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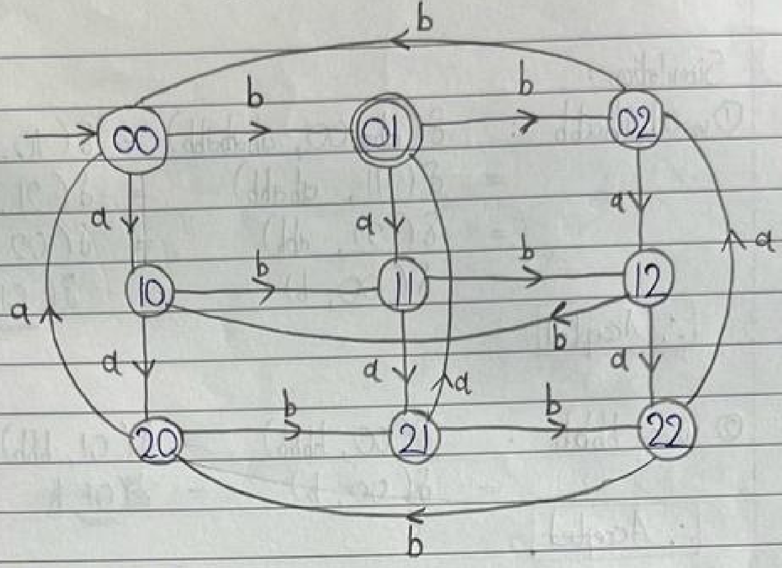
	1. a)	



Tuple definition :

$Q = \{ 00, 01, 02, \; 10, 11, 12 \; , \; 20, 21, 22 \}$

(state name formed by $n_a(\omega) \bmod 3$, $n_b(\omega) \bmod 3$)

$\Sigma = \{a, b\}$

q_0 (initial state) $= 00$

F (final state) $= 01$

$\delta : Q \times \Sigma = Q$ as shown in transition diagram and transition table below.

Transition Table :

$Q \backslash \Sigma$	a	b
→00	10	01
*01	11	02
02	12	00
10	20	11
11	21	12
12	22	10
20	00	21
21	01	22
22	02	20

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Simulation :

① $w = ababab b$: $\delta(\cancel{aba}\,00, abababb) = \delta(10, bababb)$
$= \delta(11, ababb) = \delta(21, babb)$
$= \delta(22, abb) = \delta(02, bb)$
$= \delta(00, b) = \underline{01}$

$\boxed{\therefore \text{Accepted}}$

② $w = bbbb$: $\delta(00, bbbb) = \delta(01, bbb) = \delta(02, bb)$
$= \delta(00, b) = \underline{\underline{01}}$

$\boxed{\therefore \text{Accepted}}$

③ $w = aba$: $\delta(00, aba) = \delta(10, ba) = \delta(11, a) = \underline{\underline{21}}$

$\boxed{\therefore \text{Rejected}}$

④ $w = aaa$: $\delta(\cancel{aa}\,00, aaa) = \delta(10, aa) = \delta(20, a) = \underline{\underline{00}}$

$\therefore \boxed{\text{Rejected}}$

~~8~~ ~~7~~
10

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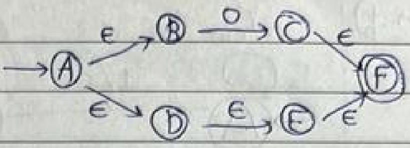
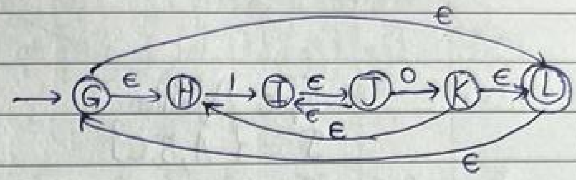
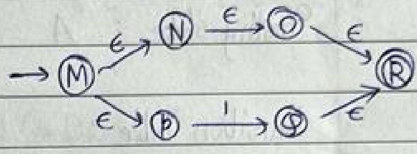
2.d) ϵ-NFA for $0/\epsilon$:



