SOMAIYA
VIDYAVIHAR UNIVERSITY

Maximum Marks: 100	Semester: Jan 2023 - May 2023 Examination: ESE Examination	Duration:3 Hrs.
Programme code: 01/03 Programme: B. Tech Computer/IT Engineering	Class: SY	Semester: IV (SVU 2020)
Name of the Constituent College: K. J. Somaiya College of Engineering		Name of the department: Computer/IT
Course Code: 116U01C401/116U03C401	Name of the Course: Probability, Statistics and Optimization Techniques	
Instructions: 1) All questions are compulsory 2) Assume suitable data wherever necessary		

Que. No.	Question	Max. Marks
Q1	Solve any **Four** of the following.	20
i)	Three machines A, B, C produce respectively 60%, 30% & 10% of the total number of items of a factory. The percentage of defective outputs of these machines are respectively 2%,3% & 4%.An item is chosen at random and found to be defective . Using Bayes theorem find the probability that it was produced by the factory A.	5

Solu^n Let E_1, E_2, E_3 be the events that items are produced by machine A, B, C respectively.
Let E be the event that item produced is defect

$P(E_1) = 60\% = 0.6$, $P(E_2) = 30\% = 0.3$, $P(E_3) = 0.1$

$P(E/E_1) = 2\% = 0.02$, $P(E/E_2) = 0.03$, $P(E/E_3) = 0.04$ → 02

$$\therefore P(E_1/E) = \frac{P(E_1)P(E/E_1)}{\sum_{i=1}^{3} P(E_i)P(E/E_i)}$$

$$= \frac{0.6 \times 0.02}{(0.6 \times 0.02)+(0.3)(0.03)+(0.1)(0.04)}$$

$$= \boxed{\cancel{0.025}} \quad \boxed{0.48}$$ → 05

ii)	Compute Rank correlation coefficient from the following data	5

x	105	104	102	101	100	99	98	96	93	92
y	101	103	100	98	95	96	104	92	97	94

Solu^n

R_1 :	1	2	3	4	5	6	7	8	9	10
R_2 :	3	2	4	5	8	7	1	10	6	9
d_i^2 :	4	0	1	1	9	1	36	4	9	1

→ 03

$$R = 1 - \frac{6\sum d_i^2}{n^3-n} = 1 - \frac{6 \times 66}{990} = \boxed{0.6}$$ → 05

iii)	A sample of 900 numbers has a mean 3.4 cms and s.d. 2.61 cms. If the population is normal, find the 95% and 98% confidence limits of the population mean.	5

Soluⁿ Given $n=900$, $\bar{x}=3.4$, $\sigma=2.61$

95% Confidence limits of the population mean

$= \left(\bar{x}-1.96\frac{\sigma}{\sqrt{n}}, \bar{x}+1.96\frac{\sigma}{\sqrt{n}}\right) = (3.2295, 3.5705)$ — 3

98% confidence limits of the population mean

$\left(\bar{x}-2.58(\sigma/\sqrt{n}), \bar{x}+2.58(\sigma/\sqrt{n})\right) = \boxed{(3.1973, 3.6027)}$ — 5

iv)	Convert the given LPP into the standard form Minimise $z=7x_1-48x_2+23x_3$ Subject to $61x_1-29x_2+12x_3 \leq 93$ $3x_1-61x_2+81x_3 \geq 9$ $x_1-33x_2+53x_3 \leq -5$ where $x_1, x_2, \geq 0$ and x_3 is unrestricted in sign	5

Soluⁿ Put $x_3 = x_3' - x_3''$

$\text{Max}(-Z) = \text{Max } Z^* = -7x_1 + 48x_2 - 23x_3' + 23x_3''$ — 2

s.t. $61x_1 - 29x_2 + 12x_3' - 12x_3'' + S_1 = 93$

$3x_1 - 61x_2 + 81x_3' - 81x_3'' - S_2 = 9$

$-x_1 + 33x_2 - 53x_3' + 53x_3'' - S_3 = 5$

where $x_1, x_2, x_3', x_3'' \geq 0$ — 5

v)	Find the average number of customers in the system and in the queue if the system is (M/M/1/∞) and $\mu=10, \lambda=8$ per hour	5

