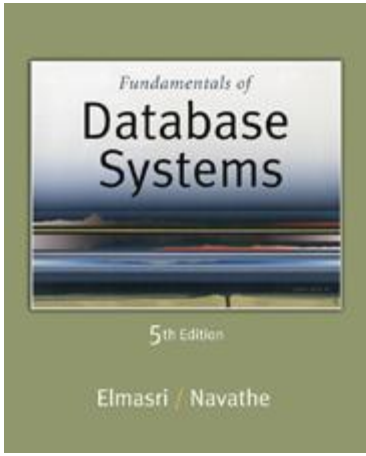
Fundamentals of
Database Systems

5th Edition

Elmasri / Navathe

Chapter 6

The Relational Algebra and Calculus



Chapter Outline

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Domain Relational Calculus
- Example Database Application (COMPANY)

Relational Algebra Overview

- Relational algebra is the basic set of operations for the relational model
- These operations enable a user to specify **basic retrieval requests** (or **queries**)
- The result of an operation is a *new relation*, which may have been formed from one or more *input* relations.

Relational Algebra Overview (continued)

- The **algebra operations** thus produce new relations
  - These can be further manipulated using operations of the same algebra
- A sequence of relational algebra operations forms a **relational algebra expression**
  - The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

Brief History of Origins of Algebra

- Muhammad ibn Musa al-Khwarizmi (800-847 CE) wrote a book titled al-jabr about arithmetic of variables
  - Book was translated into Latin.
  - Its title (al-jabr) gave Algebra its name.
- Al-Khwarizmi called variables "shay"
  - "Shay" is Arabic for "thing".
  - Spanish transliterated "shay" as "xay" ("x" was "sh" in Spain).
  - In time this word was abbreviated as x.
- Where does the word Algorithm come from?
  - Algorithm originates from "al-Khwarizmi"
  - Reference: PBS (http://www.pbs.org/empires/islam/innoalgebra.html)

Relational Algebra Overview

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations
    - SELECT (symbol: **σ** (sigma))
    - PROJECT (symbol: **π** (pi))
    - RENAME (symbol: **ρ** (rho))
  - Relational Algebra Operations From Set Theory
    - UNION (**∪**), INTERSECTION (∩), DIFFERENCE (or MINUS, **–**)
    - CARTESIAN PRODUCT (**x**)
  - Binary Relational Operations
    - JOIN (several variations of JOIN exist)
    - DIVISION
  - Additional Relational Operations
    - OUTER JOINS, OUTER UNION
    - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

Database State for COMPANY

- All examples discussed below refer to the COMPANY database shown here.

