(a) $\text{Min } f(X) = 3x_1^2 + 2x_2^2 + \frac{1}{2}x_3^2$

Subject to $x_1 + x_2 = 15$

$x_3 = 1$

$$L = 3x_1^2 + 2x_2^2 + \frac{1}{2}x_3^2 + \lambda_1(x_1 + x_2 - 15) + \lambda_2(x_3 - 1)$$

$$\begin{cases} \frac{\partial L}{\partial x_1} = 6x_1 + \lambda_1 = 0 \\ \frac{\partial L}{\partial x_2} = 4x_2 + \lambda_1 = 0 \\ \frac{\partial L}{\partial x_3} = x_3 + \lambda_2 = 0 \\ x_1 + x_2 - 15 = 0 \\ x_3 - 1 = 0 \end{cases}$$

$x_1, x_2, x_3 \geq 0$

$\lambda_2 = -1 \quad \lambda_1 = -36$

$x_1 = -\frac{\lambda_1}{6} = -\frac{-36}{6} = 6$

$x_2 = -\frac{\lambda_1}{4} = -\frac{-36}{4} = 9$

$x_3 = 1$

$\Rightarrow$

$\min z = 3 \times 36 + 2 \times 81 + \frac{1}{2}$

$= 270.5$

(b)

$$x_1 + x_2 - 15 - \Delta = 0$$

$$x_1 = -\frac{\lambda_1}{6} \quad x_2 = -\frac{\lambda_1}{4} \Rightarrow x_1 + x_2 = \frac{-2\lambda_1 - 3\lambda_1}{12}$$

$$= -\frac{5\lambda_1}{12}$$

$$-\frac{5\lambda_1}{12} = 15 + \Delta \Rightarrow \lambda_1 = \frac{(15+\Delta)12}{-5}$$

$$x_1 = -\frac{(15+\Delta)\cancel{12}^{2}}{-5 \times \cancel{6}} \qquad x_2 = -\frac{(15+\Delta)\cancel{12}^{3}}{-5 \times \cancel{4}}$$

$$= \frac{30 + 2\Delta}{5} \qquad = \frac{45 + 3\Delta}{5}$$

$$= 6 + 0.4\Delta \qquad = 9 + 0.6\Delta$$

(c)

	J_1	J_2	J_3	J_4	J_5
P_1	~~20~~ 5	~~15~~ 0	~~16~~ 1	~~17~~ 2	M
P_2	M	M	~~16~~ 0	~~20~~ 4	~~17~~ 1
P_3	~~12~~ 1	~~15~~ 4	~~11~~ 0	M	~~17~~ 6
P_4	~~13~~ 1	M	~~14~~ 2	~~20~~ 8	~~12~~ 0
P_5	0	0	0	0	0

$$\begin{array}{ccccc}
\cancel{5}\,4 & 0 & 1 & \cancel{2}\,1 & M \\
M & M & 0 & \cancel{4}\,3 & 1 \\
\cancel{1}\,0 & 4 & 0 & M & 6 \\
\cancel{1}\,0 & M & 2 & \cancel{8}\,7 & 0 \\
0 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & & 1
\end{array}$$

min = 1

$$\begin{array}{ccccc}
4 & 0 & 1 & 1 & M \\
M & M & 0 & 3 & 1 \\
0 & 4 & 0 & M & 6 \\
0 & M & 2 & 7 & 0 \\
0 & 1 & 1 & 0 & 1
\end{array}$$

15 + 16 + 12 + 12 = 55