SOMAIYA VIDYAVIHAR UNIVERSITY

08·05·2024 (E)

Semester: Jan 2024 - May 2024 **Maximum Marks: 100** **Examination: ESE Examination**		**Duration:3 Hrs.**
Programme code: 01/04 **Programme:** B. Tech Computer/IT Engineering	**Class:** SY	**Semester: IV (SVU 2020)**
Name of the Constituent College: K. J. Somaiya College of Engineering	**Name of the department: Computer/IT**	
Course Code:116U01C401/116U04C401	**Name of the Course:** Probability, Statistics and Optimization Techniques	
Instructions: 1) All questions are compulsory 2) Assume suitable data wherever necessary		

Que. No.	Question	Max. Marks
Q1	Solve any **Four** of the following.	**20**
i)	Consider 3 candidates X, Y, Z, with chances of getting appointed are 4: 2: 3 respectively. The probability that Mr X is selected and company become profitable is 0.3. The probability of Y & Z for the same is 0.5 & 0.8 respectively. If the company become profitable then what is the probability that Mr Z is appointed? (Solve by Bayes' Theorem)	5
ii)	Obtain the Spearman's rank correlation coefficient (R) from the following data. X : 20, 22, 28, 28, 25, 50 Y : 32, 38, 45, 45, 70, 45	5
iii)	A random sample of 200 observations has mean 6.5 cm. Can it be a random sample From population whose mean is 7 cm and variance is 8.5 cm at 2% LOS? (Two tailed Z test with critical value 2.33)	5
iv)	Convert the given LPP into the standard form Minimise $z = -3x_1 + 2x_2 - x_3$ Subject to $x_1 - 3x_2 + 2x_3 \geq -6,\ 3x_1 + 4x_3 \leq 3, -3x_1 + 5x_2 \leq 4$ Where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.	5
v)	Arrivals at telephone booth are considered to be Poisson with an average time of 10 min. between one arrival and the next .The length of phone calls is assumed to be distributed exponentially with a mean of 3 min. **(a)** What is the probability that a person arriving at the booth will have to wait? **(b)** What is the probability that it will take a customer more than 10 min. altogether to wait for the phone and complete his call? Assume The system is (M/M/1/∞)	5
vi)	Given $f_{xy}(x,y) = \begin{cases} cx(x-y), & 0 < x < 2, -x < y < x \\ 0, & elsewhere \end{cases}$ **(a)** Evaluate c **(b)** find $f_x(x)$ (Marginal distribution with respect to x)	5

Q2 A	Solve the following.	10
i)	If $\sigma_y = \sigma_y = \sigma$ and the angle between the lines of regression is $\tan^{-1} 3$. find the coefficient of correlation.	5
ii)	A die was thrown 132 times and the following frequencies were observed No obtained: 1 | 2 | 3 | 4 | 5 | 6 | Total Frequency: 15 | 20 | 25 | 15 | 29 | 28 | 132 Use χ^2 test to test the hypothesis that the die is unbiased at 5% LOS.	5
	OR	
Q2 A	Using Lagrange's Multiplier method solve the following NLPP $z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$ Subject to $x_1 + x_2 + x_3 = 7, x_1, x_2, x_3 \geq 0$.	10
Q 2 B	Solve any **One** of the following.	10
i)	A diesel pump has capacity to accommodate only 4 trucks including the one at the pump. On an average trucks arrive at the rate of 5 per hour and the service rate is 6 per hour. Assume that the arrival process is Poisson and the service time is an exponential random variable. **(a)** What is the average time for which a truck is at the pump? **(b)** What is the average waiting time for a truck in the queue? **(c)** What percentage of trucks will be turned away?	10
ii)	It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced item are send to the market in packets of 20, find the number of packets containing **(a)** at least 2 **(b)** exactly 2 & **(c)** at most 2 defective items in a consignment of 1000 packets using Binomial distribution & Poisson distribution.	10
Q3	Solve any **Two** of the following.	20
i)	**(a)** A manufacturer knows from his experience that the resistance of resistor he produces is normal with $\mu = 100$ ohms & standard deviation $\sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms & 102 ohms?	05
	(b) A sample of 200 fish of a particular kind taken as random from one end of a lake had mean weight of 20 lbs & standard deviation of 2 lbs. At the other end of the lake, a sample of 80 fish of the same kind had mean weight of 20.51lbs & standard deviation of 2 lbs. Is the difference between the mean weights significant at 1% level of significance?	05
ii)	Find the equations of two regression lines for the following data X: 78 | 36 | 98 | 25 | 45 | 82 | 90 | 62 | 65 | 39 Y: 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 Estimate Y when $X = 40$ and Estimate X when $Y = 80$.	10

iii)	Define the following terms: Solution, Basic solution, Basic Feasible solution and degenerate solution of LPP. Also find **(a)** All basic solutions **(b)** All feasible basic solutions **(c)** All degenerate solutions. Hence decide the optimal feasible basic for the following L.P.P. Maximise $z = 2x_1 - 2x_2 + 4x_3 - 5x_4$ Subject to $x_1 + 4x_2 - 2x_3 + 8x_4 = 2,\ -x_1 + 2x_2 + 3x_3 + 4x_4 = 1$ where $x_1, x_2, x_3, x_4 \geq 0$	10
Q4	Solve any **Two** of the following.	**20**
i)	For a normal variate X with mean 2.5 and standard deviation 3.5 find the probability that **(a)** $P(2 \leq X \leq 4.5)$ **(b)** $P(-1.5 \leq X \leq 5.5)$ **(c)** $P(\lvert X \rvert \leq 2)$ **(d)** Find c such that $P(X < c) = 0.2$	10
ii)	In a certain experiment to compare two types of pig foods A & B the following results of increasing weights were obtained Pig no: 1 2 3 4 5 6 7 8 Increase in weight by A: 49 53 51 52 47 50 52 53 Increase in weight by B: 52 55 52 53 50 54 54 53 Can we conclude that food B is better than food A if the same set of pigs were used in both cases? (Use one tailed test at 5% LOS)	10
iii)	Solve the given LPP by Penalty method (Big M) Minimize $z = 2x_1 + x_2$ Subject to $3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 3$ where $x_1, x_2 \geq 0$	10
Q5	Solve any **Four** of the following.	**20**
i)	Suppose a random number N is taken from 690 to 850 in uniform distribution .Find the probability that number N is greater than 790. Find the Mean and the Variance of the distribution.	5
ii)	Calculate the two regression coefficients and the coefficient of correlation from the data: $N = 10, \sum x = 350, \sum y = 310,\ \sum (x-35)^2 = 162,$ $\sum (y-31)^2 = 222,\ \sum (x-35)(y-31) = 35$	5
iii)	The mean value of random sample of 20 items was found to be 145 with standard deviation of 40. Find 90% confidence limits for the population mean.	5
iv)	Find the relative maximum or minimum of the function $Z = 9x_1 + 6x_3 + x_1x_2 - {x_1}^2 - {x_2}^2 - {x_3}^2$	5
v)	Find the probability that a customer has to wait in an M/M/1/∞ model if $\lambda = 8$, $\mu = 10$ per hour. Also find the probability that a customer has to wait in the queue more than 15 minutes.	5
vi)	Obtain the dual of given LPP Maximize $z = 2x_1 - x_2 + 3x_3$ Subject to $x_1 - 2x_2 + x_3 \geq 4,\ 2x_1 + x_3 \leq 10,\ x_1 + x_2 + 3x_3 = 20,\ x_1, x_2, x_3, \geq 0$	5