

(a)

$$Q_{01} = 2 \quad Q_{02} = 0 \Rightarrow Q_{00} = -2$$

$$Q_{10} = 3 \quad Q_{12} = 0.5 \Rightarrow Q_{11} = -3.5$$

$$Q_{20} = 0 \quad Q_{21} = 1 \Rightarrow Q_{22} = -1$$

$$Q = \begin{pmatrix} -2 & 2 & 0 \\ 3 & -3.5 & 0.5 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\begin{cases} \pi Q = 0 \\ \sum \pi_j = 1 \end{cases} \Rightarrow [\pi_1 \pi_2 \pi_3] \begin{bmatrix} -2 & 2 & 0 \\ 3 & -3.5 & 0.5 \\ 0 & 1 & -1 \end{bmatrix} = 0$$

$$\begin{cases} -2\pi_1 + 3\pi_2 = 0 \\ 2\pi_1 - 3.5\pi_2 + \pi_3 = 0 \\ 0.5\pi_2 - \pi_3 = 0 \end{cases} \Rightarrow \begin{cases} \pi_1 = \frac{1}{2} \\ \pi_2 = \frac{1}{3} \\ \pi_3 = \frac{1}{6} \end{cases} \quad \left[\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{6} \right]$$

$$\text{make up rate} = \frac{1}{3} \times 3 = 1$$

(b)

$$Q_{01} = 2 \quad Q_{02} = 0 \quad Q_{03} = 0 \quad Q_{04} = 0 \Rightarrow Q_{00} = -2$$

$$Q_{10} = 0 \quad Q_{12} = 3 \quad Q_{13} = 0.5 \quad Q_{14} = 0 \Rightarrow Q_{11} = -3.5$$

$$Q_{20} = 3 \quad Q_{21} = 0 \quad Q_{23} = 0 \quad Q_{24} = 0.5 \Rightarrow Q_{22} = -3.5$$

$$Q_{30} = 0 \quad Q_{31} = 1 \quad Q_{32} = 0 \quad Q_{34} = 0 \Rightarrow Q_{33} = -1$$

$$Q_{40} = 0 \quad Q_{41} = 0 \quad Q_{42} = 1 \quad Q_{43} = 0 \Rightarrow Q_{44} = -1$$

$$Q = \begin{bmatrix} -2 & 2 & 0 & 0 & 0 \\ 0 & -3.5 & 3 & 0.5 & 0 \\ 3 & 0 & -3.5 & 0 & 0.5 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{cases} -2\pi_1 + 3\pi_3 = 0 \\ 2\pi_1 - 3.5\pi_2 + \pi_4 = 0 \\ 3\pi_2 - 3.5\pi_3 + \pi_5 = 0 \\ 0.5\pi_2 - \pi_4 = 0 \\ 0.5\pi_3 - \pi_5 = 0 \\ \pi_1 + \dots + \pi_5 = 1 \end{cases}$$

$$\pi_3 = \frac{2}{3}\pi_1$$

$$\pi_2 = \frac{2}{3}\pi_1$$

$$\pi_4 = \frac{1}{3}\pi_1$$

$$\pi_5 = \frac{1}{3}\pi_1$$

$$2 + 2 + 1 + 1 + 3 \frac{\pi_1}{3} = 1 \Rightarrow \pi_1 = \frac{1}{3}$$

$$\pi_2 = \frac{2}{9}$$

$$\pi_3 = \frac{2}{9}$$

$$\pi_4 = \frac{1}{9}$$

$$\pi_5 = \frac{1}{9}$$

average make up rate

$$\frac{2}{9} \times 3 + \frac{2}{9} \times 3 = \frac{4}{3}$$

part (b) > part (a)