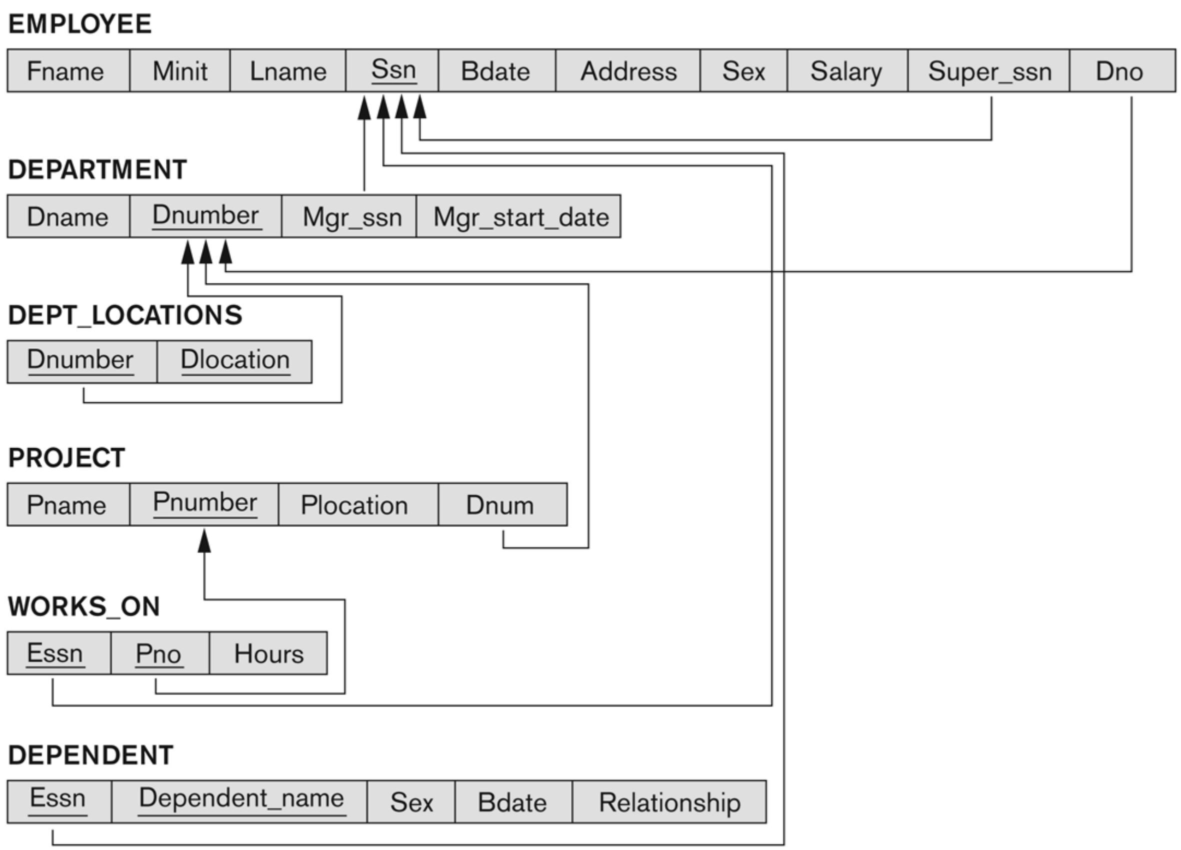


Figure 5.7
Referential integrity constraints displayed on the COMPANY relational database schema.

Unary Relational Operations: SELECT

- The SELECT operation (denoted by σ (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.
  - The selection condition acts as a **filter**
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)
- Examples:
  - Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{\text{DNO} = 4}\ (\text{EMPLOYEE})$$

  - Select the employee tuples whose salary is greater than $30,000:

$$\sigma_{\text{SALARY} > 30,000}\ (\text{EMPLOYEE})$$

Unary Relational Operations: SELECT

- In general, the *select* operation is denoted by **σ**
- **Syntax: σ** $_{\text{<selection condition>}}$(R)
- where
  - the symbol **σ** (sigma) is used to denote the *select* operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition **true** are selected
    - appear in the result of the operation
  - tuples that make the condition **false** are filtered out
    - discarded from the result of the operation

Unary Relational Operations: SELECT (contd.)

- SELECT Operation Properties
  - The SELECT operation $\sigma_{\text{<selection condition>}}(R)$ produces a relation S that has the same schema (same attributes) as R
  - SELECT σ is commutative:
    - $\sigma_{\text{<condition1>}}(\sigma_{\text{< condition2>}}(R)) = \sigma_{\text{<condition2>}}(\sigma_{\text{< condition1>}}(R))$
  - Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(\sigma_{\text{<cond1>}}(R)))$
  - A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{< cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond1> AND < cond2> AND < cond3>}}(R)$
  - The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

Unary Relational Operations: PROJECT

- PROJECT Operation is denoted by **π** (pi)
- This operation keeps certain *columns* (attributes) from a relation and discards the other columns.
  - PROJECT creates a vertical partitioning
    - The list of specified columns (attributes) is kept in each tuple
    - The other attributes in each tuple are discarded

- The general form of the *project* operation is:

$$\pi_{\text{<attribute list>}}(R)$$

  - π (pi) is the symbol used to represent the *project* operation
  - <attribute list> is the desired list of attributes from relation R.

Unary Relational Operations: PROJECT (cont.)

- The project operation *removes any duplicate tuples*
  - This is because the result of the *project* operation must be a *set of tuples*
    - Mathematical sets *do not allow* duplicate elements
- Example: To list each employee's first and last name and salary, the following is used:

$$\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$$

Unary Relational Operations: PROJECT (contd.)

- PROJECT Operation Properties
  - The number of tuples in the result of projection $\pi_{<list>}(R)$ is always less or equal to the number of tuples in R
    - If the list of attributes includes a *key* of R, then the number of tuples in the result of PROJECT is *equal* to the number of tuples in R
  - PROJECT is *not* commutative

One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Examples of applying SELECT and PROJECT operations

(a) $\sigma_{(DNO=4 \text{ AND } SALARY>25000) \text{ OR } (DNO=5 \text{ AND } SALARY>30000)}(EMPLOYEE)$.

