$98cm^3$ length: x_1 height: x_2 $\frac{x_1}{x_2} = 2.5$ \$1/cm^2

(a) breadth: x_3

min: $Z = 2(x_1x_2 + x_1x_3 + x_2x_3) \times 10$

$= 20x_1x_2 + 20x_1x_3 + 20x_2x_3$

Subject to:

$x_1x_2x_3 = 98$

$x_1 = 2.5x_2$

$x_1, x_2, x_3 \geq 0$

(b)

$$L = 20x_1x_2 + 20x_1x_3 + 20x_2x_3 + \lambda_1(x_1x_2x_3 - 98) + \lambda_2(x_1 - 2.5x_2)$$

$$\nabla L = 0$$

$$\Rightarrow \quad 20x_2 + 20x_3 + x_2x_3\lambda_1 + \lambda_2 = 0$$

$$20x_1 + 20x_3 + x_1x_3\lambda_1 - 2.5\lambda_2 = 0$$

$$20x_1 + 20x_2 + x_1x_2\lambda_1 = 0$$

$$x_1x_2x_3 = 98$$

$$x_1 = 2.5x_2$$

$$\begin{cases} 50x_2 + 20x_3 + 2.5x_2x_3\lambda_1 - 2.5\lambda_2 = 0 \\ 50x_2 + 50x_3 + 2.5x_2x_3\lambda_1 + 2.5\lambda_2 = 0 \end{cases}$$

$$50x_2 + 20x_2 + 2.5x_2^2\lambda_1 = 0$$

$$70x_2 + 2.5x_2^2\lambda_1 = 0$$

$$70 = -2.5x_2\lambda_1 \Rightarrow x_2 = \frac{70}{-2.5\lambda_1}$$

$$x_1 = \frac{70}{-\lambda_1}$$

$$100x_2 + 70x_3 + 5x_2x_3\lambda_1 = 0$$

$$100 \times \frac{70}{-2.5\lambda_1} + 70x_3 + 5 \cdot \frac{70}{-2.5}x_3 = 0$$

$$100 \times \frac{70}{-2.5\lambda_1} = 70x_3 \Rightarrow x_3 = \frac{100}{-2.5\lambda_1}$$

$$\frac{70}{-\lambda_1} \cdot \frac{70}{-2.5\lambda_1} \cdot \frac{100}{-2.5\lambda_1} = 98 \Rightarrow \lambda_1 = -9.283$$

$$x_1 = 7.5407 \quad x_2 = 3.0163 \quad x_3 = 4.309$$