

EE183DA
Team Buffalo
Lab 2: Kalman Filter State Estimation Algorithm

Iou-Sheng (Danny) Chang
UID: 804-743-003

February 4th 2019

Contents

1	Introduction and Lab Overview	3
2	Symbols and Conventions	4
2.1	Symbols	4
2.2	Assumptions	4
2.3	Systems	5
3	Robot State Update Dynamics and Linearized Model	6
3.1	Case 1: Traveling along a straight line (forward/backward)	6
3.2	Case 2: Turning about its rotational axis (clockwise/counter-clockwise)	6
3.3	MATLAB Implementation	7
4	Sensor Measurement and Linearized Model	10
4.1	Ideal Geometric Lidar Sensor Measurement	10
4.2	Equations	11
4.3	Linearization	13
4.4	Special Cases	13
4.5	MATLAB Implementation	15
5	Kalman Filter Procedure	18
5.1	Predict	18
5.2	Kalman Gain	18
5.3	Measurement Update	18
5.4	MATLAB Implementation	18
6	Results	21
6.1	Car State Estimator	21
6.2	True State	21
6.3	Comparison of State Estimates to True States	22
6.4	Discussion and Future Work	23
7	Appendix and Demonstration	24
8	References	25

1 Introduction and Lab Overview

In this lab, we use Kalman Filter to estimate the state of the two-wheeled robot with sensor measurements. By using the developed mathematical model of the complete robot system, we implement and evaluate a Kalman Filter based state estimation algorithm.

Since the process dynamics of our system are non-linear but smooth, we may use its linearized approximation for the process model. The Block diagram of the two-wheeled robot system is shown in 1.

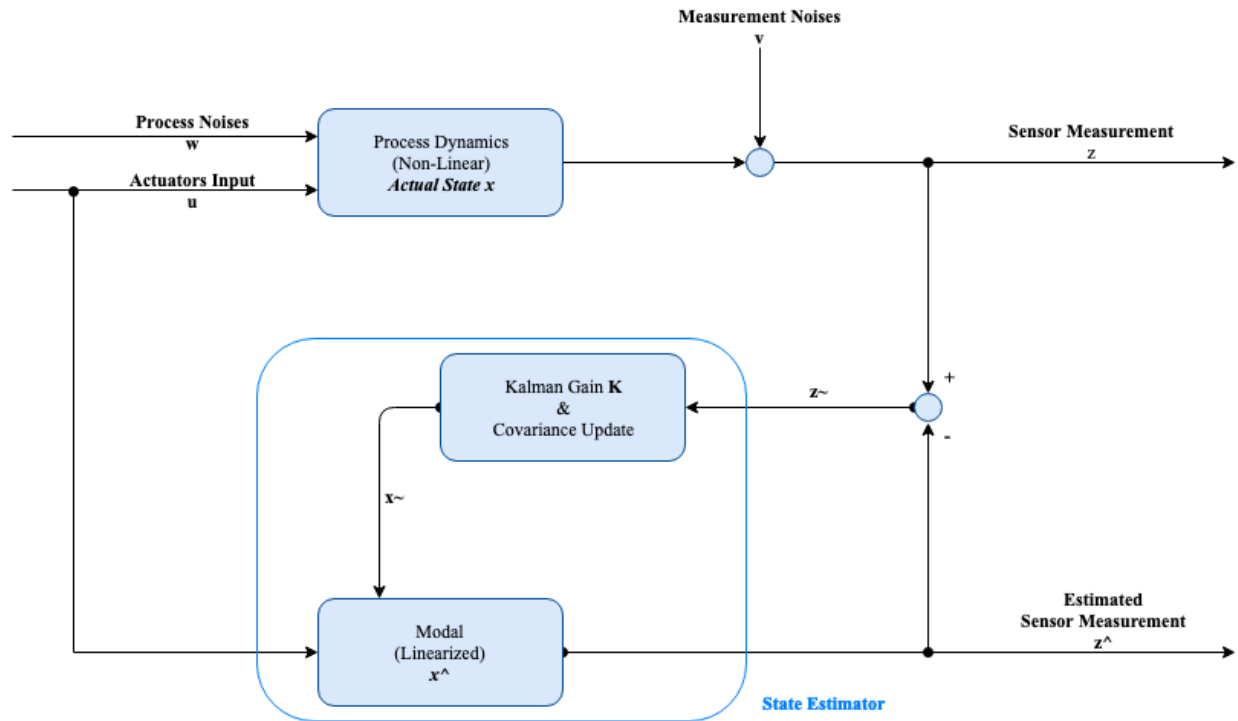


Figure 1: System Block Diagram

2 Symbols and Conventions

2.1 Symbols

List of all symbols used for deriving the theoretical, geometric model.

