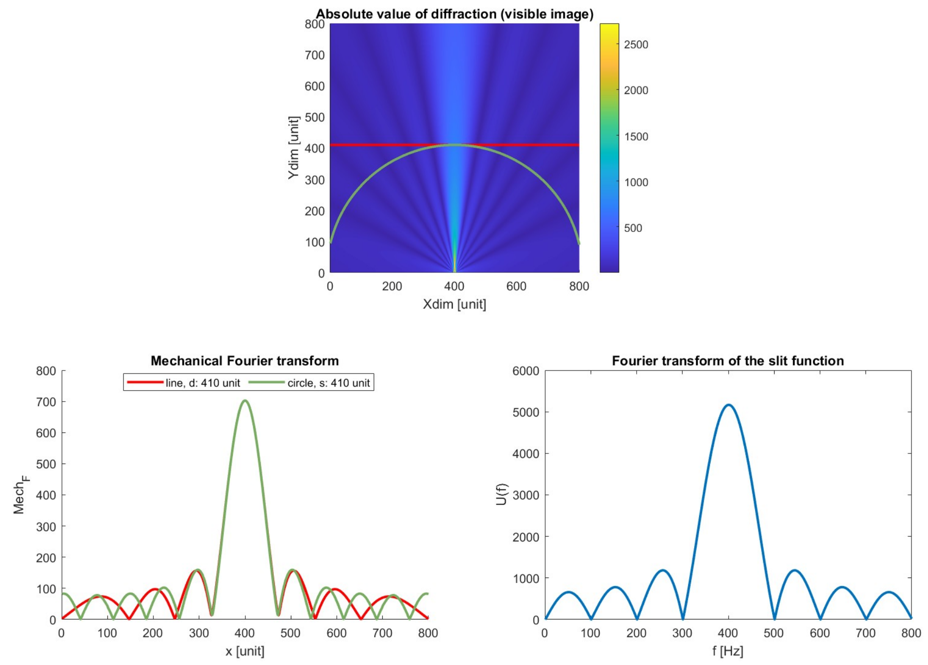

Fig. 3: Top: visible image of diffraction, Bottom left: Mechanical Fourier transform, Bottom right: Calculated Fourier Transform, using the slit width of 8 units and distance of 410 units.

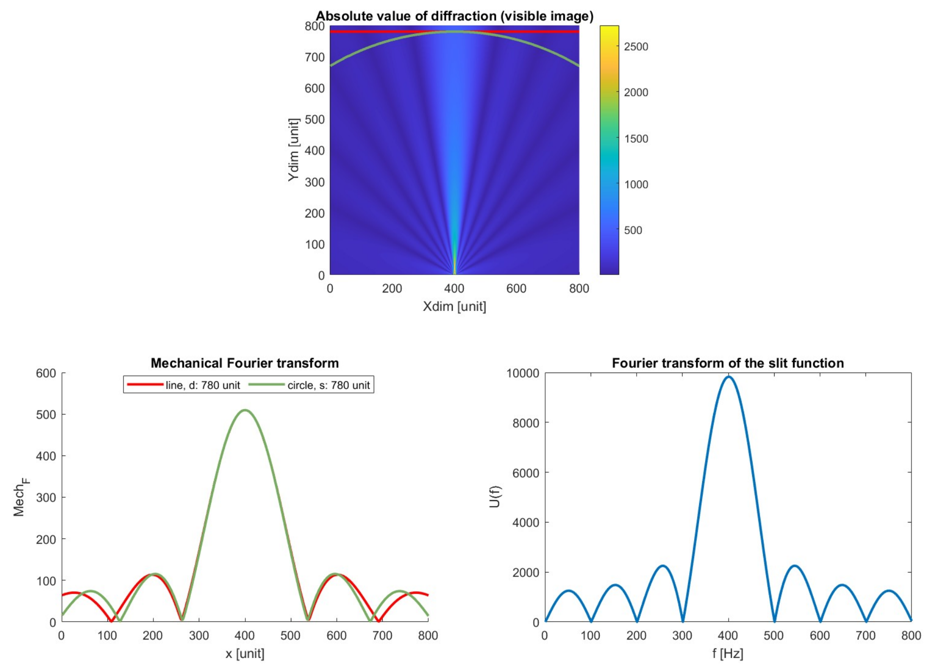

Fig. 4: Top: visible image of diffraction, Bottom left: Mechanical Fourier transform, Bottom right: Calculated Fourier Transform, using the slit width of 8 units and distance of 780 units.

