

Shipping cost (\$) per carton

Country

	1	2	3	Supply (box)
Factory 1	20	24	24	60
Factory 2	18	23	20	70
Factory 3	20	25	26	50
Demand (box)	40	65	75	

(a) x_{ij} means Factory i to Country j

$$Z = 20x_{11} + 24x_{12} + 24x_{13} + 18x_{21} + 23x_{22} + 20x_{23} \\ + 20x_{31} + 25x_{32} + 26x_{33}$$

Subject to $x_{11} + x_{12} + x_{13} = 60$

$$x_{21} + x_{22} + x_{23} = 70$$

$$x_{31} + x_{32} + x_{33} = 50$$

$$x_{11} + x_{21} + x_{31} = 40$$

$$x_{12} + x_{22} + x_{32} = 65$$

$$x_{13} + x_{23} + x_{33} = 75$$

$$x_{ij} \geq 0$$

(b)

Shipping cost (\$) per carton

Country

	1	2	3	Supply (box)
Factory	1 20	24	24	60
	2 18	23	20	70
Demand (box)	3 20	25	26	50
	40	65	75	

$$3+3-1=5$$

$$\begin{array}{ccc} \underline{20} & \underline{24} & \underline{24} \\ & 55 & 5 \\ & & u=0 \end{array}$$

$$\begin{array}{ccc} \underline{18} & \underline{23} & \underline{20} \\ & & 70 \\ & & -4 \end{array}$$

$$\begin{array}{ccc} \underline{20} & \underline{25} & \underline{26} \\ 40 & 10 & \\ & & 1 \end{array}$$

$$U = 19 \quad 24 \quad 24$$

$$C_{ij} - u_i - v_j \geq 0 \quad \text{optimal}$$

$$Z = 24 \times 55 + 24 \times 5 + 20 \times 70 + 20 \times 40 + 25 \times 10 = 3890$$

(C)

Shipping cost (\$) per carton

Country

	1	2	3	Supply (box)
	1	24	24	60
Factory	2	18	23	70
	3	20	25	26+Δ
Demand (box)	40	65	75	50

$$26 + \Delta - 24 - 1 \geq 0 \Rightarrow \Delta \geq 1$$