Time series analysis

1. Introduction

A time series is a sequence of observations recorded in discrete time points.
They can be recorded in hourly (e.g. air temperature), daily (e.g. DJI Average), monthly (e.g. sales) or yearly (e.g. GDP) time intervals.
Real-world time series data commonly contains several components or recognizable patterns.
Those time series components can be divided into

1. **Systematic components** - are those that allow modeling as they have some consistent or recurring pattern. Examples for systematic time series components are
   a. **Trend** - A trend is a consistent increase or decrease in units of the underlying data. Consistent in this context means that there are consecutively increasing peaks during a so-called uptrend and consecutively decreasing lows during a so-called downtrend. Also, if there is no trend, this can be called **stationary**.
   b. **Seasonality** - describes cyclical effects due to the time of the year. Daily data may show valleys on the weekend, weekly data may show peaks for the first week of every or some month(s) or monthly data may show lower levels for the winter or summer months respectively.
2. **Non-systematic** components - any other observation in the data is categorized as unsystematic and is called error or remainder.

The general magnitude of the time series is also referred to with “level”.

In our example, we will be focusing on sales dataset for 3 years and 3 months by month interval, and we use one dimensional time series analysis techniques and compare them.

2. Loading libraries

All necessary libraries are loaded and written all together at the beginning of the project. This way we know what packages we used, and all possible changes are made easier.
In **fpp2** are all auto model and plotting functions.
RODBC is used to create connection using ODBC driver, the most popular and basic database connection driver.
Uroot is used for later in chapter for changing model parameters .

```
#install.packages("fpp2")
library(fpp2)
#install.packages("uroot")
library(uroot)
#install.packages("RODBC")
library(RODBC)
```

3. Importing data

We will use SQL Server to import data from view object named **TEST_ProdajaPoMesecima**.
Firstly, we create connection object by giving them connection string, contained of **driver's name**, **server's name**, **database**, **uid** - username and **pwd** - password.
Then, we call stored view from server, and get raw data inside data variable.

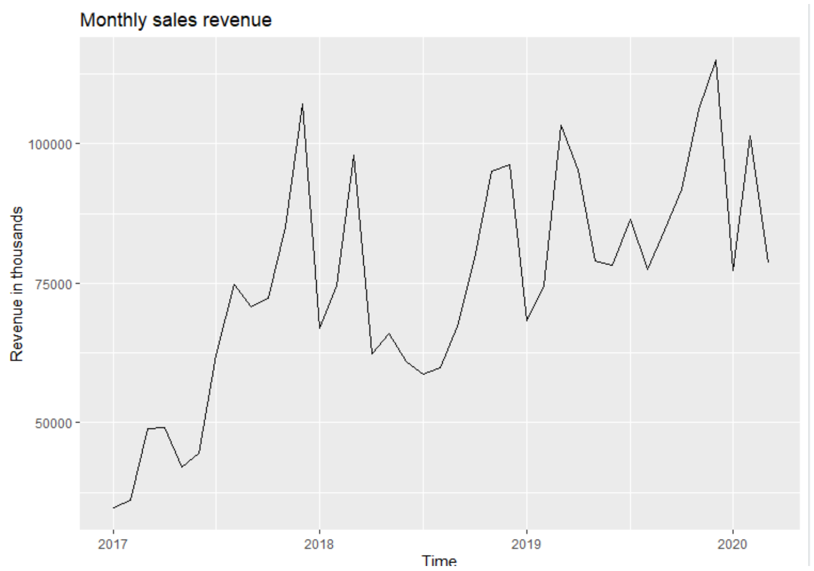
```
connection <- odbcDriverConnect(connection = "Driver={SQL Server Native Client 11.0};server=sqlrep.algrosso.com;database=AlGrosso Analitycs;uid=ReadOnly;pwd=BBRO.124578;")
?odbcDriverConnect
#getting data from View
data <- sqlFetch(connection, "dbo.Test_ProdajaPoMesecima", colnames = FALSE, rownames = TRUE)
```

Structured data is needed before procced into next step. For that we will remove last month's incomplete data, present sales in thousands values, and declare our data as time series data, by defining start period and frequency of our data, which, for month, is 12.

