

① AHP

1> reciprocal matrix (倒数矩阵) Synthesis

1. 对每一列值相加

2. 对元素除以所在列的总和

3. 对每一行求平均值 平均值是 priority vector

2> Consistency 一致性

1. criteria pairwise comparison matrix * priority vector

2. step 1 得到的矩阵元素除以 priority vector

3. 将 step 2 的矩阵值计算平均 λ_{\max}

4. 计算 $C.I. = \frac{\lambda_{\max} - n}{n-1}$

5. $CR = \frac{C.I.}{R.I.}$ 如果 $CR < 0.1$ 则可接受 Acceptable.

3> 各候选人 in reciprocal matrix

计算 step 1 得到一个每个标准的优先级 priority vector

4> 综合优先级

step 3 in matrix 乘以 step 1 in priority vector

② Decision Tree

1. 绘制 decision tree 时 没有分概率的节点用“□”

有概率分支的节点用“○”

event fork

decision forks

Utility Theory

不同人对风险有不同的态度，效用可以衡量风险代替方案。

1 > Von Neumann-Morgenstern Approach

$$E(U \text{ for } L) = \sum_{i=1}^n p_i u(r_i)$$

$$u(\text{最不利}) = 0$$

$$u(\text{最有利}) = 1$$

例1: $L_1 \xrightarrow{1} 10000$

$$L_2 \xrightarrow{0.5} \begin{cases} 30000 \\ 0 \end{cases}$$

$$L_4 \xrightarrow{0.98} \begin{cases} -10000 \\ 500 \end{cases}$$

$$L_3 \xrightarrow{1} 0$$

例1 $u(30000) = 1 \quad u(-10000) = 0$

假设 $\xrightarrow{1} 10000$ 和 $\xrightarrow{0.9} \begin{cases} 30000 \\ -10000 \end{cases}$ 无差

例1 $u(10000) = 1 \times 0.9 + 0 \times 0.1 = 0.9$

同理可以得到 $U(1500) = 0.62$ $U(0) = 0.6$

则 $E(U \text{ for } L_1) = 0.9 \times 1 = 0.9$

$$E(U \text{ for } L_2) = 0.5 \times 1 + 0.6 \times 0.5 = 0.8$$

$$E(U \text{ for } L_3) = 1 \times 0.6 = 0.6$$

$$E(U \text{ for } L_4) = 0.98 \times 0.62 = 0.6076$$

2>

$CE(L)$: Certainty equivalent of lottery L

例:



则 $CE(L) = -3400$

$RPL(L)$: risk premium of lottery L (风险溢价)

$$RPL(L) = EV(L) - CE(L)$$

$EV(L)$ 是期望

A lottery distribution diagram showing a lottery L with two possible outcomes: a win of 30000 with probability 0.5, and a loss of 10000 with probability 0.5. The lottery is represented by a bracket with two branches. The top branch is labeled 0.5 and 30000, and the bottom branch is labeled 0.5 and -10000.

$$EV(L) = 0.5 \times (30000 - 10000) = 10000$$

$$RPL(L) = 10000 - (-3400) = 13400$$

* $RPL(L) > 0 \Rightarrow$ Risk-averse 风险规避型

$RPL(L) < 0 \Rightarrow$ Risk-seeking 风险偏好型

$RPL(L) = 0 \Rightarrow$ Risk-neutral 风险中性

3> Value of Information

插入 test 的价值 : $EV(\text{after test}) - EV(\text{without test})$

Expected Value of Sample Information (EVSI)

$$EVSI = EVWSI - EVWOI$$

\downarrow \downarrow
with sample original

EVWSI: 假设 test 不花钱得到 test 后的价值

EVWOI: 不 test 的值

③ Decision Making under Uncertainty

- 1> Maximin (Criterion of pessimism) 行 { 最小值的最大值
- 2> Maximax (Criterion of optimism) 行 { 最大值的最大值
- 3> Hurwicz Criterion C_{opt} - 乐观系数

期望 payoff $H = C_{opt}(\max \text{ payoff}) + (1 - C_{opt})(\min \text{ payoff})$

期望 cost $H = C_{opt}(\min \text{ cost}) + (1 - C_{opt})(\max \text{ cost})$

- 4> Criterion of Regret (Savage criterion)

1. 计算每列最大值
2. 用最大值减各元素取绝对值
3. 取每行最大值，最大值的最小值为结果

- 5> Laplace Criterion

求每个策略的加权和（比如平均），取最大