(a)

System 1 : three M/M/1 queues $\frac{\lambda}{3}$ μ

System 2 : one M/M/1 queue λ 3μ

System 3 : one M/M/3 queue λ μ

$$W_1 = \frac{1}{\mu-\lambda} = \frac{1}{\mu-\frac{\lambda}{3}} = \frac{3}{3\mu-\lambda}$$

$$W_2 = \frac{1}{\mu-\lambda} = \frac{1}{3\mu-\lambda}$$

$$W_3 = \frac{\rho(3\rho)^3\pi_0}{3!\,\lambda(1-\rho)^2} + \frac{1}{\mu} \qquad \rho = \frac{\lambda}{\mu}$$

stable : $\rho < 1$:

1、 $\frac{\frac{\lambda}{3}}{\mu} < 1 \Rightarrow \lambda < 3\mu$

2、 $\frac{\lambda}{3\mu} < 1 \Rightarrow \lambda < 3\mu$ $\Rightarrow \lambda < 3\mu$

3、 $\frac{\lambda}{3\mu} < 1 \Rightarrow \lambda < 3\mu$

(b)

$\lambda = 1 \quad \mu = 1$

$W_1 = \frac{3}{3\times1-1} = \frac{3}{2} \qquad W_2 = \frac{1}{3-1} = \frac{1}{2}$

W_3: $\quad \pi_0 = \left[\frac{(3\rho)^3}{3!(1-\rho)} + \sum_{k=0}^{2}\frac{(3\rho)^k}{k!}\right]^{-1} \qquad \rho = \frac{\lambda}{m\mu} = \frac{1}{3}$

$\sum_{k=0}^{2}\frac{1}{k!} = 1 + 1 + \frac{1}{2} = \frac{5}{2}$

$\frac{(3\rho)^3}{3!(1-\rho)} = \frac{1}{6(1-\frac{1}{3})} = \frac{1}{6\times\frac{2}{3}} = \frac{1}{4} \qquad \Rightarrow \pi_0 = \left[\frac{5}{2}+\frac{1}{4}\right]^{-1} = \frac{4}{11}$

$W_3 = \frac{\frac{1}{3}\times\frac{4}{11}}{6\times1\times\frac{4}{9}} + 1 = \frac{\frac{4}{3\times11}}{\frac{2\times4}{3}} + 1 = \frac{23}{22}$

$W_2 < W_3 < W_1$

(c)

$$W_1 = \frac{1}{\mu-\lambda} = \frac{1}{\mu-\frac{\lambda}{3}} = \frac{3}{3\mu-\lambda}$$

$$W_2 = \frac{1}{\mu-\lambda} = \frac{1}{3\mu-\lambda}$$

$$W_3 = \frac{\rho(3\rho)^3\pi_0}{3!\,\lambda(1-\rho)^2} + \frac{1}{\mu}$$

$W_2 < W_3 < W_1$ 这题不确定