II. Problem: Fermat's principle from wave propagation

Implement the numerical solution problem we discussed in class, i. e. show that Fermat's principle (principle of least time) comes out as a case of man light beams interfering with each other. Use the geometry from the figure below. Assume the light propagates with a $k_1 = 2\pi/\lambda$ wavenumber in vacuum (top half plane) and with a $k_2 = n_2 k_1$ in the bottom plane. The complex amplitude of a ray going from the source to the detector via two straight line segments is $E = e^(jk_1 s_1) \times e^(jk_2 s_2)$ (assuming a unit amplitude at the source). The beam can be traced by splitting the x axis to 1mm length segments and superposing a few thousand beams for each segment.

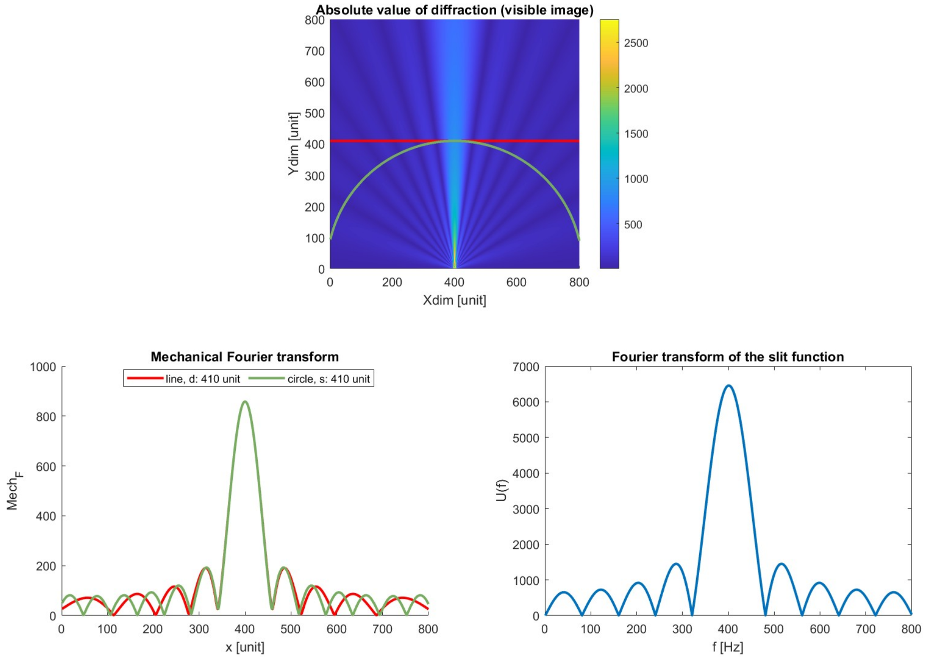

Fig. 5: Top: visible image of diffraction, Bottom left: Mechanical Fourier transform, Bottom right: Calculated Fourier Transform, using the slit width of 10 units and distance of 410 units.

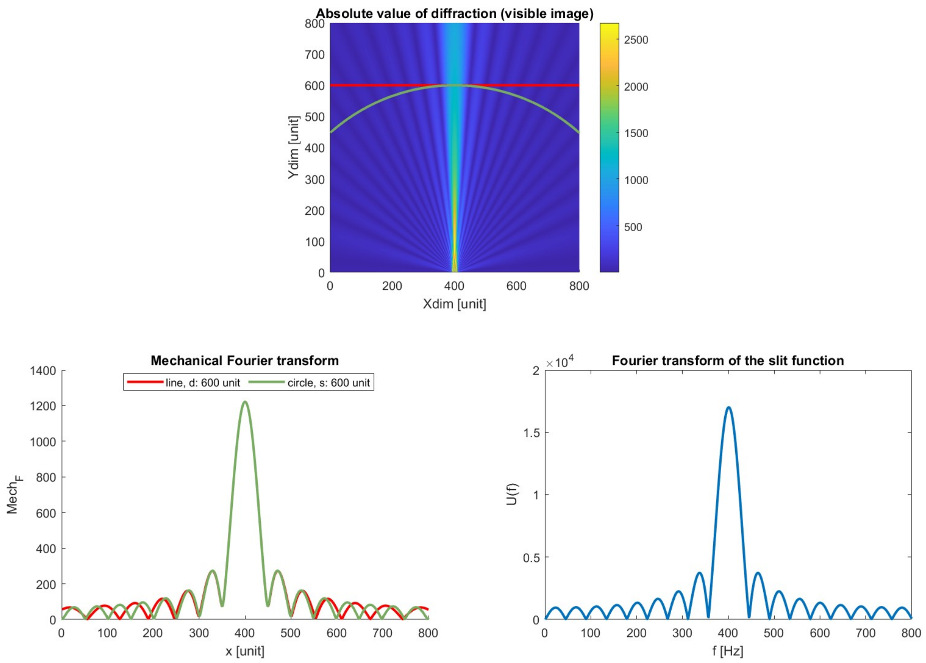

Fig. 6: Top: visible image of diffraction, Bottom left: Mechanical Fourier transform, Bottom right: Calculated Fourier Transform, using the slit width of 18 units and distance of 600 units.