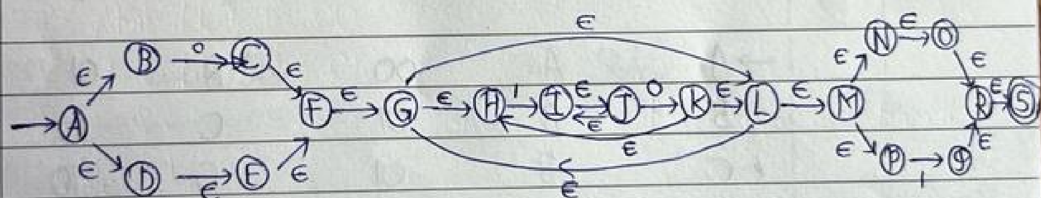
ϵ-NFA for $(10)^*$:



ϵNFA for $\epsilon/1$:



$\therefore$ ϵ-NFA for $(0/\epsilon)(10)^*(\epsilon/1)$:



Tuple Definition :

$Q = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S\}$

$\Sigma = \{\cancel{\phi}, \epsilon, 0, 1\}$

$\delta : \cancel{Q \times \Sigma} \rightarrow Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$ as shown above

Starting State : A   Final State : S

D

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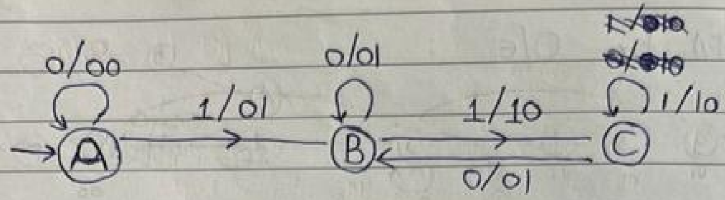
2.b)



Tuple Definition :

$Q = \{A, B, C\}$ $\Sigma = \{0, 1\}$ $O = \{00, 01, 10\}$

$\delta : Q \times \Sigma \rightarrow Q$ $\lambda : Q \times \Sigma \rightarrow O$

Starting state : A

Transition Table :

$Q \setminus \Sigma$	0		1	
	Next State	Output	Next State	Output
$\rightarrow$A	A	00	B	01
B	B	01	C	10
*C	B	01	C	10

~~Simulation~~ :

~~δ w = 0011~~ : ~~λ(A, 0011) = λ(~~

Simulation :

① w = 0011 : $\rightarrow$ ~~A $\xrightarrow{0/00}$ B $\rightarrow$~~

$\rightarrow A \xrightarrow{0/00} A \xrightarrow{0/00} A \xrightarrow{1/01} B \xrightarrow{1/10} C$

04

$\therefore$ Output = 00 00 01 10

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3.(b) By Equivalence Class Method :

0- Equivalent : $\Pi_0 = \{q0, q1, q3, q4, q5, q6, q7\}\ \{q2\}$

1- Equivalent : $\Pi_1 = \{q0, q4, q6\}\quad \{q1\}\{q3\}\{q5\}\{q7\}\quad \{q2\}$

$= \{q0, q4, q6\}\quad \{q1, q7\}\{q3\}\{q5\}\quad \{q2\}$

$= \{q0, q4, q6\}\quad \{q1, q7\}\{q3, q5\}\quad \{q2\}$

2- Equivalent : $\Pi_2 = \{q0, q4\}\quad \{q6\}\quad \{q1, q7\}\{q3, q5\}$

3. a) ~~Remove~~ Equivalent States ~~Removed~~ Combined :

q0 and q4 → q0

q1 and q7 → q1

q3 and q5 → q3

∴

Present State	Next State 0	Next State 1
→ q0	q1	q3
q1	q6	q2
* q2	q0	q2
q3	q2	q6
q6	q6	q0