Soluⁿ $\rho = \frac{\lambda}{\mu} = \frac{8}{10} = \frac{4}{5}$

$L_q = \frac{\rho^2}{1-\rho} = \frac{(16/25)}{1-(4/5)} = \frac{16/25}{1/5} = \frac{16}{5} = \boxed{3.2}$ — 2

∴ The average no. of customers in the queue $= \boxed{3}$

$L_s = \frac{\rho}{1-\rho} = \frac{(4/5)}{1-(4/5)} = \boxed{4}$

∴ The average no. of customers in the system $= 4$ — 5

vi)	The joint probability distribution function of (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}$ where $0 \le x \le 2$, $0 \le y \le 1$. Compute (a) $P(X > 1)$ (b) $P(Y < 0.5)$ (c) $P(X > 1 \mid Y < 0.5)$	5

Solu^n (a) $P(X>1) = \iint f_{XY}(x,y)\,dx\,dy$

$$= \int_{y=0}^{1}\int_{x=1}^{2} xy^2 + \frac{x^2}{8}\,dx\,dy$$

$$= \int_{y=0}^{1} y^2\left(\frac{x^2}{2}\right)_1^2 + \left(\frac{x^3}{24}\right)_1^2 dy = \int_{y=0}^{1} \frac{3}{2}y^2 + \frac{7}{24}\,dy$$

$$= 3/2\left(y^3/3\right)_0^1 + \frac{7}{24}(y)_0^1 = \boxed{19/24} \longrightarrow 01$$

(b) $P(Y<0.5) = \int_{y=0}^{1/2}\int_{x=0}^{2} xy^2 + x^2/8\,dx\,dy = \boxed{1/4} \longrightarrow 02$

(c) $P(X>1 / Y<0.5) = \frac{P(X>1, Y<0.5)}{P(Y<0.5)} = \frac{5/24}{1/4} = \boxed{\frac{5}{6}} \longrightarrow 05$

Q2 A	Solve the following.	10
i)	The regression lines of a sample are x + 6y = 6 and 3x + 2y =10 Find (a) $\overline{x}$ and $\overline{y}$ (b) correlation coefficient r. Also estimate y when x =12. (c)verify that the sum of the coefficients of regression is greater than 2r	5

Solu^n (a) $\overline{x}$ and $\overline{y}$ are obtained by solving two eq^{n}s

$$\left.\begin{array}{l} x + 6y = 6 \\ 3x + 2y = 10 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l}\overline{x} = 3 \\ \overline{y} = 1/2\end{array}} \longrightarrow 01$$

(b) Reg line of y on x : $y = -\frac{1}{6}x + 1$

$\therefore\ b_{yx} = -1/6$

Reg Line of x on y : $x = -\frac{2}{3}y + \frac{10}{3}$

$\therefore\ b_{xy} = -2/3$.

$\therefore\ r = \sqrt{b_{yx} b_{xy}} = \sqrt{1/9} = \boxed{-1/3}$ as both b_{yx} & b_{xy} are $-$ve.

when $x = 12$: $y = -1/6\,x + 1 \Rightarrow \boxed{y = -1} \longrightarrow 03$

(c) $b_{yx} + b_{xy} = -1/6 - 2/3 = -5/6 \longrightarrow 05$

ii)	A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample is from a batch of string having a mean breaking strength of 15.6 pounds & standard deviation of 2.2 pounds.	5

Solu^n $H_0 : \bar{x} = \mu$

$H_1 : \bar{x} \neq \mu$ — 2

$\alpha = 5\%$, $Z_\alpha = 1.96$

$Z_{cal} = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$ where $n = 50$ $\bar{x} = 14.5$

$\mu = 15.6$, $\sigma = 2.2$

$\therefore Z_{cal} = \dfrac{14.5 - 15.6}{2.2/\sqrt{50}} = \boxed{3.5355}$

$\therefore Z_{cal} \nless Z_\alpha \Rightarrow H_0$ is not accepted — 5

$\therefore$ We can't say the sampe is from the batch of string having a mean breaking strength of 15.6 pounds.