L	length of the box
W	width of the box
C_W	distance between left and right wheels
C_C	distance between rotational center and car reference point
C_f	distance between front lidar sensor and car reference point
C_r	distance between right lidar sensor and car reference point
PWM_L	input PWM of the left wheel
PWM_R	input PWM of the right wheel
v	linear velocity of the car
v_L	linear velocity of the left wheel
v_R	linear velocity of the right wheel
ω	angular velocity of the car
ω_L	angular velocity of the left wheel
ω_R	angular velocity of the right wheel
p_x	position: absolute x coordinate of the car
p_y	position: absolute y coordinate of the car
p_θ	absolute orientation of the car
θ_{MPU}	MPU9250 magnetometer angle measurement
l_f	front laser range sensor reading
l_r	right laser range sensor reading
δt	time traveled by the car
δt_L	time traveled by the left wheel
δt_R	time traveled by the right wheel
N_W	process noise
V	measurement noise
N_v	linear velocity noise
N_ω	angular velocity noise

2.2 Assumptions

The following assumptions are made according the the car's geometry and for the derivation of theoretical, geometric model.

1. (p_x, p_y) is taken as the car's reference point, which is in the top right corner of the car.
2. r_θ is measured from the car's rotational center, which is the mid-point between the two wheels.

3. When the car is going straight (either forward or backward), $v = v_L = v_R$, having same sign and magnitude.
4. When the car is turning (either clockwise or counter-clockwise), it is turning about its rotational center, In this case, $|v| = |v_L| = |v_R|$, however v_L and v_R have opposite sign.
5. $p_\theta = \theta_{MPU}$ assuming the MPU9250 magnetometer is calibrated.
6. $N_W = 0$, there is no process noise.

2.3 Systems

Input

$$u = \begin{bmatrix} PWM_L \\ PWM_R \end{bmatrix}$$

State

$$x = \begin{bmatrix} p_x \\ p_y \\ p_\theta \end{bmatrix}$$

Sensor Measurements

$$z = \begin{bmatrix} l_f \\ l_r \\ \theta_{MPU} \end{bmatrix}$$

3 Robot State Update Dynamics and Linearized Model

Recall the theoretical, geometric model of the robot state update dynamics, that we derived in lab 1, which has the form:

$$x'(t) = x_{t+1} = f(x_t, u_t, t) = Ax_t + Bu_t$$

In order to have complete control of the car, we split the movements of the car into two cases, traveling along a straight line (forward/backward) and turning about its rotational axis (clockwise/counter-clockwise).

Note that the following equations work for $p_{\theta_t}, \theta_{MPU} = [0, 2\pi]$, (all values of θ).

3.1 Case 1: Traveling along a straight line (forward/backward)

$$\begin{aligned} p_{x,t+1} &= p_{x,t} + (v_t)t \cdot \cos p_{\theta_t} \\ p_{y,t+1} &= p_{y,t} + (v_t)t \cdot \sin p_{\theta_t} \\ p_{\theta_{t+1}} &= p_{\theta_t} \end{aligned}$$

Expressing in matrix form yields:

$$\begin{aligned} x'(t) = x_{t+1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{x,t} \\ p_{y,t} \\ p_{\theta_t} \end{bmatrix} + \begin{bmatrix} (v_t)t \cdot \cos p_{\theta_t} \\ (v_t)t \cdot \sin p_{\theta_t} \\ 0 \end{bmatrix} \\ &= I_3 x_t + \begin{bmatrix} (v_t)t \cdot \cos p_{\theta_t} \\ (v_t)t \cdot \sin p_{\theta_t} \\ 0 \end{bmatrix} \end{aligned}$$

Note that for forward mode $v_t > 0$, while for backward mode $v_t < 0$.

3.2 Case 2: Turning about its rotational axis (clockwise/counter-clockwise)

Note that $p_{x,t+1} \neq p_{x,t}$ and $p_{y,t+1} \neq p_{y,t}$ due to the fact that rotational axis is different from the car reference point.

$$\begin{aligned} p_{x,t+1} &= p_{x,t} - C_C [\cos p_{\theta_t} - \cos p_{\theta_{t+1}}] \\ p_{y,t+1} &= p_{y,t} + C_C [\sin p_{\theta_t} - \sin p_{\theta_{t+1}}] \\ p_{\theta_{t+1}} &= p_{\theta_t} + (\omega_t)t \end{aligned}$$