$\Pi_0 = \{q0, q1, q3, q6\}\ \{q2\}$

$\Pi_1 = \{q0, q6\}\quad \{q1\}\quad \{q3\}\ \{q2\}$

$= \{q0, q6\}\quad \{q1, q2\}\quad \{q3\}$

$\Pi_2 = \{q0\}\quad \{q6\}\ \{q1, q2\}\ \{q3\}$

$\Pi_3 = \{q0\}\quad \{q6\}\quad \{q1, q2\}\ \{q3\}$ ← same as Π_2

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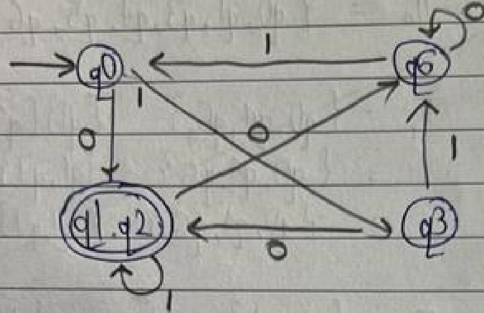
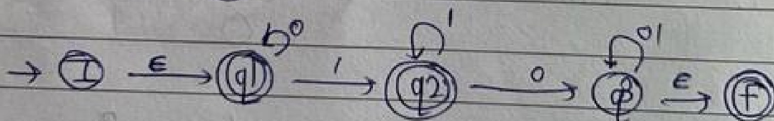
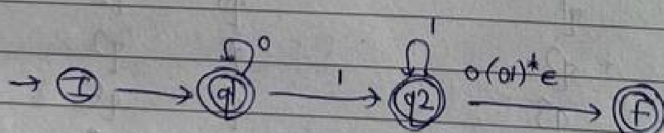
: Minimized DFA :



03

(b)





00

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	3.a	Let L be regular, and p = pumping length.
		Consider $w = a^p b^{2p} c^p$
		where $uv = a^p$ and $w = b^{2p} c^p$
		Let $v = a^p$.
		$uv^2w = a^{2p} b^{2p} c^p$
		By def. of L, this does not belong to L.
		$\therefore$ Negative Test for pumping lemma.
		$\therefore$ L Not a regular lang
0		

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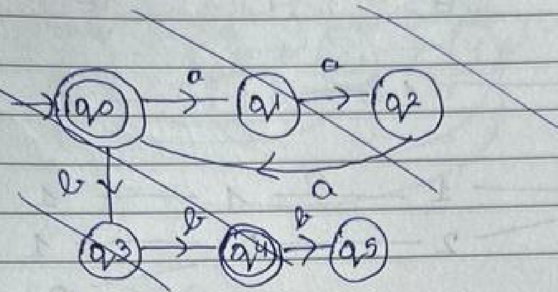
A1



A1 $\delta_2(q_K, a) = \varepsilon(\varepsilon\ \text{closure}(q_K, a))$.

~~A1~~ States ϵ^* Input 0 ϵ^*

1 — 1 — 2 — 1, 2

2 — 1 — 2 — 1, 2
2 — 2 — 3 — 2, 3

3 — 1 — 2 — 1, 2
3 — 2 — 3 — 1, 2, 3
3 — 3 — 4 — 1, 2, 3, 4

4 — 1 — 2 — 1, 2
4 — 2 — 3 — 1, 2, 3
4 — 3 — 4 — 1, 2, 3, 4
4 — 4 — 1 — 1

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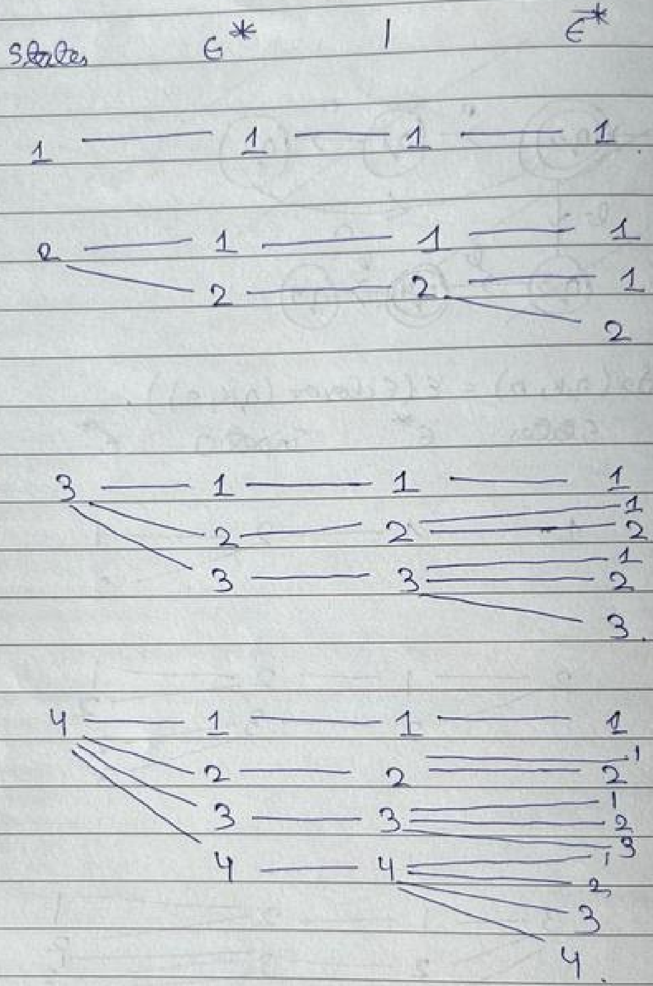