OR

Q2 A	Using Lagrange's Multiplier method solve the following NLPP $z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$ subject to $x_1 + x_2 + x_3 = 20$, $x_1, x_2, x_3 \geq 0$	10

Solu^n $L(x_1, x_2, x_3, \lambda) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100 - \lambda(x_1 + x_2 + x_3 - 20)$

$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 4x_1 + 10 - \lambda = 0$ — (1) $\frac{\partial L}{\partial x_2} = 0 \Rightarrow 2x_2 + 8 - \lambda = 0$ — (2)

$\frac{\partial L}{\partial x_3} = 0 \Rightarrow 6x_3 + 6 - \lambda = 0$ — (3) $\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_1 + x_2 + x_3 = 20$ — (4) — 5

(1)$\times 3$ + (2)$\times 6$ + (3)$\times 2$ gives $12(x_1 + x_2 + x_3) + 90 - 11\lambda = 0$ — (5)

(4) & (5) gives $\lambda = 30$

using (1), (2) & (3) we get $x_1 = 5$, $x_2 = 11$, $x_3 = 4$

$$\Delta_4 = \begin{vmatrix} 0 & \partial h/\partial x_1 & \partial h/\partial x_2 & \partial h/\partial x_3 \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda\frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda\frac{\partial^2 h}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} - \lambda\frac{\partial^2 h}{\partial x_1 \partial x_3} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda\frac{\partial^2 h}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_3} \\ \frac{\partial h}{\partial x_3} & \frac{\partial^2 f}{\partial x_3 \partial x_1} - \lambda\frac{\partial^2 h}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} - \lambda\frac{\partial^2 h}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} - \lambda\frac{\partial^2 h}{\partial x_3^2} \end{vmatrix}$$ — 8

$C_2 - C_4, C_3 - C_4$

$$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & -6 & -6 & 6 \end{vmatrix} = (-1)\begin{vmatrix} 1 & 4 & 0 \\ 1 & 0 & 2 \\ 1 & -6 & -6 \end{vmatrix} = -44$$

$$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 2 \end{vmatrix} = -6$$

$\Rightarrow \Delta_3$ & Δ_4 both are negative.

$\Rightarrow X_0 = (5, 11, 4)$ is minima

$\Rightarrow Z_{min} = \boxed{281}$ — 10

Q 2 B	Solve any **One** of the following.	10
i)	The local one person barber shop can accommodate maximum of 5 people at a time (4 waiting and 1 getting haircut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair according to an Exponential distribution at an average rate of 4 per hour. (a) What percentage of time is the barber idle? (b) What fraction of potential of customers are turned away? (c) What is the expected number of customers waiting for a haircut? (d) How much time can a customer expect to spend in the barber shop?	10

Solu^n $\rho = \frac{\lambda}{\mu} = \frac{5}{4} \cancel{=1}$, $N = 5$

(a) Idle time of barber $= P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \boxed{0.088}$ — 4

i.e. 8.8%.

(b) P (customers are turned way) = potential customers loss

$= P_n = \rho^n P_0 = \boxed{0.2711}$ — 6

(c) expected no. of customers waiting for a haircut = Lq

$= L_s - \rho = \left\{ \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} \right\} - \rho = \boxed{1.88}$ — 8

(d) time a customer expect to spend in the barber shop

$= W_s = \frac{L_s}{\lambda(1-P_N)} = \boxed{0.9466}$ — 10

ii)	Define probability mass function of Poisson distribution and Fit a Poisson distribution to the following data if the following mistakes per page were observed in a book.	10

No. of mistakes	0	1	2	3	4	Total
No. of pages	211	90	19	5	0	325

solu^n If X follows a poisson dist^n with mean = m

then $P(X=x) = \frac{e^{-m} m^x}{x!}$, $m = \frac{\sum f_i x_i}{\sum f_i} = \frac{143}{325} = 0.44$ → 02

$\therefore P(X=x) = e^{-0.44} (0.44)^x / x!$

$P(X=0) = 0.6440$ $\quad$ $P(X=3) = 0.0091$

$P(X=1) = 0.2833$ $\quad$ $P(X=4) = 0.00100$.