Expressing in matrix form yields:

$$\begin{aligned} x'(t) = x_{t+1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{x,t} \\ p_{y,t} \\ \theta_t \end{bmatrix} + \begin{bmatrix} -C_C[\cos p_{\theta_t} - \cos p_{\theta_{t+1}}] \\ +C_C[\sin p_{\theta_t} - \sin p_{\theta_{t+1}}] \\ (\omega_t)t \end{bmatrix} \\ &= I_3 x_t + \begin{bmatrix} -C_C[\cos p_{\theta_t} - \cos p_{\theta_{t+1}}] \\ +C_C[\sin p_{\theta_t} - \sin p_{\theta_{t+1}}] \\ (\omega_t)t \end{bmatrix} \end{aligned}$$

Note that for counter-clockwise (left turn) mode $\omega_t < 0$, while for clockwise (right turn) mode $\omega_t > 0$.

3.3 Matlab Implementation

Listing 1: MATLAB code for Robot State Update Dynamics

```

1 function [y] = estimate(x,mode)
2     % variables and constants
3     A = eye(3);
4     C_c = 56;
5     theta = x(3);
6     % Four modes of moving
7     switch mode
8         % Forward mode
9         case 1
10            PWM_R = 63;
11            PWM_L = 107;
12            t = 0.065;
13            input = [PWM_R^2;
14                    PWM_R;
15                    1;
16                    PWM_L^2;
17                    PWM_L;
18                    1];
19            c = [-0.1863  19.934  -382.02  0  0  0;
20                0  0  0  0.016  -0.649  20.575];
21            v = mean(c*input);
22            u = [v;
23                v;
24                theta];
25            B = [t*cosd(theta)  0  0;
26                0  -t*sind(theta)  0;
27                0  0  0];
28            y = A*x + B*u;
29            % Backward mode

```

```

30     case 2
31         PWM_R = 104;
32         PWM_L = 65;
33         t = 0.065;
34         input = [PWM_R^2;
35                 PWM_R;
36                 1;
37                 PWM_L^2;
38                 PWM_L;
39                 1];
40         c = [0.2874 -64.469 3462.5 0 0 0;
41              0 0 0 0.2789 -31.524 735.09];
42         v = mean(c*input);
43         u = [-v;
44             -v;
45             theta];
46         B = [-t*cosd(theta) 0 0;
47              0 t*sind(theta) 0;
48              0 0 0];
49         y = A*x + B*u;
50     % Left turn mode
51     case 3
52         PWM_R = 64;
53         PWM_L = 65;
54         t = 0.045;
55         input = [PWM_R^2;
56                 PWM_R;
57                 1;
58                 PWM_L^2;
59                 PWM_L;
60                 1];
61         coeff = [-0.1062, 9.9628, -51.161, -0.0245, -2.6331,
62                  428.17];
63         w = -1* mean(coeff*input);
64         C_c= C_c*0.3;
65         B = [-C_c*(cosd(theta) - cosd(theta + w*t));
66             C_c*(sind(theta) - sind(theta + w*t));
67             w*t];
68         y = A*x + B;
69     % Right turn mode
70     case 4
71         PWM_R = 114;
72         PWM_L = 105;
73         t = 0.03;
74         input = [PWM_R^2;
75                 PWM_R;
76                 1;

```



```

76         PWM_L^2;
77         PWM_L;
78         1];
79         coeff = [-0.0955, 23.74, -1287.7, -0.6161, 134.99,
80                 -7210.5];
81         w = 1*mean(coeff*input);
82         B = [C_c*(sind(theta) - sind(theta + w*t));
83             C_c*(cosd(theta) - cosd(theta + w*t));
84             w*t];
85         %w = 1*mean(coeff*input);
86         %B = [-C_c*(cosd(theta) - cosd(theta + w*t));
87             %   C_c*(sind(theta) - sind(theta + w*t));
88             %   w*t];
89         y = A*x + B;
90     otherwise
91         y = x;
92     end
93     if y(3) < 0
94         y(3) = y(3) + 360;
95     end
96     if y(3) > 360
97         y(3) = y(3) - 360;
98     end
99 end

```

4 Sensor Measurement and Linearized Model

In order to develop the idealized sensor measurement model, a geometric approach derived by the team is used.

4.1 Ideal Geometric Lidar Sensor Measurement

Process for determining the ideal geometric lidar sensor readings is listed in detail as follow.

