Bayes Classifiers

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CS 273P Machine Learning and Data Mining

Slides courtesy of Alex Ihler

Machine Learning

Bayes Classifiers

Naive Bayes Classifiers

Bayes Error

Gaussian Bayes Classifiers

A basic classifier

- Training data $D=\{x^{(i)},y^{(i)}\}$, Classifier $f(x ; D)$
  - Discrete feature vector x
  - $f(x ; D)$ is a contingency table
- Ex: credit rating prediction (bad/good)
  - X_1 = income (low/med/high)
  - How can we make the most # of correct predictions?

Features	# bad	# good
X=0	42	15
X=1	338	287
X=2	3	5

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- Ex: credit rating prediction (bad/good)
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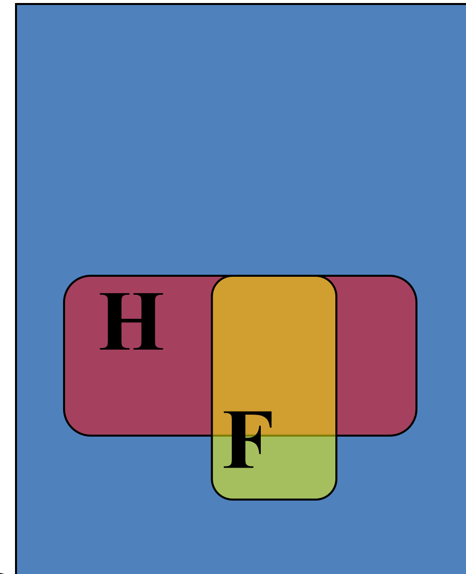
  - How can we make the most # of correct predictions?
  - Predict more likely outcome
    for each possible observation
  - Can normalize into probability:
    p(y=good | X=c)
  - How to generalize?

Features	# bad	# good
X=0	.7368	.2632
X=1	.5408	.4592
X=2	.3750	.6250

Bayes rule

- Two events: headache, flu
- p(H) = 1/10
- p(F) = 1/40
- p(H|F) = 1/2



- You wake up with a headache – what is the chance that you have the flu?

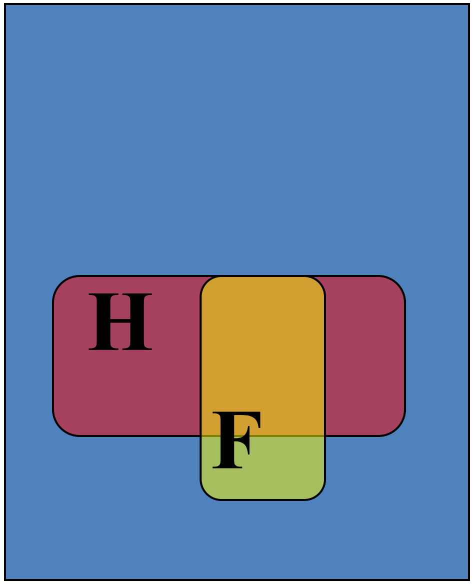
Example from Andrew Moore's slides

Bayes rule

- Two events: headache, flu
- p(H) = 1/10
- p(F) = 1/40
- p(H|F) = 1/2

- P(H & F) = ?

- P(F|H) = ?



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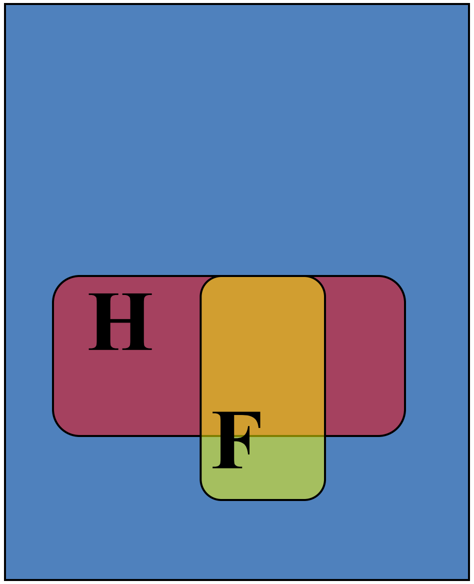


- P(H & F) = p(F) p(H|F)

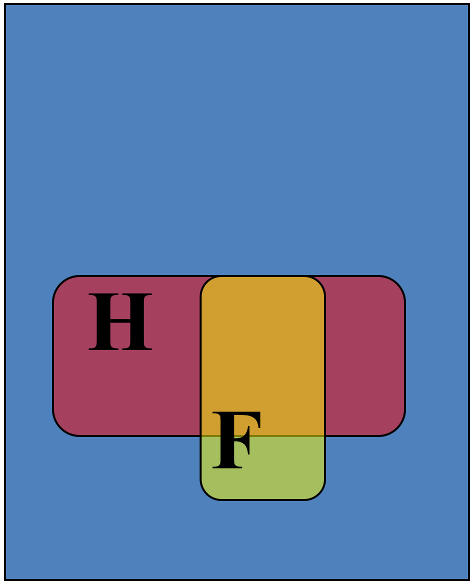
  = (1/2) * (1/40) = 1/80
- P(F|H) = ?

Example from Andrew Moore's slides

Bayes rule

- Two events: headache, flu
- $p(H) = 1/10$
- $p(F) = 1/40$
- $p(H|F) = 1/2$

- $P(H \& F) = p(F)\, p(H|F)$
  $= (1/2) * (1/40) = 1/80$
- $P(F|H) = p(H \& F) / p(H)$
  $= (1/80) / (1/10) = 1/8$



Example from Andrew Moore's slides

Classification and probability

- Suppose we want to model the data

- Prior probability of each class, p(y)
  - E.g., fraction of applicants that have good credit
- Distribution of features given the class, p(x | y=c)
  - How likely are we to see "x" in users with good credit?

- Joint distribution

$$p(y|x)p(x) = p(x,y) = p(x|y)p(y)$$

- Bayes Rule:

$$\Rightarrow \quad p(y|x) = p(x|y)p(y)/p(x)$$

$$= \frac{p(x|y)p(y)}{\sum_c p(x|y=c)p(y=c)}$$

(Use the rule of total probability to calculate the denominator!)

Bayes classifiers

- Learn "class conditional" models
  - Estimate a probability model for each class
- Training data, D
  - Split by class, $D_c = \{ x^{(j)} : y^{(j)} = c \}$
- Estimate $p(x \mid y=c)$ using D_c
- For a discrete x, this recalculates the same table…

Features	# bad	# good
X=0	42	15
X=1	338	287
X=2	3	5

$\Rightarrow$

p(x | y=0)	p(x | y=1)
42 / 383	15 / 307
338 / 383	287 / 307
3 / 383	5 / 307

$\Rightarrow$

p(y=0|x)	p(y=1|x)
.7368	.2632
.5408	.4592
.3750	.6250

p(y)	383/690	307/690