DIVIDE AND CONQUER

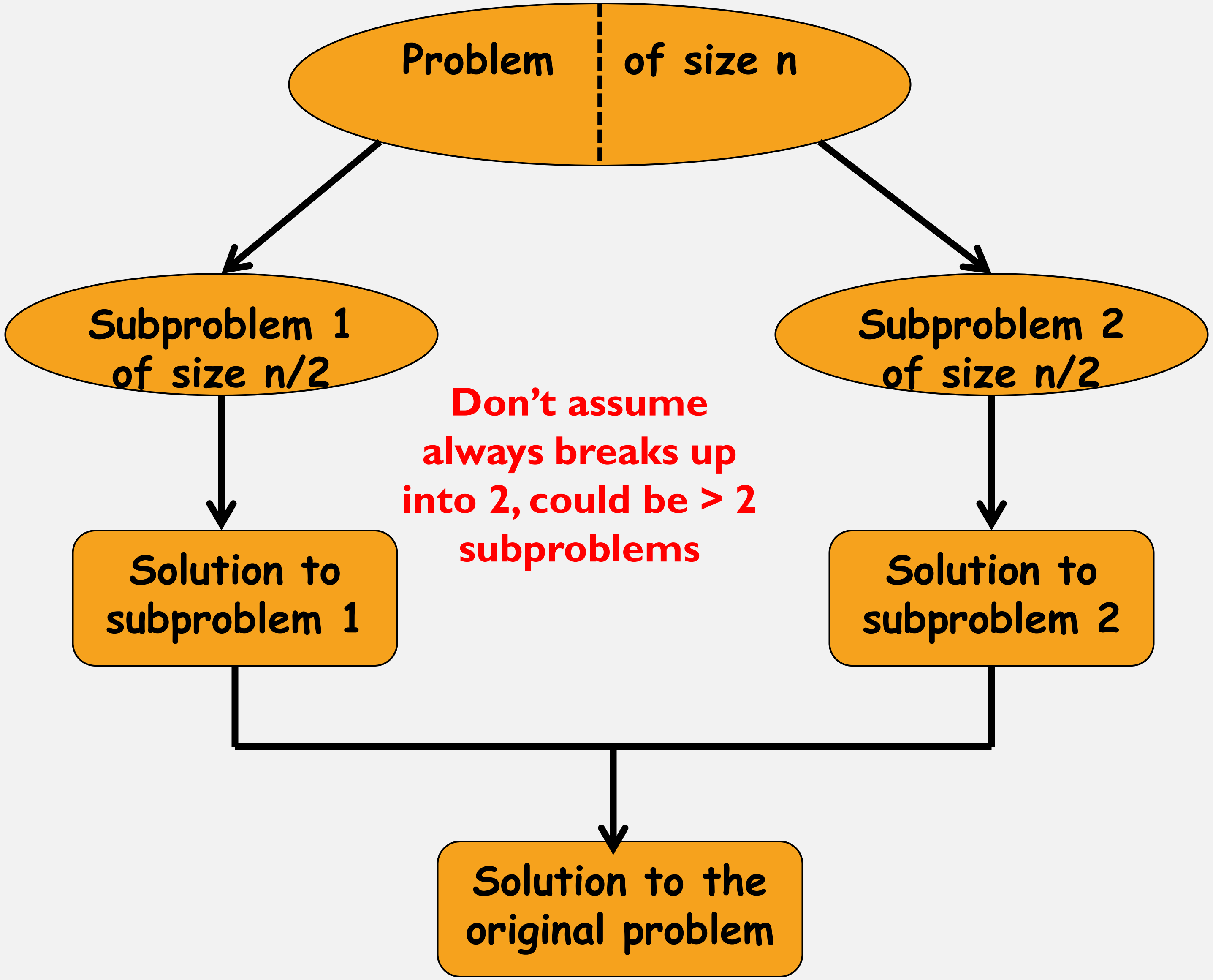
AOA : Module 2

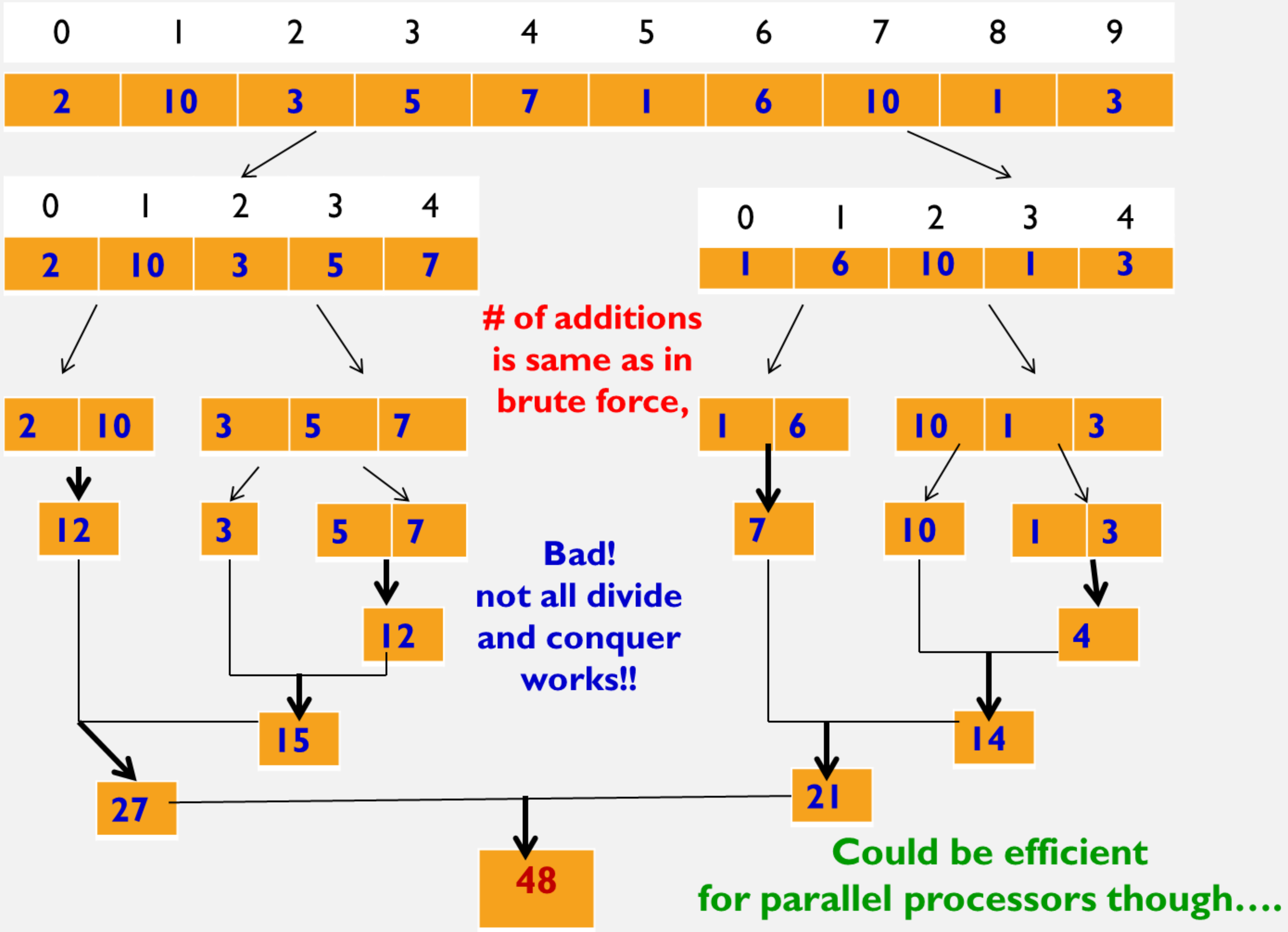
CONTENTS

INTRODUCTION

- Original problem is divided into similar kind of subproblems that are smaller in size and easy to be find.
- The solution of these small independent subproblems are combined to obtain the solution of whole problem.
- Divide and Conquer paradigm solves a problem in three steps at each level of recursion:

1. Divide
2. Conquer
3. Combine


Problem of size n
Subproblem 1 of size n/2
Subproblem 2 of size n/2
Don't assume always breaks up into 2, could be > 2 subproblems
Solution to subproblem 1
Solution to subproblem 2
Solution to the original problem


0 1 2 3 4 5 6 7 8 9
2 10 3 5 7 1 6 10 1 3
0 1 2 3 4
2 10 3 5 7
0 1 2 3 4
1 6 10 1 3
# of additions is same as in brute force,
2 10
3 5 7
1 6
10 1 3
12
3
5 7
7
10
1 3
12
4
Bad! not all divide and conquer works!!
15
14
27
21
48
Could be efficient for parallel processors though….