(b) $\pi_{LNAME,\ FNAME,\ SALARY}(EMPLOYEE)$. (c) $\pi_{SEX,\ SALARY}(EMPLOYEE)$

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	M	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Relational Algebra Expressions

- We may want to apply several relational algebra operations one after the other
  - Either we can write the operations as a single **relational algebra expression** by nesting the operations, or
  - We can apply one operation at a time and create **intermediate result relations**.
- In the latter case, we must give names to the relations that hold the intermediate results.

Single expression versus sequence of relational operations (Example)

- To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation
- We can write a *single relational algebra expression* as follows:
  - $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$
- OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation:
  - $\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO=5}}(\text{EMPLOYEE})$
  - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$

Example of applying multiple operations and RENAME

(a) $\pi_{\text{LNAME, FNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$. (b) The same expression using intermediate relations and renaming of attributes.

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2
Results of a sequence of operations.
(a) $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno=5}}(\text{EMPLOYEE}))$.
(b) Using intermediate relations and renaming of attributes.

Unary Relational Operations: RENAME

- The RENAME operator is denoted by ρ (rho)
- In some cases, we may want to *rename* the attributes of a relation or the relation name or both
  - Useful when a query requires multiple operations

Example

- Suppose we want to do cartesian product between same table then **one of the table should be renamed with another name**
- R x R

  (Ambiguity will be there)

R

A	B
α	1
β	2

- $R \times \rho_S(R)$

  (Rename R to S)

RxR

R.A	R.B	R.A	R.B
α	1	α	1
α	1	β	2
β	2	α	1
β	2	β	2

$R \times \rho_s(R)$

R.A	R.B	S.A	S.B
α	1	α	1
α	1	β	2
β	2	α	1
β	2	β	2

Unary Relational Operations: RENAME (contd.)

- The general RENAME operation ρ can be expressed by any of the following forms:
  - $\rho_{S\ (B1,\ B2,\ \ldots,\ Bn\ )}(R)$ changes both:
    - the relation name to S, *and*
    - the column (attribute) names to B1, B1, …..Bn
  - $\rho_{S}(R)$ changes:
    - the *relation name* only to S
  - $\rho_{(B1,\ B2,\ \ldots,\ Bn\ )}(R)$ changes:
    - the *column (attribute) names* only to B1, B1, …..Bn

Unary Relational Operations: RENAME (contd.)

- For convenience, we also use a *shorthand* for renaming attributes in an intermediate relation:
  - If we write:
    - RESULT $\leftarrow \pi_{\text{FNAME, LNAME, SALARY}}$ (DEP5_EMPS)
    - RESULT will have the *same attribute names* as DEP5_EMPS (same attributes as EMPLOYEE)
  - If we write:
    - RESULT (FN, LN, Sal,)$\leftarrow \pi_{\text{FNAME, LNAME, SALARY}}$ (DEP5_EMPS)
    - 3 attributes of RESULT are *renamed* to FN, LN, Sal respectively

Relational Algebra Operations from Set Theory

- Type Compatibility of operands is required for the binary set operation UNION ∪, INTERSECTION ∩, and SET DIFFERENCE –,
- R1(A1, A2, ..., An) and R2(B1, B2, ..., Bn) are type compatible if:
  - they have the same number of attributes, and
  - the domains of corresponding attributes are type compatible (i.e. dom(Ai)=dom(Bi) for i=1, 2, ..., n).
- The resulting relation for R1∪R2 (also for R1∩R2, or R1–R2, see next slides) has the same attribute names as the *first* operand relation R1 (by convention)

Relational Algebra Operations from Set Theory:

- UNION Operation
  - Binary operation, denoted by ∪
  - The result of R ∪ S, is a relation that includes all tuples that are either in R or in S or in both R and S
  - Duplicate tuples are eliminated
  - The two operand relations R and S must be "type compatible" (or UNION compatible)
    - R and S must have same number of attributes
    - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

Example of the result of a UNION operation

- UNION Example

Figure 6.3
Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2.

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Relational Algebra Operations from Set Theory: UNION

- Example:
  - To retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below)
  - We can use the UNION operation as follows:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO=5}} (\text{EMPLOYEE})$$

$$\text{RESULT1} \leftarrow \pi_{\text{SSN}}(\text{DEP5\_EMPS})$$

$$\text{RESULT2(SSN)} \leftarrow \pi_{\text{SUPERSSN}}(\text{DEP5\_EMPS})$$

$$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$$

  - The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

Relational Algebra Operations from Set Theory: INTERSECTION

- INTERSECTION is denoted by ∩
- The result of the operation R ∩ S, is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

Relational Algebra Operations from Set Theory: SET DIFFERENCE (cont.)

