SOMAIYA VIDYAVIHAR UNIVERSITY

Semester: Jan 2024-April 2024		
Maximum Marks: 30	**Examination: In-Semester Examination**	**Duration: 1hr. 15 min.**
Programme code: 01 **Programme:** B. Tech Computer Engineering	**Class:** SY	**Semester:** IV **(SVU 2020)**
Name of the Constituent College: **K. J. Somaiya College of Engineering**		**Name of the department:** COMP
Course Code: 116U01C401	**Name of the Course:** Probability, Statistics and Optimization Techniques	

Question No.		Max. Marks
Q.1	Attempt any **THREE** of the following	
a)	The joint probability distribution of X and Y is given by $P(X=x, Y=y) = \frac{2x+3y}{72}$; $x = 0,1,2,\ y = 1,2,3$ (i) Find the joint p.m.f s of X and Y (ii) Find the Marginal Probability distributions of X and Y. (iii) Find $P(X+Y \leq 2)$ Solution: The joint pmf of X and Y is as follows	06

X/Y	1	2	3	Total
0	1/24	1/12	1/8	1/4
1	5/72	1/9	11/72	1/3
2	7/72	5/36	13/72	5/12
Total	5/24	1/3	11/24	1

The Marginal Probability distribution of X is

X	0	1	2	Total
P(X=x)	¼	1/3	5/12	1

The Marginal Probability Distribution of Y is

Y	1	2	3	Total
P(Y=y)	5/24	1/3	11/24	1

$$P(X+Y \leq 2) = P(X=0, Y=1) + P(X=0, Y=2) + P(X=1, Y=1) = \frac{7}{36}$$

b)	A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operator's B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, (i) What is the probability that it was produced by A? (ii) What is the probability that it was produced by B? Solution: Let E_1 , E_2, E_3 be the respective events of the time consumed by machines A, B, and C for the job.	06

	P(E_1) = 50% = 0.50 , P(E_2) = 30%= 0.30, P(E_3) = 20% =0.20 Let X be the event of producing defective items. P(X|E_1) = 1% = 0.01, P(X|E_2) = 5%=0 .05, P(X|E_3) = 7% = 0.07 The probability that the defective item was produced by A is given by P (E_1|X). **By using Bayes' theorem, we obtain** $P(E_1/X) = \frac{P(E_1)\cdot P(X/E_1)}{P(E_1)\cdot P(X/E_1)+P(E_2)\cdot P(X/E_2)+P(E_3)\cdot P(X/E_3)} = \frac{5}{34}$ The probability that the defective item was produced by B is given by P (E_2|A). **By using Bayes' theorem, we obtain** $P(E_2/X) = \frac{P(E_2)\cdot P(X/E_2)}{P(E_1)\cdot P(X/E_1)+P(E_2)\cdot P(X/E_2)+P(E_3)\cdot P(X/E_3)} = \frac{15}{34}$	
c)	If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights (i) greater than 72 inches (ii) between 65 and 71 inches Solution: Let X be the random variable of height Given $m = 68\,, \sigma = 4 \;\therefore Z = \frac{X-68}{4}$ (i) Probability of a student height being greater than 72 = $P(X > 72) = P\left(Z > \frac{72-68}{4}\right)$ $= P(Z > 1) = 0.5 - area\ from\ Z = 0\ to\ Z = 1$ $= 0.5 - 0.3413 = 0.1587$ $\therefore$ **number of students having height greater than 72 inches** $= \mathbf{500 \times 0.1587 = 79.35 \sim 79}$ (i) Probability of a student height being between 65 and 71 inches $= P(65 < X < 71) = P\left(\frac{65-68}{4} < Z < \frac{71-68}{4}\right)$ $= P(-0.75 < Z < 0.75) = 2 \times P(0 < Z < 0.75)$ $= 2 \times area\ from\ Z = 0\ to\ Z = 0.75 = 2 \times 0.2734 = 0.5468$ $\therefore$ **number of students having height between 65 and 71 inches** $= \mathbf{500 \times 0.5468 = 273.4 \sim 273}$	06
d)	A transmission channel has a per digit error probability $p = 0.01$. calculate the probability of 1 error in 10 received digits using (i) Binomial distribution (ii) Poisson Distribution Solution: (i) Using Binomial Distribution $n = 10,\ p = 0.01,\ q = 0.99$ $P(X = x) = {}^nC_x p^x q^{n-x} = {}^{10}C_x (0.01)^x (0.99)^{10-x}$ $P(X = 1) = {}^{10}C_1 (0.01)^1 (0.99)^9 = 0.0913$ (ii) Using Poisson Distribution $m = np = 10 \times 0.01 = 0.1$ $P(X = x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-0.1}(0.1)^x}{x!}$ $P(X = 1) = \frac{e^{-0.1}(0.1)^1}{1!} = 0.0904$ **the probability of 1 error in 10 received digits using** **(i) Binomial distribution =** $\mathbf{0.0913}$ **(ii) Poisson Distribution =** $\mathbf{0.0904}$	06
e)	A continuous random variable X has the probability density function given by $f(x) = \begin{cases} 2ax + b & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$ If the mean of the distribution is 3, find the constants a and b. Solution: By property of pdf $\int_0^2 f(x)dx = 1$	06