INTRODUCTION

- Time complexity to solve "Divide & Conquer" problem is given by recurrence relations.
- Recurrence relation is derived from algorithm and solved to calculate complexity.
- The general recurrence relation for divide and conquer is given as follows:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Where, T(n/b) : time required to solve each subproblem

f(n) : time required to combine the solutions of all subproblems

Where,

n: size of original problem.

a: number of subproblems.

b: size of each subproblem.

f(n): time to divide and combine subproblems

INTRODUCTION

- Usually in div. & conq., a problem instance of size n is divided into two instances of size n/2
- More generally, an instance of size n can be divided into b instances of size n/b, with a of them needing to be solved
- $T(n) = aT(n/b) + f(n)$
- Here, f(n) accounts for the time spent in dividing an instance of size n into subproblems of size n/b and combining their solution
- For adding n numbers, $a = b = 2$ and $f(n) = 1$

BINARY SEARCH

- There are two approaches:
  1. Iterative or Non-recursive
  2. Recursive
- There is a linear Array ‘a’ of size ‘n’.
- Binary Search is one of the fastest searching algorithm.
- Binary Search can only be applied on “Sorted Arrays”- either ascending or descending order.
- We compare “key” with item in the middle position. If they are equal, search ends successfully.
- Otherwise,

  if key is less than element present in the middle position,

  then apply binary search on lower half,

  else apply BINARY SEARCH on upper half of the array.
- Same process is applied to remaining half until match is found or there are no more elements left

BINARY SEARCH

Iterative Approach:

```
Algorithm IBinaryS(arr[ ], start, end, key){
        int mid;
        while(start<=end){
                mid = (start + end)/2;
                if (arr[mid] == key)
                        return 1;
                if (arr[mid]<key)
                        start = mid+1;
                else
                        end = mid-1;
        }
        return 0;
}
```

Recursive Approach:

```
Algorithm RBinaryS(arr[ ], start, end, key){
        int mid;
        if (start > end) { return 0; }
else
        mid = (start + end)/2;
        if (key == arr[mid])
        return (mid);
        else
        if (key < arr[mid]){
        RBinaryS(arr[],key, start, mid-1)
        else
        RBinaryS(arr[],key, mid+1, end)
        }
}
```

FINDING MINIMUM AND MAXIMUM

Iterative Approach:

```
Algorithm MinMax(a[ ], n, max, min){
        max=min=a[1];
        for(i=2 to n)do
{
        if(a[i]> max) then max=a[i];
        if (a[i]< min) then min=a[i];
}
}
```

Recursive Approach:

```
Algorithm MinMax(a[ ], l, h, max, min) {
            if(l==h) then
                        max=min=a[l];
            else if      (h-l==1), then
            {
            if(a[l]>=a[h]), then
                        max=a[l];
                        min=a[h];
            else{
                        max=a[h];
                        min=a[l];
}
else{
            Mid = (l+h)/2;
            MinMax(l,, mid, max, min);
            MinMax(mid+1, h, max, min);
            if(max< max1)  then max=max1;
            if (min>min1)then, min = min1;
}
}
```

