Isaac Larson

1. a) $0 + 2 = 2$
   $1 + 2 = 3$
   $4 + 2 = 6$
   $9 + 2 = 11$
   } Not perfect squares

   b) Yes, seems very likely

   c) Proof (direct): Assume $\sqrt{s} \in \mathbb{Z}$, $s \geq 4$
   Will prove $s+2$ is not a perfect square by showing $s+2$ is less than the proceeding perfect square of s.
   Let $s = x^2$ so $x = \sqrt{s}$
   $s + 2 < (x+1)^2 \Rightarrow x^2 + 2 < (x+1)^2$
   $x^2 + 2 < x^2 + 2x + 1 \Rightarrow 2 < 2x + 1$
   s is ≥ 4 which means $x \geq 2$. 2 multiplied by a value ≥ 2 will always be larger than 2.
   Therefore, for a perfect square $s \geq 4$, $s+2$ is not a perfect square.

2. Proof (direct): Assume $\sqrt[3]{c} \in \mathbb{Z}$
   Every integer can be represented as $3k$, $3k+1$, or $3k-1$
   Let $c = x^3$, $x \in \mathbb{Z}$
   Case 1: x is a multiple of 3
   $c = (3k)^3 = 27k^3 = 9(3k^3)$
   Case 2: x is one more than a multiple of 3
   $c = (3k+1)^3 = 27k^3 + 27k^2 + 9k + 1$
   $= 9(3k^3 + 3k^2 + k) + 1$
   Case 3: x is one less than a multiple of 3
   $c = (3k-1)^3 = 27k^3 - 27k^2 + 9k - 1$
   $= 9(3k^3 - 3k^2 + k) - 1$
   Therefore, every perfect cube c is a multiple of 9, one less than a multiple of 9, or one more than a multiple of 9.

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3. Let $x = 6$, $y = 4$, and $z = 9$
   $6 \mid 4(9) \rightarrow 6 \mid 36$ ← if condition is met
   then $6 \mid 4$ = False

4. a) $\{x \mid x$ is a greek alphabet between α and ϵ inclusive. $\}$

   b) $\{x \mid x = 4k,\ k \in Z^{*}\}$

5. a) False, $\{b\}$ is not an element of second set.
   b) False, the two sets are equal
   c) False, should be $\{1, 3, 5, 7, 9\}$
   d) $|P(\{a,b,c,d,e,f,g,h,i,j\})| = 2^{10} = 1024$
   e) $P(\{0, 2, 4, 8\})$
   $= \{ \{\},$
   $\{0\}, \{2\}, \{4\}, \{8\},$
   $\{0,2\}, \{0,4\}, \{0,8\}, \{2,4\}, \{2,8\}, \{4,8\},$
   $\{0,2,4\}, \{0,2,8\}, \{0,4,8\}, \{2,4,8\},$
   $\{0,2,4,8\} \}$

7.