A/S/m/K/L :

A = arrival process 到达

S = service distribution for the arriving customers 服务分布

m = no. of servers 服务器数量

K = storage space 储存空间

L = customer population

$$W_j = D_j + S_j$$

waiting time

waiting time

service time

2. Little's Result

Let

$\lambda =$ **arrival rate**

$W =$ **mean waiting time in the system**

$D =$ **mean waiting time in the queue**

$L =$ **mean no. of customers in the system**

$Q =$ **mean no. of customers in the queue**

In the steady state $(t \to \infty)$

$$L = \lambda W, \qquad Q = \lambda D.$$

M/M/1 Queue

M/M/1 Queue with Arrival Rate λ and Service Rate μ:

λ : arrival rate

μ : service time of customers

$$
\begin{aligned}
\rho &= \frac{\lambda}{\mu} \quad = U \text{ (utilization 使用)} \\
\pi_0 &= 1-\rho \\
\pi_k &= \rho^k(1-\rho), \qquad k \geq 1 \\
L &= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \qquad L = Q + \rho \\
Q &= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \\
W &= \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \qquad W = D + \frac{1}{\mu} \\
D &= D = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}
\end{aligned}
$$

$L = \lambda W$

$\Rightarrow W = \frac{L}{\lambda}$

$$
F_d(t) = \begin{cases} 1-\rho & t=0 \\ 1-\rho e^{-\mu(1-\rho)t} & t>0 \end{cases}
$$

$$
F_w(t) = 1 - e^{-(\mu-\lambda)t} \quad \text{(Exp)}
$$

$\Rightarrow \quad W = \frac{1}{\mu-\lambda}$

M/M/1/N Queue

M/M/1/N Queue with Arrival Rate λ and Service Rate μ:

$$\lambda_k = \begin{cases} \lambda & 0 \le k \le N-1 \\ 0 & k \ge N \end{cases}$$

$$\mu_k = \mu \quad k \ge 1$$

$$\rho = \frac{\lambda}{\mu} \qquad \boxed{U = 1 - \pi_0} = \frac{\rho(1-\rho^N)}{1-\rho^{N+1}}$$

$$\pi_0 = \left(\sum_{k=0}^{N} \rho^k\right)^{-1} = \frac{1-\rho}{1-\rho^{N+1}}$$

$$\pi_k = \rho^k \pi_0 = \frac{\rho^k(1-\rho)}{1-\rho^{N+1}}, \qquad 0 \le k \le N$$

$$L = \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})}$$

$$Q = \frac{\rho^2[1-\rho^N - N\rho^{N-1}(1-\rho)]}{(1-\rho)(1-\rho^{N+1})}$$

$$W = \frac{1-\rho^N - N\rho^N(1-\rho)}{\mu(1-\rho)(1-\rho^{N+1})}$$

$$D = \frac{\rho[1-\rho^N - N\rho^{N-1}(1-\rho)]}{\mu(1-\rho)(1-\rho^{N+1})}$$

M/M/m Queue

如果是 average no. of busy server

则是 $\frac{\lambda}{\mu}$

M/M/m Queue with Arrival Rate λ and Service Rate μ:

$\lambda_k = \lambda \quad k \geq 0$

$$\mu_k = \begin{cases} k\mu & 0 \leq k \leq m-1 \\ m\mu & k \geq m \end{cases}$$

$$\rho = \frac{\lambda}{m\mu} = U = \frac{\lambda}{m\mu}$$

每个 server

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1}$$

$$\pi_k = \pi_0 \begin{cases} \frac{(m\rho)^k}{k!}, & 0 \leq k \leq m-1 \\ \frac{m^m\rho^k}{m!}, & k \geq m \end{cases}$$

$$L = \frac{\rho(m\rho)^m\pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$Q = \sum_{k=m}^{\infty}(k-m)\pi_k = \frac{\rho(m\rho)^m\pi_0}{m!(1-\rho)^2}$$

$$W = \frac{L}{\lambda} = \frac{\rho(m\rho)^m\pi_0}{m!\lambda(1-\rho)^2} + \frac{1}{\mu}$$

$$D = W - \frac{1}{\mu} = \frac{\rho(m\rho)^m\pi_0}{m!\lambda(1-\rho)^2}$$

$M^b/M/1$

$M^b/M/1$ Queue with Arrival Rate λ and Service Rate μ:

b是批量 batch

$U=1-\pi_0=\rho$

$$
\begin{aligned}
\rho &= \frac{b\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \begin{cases} \left(\dfrac{\lambda+\mu}{\mu}\right)^{k-1} \dfrac{\lambda}{\mu}\pi_0 & 1 \le k \le b \\[2ex] \dfrac{\lambda+\mu}{\mu}\pi_{k-1} - \dfrac{\lambda}{\mu}\pi_{k-b-1} & k \ge b+1 \end{cases} \\
L &= \frac{\rho(1+b)}{2(1-\rho)} \\
Q &= L-\rho = \frac{\rho(b-1+2\rho)}{2(1-\rho)} \\
W &= \frac{L}{\lambda b} = \frac{1+b}{2\mu(1-\rho)} \\
D &= W - \frac{1}{\mu} = \frac{b+2\rho-1}{2\mu(1-\rho)}
\end{aligned}
$$