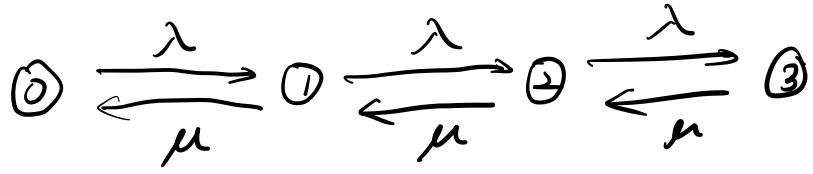
(9) M/M/1/3



rate balance equation

$$Q=\begin{bmatrix} -\lambda & \lambda & 0 & 0 \\ \mu & -(\mu+\lambda) & \lambda & 0 \\ 0 & \mu & -(\mu+\lambda) & \lambda \\ 0 & 0 & \mu & -\mu \end{bmatrix}$$

$$\begin{cases} \lambda\pi_0=\mu\pi_1 & \pi_0=\frac{\mu}{\lambda}\pi_1 \\ \lambda\pi_1+\mu\pi_1=\lambda\pi_0+\mu\pi_2 & \\ \lambda\pi_2+\mu\pi_2=\lambda\pi_1+\mu\pi_3 & \\ \lambda\pi_2=\mu\pi_3 \Rightarrow \pi_3=\frac{\lambda}{\mu}\pi_2=\left(\frac{\lambda}{\mu}\right)^3\pi_0 & \end{cases}$$

$$\pi Q=0$$

$$\lambda\pi_1=\mu\pi_2 \Rightarrow \pi_1=\frac{\mu}{\lambda}\pi_2$$

$$\pi_0=\left(\frac{\mu}{\lambda}\right)^2\pi_2$$

$$\pi_0+\pi_1+\pi_2+\pi_3=1 \Rightarrow \pi_0+\frac{\lambda}{\mu}\pi_0+\left(\frac{\lambda}{\mu}\right)^2\pi_0+\left(\frac{\lambda}{\mu}\right)^3\pi_0=1$$

$$\pi_0\left(\frac{\mu^3}{\mu^3}+\frac{\lambda\mu^2}{\mu^3}+\frac{\lambda^2\mu}{\mu^3}+\frac{\lambda^3}{\mu^3}\right)=1$$

$$\pi_0=\frac{\mu^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}$$

$$\pi_1=\frac{\mu^2\lambda}{\cdots}$$

$$\pi_2=\frac{\mu\lambda^2}{\cdots}$$

$$\pi_3=\frac{\lambda^3}{\cdots}$$

$\lambda>\mu$ $\pi_3>\pi_2>\pi_1>\pi_0$ $\lambda<\mu$ $\pi_0>\pi_1>\pi_2>\pi_3$

(b)

① $M/M/1$

② $M/M/1/N$

③ $M/M/4$

④ $M^b/M/1$