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of R – S, is a relation that includes all tuples that are in R but not in S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are *commutative* operations; that is
  - $R \cup S = S \cup R$, and $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is
  - $R \cup (S \cup T) = (R \cup S) \cup T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R – S \neq S – R$

Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

Illustrating the set operations union, intersection, and difference.

(a) Two union compatible relations.

(b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR.
(d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT.

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Figure 6.4
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT.

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- CARTESIAN (or CROSS) PRODUCT Operation
  - This operation is used to combine tuples from two relations in a combinatorial fashion.
  - Denoted by R(A1, A2, . . ., An) x S(B1, B2, . . ., Bm)
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then R x S will have $n_R * n_S$ tuples.
  - The two operands do NOT have to be "type compatible”

One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Example of applying CARTESIAN PRODUCT

Figure 6.5
The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

Example: Department X Employee

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

Fname	Minit	Lname	Ssn
John	B	Smith	123456789
Franklin	T	Wong	333445555

Dname	Dnumber	Mgr_ssn	Fname	Lname	Ssn
Research	5	333445555	John	Smith	123456789
Research	5	333445555	Franklin	Wong	333344555
Administrati on	4	987565757	John	Smith	123456789
Administrati on	4	987565757	Franklin	Wong	333344555

Binary Relational Operations: JOIN

- JOIN Operation (denoted by $\bowtie$)
  - The sequence of CARTESIAN PRODECT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - A special operation, called JOIN combines this sequence into a single operation
  - This operation is very important for any relational database with more than a single relation, because it allows us *combine related tuples* from various relations
  - The general form of a join operation on two relations R(A1, A2, . . ., An) and S(B1, B2, . . ., Bm) is:

Syntax: $R \bowtie_{\text{<join condition>}} S$

  - where R and S can be any relations that result from general *relational algebra expressions*.

Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.
  - To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
  - We do this by using the join $\bowtie$ operation.

  - DEPT_MGR $\leftarrow$ DEPARTMENT $\bowtie_{\text{MGRSSN=SSN}}$ EMPLOYEE
- MGRSSN=SSN is the join condition
  - Combines each department record with the employee who manages the department
  - The join condition can also be specified as DEPARTMENT.MGRSSN= EMPLOYEE.SSN

Example of applying the JOIN operation

DEPARTMENT	DName	DNo	Mgr_SSN
	Research	2	553621425
	Finance	5	996856974

EMPLOYEE	SSN	FName	LName	DNo
	123658974	Alex	Smith	2
	553621425	Fred	Scott	5
	996856974	Elsa	David	5
	859689742	Peter	Williams	2

$$\text{Temp} \leftarrow \text{DEPARTMENT X EMPLOYEE}$$

$$\text{Dept\_Mgr} \leftarrow \sigma_{(\text{MgrSSN} = \text{SSN})} (\text{Temp})$$

Dept_Mgr	DName	DNo	Mgr_SSN	SSN	FName	LName	DNo
	Research	2	553621425	553621425	Fred	Scott	2
	Finance	5	996856974	996856974	Elsa	David	5

Some properties of JOIN

- Consider the following JOIN operation:
  - $R(A1, A2, . . ., An) \bowtie_{R.Ai=S.Bj} S(B1, B2, . . ., Bm)$
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—r from R and s from S, but *only if they satisfy the join condition* r[Ai]=s[Bj]
  - Hence, if R has n_R tuples, and S has n_S tuples, then the join result will generally have *less than* $n_R * n_S$ tuples.
  - Only related tuples (based on the join condition) will appear in the result

Theta JOIN

- The general case of JOIN operation is called a Theta-join: $R \underset{\textit{theta}}{\bowtie} S$
- The join condition is called *theta*
- *Theta* can be any general boolean expression on the attributes of R and S; for example:
  - R.Ai<S.Bj AND (R.Ak=S.Bl OR R.Ap<S.Bq)
- Most join conditions involve one or more equality conditions “AND”ed together; for example:
  - R.Ai=S.Bj AND R.Ak=S.Bl AND R.Ap=S.Bq

Example

<join condition> : $A_i \ \theta \ B_j$

$A_i \Rightarrow$ Attribute of R
$B_j \Rightarrow$ Attribute of S
$\theta \Rightarrow \{ =, <, <=, >, >=, \neq \}$