FOR NFA

$Q = \{1, 2, 3, 4\}$.

$\Sigma = \{0, 1\}$.

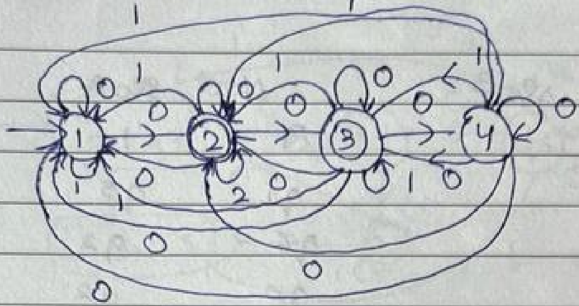
$q_0 = \{1\}$

$F = \{2, 3\}$.

$\delta =$

$Q \backslash \Sigma$	0	1
$\rightarrow 1$	$\{1, 2\}$	$\{1\}$
2	$\{1, 2, 3\}$	$\{1, 2\}$
3	$\{1, 2, 3, 4\}$	$\{1, 2, 3\}$
4	$\{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$.

Space for Marks	Question No.	START WRITING HERE



For DFA

$\delta =$

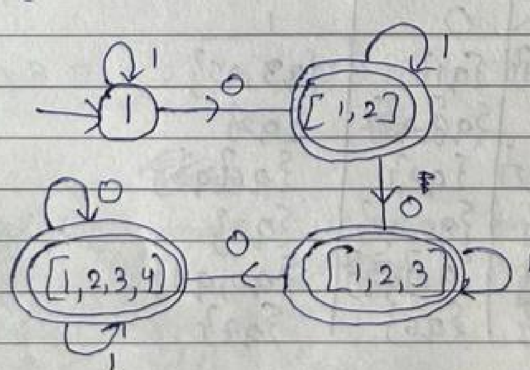
$Q \setminus \Sigma$	0	1
$\rightarrow 1$	$\{1,2\}$	$\{1\}$.
$\{1,2\}$	$\{1,2,3\}$	$\{1,2\}$.
$\{1,2,3\}$	$\{1,2,3,4\}$	$\{1,2,3\}$.
$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$.

$Q = \{1, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}\}$.

$\Sigma = \{0,1\}$.

$q^0 = \{1\}$.

$F = \{\{1,2\}, \{1,2,3\}, \{1,2,3,4\}\}$.



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A 3(a)

Present state	Next state 0	Next state 1
$\rightarrow q0$	$q1$	$q5$
$q1$	$q6$	$q2$
$* q2$	$q0$	$q2$
$q3$	$q2$	$q6$
$q4$	$q1$	$q5$
$q5$	$q2$	$q6$
$q6$	$q6$	$q4$
$q7$	$q6$	$q7$