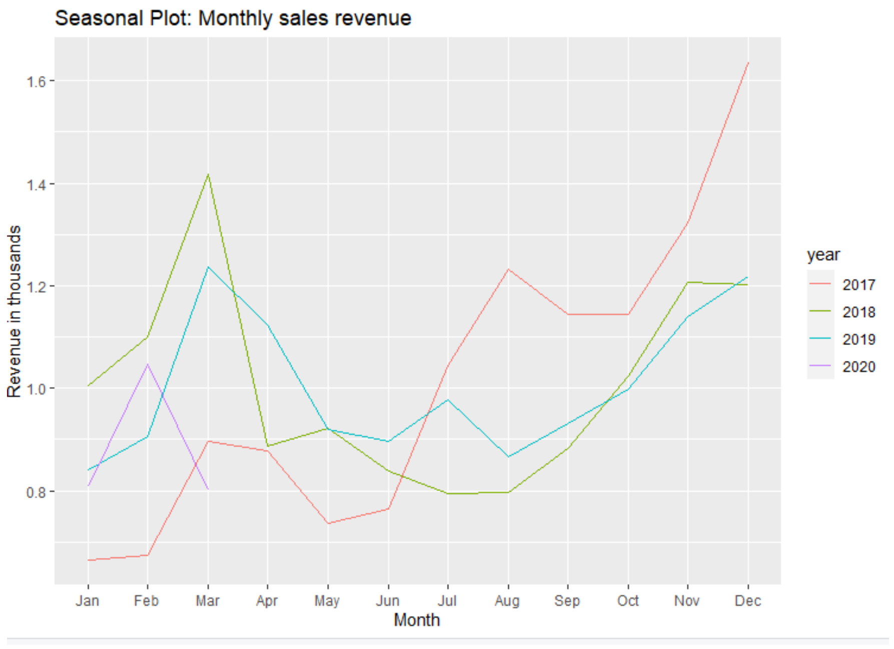
```
#removing last month from data frame, which is incomplete
data <- head(data, -1)

#dividing revenue by 1000 for tracking convinience in plots
data[,3] <- round(data[,3]/1000, digits = 0)

#declare data as time series data
tdata <- ts(data[,3], start = c(2017,1), frequency = 12)
```

4. Exploratory analysis

Our time series looks like this:



From graph, we can fill there is good positive trend. There are also variations in our values, but in this analysis, we will not focus on them.
To inspect seasonality we will **ggseasonplot** function.



We call see that in months of December, in mid-March we have strong seasonality and weak in July-August.
We need to be concern of this and use appropriate models, but I will show all models, regards of these components.

5. Splitting data

I choose proportion of 85:15 because of lack of data records. For each data partition, we need to recreate time series copy.

```
train <- data[1:33,]
test <- data[34:39,]

ttrain <- ts(train[,3], start = c(2017, 1), end = c(2019, 9), frequency = 12)
ttest <- ts(test[,3], start = c(2019, 10), end = c(2020, 3), frequency = 12)
```

6. Define model indicators

We will use tree indicators:

1. **Residual Standard Deviation** of training data **(SD res, or √sigma^2)** – lower values are good sign, but not enough
2. **Mean Absolute Error (MAE)** of test data – absolute prediction error, lower is better.
3. **Root Mean Square Deviation (RMSE)** of test data– prediction square deviation error, lower is better.
4. **Graphical presentation of predicted vs real value**
5. **For ARIMA models, ADF function is also good**

Indicator number 2. and 3. are crucial for model grading.

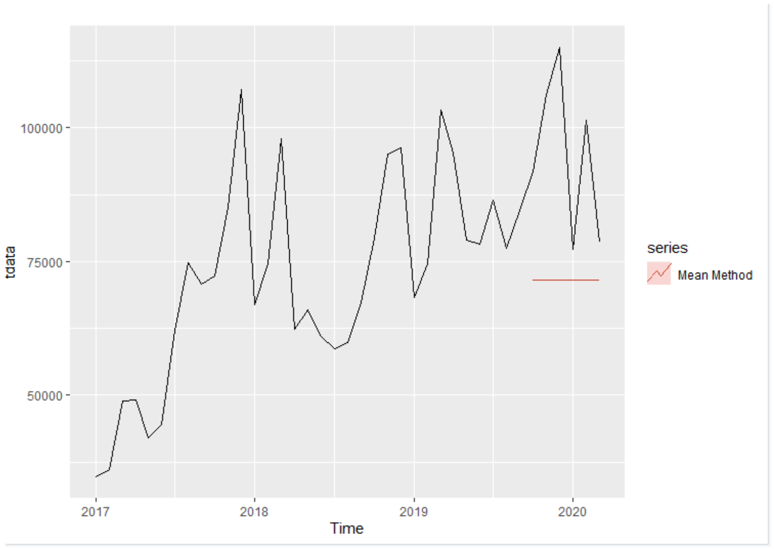
7. Modelling – simple techniques

Simple forecasting techniques are used as benchmarks. They provide a general understanding of historical data and to build intuition upon which to add additional layers complexity and are good to test basic nature about time series.
Several such techniques are common in literature such as: **mean model, naive forecast, random walk, drift method.**

7.1 Mean model

A mean model takes the mean of previous observations and uses that for forecasting.
Formula: $\mathbf{y_t = mean(y_{t-x})}$ x- observation of model for whole period
Model: **meanf <- meanf(ttrain, h=6)** h- period for prediction