Syntax: $R \bowtie_{A_i \theta B_j} S$

$R \bowtie_{A_2 = B_1} S$ | $R \bowtie_{A_2 > B_1} S$
$R \bowtie_{A_2 < B_1} S$ | $R \bowtie_{A_2 >= B_1} S$
$R \bowtie_{A_2 <= B_1} S$ | $R \bowtie_{A_2 \neq B_1} S$

R

A_1	A_2
20	25
80	40

S

B_1
50
35

$R \bowtie_{A_2 > B_1} S$

A1	A2	B1
80	40	35

Result of "$\bowtie$"

A1	A2	B1
20	25	50
20	25	35
80	40	50
80	40	35

Result of "X"

Binary Relational Operations: EQUIJOIN

- EQUIJOIN Operation
- The most common use of join involves join conditions with *equality comparisons* only
- Such a join, where the only comparison operator used is =, is called an EQUIJOIN.
  - In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - The JOIN seen in the previous example was an EQUIJOIN.

Binary Relational Operations: NATURAL JOIN Operation

- NATURAL JOIN Operation
  - Another variation of JOIN called NATURAL JOIN — denoted by * — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
    - because one of each pair of attributes with identical values is superfluous
  - The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, *have the same name* in both relations
  - If this is not the case, a renaming operation is applied first.

Binary Relational Operations NATURAL JOIN (contd.)

- Example:

PROJECT	PID	PName	DNum
	101	ProjectX	1
	102	ProjectY	2
	103	ProjectZ	2

DEPARTMENT	DNo	Mgr_SSN
	1	553621425
	2	996856974

$$\text{Proj\_Dept} \leftarrow \text{PROJECT} * \rho_{(\text{DNum},\ \text{Mgr\_SSN})}(\text{DEPARTMENT})$$

$$\text{DEPT} \leftarrow \rho_{(\text{DNum},\ \text{Mgr\_SSN})}(\text{DEPARTMENT})$$

$$\text{Proj\_Dept} \leftarrow \text{PROJECT} * \text{DEPT}$$

Proj_Dept	PID	PName	DNum	Mgr_SSN
	101	ProjectX	1	553621425
	102	ProjectY	2	996856974
	103	ProjectZ	2	996856974

Result of " * "

Example:
Operation: (EMPLOYEE * SALARY)

EMPLOYEE

EMP_CODE	EMP_NAME
101	Stephan
102	Jack
103	Harry

SALARY

EMP_CODE	SALARY
101	50000
102	30000
103	25000

EMP_CODE	EMP_NAME	SALARY
101	Stephan	50000
102	Jack	30000
103	Harry	25000

Example of NATURAL JOIN operation

(a)

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Figure 6.7
Results of two NATURAL JOIN operations.
(a) PROJ_DEPT ← PROJECT * DEPT.
(b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

Complete Set of Relational Operations

- The set of operations including SELECT σ, PROJECT π , UNION ∪, DIFFERENCE - , RENAME ρ, and CARTESIAN PRODUCT X is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- For example:
  - $R \cap S = (R \cup S) – ((R - S) \cup (S - R))$
  - $R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (R \times S)$

Binary Relational Operations: DIVISION

- DIVISION Operation
  - The division operation is applied to two relations R and S
  - $R(Z) \div S(X)$, where X is subset of Z.

- The division operator is used for queries which involve the 'all'.
  - $R1 \div R2$ = tuples of R1 associated with all tuples of R2.
- Example
  - Retrieve the name of the subject that is taught in all courses.

Name	Course
System	Btech
Database	Mtech
Database	Btech
Algebra	Btech

÷

Course
Btech
Mtech

=

Name
database

- To retrieve the Employee ID(EID) of all employees works on all projects.

EMPLOYEE

EID	PID
1001	1
1002	1
1002	2
1003	2

PROJECT

PID
1
2

$$Res \leftarrow EMPLOYEE \div PROJECT$$

Res

EID
1002

Result of the " ÷ " Operation

Example: R and S Relation

R: EMPLOYEE

EID (x)	PID (y)
1001	1
1002	1
1002	2
1003	2

S: PROJECT

PID
1
2

$$T1 \leftarrow \pi_x(R)$$
$$T2 \leftarrow \pi_x\boxed{((T1 \times S) - R)}$$
$$Res \leftarrow T1 - T2$$

T1

EID
1001
1002
1003

(T1 × S)

EID	PID
1001	1
1001	2
1002	1
1002	2
1003	1
1003	2

(T1 × S) – R

EID	PID
1001	2
1003	1

T2

EID
1001
1003

Res

EID
1002

Example of DIVISION

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

Figure 6.8
The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

Recap of Relational Algebra Operations

Table 6.1
Operations of Relational Algebra

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{(\text{<join attributes 1>}),\ (\text{<join attributes 2>})} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{(\text{<join attributes 1>}),\ (\text{<join attributes 2>})} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Additional Relational Operations: Aggregate Functions and Grouping

- A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
- Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
  - These functions are used in simple statistical queries that summarize information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values.