$\Pi_0 = \{q0, q1, q3, q4, q5, q6, q7\}\ \{q2\}.$

$\Pi_1 = \{q0, q4, q6, q7\}\ \{q1\}\ \{q3, q5\}\ \{q2\}.$

$\Pi_2 = \{q0, q4\}\ \{q6\}\{q7\}\ \{q1\}\ \{q3, q5\}\ \{q2\}$

$Q \backslash \Sigma$	0	1
$\rightarrow \{q0, q4\}$	$\{q1\}$	$\{q3, q5\}$
$\{q1\}$	$\{q6\}$	$\{q2\}$
$\{q3, q5\}$	$\{q2\}$	$\{q6\}$
$* \{q2\}$	$\{q0, q4\}$	$\{q2\}$
$\{q6\}$	$\{q6\}$	$\{q0, q4\}$
$\{q7\}$	$\{q6\}$	$\{q7\}$

Space for Marks | Question No.

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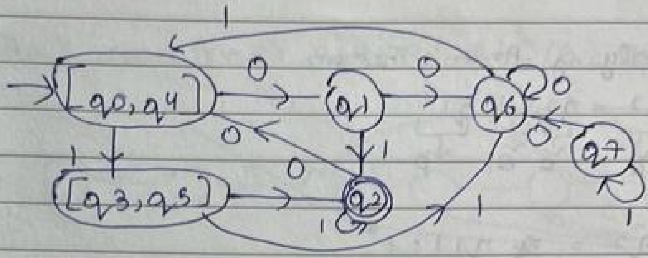

Minimized FA

$Q = \{[q_0,q_4], q_1, q_2, [q_3,q_5], q_6, q_7\}$.

$\Sigma = \{0,1\}$.

$q_0 = \{[q_0,q_4]\}$.

04

$F = \{q_2\}$.

(b) $q_1 = q_1 0 + \epsilon$ —(1)

$q_2 = q_1 1 + q_2 1$ —(2)

$q_3 = q_2 0 + q_3 0 + q_3 1$ —(3)

Applying arden's theorem on 1.

$$\underbrace{q_1}_{R} = \underbrace{q_1}_{R}\underbrace{0}_{Q} + \underbrace{\epsilon}_{P}$$

$R = RQ + P$

$R = PQ^*$ $\qquad$ $q_1 = \epsilon 0^*$ —(4).

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Applying Arden's Theorem.

$\underbrace{q_2}_{R} = \underbrace{q_2}_{R}\underbrace{1}_{Q} + \underbrace{q_1 1}_{P}$.

$q_2 = q_1 1 \cdot 1^*$

$q_2 = q_1 1^*$

Substituting q_1 in q_2

$q_2 = 0^* 1^*$.

~~taking~~ ~~using~~ $q_3 = q_2 0 + q_3 0 + q_3 1$.

$q_3 = q_3 (0+1) + q_2 0$.

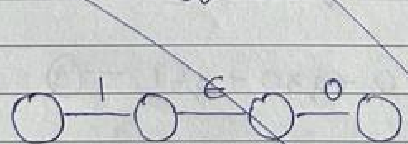
$q_3 = q_2 0 (0+1)^*$.

$q_3 = 0^* 1^* (0+1)^*$

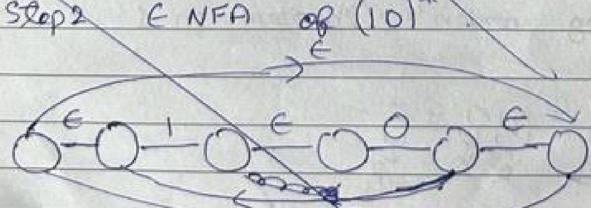
05

A2. (a) $(0 / \epsilon)(10)^*(\epsilon / 1)$.

Step 1 ϵNFA of 10.



Step 2 ϵ NFA of $(10)^*$.