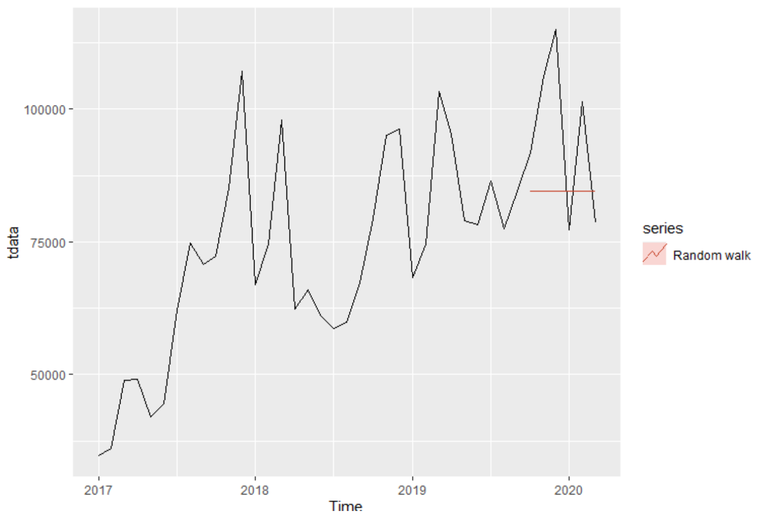
SD RES	MAE	RMSE
18876.63	23513.24	27318.80



7.2 Naive method – Random walk

A random walk, on the other hand, would predict the next value, Ŷ(t), that equals to the previous value plus a constant change.
Formula: $\mathbf{y_t = mean(y_{t-1}) + C}$ C- constant
Model: **rwf <- rwf(ttrain, h=6)** or **naive(ttrain, h=6)** h- period for prediction

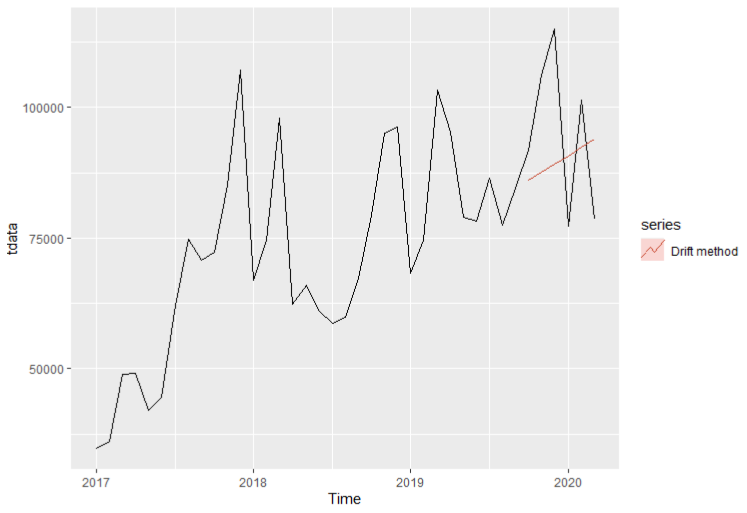
SD RES	MAE	RMSE
15495.68	14908.00	17422.21



7.3 Drift method

Equivalent to extrapolating a line drawn between first and last observations. Forecast equal to last value plus average change. It is first dynamic model.
Formula: **$y_t = y_{t-1}$ + mean(rest data)**
Model: **rwd <- rwf(ttrain, h=6, drift=TRUE)** h- period for prediction

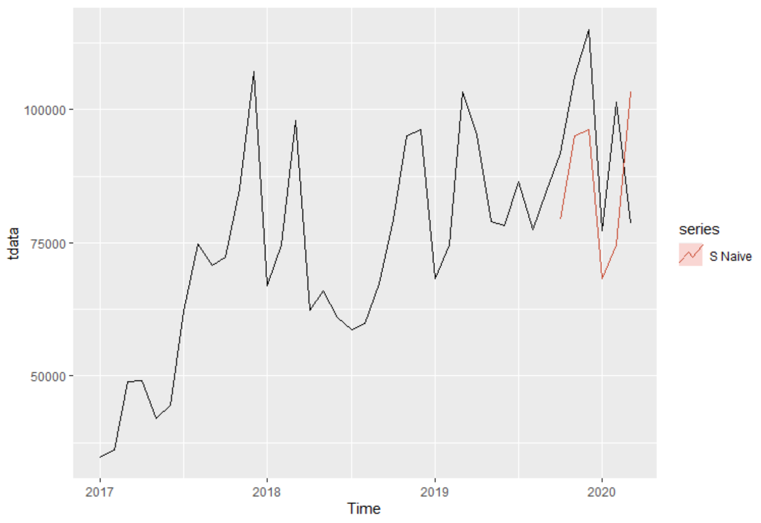
SD RES	MAE	RMSE
15495.68	14648.55	16021.92



7.4 Seasonal naïve method

Forecasts are equal to last value from same season. Good for isolating and testing seasonal effects on time series.
Formula: **$y_t = y_{t-s}$**
Model: **snaive <- snaive(ttrain, h=6)** h- period for prediction

