(a)

poisson $p(X(t)=k) = \dfrac{e^{-\lambda t}(\lambda t)^k}{k!}$ $\lambda = 1$

$L_1 = \{0, 1, 2\}$ $p(X=k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

$P_{01} = e^{-1}$ $P_{02} = e^{-1}$ $P_{00} = 1 - e^{-1} - e^{-1} = 1-2e^{-1}$

$P_{10} = 1-e^{-1}$ $P_{11} = e^{-1}$ $P_{12} = 0$

$P_{20} = 1-2e^{-1}$ $P_{21} = e^{-1}$ $P_{22} = e^{-1}$

$$p = \begin{bmatrix} 1-2e^{-1} & e^{-1} & e^{-1} \\ 1-e^{-1} & e^{-1} & 0 \\ 1-2e^{-1} & e^{-1} & e^{-1} \end{bmatrix}$$

Sojourn time $T_{00} = \dfrac{1}{1-P_{00}} = \dfrac{1}{2e^{-1}} = 0.5e$

$T_{11} = \dfrac{1}{1-P_{11}} = \dfrac{1}{1-e^{-1}}$

$T_{22} = \dfrac{1}{1-P_{22}} = \dfrac{1}{1-e^{-1}}$

(b) $YP = Y \quad \Sigma y = 1$

$$[y_1\ y_2\ y_3]\begin{bmatrix} 1-2e^{-1} & e^{-1} & e^{-1} \\ 1-e^{-1} & e^{-1} & 0 \\ 1-2e^{-1} & e^{-1} & e^{-1} \end{bmatrix} = [y_1\ y_2\ y_3]$$

$$\begin{cases} (1-2e^{-1})y_1 + (1-e^{-1})y_2 + (1-2e^{-1})y_3 = y_1 \\ (y_1+y_2+y_3)e^{-1} = y_2 \\ (y_1+y_3)e^{-1} = y_3 \\ y_1+y_2+y_3 = 1 \end{cases}$$

$$y_3 + y_2 e^{-1} = y_2 \Rightarrow y_3 = (1-e^{-1})y_2$$

$$(y_1 + (1-e^{-1})y_2)e^{-1} = (1-e^{-1})y_2$$

$$y_1 e^{-1} + e^{-1}(1-e^{-1})y_2 = (1-e^{-1})y_2$$

$$y_1 = e(1-e^{-1})^2 y_2$$

$$e(1-e^{-1})^2 y_2 + y_2 + (1-e^{-1})y_2 = 1 \Rightarrow y_2 \doteq 0.368$$

$$y_1 = 0.400$$

$$y_3 = 0.233$$

(c)

$$\frac{e^{-\lambda}\lambda^{k}}{k!}$$

$$L_k = \{0, 1, 2, 3, \cdots\cdots M\}$$

$$P_{00} = P(N \geq M) = 1 - P_{01} - P_{02} - P_{03} - \cdots\cdots - P_{0(M-1)} - P_{0M}$$

$$= 1 - \frac{e^{-1}}{(M-1)!} - \frac{e^{-1}}{(M-2)!} - \frac{e^{-1}}{(M-3)!} - \cdots \frac{e^{-1}}{1!} - \frac{e^{-1}}{0!}$$

$$= 1 - \sum_{n=0}^{M-1} \frac{e^{-1}}{n!}$$

$$P_{11} = \frac{e^{-1}}{0!} = e^{-1} \Rightarrow P_{ii} = e^{-1}$$