(a) show that light propagates in a straight path if $n_1 = n_2 = 1$
(b) demonstrate with a few examples that the light ray refracts the way dictated by Snell's law for $n_2 > n_1$

Solution:

In Figure 8. the black line represents the border between the two media with different refractive indices. The border is divided into 100 parts, each with a width of 0.02 mm. Additionally, each of these parts is further divided into 1000 subparts. I calculated the complex amplitudes of these subparts for each of the border parts and summed them up. This resulted in a summed amplitude value for each of the 100 border parts. By plotting the absolute values of these summed amplitudes (which

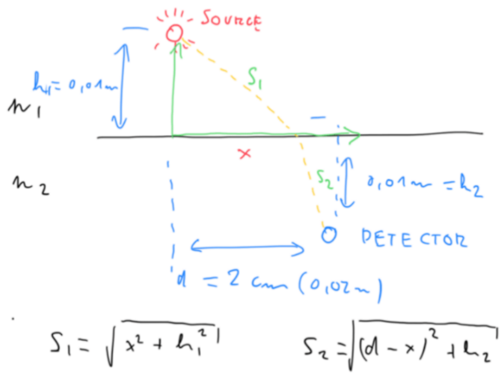

Fig. 7: Representation of task.

represent the light intensity) against the distances of the parts, we can determine the path of the light at the distance where the highest intensity value is located. Mathematical formula for the calculation:

$$
\begin{aligned}
E_{x_i} &= \sum_{n=1}^{1000} e^{jk_1 s_{1in}} e^{jk_2 s_{2in}}, \quad x_i : 1, 2, ...100, \\
s_{1in} &= \sqrt{(h_1)^2 + (n\frac{x_{i+1} - x_i}{1000} + x_i)^2}, \\
s_{2in} &= \sqrt{(h_2)^2 + (d - [n\frac{x_{i+1} - x_i}{1000} + x_i])^2}, \\
k_1 &= \frac{2\pi}{\lambda}, \\
k_2 &= \frac{n_2}{n_1} k_1,
\end{aligned}
$$

where n1, n2 are the refractive indices; h1: distance between source and border, h2: distance between border and detector; k1, k2: wavenumbers; s1, s2: the distances travelled by the light in two different media; d length of the border x; x_i is the i th $i^t h$ part of the border; λ is the wavelength of the light.

My results of the MATLAB simulation are represented by Figure 9.-11., which contain the path of the light between source and target in different media (left) and the intensity values of light in the function of distance (right) with different ratios of the refractive indices ($\frac{n2}{n1}$).

