

LPP

**STANDARD FORM
OF LPP**

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CHARACTERISTICS OF THE STANDARD FORM:

- (i) The objective function is of maximization type
- (ii) All constraints are expressed as equations
- (iii) Right hand side of each constraint is non negative
- (iv) All variables are non negative

NOTE :

- (i) Minimization of a function Z is equivalent to Maximization of the negative expression of this function
i.e. $\text{Min } Z = -\text{Max } (-Z)$
- (ii) If RHS of any constraint is negative, multiply that constraint by -1 to convert the RHS to positive.

- (iii) If a variable is unrestricted in sign, then it can be expressed as difference of two non-negative variable
i.e. X_1 is unrestricted in sign, then
 $X_1 = X_1' - X_1''$, where X_1', X_1'' are ≥ 0

- (v) In standard form, all the constraints are expressed in equation, which is possible by introducing some additional variables called **slack variables** and **surplus variable** so that a system of simultaneously linear equations is obtained.

Convert the following LPP in the standard form

EX 1. Maximize $z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \leq 15,$

$2x_1 + 5x_2 \geq 12,$

$x_1, x_2 \geq 0$

Solution: Introducing the slack variables the problem can be converted to standard form as:

$$\text{Maximize } z = 3x_1 + 5x_2 + 0s_1 + 0s_2$$

$$\text{Subject to } 3x_1 + 2x_2 + s_1 + 0s_2 = 15,$$

$$2x_1 + 5x_2 + 0s_1 - s_2 = 12,$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Convert the following LPP in the standard form

EX 2. Minimize $z = -3x_1 + 2x_2 - x_3$

Subject to $x_1 - 3x_2 + 2x_3 \geq -6,$

$$3x_1 + 4x_3 \leq 3,$$

$$-3x_1 + 5x_2 \leq 4,$$

$x_1, x_2 \geq 0, x_3$ is unrestricted

Solution: Since the problem is of minimization type we write $z' = -z$, so that the objective function is of maximization type.

Since in the first constraints the right hand side is negative, we multiply it by (-1) , so that it becomes positive and of less than type. Hence, we add slack variable $s_1 (\geq 0)$.

Since the second and the third constraints are of less than type we add slack variables s_2 and s_3 both (≥ 0) .

Since x_3 is unrestricted we write

$$x_3 = x_3' - x_3'' \text{ where } x_3' \geq 0, x_3'' \geq 0$$

Now the problem becomes

Maximize

$$z' = -z = 3x_1 - 2x_2 + x_3' - x_3'' + 0s_1 + 0s_2 + 0s_3$$

Subject to

$$-x_1 + 3x_2 - 2x_3' + 2x_3'' + s_1 + 0s_2 + 0s_3 = 6,$$

$$3x_1 + 0x_2 + 4x_3' - 4x_3'' + 0s_1 + s_2 + 0s_3 = 3,$$

$$-3x_1 + 5x_2 + 0x_3' - 0x_3'' + 0s_1 + 0s_2 + s_3 = 4,$$

$$x_1, x_2, x_3', x_3'', s_1, s_2, s_3 \geq 0.$$

Convert the following LPP in the standard form

EX 3. Minimize $z = 2x_1 + 3x_2$

Subject to $2x_1 - 3x_2 - x_3 = -4,$

$$3x_1 + 4x_2 - x_4 = -6,$$

$$2x_1 + 5x_2 + x_5 = 10,$$

$$4x_1 - 3x_2 + x_6 = 18$$

$$x_3, x_4, x_5, x_6 \geq 0$$

Solution:

Let $x_1 = y_1 - y_2$ and $x_2 = y_3 - y_4$,

$x_3 = y_5, x_4 = y_6, x_5 = y_7, x_6 = y_8$

The problem changes to

Maximize $z' = -z = -2y_1 + 2y_2 - 3y_3 + 3y_4$

Subject to $-2y_1 + 2y_2 + 3y_3 - 3y_4 + y_5 = 4$

$$-3y_1 + 3y_2 - 4y_3 + 4y_4 + y_6 = 6$$

$$2y_1 - 2y_2 + 5y_3 - 5y_4 + y_7 = 10$$

$$4y_1 - 4y_2 - 3y_3 + 3y_4 + y_8 = 18$$

$$y_i \geq 0 \text{ for all } i = 1, 2, \dots, 8$$

Covert the following LPP to standard form

EX 4. Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subject to $3x_1 - 2x_2 + 4x_3 \leq 5,$

$$2x_1 + 5x_2 = 10,$$

$$x_1 + 2x_2 + x_3 \leq 2,$$

$$x_1, x_2, x_3 \geq 0$$

Also put the problem in matrix form.

Solution: Introducing slack variables s_1 and s_3 both (≥ 0), we have

$$\text{Maximize } z = 2x_1 + 3x_2 + 6x_3 + 0s_1 + 0s_2$$

$$\text{Subject to } 3x_1 - 2x_2 + 4x_3 + s_1 + 0s_2 = 5$$

$$2x_1 + 5x_2 + 0x_3 + 0s_1 + 0s_2 = 10$$

$$x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 = 2$$

Thus, the problem in the matrix form becomes:

Maximize $z = CX$

Subject to $AX = B, X \geq 0$

Where, $A = \begin{bmatrix} 3 & -2 & 4 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & -1 \end{bmatrix},$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix}, C = [2 \quad 3 \quad 6 \quad 0 \quad 0]$$