1. Given the robot's current state (p_x, p_y, p_θ) .
2. Draw a line perpendicular to the respective edge of the car that each of the lidar sensors sit on (front and right edge).
3. The line drawn will intersect with two out of the four edges of the box, assuming that these lines continue infinitely in both directions outside the box. Note that the equations of the edges of the box are:

$$\begin{aligned} x &= 0 \text{ (left edge of the box)} \\ x &= L \text{ (right edge of the box)} \\ y &= 0 \text{ (bottom edge of the box)} \\ y &= W \text{ (top edge of the box)} \end{aligned}$$

4. Figure 2 shows that out of the two intersections, one will occur right at the edge of the actual box (inside the boundary of the box), while the other will occur outside the boundary of the box.
5. Knowing the two intersection points, and the current state of the car, the distance between the car and each of the points is then calculated.
6. After the distances are calculated, we take the minimum of the two distances as the distance between the lidar sensor and the edge/wall of the box.
7. This gives us the ideal geometric lidar sensor reading.

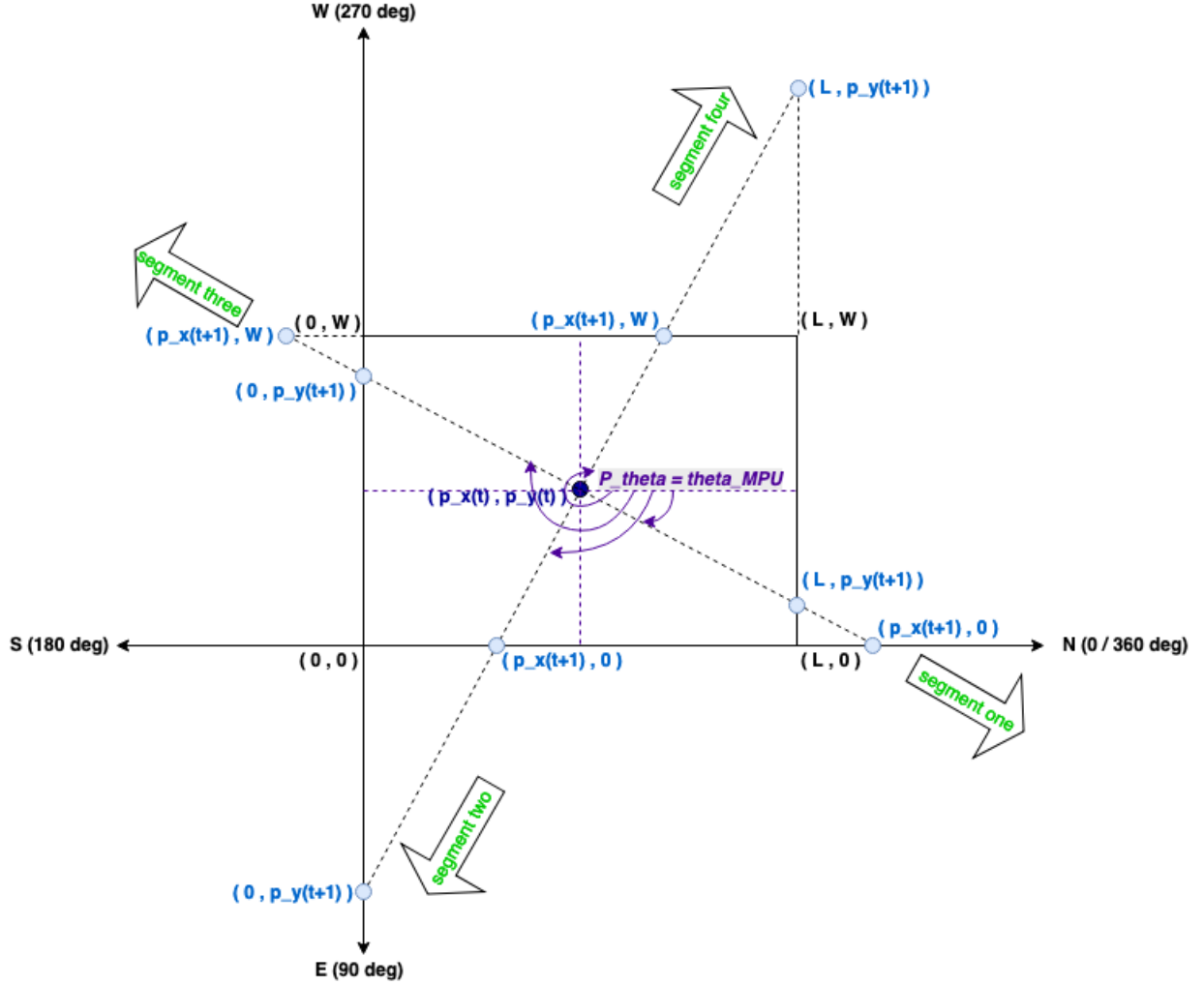


Figure 2: Sensor Geometric Model

4.2 Equations

In order to take into account of all cases ($p_{\theta_t}, \theta_{MPU} = [0, 2\pi]$), we separate into 4 cases.