SD RES	MAE	RMSE
16453.78	17185.17	18487.01



8. Exponential smoothing models – ETS

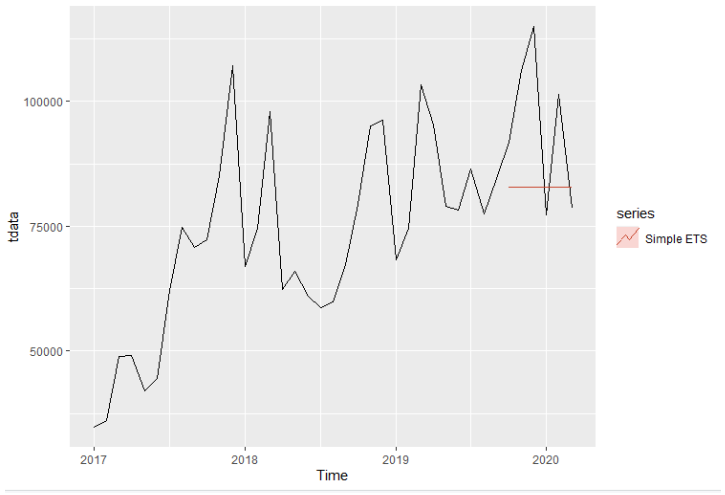
Exponential smoothing is a popular forecasting method for short-term predictions. Such forecasts of future values are based on past data whereby the most recent observations are weighted more than less recent observations. As part of this weighting, constants are being ***smoothed***. Exponential smoothing introduces the idea of building a forecasted value as the average figure from differently weighted data points for the average calculation.

There are different exponential smoothing methods that differ from each other in the components of the time series that are modeled.

8.1 Simple ETS method

Simple exponential smoothing assumes that the time series data has only a level and some error (or remainder) but no trend or seasonality. It uses only one smoothing constant. The smoothing parameter α determines the distribution of weights of past observations and with that how heavily a given time period is factored into the forecasted value. If the smoothing parameter is close to 1, recent observations carry more weight and if the smoothing parameter is closer to 0, weights of older and recent observations are more balanced.
Formula: $\mathbf{F_t = \alpha y_{t-1} + (1-\alpha)F_{t-1}}$ where
F_t , F_{t-1} - forecast values for time t, t-1; y_{t-1} actual value for time t-1; α - smoothing constant
Model: **sets <- ses(ttrain, h=6)** h- period for prediction

SD RES	MAE	RMSE
14205.04	15507.66	18561.26

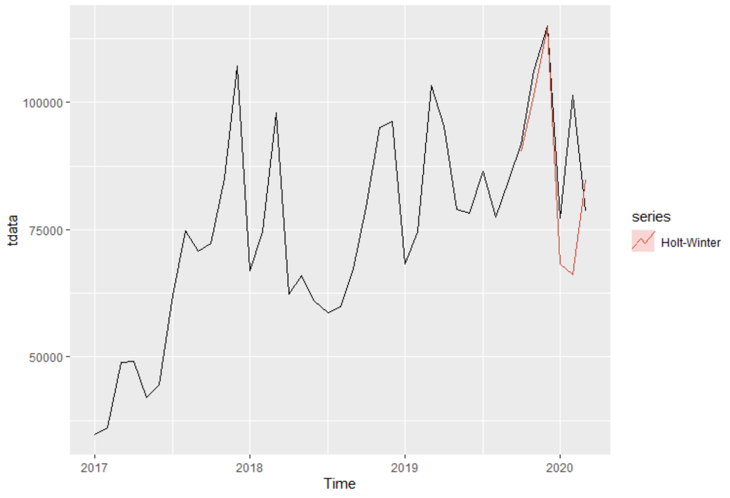


8.2 Holt-Winters exponential smoothing – HW

Holt-Winters exponential smoothing is a time series forecasting approach that takes the overall level, trend and seasonality of the underlying dataset into account for its forecast.

Hence, the Holt-Winters model has three smoothing parameters indicating the exponential decay from most recent to older observations:
α (alpha) for the level component,
β (beta) for the trend component,
γ (gamma) for the seasonality component.
Model: **hw <- hw(ttrain, h = 6, seasonal = "additive")** h- period for prediction

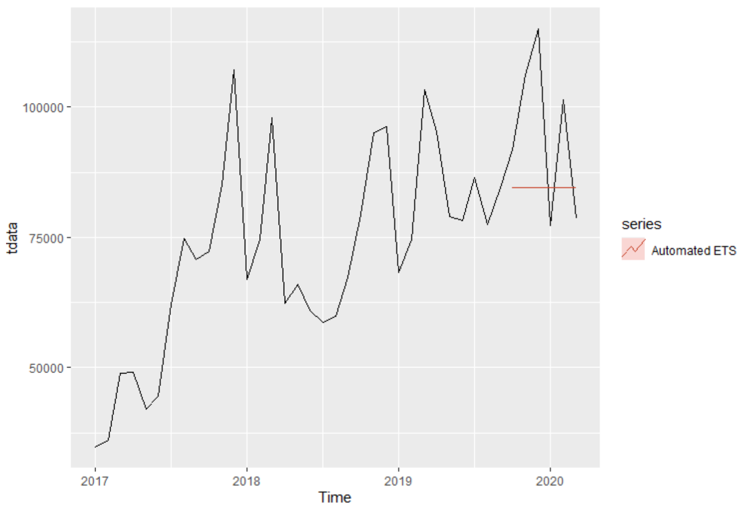
SD RES	MAE	RMSE
14830.10	9482.57	15179.86



