(a)

Demand

Product	3	4	5	6	
3	40 / 50	40.5	41	42	
4	42	40 / 180	40.5	41	
5	44 / 50	42 / 20	40 / 180	40.5 / 30	
6	46	44	42	40 / 270	

~~50~~ $u=0$

~~180~~ $u=2$

1.5 ~~280~~ 100 $u=4$

~~270~~ 0 $u=3.5$

44	42		40.5
~~100~~	~~200~~	~~180~~	300
50	20	0	30

$V = 40 \quad 38 \quad 36 \quad 36.5$

$C_i - u_i - v_i$ 均 ≥ 0 故为最优

总价为 31455

(b)

$$\text{Min} \quad f(x) = x_1^2 + x_2^2 + x_3^2$$

Subject to $\quad h_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$$h_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$$

$$L = x_1^2 + x_2^2 + x_3^2 + \lambda_1(x_1 + x_2 + 3x_3 - 2) + \lambda_2(5x_1 + 2x_2 + x_3 - 5)$$

$$\begin{cases} \nabla L = 0 \\ h_1(x) = 0 \\ h_2(x) = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 + \lambda_1 + 5\lambda_2 = 0 \\ 2x_2 + \lambda_1 + 2\lambda_2 = 0 \\ 2x_3 + 3\lambda_1 + \lambda_2 = 0 \\ 2x_1 + 2x_2 + 6x_3 - 4 = 0 \\ 10x_1 + 4x_2 + 2x_3 - 10 = 0 \end{cases}$$

$$\begin{cases} -\lambda_1 - 5\lambda_2 - \lambda_1 - 2\lambda_2 - 3(3\lambda_1 + \lambda_2) - 4 = 0 \\ -5(\lambda_1 + 5\lambda_2) - 2(\lambda_1 + 2\lambda_2) - 3\lambda_1 - \lambda_2 - 10 = 0 \end{cases}$$

$$\Rightarrow \quad -11\lambda_1 - 10\lambda_2 - 4 = 0 \qquad -33\lambda_1 - 30\lambda_2 - 12 = 0$$

$$-10\lambda_1 - 30\lambda_2 - 10 = 0 \qquad -33\lambda_1 - 12 = -10\lambda_1 - 10$$

$$-23\lambda_1 = 2$$

$$\lambda_1 = -\frac{2}{23} \quad \lambda_2 = -\frac{7}{23}$$

$$x_1 = \frac{37}{46} \quad x_2 = \frac{8}{23} \quad x_3 = \frac{13}{46} \quad f(x) = \frac{39}{46}$$

verify:

$y^T \nabla_{xx}^2 L\, y$ $\quad \nabla h(x) y = 0$

$\nabla h_1(x) = [1 \ 1 \ 3]$

$\nabla h_2(x) = [5 \ 2 \ 1]$

$$\begin{cases} y_1 + y_2 + 3y_3 = 0 \\ 5y_1 + 2y_2 + y_3 = 0 \end{cases}$$

$$\nabla_{xx} L = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$[y_1 \ y_2 \ y_3] \begin{bmatrix} 2 & & \\ & 2 & \\ & & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= 2y_1^2 + 2y_2^2 + 2y_3^2 > 0$$

SO positive definition $\Rightarrow$ minimum