We first define the intersection points for different cases as:

Y_f = Front lidar sensor intersection's Y parameter

X_f = Front lidar sensor intersection's X parameter

Y_r = Right lidar sensor intersection's Y parameter

X_r = Right lidar sensor intersection's X parameter

$$\forall p_{\theta_t} \in (0^\circ, 90^\circ)$$

$$\begin{aligned} Y_f &= 0 \\ X_f &= L \\ Y_r &= 0 \\ X_r &= 0 \end{aligned}$$

$$\forall p_{\theta_t} \in (90^\circ, 180^\circ)$$

$$\begin{aligned} Y_f &= 0 \\ X_f &= 0 \\ Y_r &= W \\ X_r &= 0 \end{aligned}$$

$$\forall p_{\theta_t} \in (180^\circ, 270^\circ)$$

$$\begin{aligned} Y_f &= W \\ X_f &= 0 \\ Y_r &= W \\ X_r &= L \end{aligned}$$

$$\forall p_{\theta_t} \in (270^\circ, 360^\circ)$$

$$\begin{aligned} Y_f &= W \\ X_f &= L \\ Y_r &= 0 \\ X_r &= L \end{aligned}$$

The Front lidar sensor's two intersections can then be calculated using the following equations.

$$\begin{aligned} p'_x &= p_x + \frac{p_y - Y_f}{\tan(p_\theta)} \\ p'_y &= p_y + (p_x - X_f)\tan(p_\theta) \end{aligned}$$

The Right lidar sensor's two intersections can then be calculated using the following equations.

$$\begin{aligned} p'_x &= p_x + (Y_r - p_y)\tan(p_\theta) \\ p'_y &= p_y + \frac{X_r - p_x}{\tan(p_\theta)} \end{aligned}$$

Distances of the two intersections in each segment as shown in Figure 2, can be obtained by the following equations.

$$\begin{aligned} r_{x_f} = r_{x_r} &= \sqrt{(p'_x - p_x)^2 + (Y - p_y)^2} \\ r_{y_f} = r_{y_r} &= \sqrt{(X - p_x)^2 + (p'_y - p_y)^2} \end{aligned}$$

4.3 Linearization

Assume that from previous session, we found the minimum distance of the two intersections, therefore knowing the correct point of intersection with the box and the actual lidar reading.

We define some more new symbols to help simplify the linearization cases.

(X_I, Y_I) Actual intersection point
 L_I Actual lidar reading (distance to the box/wall)

where

$$L_I = \sqrt{(p_{x_0} - X_I)^2 + (p_{y_0} - Y_I)^2}$$

Linearizing the equations from previous session about $(p_{x_0}, p_{y_0}, p_{\theta_0})$ gives us $z_{3 \times 1} = H_{3 \times 3} x_{3 \times 1} + D_{3 \times 1}$ for Extended Kalman Filter.

Expressing in matrix form yields:

$$\begin{aligned} \begin{bmatrix} l_f \\ l_r \\ \theta_{MPU} \end{bmatrix} &= z = \begin{bmatrix} \frac{p_{x_0} - X_I}{L_I} & \frac{p_{y_0} - Y_I}{L_I} & 0 \\ \frac{p_{x_0} - X_I}{L_I} & \frac{p_{y_0} - Y_I}{L_I} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_\theta \end{bmatrix} + \begin{bmatrix} -p_{x_0} \frac{p_{x_0} - X_I}{L_I} \\ -p_{x_0} \frac{p_{y_0} - Y_I}{L_I} \\ L_I \end{bmatrix} \\ &= Hx + D \\ H &= \begin{bmatrix} \frac{p_{x_0} - X_I}{L_I} & \frac{p_{y_0} - Y_I}{L_I} & 0 \\ \frac{p_{x_0} - X_I}{L_I} & \frac{p_{y_0} - Y_I}{L_I} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ D &= \begin{bmatrix} -p_{x_0} \frac{p_{x_0} - X_I}{L_I} \\ -p_{x_0} \frac{p_{y_0} - Y_I}{L_I} \\ L_I \end{bmatrix} \end{aligned}$$

By plugging in different intersection points into the linearized equation, we can derive the $H_{3 \times 3}$ and $D_{3 \times 1}$ matrices for all different cases as listed in previous sessions.