Given in next Page

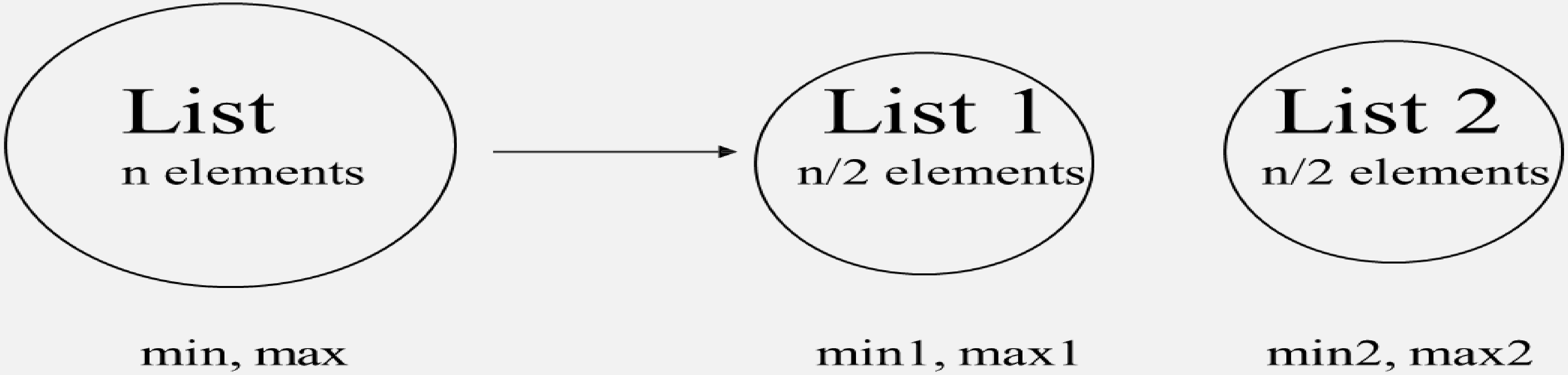
FINDING MINIMUM AND MAXIMUM

Recursive Approach:

```
Algorithm MinMax(a[ ], l, h, max,
min) {
      if(l==h) then
            max=min=a[l];
      else if(h-l==1), then
      {
      if(a[l]>=a[h]), then
            max=a[l];
            min=a[h];
      else{
            max=a[h];
            min=a[l];
}
```

```
else{
      Mid = (l+h)/2;
      MinMax(l, mid, max, min);
      MinMax(mid+1, h, max1, min1);
      if(a[max]< a[max1])  then
            max=max1;
      if (a[min]>a[min1])then,
            min = min1;
}
}
```

FINDING MINIMUM AND MAXIMUM



min = MIN (min1, min2)
max = MAX (max1, max2)

FINDING MINIMUM AND MAXIMUM

Analysis: *For algorithm containing recursive calls, we can use recurrence relation to find its complexity*

T(n) - # of comparisons needed for Rmaxmin

Recurrence relation:

$$\begin{cases} T(n) = 0 & n = 1 \\ T(n) = 1 & n = 2 \\ T(n) = 2T(\frac{n}{2}) + 2 & n > 2 \end{cases}$$

FINDING MINIMUM AND MAXIMUM

When n is a power of two, $n = 2^k$ for some positive integer k, then

$$\begin{aligned} T(n) &= 2T(n/2) + 2 \\ &= 2(2T(n/4) + 2) + 2 \\ &= 4T(n/4) + 4 + 2 \\ &\vdots \\ &= 2^{k-1}T(2) + \textstyle\sum_{1 \le i \le k-1} 2^i \end{aligned}$$