$P(X=2) = 0.0623$ → 06

$\therefore$ Exp. freq $= NP(X=x) = 325 \times P(X=x)$

No of mistakes :	0	1	2	3	4
obs. f :	211	90	19	5	0
Exp. f :	209	92	20	3	1

$\rightarrow$ (10)

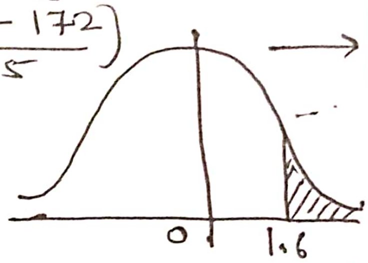
Q3	Solve any **Two** of the following.	20
i)	(a) The height of 1000 soldiers in a regiment are distributed normally with mean 172 cm and standard deviation 5 cm. how many soldiers have height greater than 180 cm.	05

Solun Let X = height of soldiers, $N = 1000$.

$m = \mu = 172,\ \sigma = 5,\ Z = \frac{X-m}{\sigma} = \frac{X-172}{5}$

$P(X > 180) = P\left(\frac{X-172}{5} > \frac{180-172}{5}\right)$ $\rightarrow$ (02)

$= P(Z > 1.6).$



$= 0.5 - P(0 < Z < 1.6)$

$= 0.5 - 0.4452 = \boxed{0.0548}$ $\rightarrow$ (04)

$\therefore$ No. of soldiers having height > 180 cm

$= NP(X > 180) = 1000 \times 0.0548 \approx \boxed{55}$ $\rightarrow$ (05)

(b) Two groups A & B of patients each consisting of 200 people are used to test effectiveness of a new serum. Group A is given serum while group B not. It is found that mean of two groups of A & B are 140 & 120 respectively and standard deviation of 14 & 12 respectively. Test at 1% LOS whether the new serum helps to cure the disease. 05

Solun Given: $\bar{X}_1 = 140,\ \bar{X}_2 = 120,\ n_1 = 200 = n_2$

$s_1 = 14,\ s_2 = 12$

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

$S.E = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 1.3038$

$Z_{cal} = \frac{\bar{X}_1 - \bar{X}_2}{S.E} = \frac{20}{1.3038} = \boxed{15.339}$ $\rightarrow$ (03)

$Z_\alpha = 2.58$ $\because$ $Z_{cal} > Z_\alpha$, Reject H_0

$\therefore$ serum do help to cure the disease $\rightarrow$ (05)

ii) Find the lines of regression for the following data to estimate y corresponding to $x = 155$ and value of x corresponding to $y = 152$ 10

x	100	110	120	130	140	150	160	170	180	190
y	45	51	54	61	66	70	74	78	85	89

Soln $\bar{x} = \frac{1450}{10} = 145$, $\bar{y} = \frac{673}{10} = 67.3$

xy	x^2	y^2
4500	10000	2025
5610	12100	2601
6480	14400	2916
7930	16900	3721
9240	19600	4356
10500	22500	4900
11840	25600	5476
13260	28900	6084
15300	32400	7225
16910	36100	7921

$\Sigma xy = 101570$

$\Sigma x^2 = 218500$ → 05

$\Sigma y^2 = 47225$

$b_{yx} = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{3985}{8250} = \boxed{0.4830}$

$b_{xy} = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma y^2 - n\bar{y}^2} = \frac{3985}{19320.1} = \boxed{2.0625}$ → 08

① $y - 67.3 = 0.4830(x - 145)$

when $x = 155$, $\boxed{y = 72.13}$

② $x - 145 = 2.0625(y - 67.3)$

when $y = 152$, $\boxed{x = 319.69}$ → 10

iii) Define the following terms 10

Solution of LPP. Basic solution of LPP. Feasible solution and degenerate solution of LPP. Also

Find (a) All basic solutions (b) All feasible basic solutions (c) All degenerate solutions hence decide the optimal feasible basic for the following L.P.P.

Maximise $z = 2x_1 + 3x_2 + x_3 + x_4$

Subject to $x_1 - 3x_2 + 2x_3 + x_4 = 5$

$x_1 + x_2 + 3x_3 - 2x_4 = 4$

where $x_1, x_2, x_3, x_4 \geq 0$

① The set of values of decision variables which satisfy all constraints of L.P.P. is called a solution to the given L.P.P.