4.4 Special Cases

As the intersection points are found using the slope ($\tan(p_\theta)$) of the line that passes through the front lidar sensor, there are four special cases that we need to be aware of.

Due to the fact that

$$\tan(0^\circ) = \tan(180^\circ) = \tan(360^\circ) = 0$$

$$\tan(90^\circ) = \tan(270^\circ) = \text{undefined}$$

the above equations will not hold for these special cases, hence we derive new $H_{3 \times 3}$ and $D_{3 \times 1}$ matrices for these special cases.

$\forall p_{\theta_t} = 0^\circ \text{ and } 360^\circ$

$$H = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}$$

$\forall p_{\theta_t} = 90^\circ$

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\forall p_{\theta_t} = 180^\circ$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ W \\ 0 \end{bmatrix}$$

$\forall p_{\theta_t} = 270^\circ$

$$H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} W \\ L \\ 0 \end{bmatrix}$$

4.5 Matlab Implementation

Listing 2: MATLAB code for Sensor Measurement Linearized Model

```

1 function [H, D] = linearize(x)
2     % Symbols and Constants
3     L = 762.5; % Length of the box [mm]
4     W = 495;   % Width of the box [mm]
5     Lf_c = 50; % front lidar sensor to the car sensor reference
6                 point [mm]
7     Lr_c = 53; % right lidar sensor to the car sensor reference
8                 point [mm]
9
10    % MPU9250
11    p_theta = x(3);
12    % Lidar
13    p_x = x(1);
14    p_y = x(2);
15
16    if p_theta < 0
17        p_theta = p_theta + 360;
18    end
19    if p_theta > 360
20        p_theta = p_theta - 360;
21    end
22
23    % Cases
24    switch p_theta
25    case 0
26        H = [-1 0 0;
27              0 1 0;
28              0 0 1];
29        D = [L;
30              0;
31              0];
32    case 360
33        H = [-1 0 0;
34              0 1 0;
35              0 0 1];
36        D = [L;
37              0;
38              0];
39    case 180
40        H = [1 0 0;
41              0 -1 0;
42              0 0 1];
43        D = [0;
44              W;

```

```

43         0];
44     case 90
45         H = [0  1  0;
46              1  0  0;
47              0  0  1];
48         D = [0;
49              0;
50              0];
51     case 270
52         H = [0  -1  0;
53              -1  0  0;
54              0  0  1];
55         D = [0;
56              0;
57              0];
58     otherwise
59         if p_theta < 0
60             p_theta = p_theta + 360;
61         end
62         if p_theta < 90
63             xf = L;
64             yf = 0;
65             xr = 0;
66             yr = 0;
67         elseif p_theta <180 && p_theta>90
68             xf = 0;
69             yf = 0;
70             xr = 0;
71             yr = W;
72         elseif p_theta <270 && p_theta>180
73             xf = 0;
74             yf = W;
75             xr = L;
76             yr = W;
77         elseif p_theta <360 && p_theta>270
78             xf = L;
79             yf = W;
80             xr = L;
81             yr = 0;
82         end
83
84         % Lidar (Test in process)
85         %p_y = x(2) + Lf_c*cosd(p_theta);
86         %f_p_x = x(1) + Lf_c*sind(p_theta);
87         %r_p_x = x(1) - Lf_c*sind(p_theta);
88         %fx = f_p_x + (p_y - yf) / tan(p_theta);
89         %fy = p_y + (f_p_x - xf) * tan(p_theta);

```



```

90     %rx = r_p_x + (yr - p_y) * tan(p_theta);
91     %ry = p_y + (xr - r_p_x) / tan(p_theta);
92
93     fx = p_x + (p_y - yf) / tan(p_theta);
94     fy = p_y + (p_x - xf) * tan(p_theta);
95     rx = p_x + (yr - p_y) * tan(p_theta);
96     ry = p_y + (xr - p_x) / tan(p_theta);
97
98     points = [xf fy;
99              fx yf;
100             rx yr;
101             xr ry];
102
103     for i = 1:2
104         % (Test in process)
105         %     if i == 1
106         %         p_x = f_p_x;
107         %     else if i == 2
108         %         p_x = r_p_x;
109         %     end
110         tempA = pdist([p_x, p_y; points(2*i-1,:)]);
111         tempB = pdist([p_x, p_y; points(2*i,:)]);
112         if tempA < tempB
113             p = [points(2*i-1,:) tempA];
114         else
115             p = [points(2*i,:) tempB];
116         end
117         H(i,:) = [(p_x - p(1))/p(3), (p_y - p(2))/p(3), 0];
118         D(i,:) = [(-p_x + p(1))/p(3)*p_x - (p_y - p(2))/p(3)*p_y
119                  + p(3)];
120     end
121     H(3,:) = [0 0 1];
122     D(3) = 0;
123 end
end

```

5 Kalman Filter Procedure

Since the process dynamics of our system are non-linear but smooth, we may use its linearized approximation for the process model, hence applying to Extended Kalman Filter, which is the non-linear version of Kalman Filter that linearizes about the current mean and covariance.