$\int_0^2 (2ax+b)dx = 1$

$\therefore 4a+2b = 1$ ………….. (1)

Mean = $\int_0^2 x\, f(x)dx \ = 3$

$\therefore \int_0^2 (2ax^2+bx)dx = 3$

$\therefore 16a+6b = 9$ ………….. (2)

Solving (1) and (2), we get $\boldsymbol{a = \frac{3}{2}}$, $\boldsymbol{b = \frac{-5}{2}}$

Q.2 Attempt any **TWO** of the following

(a) Calculate the value of rank correlation coefficient from the following data regarding marks of 6 students in statistics and accountancy in a test: 0

Marks in Statistics: 40, 42, 45, 35, 36, 39

Marks in Accountancy: 46, 43, 44, 39, 40, 43

Solution:

Marks in Statistics	**Marks in Accountancy**	$\boldsymbol{R_1}$	$\boldsymbol{R_2}$	$\boldsymbol{d = R_1 - R_2}$	$\boldsymbol{d^2}$
40	46	4	6	-2	4
42	43	5	3.5	1.5	2.25
45	44	6	5	1	1
35	39	1	1	0	0
36	40	2	2	0	0
39	43	3	3.5	-0.5	0.25
Total				**0**	$\boldsymbol{\Sigma d_i^2}$ **=7.5**

$$\therefore R = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1)\right]}{n^3 - n}$$

Since, $\Sigma d_i^2 = 7.50,\ m_1 = 2,\quad n = 6$

$$\therefore R = 1 - \frac{6\left[7.5 + \frac{1}{12}(2^3 - 2)\right]}{6^3 - 6} = 1 - \frac{6[7.5 + 0.5]}{210} = 0.771$$

(b) Find equation of both the regression lines from the following data where x, y denote the actual values. Also Estimate x when $y = 15$ and estimate y when $x = 8$. 06

$N = 12, \sum x = 120\,, \sum y = 432\,, \sum xy = 4992,\quad \sum x^2 = 1392\ ,\ \sum y^2\ = 18252$

Solution:

The coefficients of regression are given by

$$b_{yx} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{N}}{\sum x^2 - \frac{(\sum x)^2}{N}} = \frac{4992 - \frac{120 \times 432}{12}}{1392 - \frac{(120)^2}{12}} = \frac{672}{192} = 3.5$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{N}}{\sum y^2 - \frac{(\sum y)^2}{N}} = \frac{4992 - \frac{120 \times 432}{12}}{18252 - \frac{(432)^2}{12}} = \frac{672}{2700} = 0.249$$

The equation of the line of regression of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$$y - 36 = 3.5(x - 10) \ \Rightarrow y = 3.5x + 1$$

When $x = 8$ $we\ get\ y = 3.5 \times 8 + 1 = 29$

The equation of the line of regression of x on y is $x - \bar{x} = b_{xy}(y - \bar{y})$

$$x - 10 = 0.249(y - 36) \ \Rightarrow x = 0.249y + 1.036$$

When $y = 15$ $we\ get\ x = 0.249 \times 15 + 1.036 = 4.771$

	(c)	Calculate the Karl Pearson coefficient of correlation between price and demand Price: 2, 3, 4, 7, 4 Demand: 8, 7, 3, 1, 1 Solution:	06

x	y	x^2	y^2	xy
2	8	4	64	16
3	7	9	49	21
4	3	16	9	12
7	1	49	1	7
4	1	16	1	4
$\sum x = 20$	$\sum y = 20$	$\sum x^2 = 94$	$\sum y^2 = 124$	$\sum xy = 60$

$\bar{x} = \frac{\sum x}{N} = \frac{20}{5} = 4 \quad \bar{y} = \frac{\sum y}{N} = \frac{20}{5} = 4$

$$r = \frac{\sum xy - N\bar{x}\bar{y}}{\sqrt{\sum x^2 - N\bar{x}^2}\sqrt{\sum y^2 - N\bar{y}^2}} = \frac{60 - 5\times4\times4}{\sqrt{94 - 5\times16}\,\sqrt{124 - 5\times16}} = \frac{-20}{\sqrt{14}\sqrt{44}} = -0.8058$$