② If a L.P.P. has "m" equations & "n" unknowns then it can be written matrix form $AX = b$ where A is $m \times n$ matrix of rank m. Let B be any $m \times m$ submatrix formed by m linearly independent columns of A. Then a solution obtained by setting $(n-m)$ variables equal to zero & solving the resulting system of equations is called a Basic Solution of L.P.P.

③ The set of values of decision variables which satisfies all the constraints & non-negative constraints of L.P.P. is called the feasible solution.

④ If value of atleast one basic variable is zero then the solution is called as degenerate B.F.S.

— 4

P.T.O

No. of Basic Solutions	Non-basic variables $=0$	Basic Variables	Equations & the values of the basic variables.	Is the Solution feasible	Is the solution degenerate	Value of Z	Is the solution optimal
1	$x_3 = x_4 = 0$	x_1, x_2	$\left.\begin{array}{l} x_1 - 3x_2 = 5 \\ x_1 + x_2 = 4 \end{array}\right\}$ $x_1 = 17/4$, $x_2 = -1/4$	No	–	–	No
2	$x_2 = x_4 = 0$	x_1, x_3	$\left.\begin{array}{l} x_1 + 2x_3 = 5 \\ x_1 + 3x_3 = 4 \end{array}\right\}$ $x_1 = 7$, $x_3 = -1$	NO	–	–	NO
3	$x_1 = x_4 = 0$	x_2, x_3	$\left.\begin{array}{l} -3x_2 + 2x_3 = 5 \\ x_2 + 3x_3 = 4 \end{array}\right\}$ $x_2 = -7/11$, $x_3 = 17/11$	NO	–	–	NO
4	$x_2 = x_3 = 0$	x_1, x_4	$\left.\begin{array}{l} x_1 + x_4 = 5 \\ x_1 - 2x_4 = 4 \end{array}\right\}$ $x_1 = 14/3$, $x_4 = 1/3$	yes	No	$29/3$	Yes
5	$x_1 = x_3 = 0$	x_2, x_4	$\left.\begin{array}{l} -3x_2 + x_4 = 5 \\ x_2 + 2x_4 = 4 \end{array}\right\}$ $x_2 = -14/5$, $x_4 = -17/5$	NO	–	–	NO
6	$x_1 = x_2 = 0$	x_3, x_4	$\left.\begin{array}{l} 2x_3 + x_4 = 5 \\ 3x_3 - 2x_4 = 4 \end{array}\right\}$ $x_3 = 2$, $x_4 = 1$	yes	No	5	NO

10

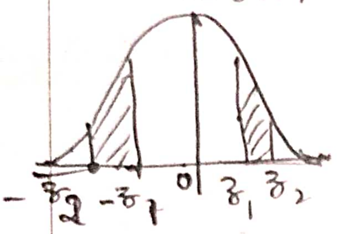
Q4	Solve any **Two** of the following.	20
i)	The probability that an electronic component will fail in less than 1200 hours of continuous use is 0.25 Use Normal approximations to find the probability that among 200 such components exactly 45 will fail in less than 1200 hours of continuous use.	10

Solu^n: Given: $p = 0.25$, $n = 200$

Let X = no. of components fail in < 1200 hrs of use

$m = np = 50$; $\sigma = \sqrt{npq} = 6.12$, $Z = \frac{X-50}{6.12}$ (02)

we want $P(X=45) = P(44.5 \le X \le 45.5)$



$= P(-0.8986 \le Z \le -0.7353)$

z_2 z_1

$= P(0 \le z \le 0.8986)$ (06)

$- P(0 \le z \le 0.7353)$

$= 0.3133 - 0.2673$

$= \boxed{0.046}$ (10)

ii)	A certain drug is claimed to be effective in curing cold in an experiment on 500 persons with cold. Half of them were given drug and half of them were given the sugar pills. The patients reaction to the treatment are recorded in the following table using χ^2-test (use 5% LOS)	10

Solu^n

	Helped	Harmed	No Effect	Total
Drug	150	30	70	250
Sugar pills	130	40	80	250
Total	280	70	150	500

On the basis of this data, can it be concluded that the drug and sugar pills differ significantly in curing cold.