8.3 Automated exponential smoothing

There is also an automated exponential smoothing forecast. In this case, it is automatically determined whether a time series component is additive or multiplicative.
The ets function takes several arguments including model. The model argument takes three letters as input.
The first letter denotes the error type (A, M or Z),
the second letter denotes the trend type (N, A, M or Z),
and the third letter denotes the seasonality type (N, A, M or Z).
The letters stand for N = none, A = additive, M = multiplicative and Z = automatically selected.
Model: **hw <- aets <- ets(ttrain) - It is suggesting ETS (M,N,N)**

SD RES	MAE	RMSE
0.1913	14908.23	17422.64



9. ARIMA family models

Auto Regressive Integrated Moving Average (ARIMA) is arguably the most popular and widely used statistical forecasting technique. As the name suggests, this family of techniques has 3 components:
a) an "**autoregression**" component that models the relationship between the series and it's lagged observations;
b) a "**moving average**" model that models the forecast as a function of lagged forecast errors; and
c) an "**integrated**" component that makes the series stationary.

The model takes in the following parameter values:

p that defines the number of lags;

d that specifies the number of differences used; and

q that defines the size of moving average window

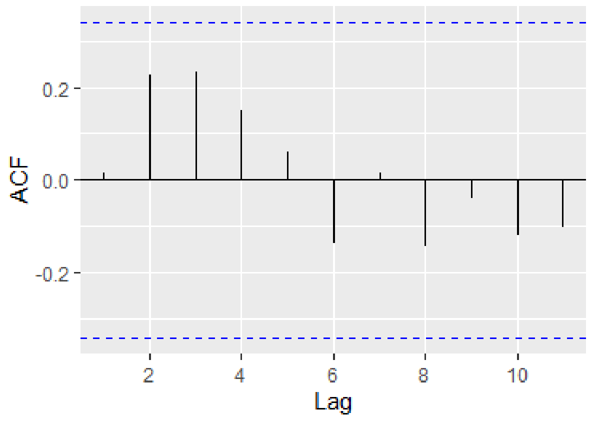
9.1 Moving average model

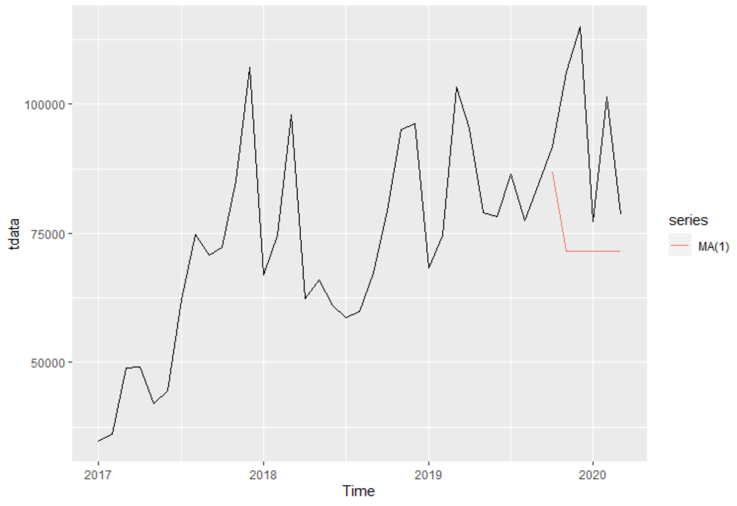
The moving-average model specifies that the output variable depends linearly on the current and various past values of a stochastic (imperfectly predictable) term.

The moving-average model is a special case and key component of the more general ARMA and ARIMA (0, 0, q) models of time series, which have a more complicated stochastic structure.
Model: **MA(q): $X_t = \mu + \epsilon_t + \epsilon_{t-1}\,\theta_1 + \ldots + \epsilon_{t-q}\,\theta_q$** - where **$\mu$** is the mean of the series, the ***$\vartheta_1, \ldots, \vartheta_q$*** are the parameters of the model and the ***$\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$*** are white noise error terms. The value of ***q*** is called the order of the MA model. **We will use MA(1).**

SD RES	MAE	RMSE
12806.54	20977.35	26129.44

ACF function is inside dashed lines, which is positive sign.





