

(a)

state space for $\{u_k\}$ $\{0, 1, 2\}$

DTMC Poisson $P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$

$$P_{22} = e^{-0.1} \quad P_{21} = e^{-0.1} \times 0.1 \quad P_{20} = 1 - P_{22} - P_{21} \\ = 1 - e^{-0.1} - 0.1e^{-0.1} \\ = 1 - 1.1e^{-0.1}$$

$$P_{12} = e^{-0.1} \quad P_{11} = 0.1e^{-0.1} \quad P_{10} = 1 - 1.1e^{-0.1}$$

$$P_{02} = e^{-0.1} \quad P_{01} = 0.1e^{-0.1} \quad P_{00} = 1 - 1.1e^{-0.1}$$

$$P = \begin{bmatrix} 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \end{bmatrix}$$

$$b) E(T_0) = \frac{1}{1-p_{00}} = \frac{1}{1.1e^{-0.1}}$$

$$E(T_1) = \frac{1}{1-p_{11}} = \frac{1}{1-0.1e^{-0.1}}$$

$$E(T_2) = \frac{1}{1-p_{22}} = \frac{1}{1-e^{-0.1}}$$

T_2 stay the longest among of time No

c)

$$\begin{cases} Y = YP \\ \sum y_i = 1 \end{cases} \Rightarrow \begin{aligned} (y_1 + y_2 + y_3)(1 - 1.1e^{-0.1}) &= y_1 \\ (y_1 + y_2 + y_3)0.1e^{-0.1} &= y_2 \\ (y_1 + y_2 + y_3)e^{-0.1} &= y_3 \\ y_1 + y_2 + y_3 &= 1 \end{aligned}$$

$$\Rightarrow y_1 = 1 - 1.1e^{-0.1} \quad y_2 = 0.1e^{-0.1} \quad y_3 = e^{-0.1}$$

d)

$$\pi_4 = \pi_0 p^4 \quad \pi_4 = [1 - 1.1e^{-0.1} \quad 0.1e^{-0.1} \quad e^{-0.1}]$$

$$p = 0.1e^{-0.1}$$