Ho: The drug & sugar pills don't differ

H1: The drug & sugar pills differ significantly. (2)

$E(150) = \frac{(280)(250)}{500} = 140$	$E(30) = \frac{(70)(250)}{500} = 35$	$E(70) = \frac{(150)(250)}{500} = 75$
$E(130) = \frac{(280)(250)}{500} = 140$	$E(40) = \frac{(70)(250)}{500} = 35$	$E(80) = \frac{(150)(250)}{500} = 75$

$\therefore \chi^2_{cal} = \frac{(150-140)^2}{140} + \frac{(30-35)^2}{35} + \frac{(70-75)^2}{35} + \frac{(130-140)^2}{140} + \frac{(40-35)^2}{35}$ (6)

$+ \frac{(80-75)^2}{75} = \boxed{3.5238}$

D.o.f $= (r-1)(c-1) = (2-1)(3-1) = 2 \Rightarrow \chi^2_{5\%} = \boxed{5.991}$ (8)

$\chi^2_{cal} < \chi^2_{5\%} \Rightarrow$ Ho is accepted

i.e. the drug & sugar pills don't differ (10)

iii) Solve the given LPP by Simplex method 10

Maximise $z = 4x_1 + 3x_2 + 6x_3$

Subject to

$2x_1 + 5x_2 \leq 430$

$4x_1 + 3x_3 \leq 470$

$2x_1 + 3x_2 + 2x_3 \leq 440$

where $x_1, x_2, x_3 \geq 0$

Standard form: Max $Z = 4x_1 + 3x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3$

i.e. $Z - 4x_1 - 3x_2 - 6x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + 5x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 430$

$4x_1 + 0x_2 + 3x_3 + 0s_1 + 1s_2 + 0s_3 = 470$

$2x_1 + 3x_2 + 2x_3 + 0s_1 + 0s_2 + s_3 = 440$

It No. 0	Basic variables	x_1	x_2	x_3 ↓	S_1	S_2	S_3	R.H.S. Soln.	Ratio
R_0	Z	−4	−3	−6	0	0	0		
R_1	S_1	2	5	0	1	0	0	430	–
R_2	S_3	4	0	$\boxed{3}$	0	1	0	470	156.67 →
R_3	S_3	2	3	2	0	0	1	440	220
$R_0' = R_0 + 2R_2'$ I	Z	4	−3 ↓	0	0	2	0	940	
$R_1' = R_1$	S1	2	5	0	1	0	0	430	86
$R_2' = R_2/3$	x_3	4/3	0	1	0	1/3	0	470/3	–
$R_3' = R_3 - 2R_2'$	S3	−2/3	$\boxed{3}$	0	1	−2/3	0	380/3	380/9 →
$R_0'' = R_0' + 3R_3''$ II	Z	10/3	0	0	1	4/3	0		
$R_1'' = R_1' - 5R_3''$	S_1	28/9	0	0	−5/3	10/9	1	1970/9	
$R_2'' = R_2'$	x_3	4/3	0	1	0	1/3	0	470/3	
$R_3'' = R_3'/3$	x_2	−2/9	1	0	1/3	−2/9	0	380/9	

5

7

9

∴ $x_1 = 0$, $x_2 = \frac{380}{9}$, $x_3 = \frac{470}{3}$ & $Z_{max} = \boxed{\frac{3200}{3}}$ 10

Q5 (i) X follows a Uniform dist over $(2, b)$ & $P(3 < X < 6) = 0.3$

$\Rightarrow f(x) = \frac{1}{b-2}$ & $\int_3^6 f(x)dx = 0.3 \Rightarrow \frac{3}{b-2} = 0.3 = \frac{3}{10}$

$\Rightarrow 30 = 3b - 6 \Rightarrow 3b = 36 \Rightarrow b = \cancel{\frac{48}{9}}\ 12$ 2

Mean $= \frac{a+b}{2} = \frac{2+b}{2} = \cancel{\boxed{13/9}}\ \boxed{7}$

var $= \frac{(b-a)^2}{12} = \frac{100/9}{12} = \cancel{\boxed{\frac{100}{108}}}\ \boxed{\frac{25}{3}}$ 5

ii)	If the tangent of the angle made by the lines of regression of y on x is 0.6 and $\sigma_y = 2\sigma_x$. find the correlation coefficient between x and y.	5