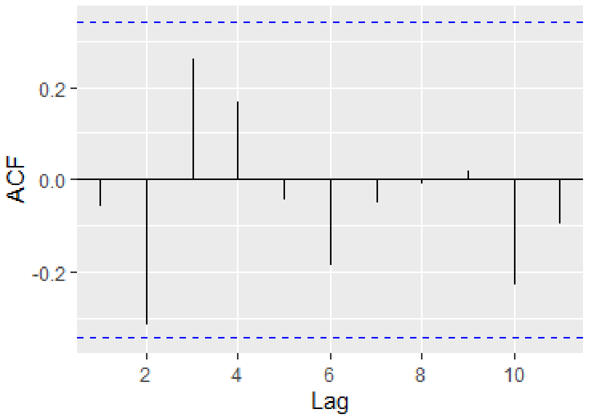
9.2 Autoregressive model

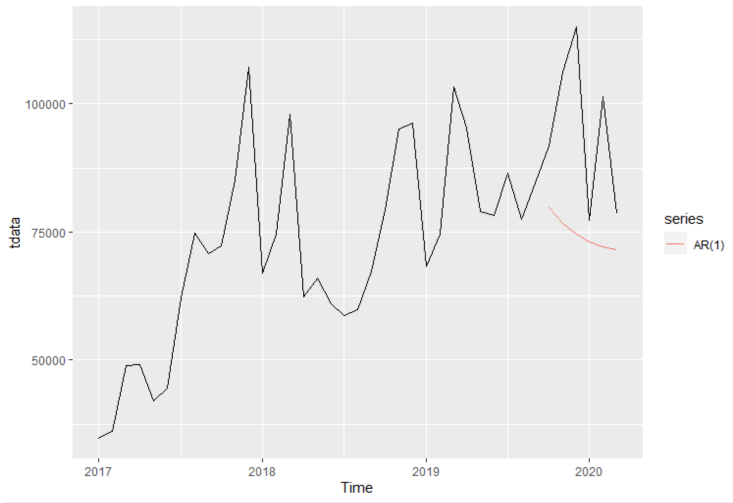
The autoregressive model specifies that the output variable depends linearly on its own previous values and on a stochastic. Together with the moving-average (MA) model, it is a special case and key component of the more general autoregressive–moving-average (ARMA) and autoregressive integrated moving average (ARIMA(p,0,0)) models of time series. It is also a special case of the

vector autoregressive model (VAR), which consists of a system of more than one interlocking stochastic difference equation in more than one evolving random variable.
Model: AR(p): **$Y_t = \beta_0+\beta_1 y_{t-1} +\ldots+ \beta_p y_{t-p} +\epsilon_t$** - where **$\beta_0$** is constant, the **$\beta_1$**, ..., **$\beta_p$** are the parameters of the model and the ***ε_t*** is white noise. The value of ***p*** is called the order of the AR model. **We will use AR(1).**

SD RES	MAE	RMSE
14152.43	20378.13	24385.42

ACF function is inside dashed lines, which is positive sign.



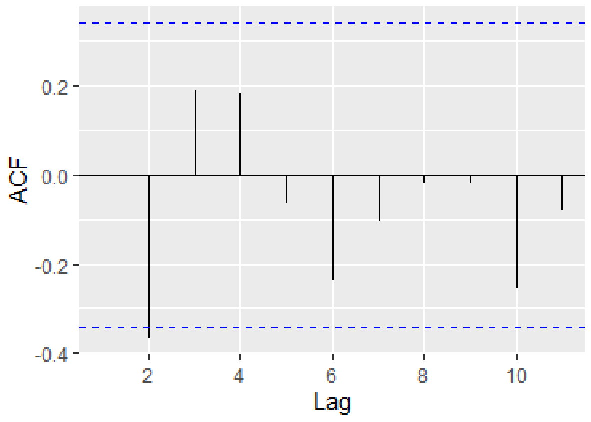


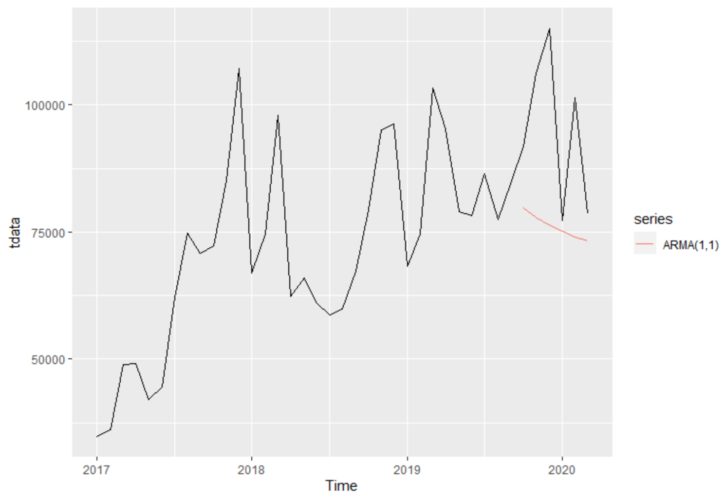
9.3 Autoregressive–moving-average model - ARMA

The ARMA model consist of two parts. The AR part involves regressing the variable on its own lagged (i.e., past) values. The MA part involves modeling the error term as a linear combination of error terms occurring contemporaneously and at various times in the past.
Model: ARMA(p, q): **$Y_t = \mu +\beta_0 +\beta_1 y_{t-1} +\ldots+ \beta_p y_{t-p} +\epsilon_t + \epsilon_{t-1}\,\theta_1+\ldots+ \epsilon_{t-q}\,\theta_q$**
We will use ARMA(1,1).