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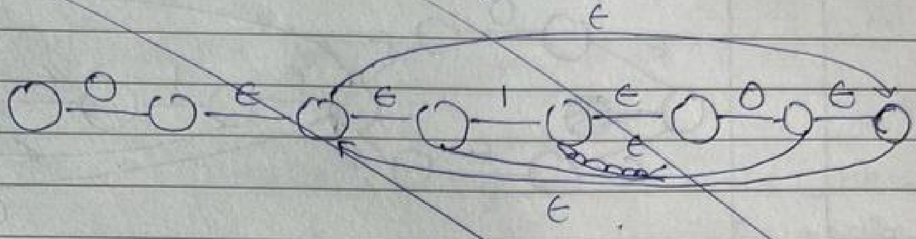
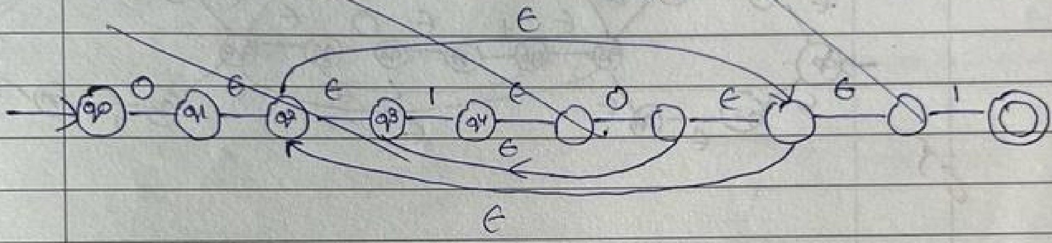
~~Step 3 concatenate ∈ NFA of (0/∈) to 10^*.~~

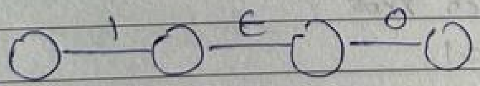


~~Step 4 concatenate ∈ NFA of (0/∈)10^* to (ε/1).~~

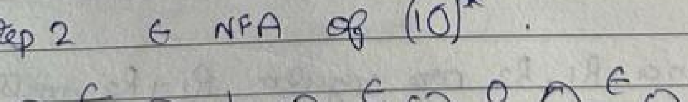


A2 (a) (0/∈) $(10)^*$ (∈/1)

Step 1 ∈ NFA of (10).



Step 2 ∈ NFA of $(10)^*$.



~~Step 3~~

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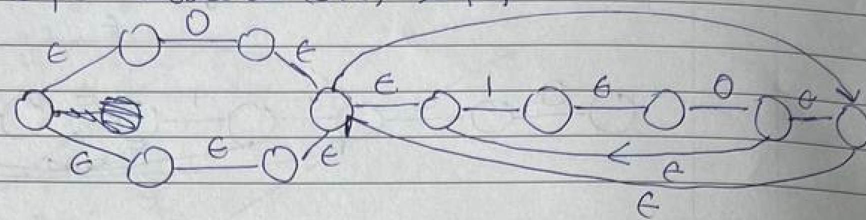
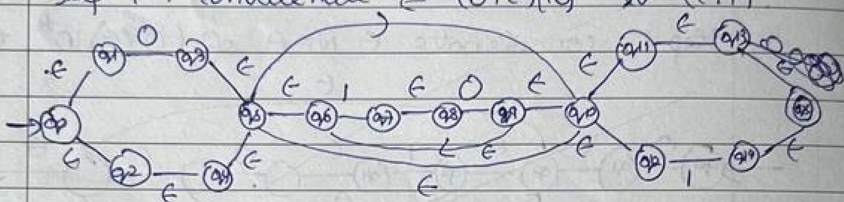
Step 3: Concatenate $(0/\epsilon)$ to $(10)^*$



Step 4: concatenate ϵ $(0/\epsilon)(10)^*$ to $(\epsilon/1)$



05

A2 (b) Regular languages under closure can be closed under union, concatenation, intersection.

Closure under union.
Consider two languages L_1 & L_2 under Regular Expressions R_1, R_2. such that

$L(R_1) = L_1$

$L(R_2) = L_2$.

Since R_1, R_2 are regular $R_1 + R_2$ will also be regular
$\therefore L_1 + L_2$ will also be regular.

Concatenation.

$L(R_1) = L_1$

$L(R_2) = L_2$.

$R_1 \cdot R_2$ will be regular & $\therefore L_1 \cdot L_2$ will also be regular.

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Intersection.

$L(R_1) = L_1$
$L(R_2) = L_2$.

Since $R_1 - R_2$ will be regular $\therefore L_1 - L_2$ will also be regular.

67