Solun Given : $b_{yx} = 0.6$, $\sigma_y = 2\sigma_x$

we know that $b_{yx} = r\frac{\sigma_y}{\sigma_x}$ → 02

$\Rightarrow 0.6 = r\frac{2\cancel{\sigma_x}}{\cancel{\sigma_x}}$

$= 2r$ → 04

$\therefore r = \frac{0.6}{2} = \boxed{0.3}$ → 05

iii)	A random sample of 400 items gives the mean 4.45 & variance 4. Can it be regarded as drawn from a normal population with mean 4 at 5% level of significance?	5

Solun Given : $n = 400$, $\bar{x} = 4.45$, $\sigma^2 = 4$, $\mu = 4$

Let $H_0 : \mu = 4$

$H_a : \mu \neq 4$

$Z_{cal} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{4.45 - 4}{2/\sqrt{400}} = \boxed{4.5}$ → 03

at 5% LOS $Z_{table} = 1.96$

$\because Z_{cal} > Z_{tab}$. we Reject H_0

$\therefore$ Sampl is not drawn from normal popu. with $\mu = 4$ → 05

iv)	Find the relative maximum or minimum of the function $z = 20 + x_1 + 2x_3 + x_2x_3 - x_1{}^2 - x_2{}^2 - x_3{}^2$	5

Solun $f(x_1, x_2, x_3) = 20 + x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$

$\frac{\partial f}{\partial x_1} = 0 \Rightarrow 1 - 2x_1 = 0 \Longrightarrow x_1 = 1/2$

$\frac{\partial f}{\partial x_2} = 0 \Rightarrow x_3 - 2x_2 = 0 \Rightarrow$

$\frac{\partial f}{\partial x_3} = 0 \Rightarrow 2 + x_2 - 2x_3 = 0 \Rightarrow$ $\Big\}$ $x_3 = 4/3$, $x_2 = 2/3$ → 02

at $X_0(1/2, 2/3, 4/3)$.

$$H = \begin{bmatrix} f_{x_1x_1} & f_{x_1x_2} & f_{x_1x_3} \\ f_{x_2x_1} & f_{x_2x_2} & f_{x_2x_3} \\ f_{x_3x_1} & f_{x_3x_2} & f_{x_3x_3} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$ → 04

05

Since principal minors are $-2, 4, -6$ which are alternately $-$ve & $+$ve $\therefore X_0$ is pt of max & $Z_{max} = \boxed{\frac{259}{12}}$

v) Find the traffic intensity of the system (M/M/1/∞) model if μ =11 *per hour*, λ =8 per hour. Also find the probability that a customer has to wait for more than 20 minutes to be out of the service station. 5

$\rho = \frac{\lambda}{\mu} = \frac{8}{11}$

$t = 20 \text{ min} = \frac{20}{60} \text{ hrs}$ — 2

A customer has to wait for more than 20 mins to be out of the service station $= P(W_s > t) = e^{-\mu(1-\rho)t}$

$= P(W_s > \frac{1}{3}) = e^{-11(1-\frac{8}{11})\frac{1}{3}}$

$= 0.3679$ — 5

vi) Obtain the dual of the following LPP 5

Minimise z =$3x_1 + 17x_2 + 9x_3$

Subject to

$-x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 = 1$

where $x_1, x_2, x_3 \geq 0$

Solu^n^

Min, $Z = 3x_1 + 17x_2 + 9x_3$

s.t $-x_2 + x_3 \geq 3$

$3x_1 - 2x_3 \geq -1$

$2x_1 + x_2 - 5x_3 \geq 1$

$-2x_1 - x_2 + 5x_3 \geq -1$.

$x_1, x_2, x_3 \geq 0$. → 02

∴ Dual of the given prob is

Max $w = 3y_1 - y_2 + y_3 - y_4$

s.t $0y_1 + 3y_2 + 2y_3 - 2y_4 \leq 3$

$-y_1 + 0y_2 + y_3 - y_4 \leq 17$

$y_1 - 2y_2 - 5y_3 + 5y_4 \leq 9$

where y_1, y_2, y_3 & y_4 are ≥ 0 which are dual variables. → 03