SD RES	MAE	RMSE
14078.28	18957.66	23145.57

ACF function is not fully inside dashed lines, which is sign there are probably better models





9.4 Autoregressive integrated moving average- ARIMA

ARIMA models are applied in some cases where data show evidence of non-stationarity, where an initial differencing step (corresponding to the "integrated" part of the model) can be applied one or more times to eliminate the non-stationarity.
The I (for "integrated") indicates that the data values have been replaced with the difference between their values and the previous values (and this differencing process may have been performed more than once). The purpose of each of these features is to make the model fit the data as well as possible. There are two types of models:

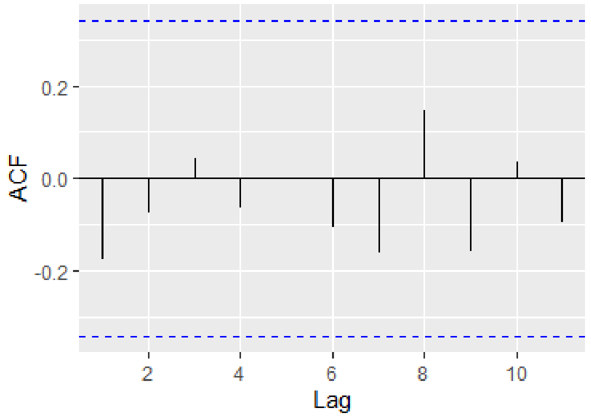
1. Non-seasonal ARIMA models are generally denoted ARIMA(p,d,q) where parameters p, d, and q are non-negative integers, p is the order (number of time lags) of the autoregressive model, d is the degree of differencing (the number of times the data have had past values subtracted), and q is the order of the moving-average model;
2. Seasonal ARIMA models are usually denoted ARIMA(p,d,q)(P,D,Q)m, where m refers to the number of periods in each season, and the uppercase P,D,Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model.

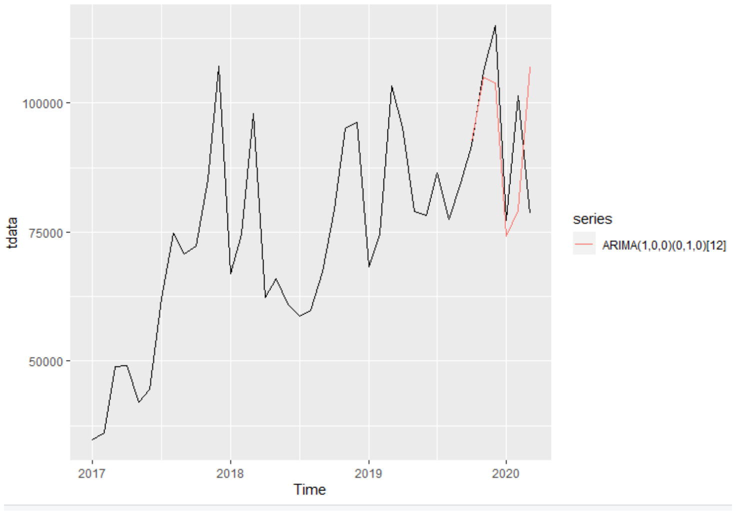
9.4.1 ARIMA (1,0,0)(0,1,0)[12]

I used auto.arima function from forecast package, which is good function for exploring various types of fitted ARIMA models for particular dataset. It will try all possible combinations of ARIMA models by changing parameter values. We will start by first model, that was suggested, seasonal ARIMA.

SD RES	MAE	RMSE
13981.54	11087.77	15449.20

ACF function is inside dashed lines, which is positive sign.



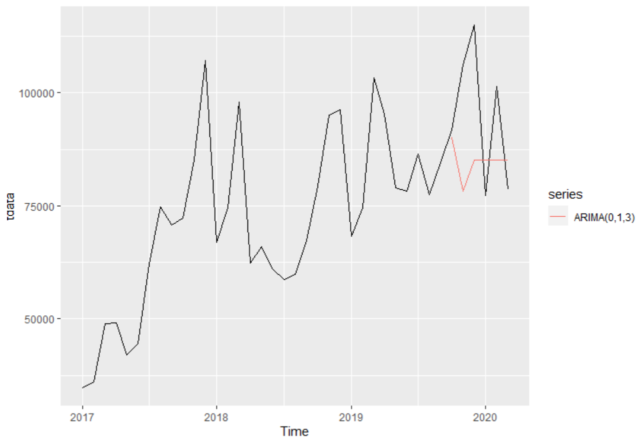


9.4.2 ARIMA (0,1,3)

Last model was suggested with few error messages. That errors pointed that chosen seasonal unit root test encountered an error when testing for the second difference (insufficient number of data after 1st difference). By default, it used one seasonal difference.