Assume $n = 2^k$ for some integer k $\quad 2^{k-1} = \frac{n}{2}$

$$= 2^{k-1} \cdot T(2) + (2^k - 2) = \frac{n}{2} \cdot 1 + n - 2$$

$$= 1.5n - 2$$

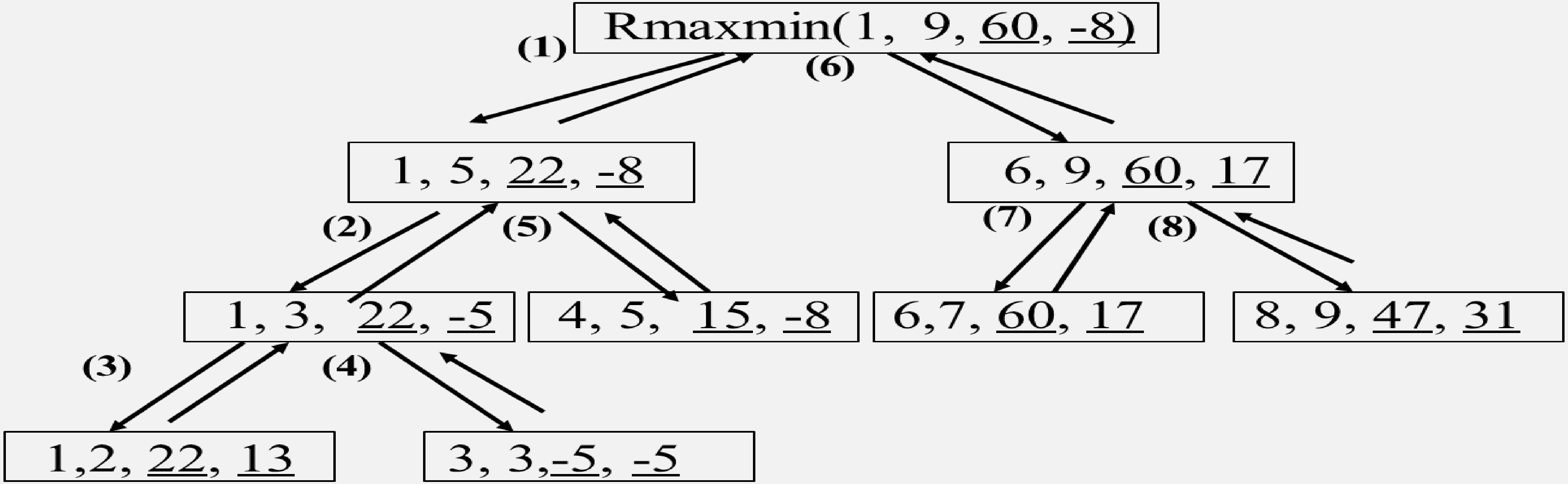
FINDING MINIMUM AND MAXIMUM

FINDING MINIMUM AND MAXIMUM

Example: find max and min in the array:

22, 13, -5, -8, 15, 60, 17, 31, 47 (n = 9)

Index:	1	2	3	4	5	6	7	8	9
Array:	22	13	-5	-8	15	60	17	31	47



MERGE SORT

- Simple and efficient algorithm for sorting a list of numbers
- Based on divide and Conquer paradigm
- Performed in three steps:

1. <u>Divide:</u>
   i. List of n elements is divided into 2 sub-lists of n/2 elements
   ii. Computes middle of the array, so it takes constant time O(1).
2. <u>Conquer:</u>
   1. Each half is sorted independently.
   2. Merge sort is recursively used to sort elements of smaller sub-lists.
   3. This step contributes T(n/2) + T(n/2) to running time.

MERGE SORT

3. Combine:
   1. Two sorted halves are merged to obtain a sorted sequence
   2. This requires merging of n elements into 1 list.
   3. It contributes O(n) to running time.

NOTE: The Key operation of merge sort is Merging

MERGE SORT ALGORITHM

```
mergeSort(arr[ ],low, high)
//arr is array, low is left sub-list, high is right sub-list
{
   if(low<high)
   {
      mid = (low+high)/2;
      mergeSort(arr, low, mid);
      mergeSort(arr,mid+1,high);
      merge(arr, low, mid, high);
   }
}
```

MERGE ALGORITHM

```
void merge(int arr[ ], int low, int mid,
int high) {
  int i = low;
  int j = mid + 1;
  int k = low;

  /* create temp array */
  int temp[5];
```

1

```
while (i <= mid && j <= high) {
    if (arr[i] <= arr[j]) {
      temp[k] = arr[i];
      i++;
      k++;
    }
else {
      temp[k] = arr[j];
      j++;
      k++;
    }
 }
```

2

MERGE ALGORITHM

```
/* Copy the remaining elements
of first half, if there are any */
  while (i <= mid) {
    temp[k] = arr[i];
    i++;
    k++;
  }
```

3

```
/* Copy the remaining elements
of 2nd half, if there are any */
  while (j <= high) {
    temp[k] = arr[j];
    j++;
    k++;
  }
```

4

```
/* Copy the temp array to original array */
  for (int k = low; k <= high; k++) {
    arr[k] = temp[k];
  }
}
```

5

MERGE SORT EXAMPLE

Example:

54	26	93	17	77	31	44	55