Aggregate Function Operation

- Use of the Aggregate Functional operation $\mathcal{F}$
  - $\mathcal{F}_{\text{MAX Salary}}$ (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{MIN Salary}}$ (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{SUM Salary}}$ (EMPLOYEE) retrieves the sum of the Salary from the EMPLOYEE relation
  - $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE) computes the count (number) of employees and their average salary
    - Note: count just counts the number of rows, without removing duplicates

Using Grouping with Aggregation

- The previous examples all summarized one or more attributes for a set of tuples
  - Maximum Salary or Count (number of) Ssn
- Grouping can be combined with Aggregate Functions
- Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY
- A variation of aggregate operation $\mathcal{F}$ allows this:
  - Grouping attribute placed to left of symbol
  - Aggregate functions to right of symbol
  - $_{\text{DNO}}\ \mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE)
- Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department

Figure 5.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Examples of applying aggregate functions and grouping

The aggregate function operation.

(a) $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}} \Im_{\text{COUNT Ssn, AVERAGE Salary}} (\text{EMPLOYEE}))$.

(b) $_{\text{Dno}} \Im_{\text{COUNT Ssn, AVERAGE Salary}} (\text{EMPLOYEE})$.

(c) $\Im_{\text{COUNT Ssn, AVERAGE Salary}} (\text{EMPLOYEE})$.

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

Additional Relational Operations (cont.)

- The OUTER JOIN Operation
  - In EQUIJOIN, tuples without a *matching* (or *related*) tuple are eliminated from the join result
    - Tuples with null in the join attributes are also eliminated
    - This amounts to loss of information.
  - A set of operations, called OUTER joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.

Additional Relational Operations (cont.)

- The left outer join operation keeps every tuple in the first or left relation R in R ⟕ S; if no matching tuple is found in S, then the attributes of S in the join result are filled or "padded" with null values.
- A similar operation, right outer join, keeps every tuple in the second or right relation S in the result of R ⟖ S.
- A third operation, full outer join, denoted by ⟗ keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

Outer join operations

⟕ Left Outer Join ⟖ Right Outer Join ⟗ Full Outer Join

- Example:

R	A	B
	1	a
	2	c

S	C	D
	1	b
	3	d

Res ← R ⟗$_{A = C}$ S

Res	A	B	C	D
	1	a	1	b
	2	c	null	null
	null	null	3	d

Full Outer Join Operation

Res ← R ⟖$_{A = C}$ S

Res	A	B	C	D
	1	a	1	b
	null	null	3	d

Right Outer Join Operation

Res ← R ⟕$_{A = C}$ S

Res	A	B	C	D
	1	a	1	b
	2	c	null	null

Left Outer Join Operation

Examples of Queries in Relational Algebra

- **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

  RESEARCH_DEPT $\leftarrow \sigma_{\text{DNAME='Research'}}$ (DEPARTMENT)

  RESEARCH_EMPS $\leftarrow$ (RESEARCH_DEPT $\bowtie_{\text{DNUMBER= DNOEMPLOYEE}}$ EMPLOYEE)

  RESULT $\leftarrow \pi_{\text{FNAME, LNAME, ADDRESS}}$ (RESEARCH_EMPS)

- **Q2: Retrieve the names of employees who have no dependents.**

  ALL_EMPS $\leftarrow \pi_{\text{SSN}}$(EMPLOYEE)

  EMPS_WITH_DEPS(SSN) $\leftarrow \pi_{\text{ESSN}}$(DEPENDENT)

  EMPS_WITHOUT_DEPS $\leftarrow$ (ALL_EMPS - EMPS_WITH_DEPS)

  RESULT $\leftarrow \pi_{\text{LNAME, FNAME}}$ (EMPS_WITHOUT_DEPS * EMPLOYEE)

Chapter Summary

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra