SOMAIYA VIDYAVIHAR UNIVERSITY

Semester: Jan 2023 - May 2023 **Maximum Marks: 100** **Examination: ESE Examination** **Duration:3 Hrs.**		
Programme code: 01/03 **Programme:** B. Tech Computer/IT Engineering	**Class:** SY	**Semester: IV (SVU 2020)**
Name of the Constituent College: K. J. Somaiya College of Engineering	**Name of the department: Computer/IT**	
Course Code: 116U01C401/116U03C401	**Name of the Course:** Probability, Statistics and Optimization Techniques	
Instructions: 1) All questions are compulsory 2) Assume suitable data wherever necessary		

Que. No.	Question	Max. Marks
Q1	Solve any **Four** of the following.	**20**
i)	Three machines A, B, C produce respectively 60%, 30% & 10% of the total number of items of a factory. The percentage of defective outputs of these machines are respectively 2%,3% & 4%.An item is chosen at random and found to be defective . Using Bayes theorem find the probability that it was produced by the factory A	5
ii)	Compute Rank correlation coefficient from the following data x: 105, 104, 102, 101, 100, 99, 98, 96, 93, 92 y: 101, 103, 100, 98, 95, 96, 104, 92, 97, 94	5
iii)	A sample of 900 numbers has a mean 3.4 cms and s.d. 2.61 cms. If the population is normal, find the 95% and 98% confidence limits of the population mean.	5
iv)	Convert the given LPP into the standard form Minimise z =$7x_1 - 48x_2 + 23x_3$ Subject to $61x_1 - 29x_2 + 12x_3 \leq 93$ $3x_1 - 61x_2 + 81x_3 \geq 9$ $x_1 - 33x_2 + 53x_3 \leq -5$ where $x_1, x_2, \geq 0$ and x_3 is unrestricted in sign	5
v)	Find the average number of customers in the system and in the queue if the system is (M/M/1/∞) and $\mu = 10, \lambda = 8$ per hour	5
vi)	The joint probability distribution function of (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}$ where $0 \leq x \leq 2$, $0 \leq y \leq 1$.Compute(a)$P(X > 1)$ (b)$P(Y < 0.5)$ (c)$P(X > 1 \mid Y < 0.5)$	5
Q2 A	Solve the following.	**10**
i)	The regression lines of a sample are x + 6y = 6 and 3x + 2y =10 Find (a) $\overline{x}$ and $\overline{y}$ (b) correlation coefficient r. Also estimate y when x =12. (c)verify that the sum of the coefficients of regression is greater than 2r	5
ii)	A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample is from a batch of string having a mean breaking strength of 15.6 pounds & standard deviation of 2.2 pounds.	5

OR

Q2 A	Using Lagrange's Multiplier method solve the following NLPP $z = 2{x_1}^2 + {x_2}^2 + 3{x_3}^2 + 10x_1 + 8x_2 + 6x_3 - 100$ subject to $x_1 + x_2 + x_3 = 20$, $x_1, x_2, x_3 \geq 0$	10
Q 2 B	Solve any **One** of the following.	**10**
i)	The local one person barber shop can accommodate maximum of 5 people at a time (4 waiting and 1 getting haircut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair according to an Exponential distribution at an average rate of 4 per hour. (a) What percentage of time is the barber idle? (b) What fraction of potential of customers are turned away? (c) What is the expected number of customers waiting for a haircut? (d) How much time can a customer expect to spend in the barber shop?	10
ii)	Define probability mass function of Poisson distribution and Fit a Poisson distribution to the following data if the following mistakes per page were observed in a book.	10

No. of mistakes	0	1	2	3	4	Total
No. of pages	211	90	19	5	0	325

Q3	Solve any **Two** of the following.	**20**
i)	(a) The height of 1000 soldiers in a regiment are distributed normally with mean 172 cm and standard deviation 5 cm. how many soldiers have height greater than 180 cm.	05
	(b) Two groups A & B of patients each consisting of 200 people are used to test effectiveness of a new serum. Group A is given serum while group B not. It is found that mean of two groups of A & B are 140 & 120 respectively and standard deviation of 14 & 12 respectively. Test at 1% LOS whether the new serum helps to cure the disease.	05
ii)	Find the lines of regression for the following data to estimate y corresponding to $x = 155$ and value of x corresponding to $y = 152$	10

x	100	110	120	130	140	150	160	170	180	190
y	45	51	54	61	66	70	74	78	85	89

iii)	Define the following terms Solution of LPP, Basic solution of LPP, Feasible solution and degenerate solution of LPP. Also Find (a) All basic solutions (b) All feasible basic solutions (c) All degenerate solutions hence decide the optimal feasible basic for the following L.P.P. Maximise $z = 2x_1 + 3x_2 + x_3 + x_4$ Subject to $x_1 - 3x_2 + 2x_3 + x_4 = 5$ $x_1 + x_2 + 3x_3 - 2x_4 = 4$ where $x_1, x_2, x_3, x_4 \geq 0$	10

<table>
<tr><td>Q4</td><td>Solve any Two of the following.</td><td>20</td></tr>
<tr><td>i)</td><td>The probability that an electronic component will fail in less than 1200 hours of continuous use is 0.25 Use Normal approximations to find the probability that among 200 such components exactly 45 will fail in less than 1200 hours of continuous use</td><td>10</td></tr>
<tr><td>ii)</td><td>A certain drug is claimed to be effective in curing cold in an experiment on 500 persons with cold. Half of them were given drug and half of them were given the sugar pills. The patients reaction to the treatment are recorded in the following table using χ^2 -test (use 5% LOS)
<table><tr><td></td><td>Helped</td><td>Harmed</td><td>No Effect</td><td>Total</td></tr><tr><td>Drug</td><td>150</td><td>30</td><td>70</td><td>250</td></tr><tr><td>Sugar pills</td><td>130</td><td>40</td><td>80</td><td>250</td></tr><tr><td>Total</td><td>280</td><td>70</td><td>150</td><td>500</td></tr></table>On the basis of this data, can it be concluded that the drug and sugar pills differ significantly in curing cold.</td><td>10</td></tr>
<tr><td>iii)</td><td>Solve the given LPP by Simplex method
Maximise z =$4x_1 + 3x_2 + 6x_3$
Subject to
$2x_1 + 5x_2 \leq 430$
$4x_1 + 3x_3 \leq 470$
$2x_1 + 3x_2 + 2x_3 \leq 440$
where $x_1, x_2, x_3 \geq 0$</td><td>10</td></tr>
<tr><td>Q5</td><td>Solve any Four of the following.</td><td>20</td></tr>
<tr><td>i)</td><td>X follows a Uniform Distribution over the range (2,b) such that P(3<X<6)=0.3 Find mean and variance of X.</td><td>5</td></tr>
<tr><td>ii)</td><td>If the tangent of the angle made by the lines of regression of y on x is 0.6 and $\sigma_y = 2\sigma_x$, find the correlation coefficient between x and y.</td><td>5</td></tr>
<tr><td>iii)</td><td>A random sample of 400 items gives the mean 4.45 & variance 4. Can it be regarded as drawn from a normal population with mean 4 at 5% level of significance?</td><td>5</td></tr>
<tr><td>iv)</td><td>Find the relative maximum or minimum of the function
$z = 20 + x_1 + 2x_3 + x_2x_3 - {x_1}^2 - {x_2}^2 - {x_3}^2$</td><td>5</td></tr>
<tr><td>v)</td><td>Find the traffic intensity of the system (M/M/1/∞) model if $\mu = 1\ per\ hour, \lambda$ =8 per hour. Also find the probability that a customer has to wait for more than 20 minutes to be out of the service station.</td><td>5</td></tr>
<tr><td>vi)</td><td>Obtain the dual of the following LPP
Minimise z =$3x_1 + 17x_2 + 9x_3$
Subject to
$-x_2 + x_3 \geq 3$
$-3x_1 + 2x_3 \leq 1$
$2x_1 + x_2 - 5x_3 = 1$
where $x_1, x_2, x_3 \geq 0$</td><td>5</td></tr>
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