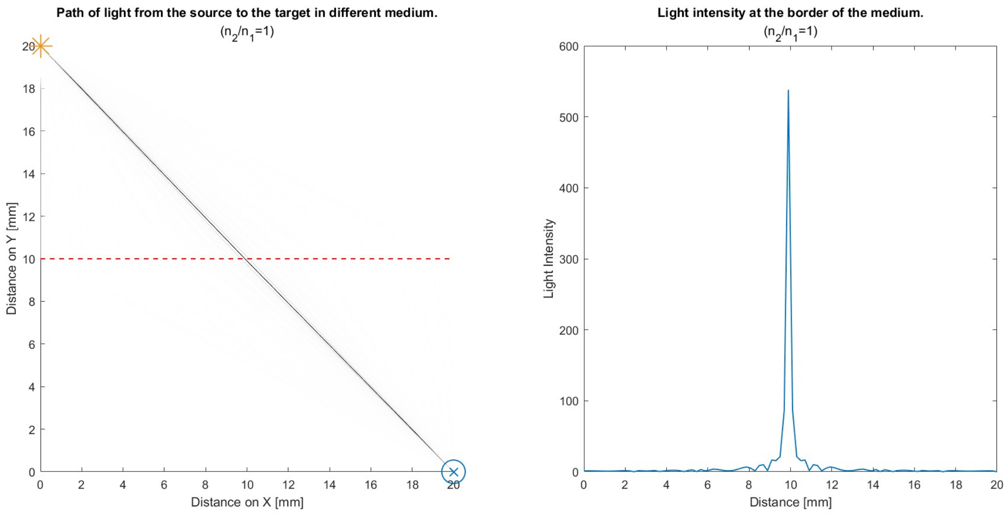

Fig. 8: Representation of Fermat's principle in case of $\frac{n2}{n1} = 1$

As the ratio of the refractive indices equals to 1 (as seen on Figure 9.), the light will propagate from source to detector in a straight line, without getting refracted at the border. This means the light rays travel the same distance in both medium. With a ratio smaller than 1 (Figure 10.), the light propagates less in the first denser medium, and along the border the refraction occurs closer to the source. With a ratio greater than 1 (Figure 11.), the light propagates a longer distance in the less dense first medium and travels less in the second denser medium. With my results I could succesfully demonstrate that light rays get refracted the way as it is dictated by Snell's law and they correspond to the theory of Fermat's principle (fastest propagation).

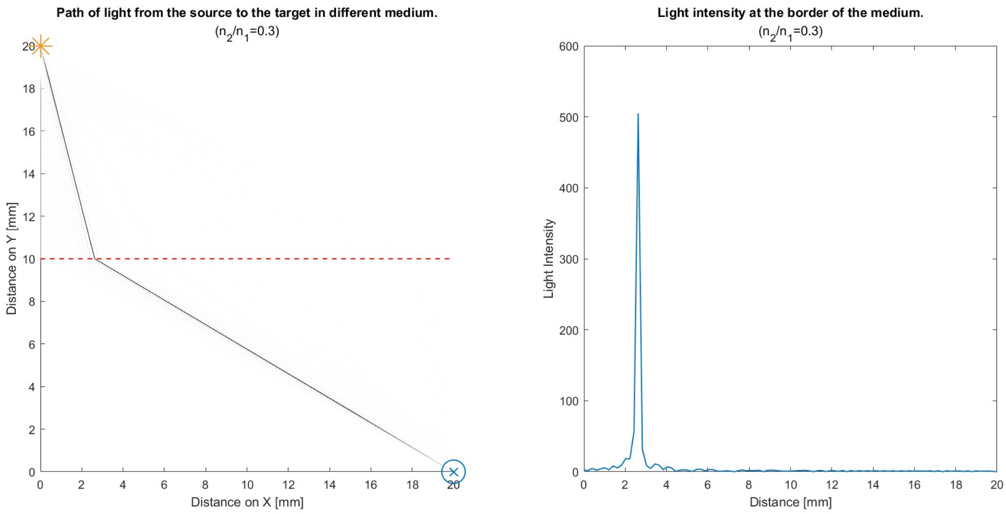

Fig. 9: Representation of Fermat's principle in case of $\frac{n2}{n1} = 0.3$

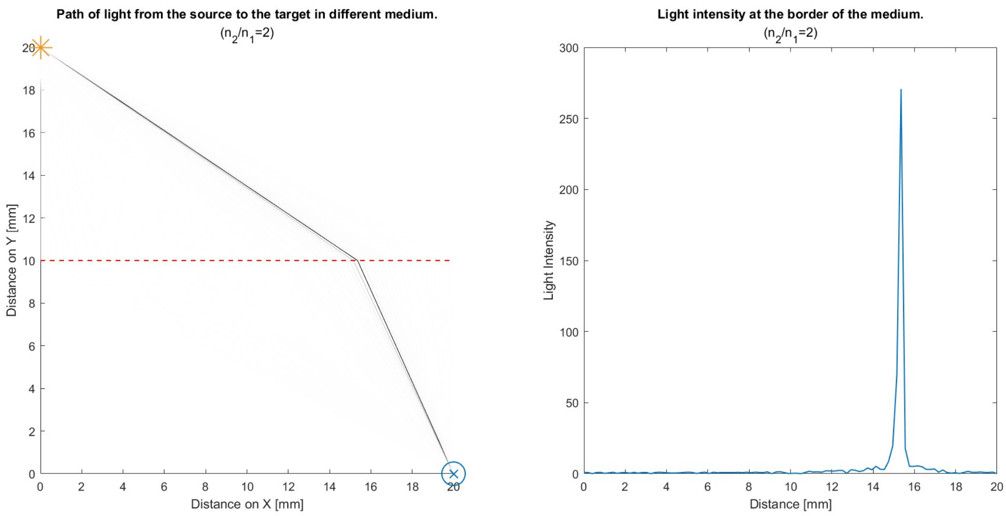

Fig. 10: Representation of Fermat's principle in case of $\frac{n2}{n1} = 2$