$V = 98\,cm^3$ $L = 2.5\,H$ cost $10/cm^2$

$$h = \frac{x_1}{2.5}$$

(a) length x_1 breadth x_2

Min: $Z = \left(x_1 x_2 + x_1 \cdot \frac{x_1}{2.5} + x_2 \cdot \frac{x_1}{2.5}\right) \times 20$

$= 20x_1x_2 + 8x_1^2 + 8x_1x_2$

Subject to:

$$x_1 x_2 \cdot \frac{x_1}{2.5} = 98$$

$x_1, x_2 \geq 0$

(b) Min: $Z = 20x_1x_2 + 8x_1^2 + 8x_1x_2 = 28x_1x_2 + 8x_1^2$

Subject to:

$$x_1x_2 \cdot \frac{x_1}{2.5} = 98 \Rightarrow x_1^2x_2 - 245 = 0$$

$$x_1, x_2 > 0$$

Lagrange:

$$L = 20x_1x_2 + 8x_1^2 + 8x_1x_2 + \lambda(x_1^2x_2 - 245)$$

$$\begin{cases} \nabla_x L = 0 \\ h(x) = 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} 20x_2 + 16x_1 + 8x_2 + 2\lambda x_1x_2 = 0 \\ 20x_1 + 8x_1 + x_1^2\lambda = 0 \\ x_1^2x_2 - 245 = 0 \end{cases}$$

$$28x_2 + 16x_1 + 2x_1x_2\lambda = 0$$

$$28x_1 + x_1^2\lambda = 0 \Rightarrow 28 + x_1\lambda = 0$$

$$x_1 = -\frac{28}{\lambda}$$

$$28x_2 - 16\frac{28}{\lambda} + 2\left(-\frac{28}{\lambda}\right)\lambda x_2 = 0$$

$$28x_2 - \frac{448}{\lambda} - 56x_2 = 0 \Rightarrow x_2 = -\frac{16}{\lambda}$$

$$x_1^2x_2 = 245 \Rightarrow \left(\frac{28}{\lambda}\right)^2 \cdot \frac{-16}{\lambda} = 245 \Rightarrow \lambda = -3.713$$

$$x_1 = 7.541 \qquad x_2 = 4.309$$

minimum cost 1364.77

(C)

new volume: $98 \times (1-5\%) = 93.1 cm^2$

$\Delta = 93.1 - 98 = -4.9 cm^2$

$\Delta f = -\lambda \Delta = -(-3.773 \times (-4.9)) = -18.1937$

$f^* = f + \Delta f = 1364.77 - 18.1937 = 1346.5763$