

(a)

(1)

$$M/M/1: \mu=3 \quad \lambda=2 \quad L = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = 2$$

$$M/M/1/3: \mu=3 \quad \lambda=2 \quad L = \frac{\rho[1-\rho^3-3\rho^3(1-\rho)]}{(1-\rho)(1-\rho^4)}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3} \Rightarrow L = \frac{60}{65}$$

$$M^3/M/1: \mu=3 \quad \lambda=\frac{2}{3} \quad L = \frac{\rho(1+b)}{2(1-\rho)}$$

$$\rho = \frac{b\lambda}{\mu} = \frac{3 \times \frac{2}{3}}{3} = \frac{2}{3} \Rightarrow L = \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})} = 4$$

$$M/M/3: \mu=2 \quad \lambda=2 \quad L = \frac{\rho(3\rho)^3 \pi_0}{6(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3 \times 2} = \frac{1}{3} \quad \pi_0 = \left[ \frac{(3\rho)^3}{6(1-\rho)} + \sum_{k=0}^2 \frac{(3\rho)^k}{k!} \right]^{-1}$$
$$= \left[ \frac{1}{6 \times \frac{2}{3}} + \sum_{k=0}^2 \frac{1}{k!} \right]^{-1} = \frac{4}{11}$$

$$L = \frac{\frac{1}{3} \pi_0}{6 \left(\frac{2}{3}\right)^2} + 1 = \frac{23}{22}$$

d i)

$$w = \frac{L}{\lambda}$$

$$w_1 = \frac{L_1}{\lambda} = \frac{2}{2} = 1$$

$$w_2 = \frac{L_2}{\lambda} = \frac{66}{\frac{65}{2}} = \frac{33}{65}$$

$$w_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3} \times 3} = 2$$

$$w_4 = \frac{L}{\lambda} = \frac{\frac{23}{22}}{\frac{2}{2}} = \frac{23}{44}$$

d ii)

$$m/m/1: \frac{100}{40-20} = 5$$

$$m/m/1/3 \quad \frac{300}{120-90} = 10$$

$$m^3/m/1 \quad \frac{100}{30-18} = \frac{50}{6}$$

$$m/m/3 \quad \frac{150}{100-60} = \frac{15}{4}$$

(b)

choose  $m/m/3$

- ① short waiting time
- ② quick break-even time
- ③ highest monthly profit