18/5/2022 (E)

SOMAIYA VIDYAVIHAR UNIVERSITY

Semester: January 2022 – May 2022 **Maximum Marks: 100** **Examination: ESE Examination** **Duration: 3hrs**		
Programme code: 01&04 **Programme: B Tech Comp/IT**	**Class: SY**	**Semester: IV (SVU 2020)**
Name of the Constituent College: **K. J. Somaiya College of Engineering**	**Name of the department:** COMP/IT	
Course Code: 116U01C401 /116U04C401	**Name of the Course: Probability, Statistics and Optimization Techniques**	
Instructions: 1)Draw neat diagrams 2)Assume suitable data if necessary		

Q No			MAX MARKS
Q1	a	If X_1 has mean 5 and variance 5, X_2 has mean -2 and variance 3, find $E(2X_1+3X_2-5)$, $V(2X_1+3X_2-5)$	5
	b	**Solve any THREE of the following**	21
	(i)	Determine the constant 'a' and find mean, $P(4 \le x \le 7)$ if the distribution function of a continuous random variable is defined as $f(x)=\frac{a}{x^5}, 2 \le x \le 10$	
	(ii)	If the height of 1000 students is normally distributed with mean 69 inches and standard deviation 4 inches. Find the expected number of students having heights: i) greater than 67 inches, ii) less than 68 inches, iii) between 65 & 71 inches	
	(iii)	The number of phone calls coming in to a telephone exchange between 2 & 4 P.M. say X is a random variable has Poisson distribution with parameter 2. Similarly the number of phone calls coming between 4 & 6 P.M. say Y is a random variable has Poisson distribution with parameter 6. If X& Y are independent Poisson random variables find the probability that during 2&6 P.M. there will be i) no phone calls at all ii) more than 3 calls. (iii) at most two calls	
	(iv)	A box to be constructed so that its height is 12 inches and its base is X inches by X inches. If X has a uniform distribution over the interval (2, 10), then what is the expected volume of the box in cubic inches?	
	(v)	The joint probability distribution function of (X,Y) is given by $f(x,y)=e^{-(x+y)} \quad 0 \le x, \ 0 \le y$ Compute $P(X>2), P(1<X+Y<3))$	

Q2 **a** A data for selection of students regarding placement is given below. Find the probability that a boy is selected for the placement and log of odds of this probability — **5**

Students	Selection in placement		Total
	yes	no	
Girls	753	102	855
Boys	382	158	540
Total	1145	250	135

b **Solve any ONE of the following** — **7**

(i) Calculate the correlation coefficient from the following data.

x	23	27	28	29	30	31	33	35	36	39
y	18	22	23	24	25	26	28	29	30	32

(ii) Obtain two lines of regression and coefficient of correlation from the following data-

X	65	66	67	68	69	70	72	67
y	67	68	65	72	72	69	71	66

Q3 **a** Two samples are drawn from two different population gave the following results. Find 95% confidence limits for the difference between the population means. — **5**

	Size	Mean	S.D
Sample I	400	124	14
Sample II	250	120	12

b **Solve any TWO of the following** — **14**

(i) Intelligence tests of two groups of boys & girls obtained from two normal populations having the same standard deviations gave the following results. Test at 1% level of significance whether the boys perform better than the girls.

	Size	Mean	S.D
Girls	121	84	10
Boys	181	81	12

(ii) A certain injection administered to 12 patients resulted in the following change of blood pressure 5,2,8, -1,3,0,6, -2,1,5,0,4. Can be concluded that the injection will be in general accompanied by an increase in blood pressure at 5% LOS?

(iii) From the following table, showing the number of plants having certain character, test the hypothesis that the flower colour is independent of flatness of leaf.

				Flat leaves	Curved leaves	Total	
			White Flowers	99	36	135	
			Red Flowers	20	5	25	
			Total	119	41	160	
Q4	a	Construct the Dual of the following LPP Maximize $z = 5x_1 + 2x_2 - 3x_3$ Subject to $2x_1 - 2x_2 + x_3 \geq 4$ $2x_1 + x_3 \leq 8$ $x_1 + x_2 + 3x_3 = 20$ $x_1, x_3 \geq 0$, x_2 unrestricted					5
	b	**Solve any THREE of the following**					21
	(i)	Using Simplex method solve the following LPP Maximize $z = 3x_1 + 2x_2 + 5x_3$ Subject to $x_1 + x_2 + x_3 \leq 9$ $2x_1 + 3x_2 + 5x_3 \leq 30$ $2x_1 - x_2 - x_3 \leq 8$ $x_1, x_2, x_3 \geq 0$					
	(ii)	Using Big M method solve the following LPP Maximize $z = 6x_1 + 4x_2$ Subject to $2x_1 + 3x_2 \leq 30$, $3x_1 + 2x_2 \leq 24, x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$					
	(iii)	Using Duality Solve the following linear programming problem Minimize $z = 4x_1 + 3x_2 + 6x_3$ Subject to $x_1 + x_3 \geq 2$, $x_2 + x_3 \geq 5$, $x_1, x_2, x_3 \geq 0$					
	(iv)	Using Dual simplex method Solve the following linear programming problem Minimize $z = 2x_1 + 2x_2 + 4x_3$ Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$, $3x_1 + x_2 + 7x_3 \leq 3$, $x_1 + 4x_2 + 6x_3 \leq 5$ $x_1, x_2, x_3 \geq 0$					
	(v)	Solve the following NLPP Maximize $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$ Subject to $2x_1 + 5x_2 \leq 98$, $x_1, x_2 \geq 0$					
Q5	a	In a bank cheques are cashed at a single 'teller' counter. Customers arrive at the counter in a Poisson manner at an average rate of 30 customers per hour. The teller takes, on an average, a minute and a half to cash a cheque The service time has been shown to be exponentially distributed. Calculate the % of time the teller is busy.					3

	b	**Solve any TWO of the following**	14
	(i)	Patients arrive at a clinic according to Poissondistribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. The examination time per patient is exponential with mean rate of 20 per hour. (a) Find number of patients in the clinic before the examination (b) What is the probability that an arriving patient will not wait. (c) What is the expected waiting time until a patient is discharged from the clinic ?	
	(ii)	Trucks arrival at a factory is for collecting finished goods that are supposed to be transported to distant markets. As and when they come they are required to join awaiting line and are served on first come, first served basis. Trucks arrive at the rate of 10 per hour where as the loading rate is 15 per hour. It is also given that arrivals are Poisson and loading is exponentially distributed. (a) Transporters have complained that their trucks have to wait for nearly 12 minutes at the plant. Examine whether the complaint is justified. (b) Determine the number of trucks waiting in the queue before getting loaded. (c) Find the probability that a truck cannot be loaded immediately.	
	(iii)	Customer arrives at a box office window, being manned by a single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds Find the average time spent by a customer. Also determine the average number of customers in the system and the average queue length	