5.1 Predict

1. $\hat{x}_{t+1}^- = A\hat{x}_t^+ + Bu_t$
2. $P_{t+1}^- = AP_t^+A^T + Q$

5.2 Kalman Gain

1. $K_{t+1} = P^-C^T(CP^-C^T + R)^{-1}$

5.3 Measurement Update

1. $\hat{x}^+ = \hat{x}^- + K(z - H\hat{x}^-)$
2. $P^+ = (I - KH)P^-$

where

$$\begin{aligned}
 e &= x(t) - \hat{x}^+(t) \\
 P &= E[ee^T] = \text{Error Covariance} \\
 R &= E[vv^T] \\
 Q &= E[ww^T] = 0
 \end{aligned}$$

5.4 Matlab Implementation

Listing 3: MATLAB code for Kalman Filter

```

1 %% EE 183DA
2 % Team Buffalo
3 % Lab 2 - Kalman Filter state estimator
4 clc; clear all; close all;
5
6 %% Load and subtract data
7 exp1 = load ('measure.txt');
8 exp2 = load ('measure_NEW.txt');
9 SENSOR = exp2(1:3,:);
10 LIDAR_F = exp2(1,:);

```

```

11 LIDAR_R = exp2(2,:);
12 MPU = exp2(3,:);
13 MODE = exp2(4,:);
14
15 %% Initial State
16 % exp1
17 %p_x = 150;
18 %p_y = 100;
19 %p_theta = 333;
20 % exp2
21 p_x = 50;
22 p_y = 200;
23 p_theta = 180;
24
25 x = [p_x; p_y; p_theta];
26
27 %% Sensor offset
28 offset = [28.236;
29           63.228;
30           0];
31 PtoL = [1.0065  0  0;
32         0  1.0547  0;
33         0  0  1];
34
35 %% Kalman Filter
36 % Variables and constant
37 I = eye(3);
38 A = I;
39 R = 0.1* eye(3); % Measurement Noise
40 % Measurement Noise (Test in process)
41 %v = [];
42 %for i = 1:1000
43 %    v = [v, wgn(3,1,1)];
44 %end
45 %R = 0.01*v*v';
46
47 for i = 1:length(exp2)
48     %estimation update
49     x(:,i+1) = estimate(x(:,i),MODE(i));
50
51     % z = Hx + V
52     %mid = findmiddle(x(:,i))';
53     [H,D] = linearize(x(:,i));
54     %mid = findmiddle(x(:,i+1))';
55     z(:,i) = PtoL * (H*x(:,i+1) + D) + offset;
56
57     % (Test in process)

```

```
58     %e = x(:,i) - x(:,i+1);
59     %p = 0.01*e*e';
60     p = 0.0001 * eye(3);
61
62     % measurement unpade
63     K = p*H.'*inv(H*p*H.'+R);
64     x(:,i+1) = x(:,i+1) + K*(SENSOR(i)- z(:,i));
65     p = (I - K*H) * p;
66 end
67
68 %% Plot the car state
69 plot_car_state(x);
```

6 Results

6.1 Car State Estimator

The resulted state estimator is plotted in a MATLAB figure, as shown in Figure 3.

In the MATLAB figure, the x axis limit and y axis limit is set to be the dimension of the box that the car is running in. The green line shows the complete path the car is following, and the red * indicates the car reference point. To easier visualize how the car is performing in the box, we further plotted a rectangle indicating the car's body, while the dark blue line indicates the head of the car.