We will consider using a different seasonal unit root test (for example Canova and Hansen (CH) test statistic)

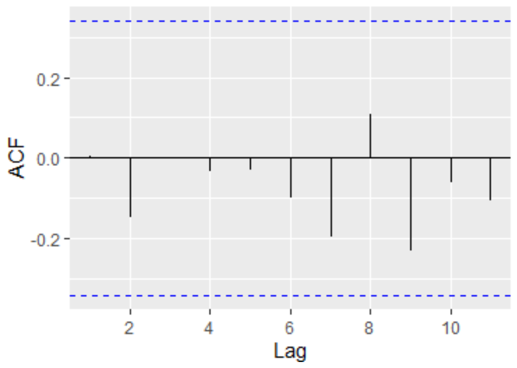
SD RES	MAE	RMSE
12806.54	15033.53	18463.70

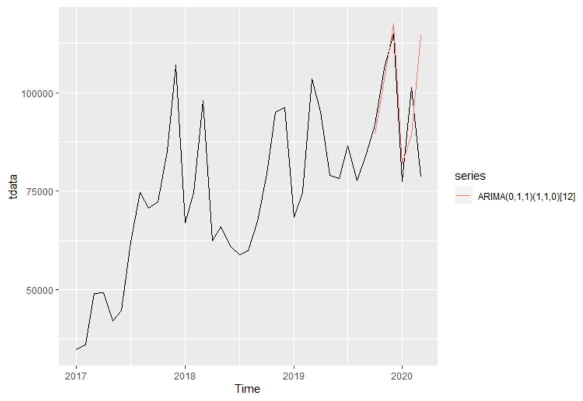


9.4.3 ARIMA (0,1,1)(1,1,0)[12]

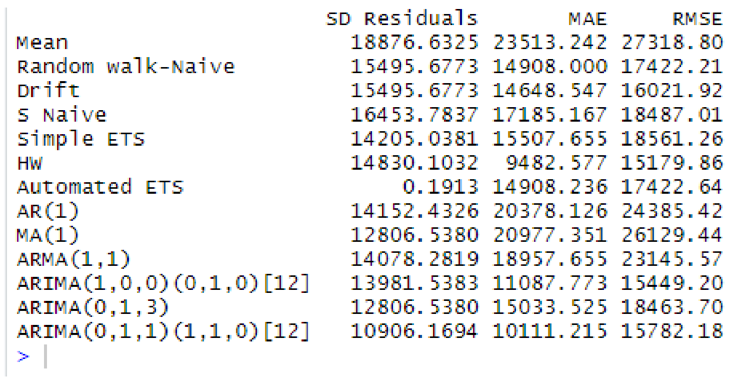
Last model that i will be using came from idea to extend and explore model choosing capabilities with whole data, and then to use that newly found model on training data only.

SD RES	MAE	RMSE
10906.17	10111.22	15782.18





10. Conclusion - choosing best fitting model for forecasting



```
                          SD Residuals       MAE     RMSE
Mean                        18876.6325 23513.242 27318.80
Random walk-Naive           15495.6773 14908.000 17422.21
Drift                       15495.6773 14648.547 16021.92
S Naive                     16453.7837 17185.167 18487.01
Simple ETS                  14205.0381 15507.655 18561.26
HW                          14830.1032  9482.577 15179.86
Automated ETS                   0.1913 14908.236 17422.64
AR(1)                       14152.4326 20378.126 24385.42
MA(1)                       12806.5380 20977.351 26129.44
ARMA(1,1)                   14078.2819 18957.655 23145.57
ARIMA(1,0,0)(0,1,0)[12]     13981.5383 11087.773 15449.20
ARIMA(0,1,3)                12806.5380 15033.525 18463.70
ARIMA(0,1,1)(1,1,0)[12]     10906.1694 10111.215 15782.18
> 
```

First four benchmark models, are good to test certain things about time series, but are not used as final models. AR and MA models are good when there are non-seasonal effects.

HW is clear winner here, and there are two main reasons(mainly against ARIMA model):

1. **Short time frame** – 39 observation only in total;
2. **Existence of structural fracture movements** - (cause is Corona virus).

In more general occasions, ARIMA models are preferred. They are superior models, and are widely used in practice.
Also, there are many more complex model structures that are not part of this research project, and for more in depth problem-oriented area (VAR, GARCH, LSTM…). It would be pleasure for me to continue researching, working and learning them 😊.