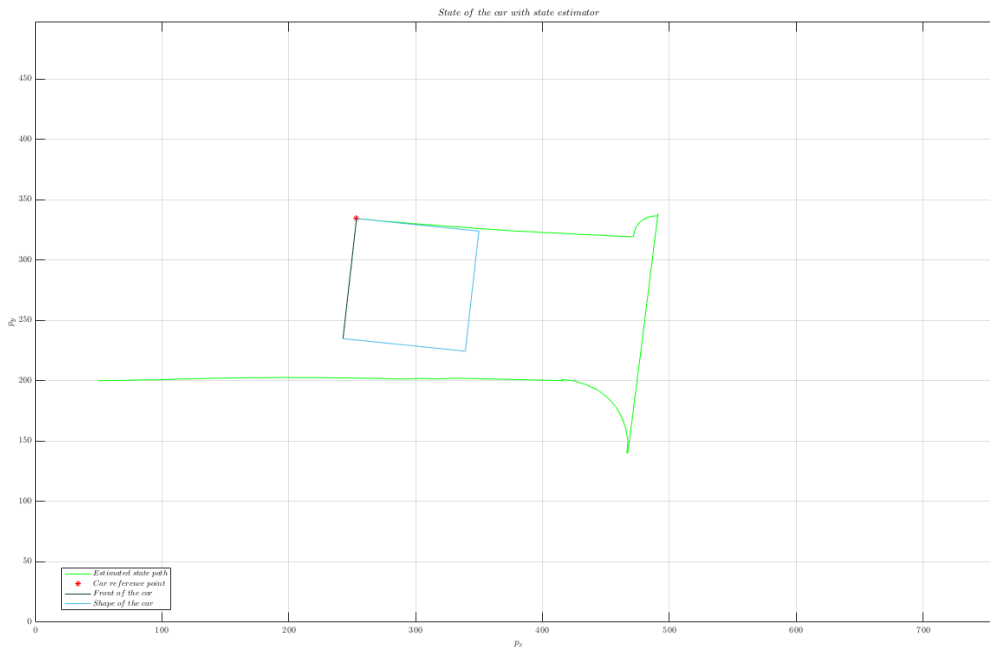


Figure 3: MATLAB Car State estimator

6.2 True State

In order to determine the true state of the car observed by an external observer, we used a birds-eye camera and created a box with grids drawn as shown in Figure 4. Each grid is precisely drawn to be 50×50 [mm], therefore from the birds-eye view we can tell if the state estimated result is close to the true state.



Figure 4: Box with Grids for True State Determination

6.3 Comparison of State Estimates to True States

From the MATLAB plot and the birds-eye view video (please refer to appendix and demonstration for video link and detailed information), we can tell that the path the state estimator predicted and the true path have the same general shape, meaning that the mathematical model we derived is accurate.

However, while we increase the noise and uncertainty in the Kalman Filter Algorithm, as well as the case where the initial state is unknown, the path doesn't quite follow the true state anymore, even though still having the same starting and ending position (state) in the box.

The factor leading to the error in the Kalman Filter Algorithm is the Error Covariance. When the error covariance is measured, and determined to be small (on the order of 0.1 or less), the Kalman filter performs as it should, correcting the state model as the robot experiences noise and deviates from the path expected by the model. However, as error covariance grows, the Kalman Filter exhibits undefined behavior, and instead of correcting erroneous state estimates by the model, it actually causes the model to be less accurate, as the path it depicts does not in any way resemble the actual state.

In cases where the error covariance is small, the closed loop state estimator, with the Kalman

Filter implemented, was more accurate than the open loop state estimator. In cases where the error covariance is large, the closed loop state estimator is less accurate than the open loop state estimator, because, as discussed above, the closed loop state estimator makes the model less accurate.

6.4 Discussion and Future Work

While we discovered that when increasing the noise and uncertainty will lead to a under performed state estimate result, the team discussed that a possible reason leads to this error may be due to the cable connected to the ESP WiFi chip while collecting experimental data.

After few trials of testing, we figured when the cable is connected to the ESP WiFi chip, the car won't follow the desired route, as the tension in the cable is doing work on the car even though we treat it carefully while collecting data. We then verified our assumption with wirelessly controlling the car and it performs just fine as it should be.

Future work and improvement will include writing a code that allows wireless data collection and allow the data collected to be sent and save to MATLAB. This way we get rid of the possible source of error. The team should also further look into what is the reason that causes the state estimator to underperform while increasing the noise and uncertainty in our Kalman Filter Algorithm.

7 Appendix and Demonstration

All class materials and documents can be found on [GitHub](#).

All codes, videos, excel sheets and related documents for Lab 2 can be found on [GitHub](#).

For specific experiment videos and demonstrations please visit the links attached to the topics:

- [Video: MATLAB Live Car State Estimates](#)
- [Video: True State - Birds-eye view](#)
- [MATLAB Code](#)
- [Images and Figures](#)
- [Experimental Data](#)

8 References

1. Mehta, Ankur. "Design of Robotic Systems I." EE183DA. University of California Los Angeles, Los Angeles, California. Lectures.
2. Asada, and Harry. "Lecture Notes." MIT Open CourseWare. Massachusetts Institute of Technology, n.d. Web. 31 Jan. 2019.