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# Topological Sort

*Robert W. Kramer*

A directed graph  $G$  is a *Directed Acyclic Graph* (DAG) if it has no cycles

DFS can determine if  $G$  is a DAG

Observation: When  $v$  is discovered...

- ▷  $v$ 's ancestors are gray
- ▷ all other vertices are white or black

If  $v$  is active,  $v \rightarrow w \in G$ , and  $w$  is gray...

- ▷  $w$  is an ancestor of  $v$
- ▷  $v \rightarrow w$  forms a cycle

To detect cycles:

PRESWEEP:  $hasCycle \leftarrow false$

PREVERTEXCOLORCHECK: if  $w.color = gray$  then  $hasCycle \leftarrow true$

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Vertices in a DAG can be ordered so that if  $v \rightarrow w \in G$ , then  $v$  appears before  $w$  in the order

Real-life examples:

- ▷ Prerequisite chains and order of classes
- ▷ Order of spreadsheet cell calculations

**Observation:** If  $v \rightarrow w \in G$ , then  $v.finish > w.finish$

When  $v$  is discovered,  $w$  will either be:

- ▷ white,  $w$  is a descendant and will be processed before  $v$  finishes
- ▷ gray, we have a loop and  $G$  is not a DAG
- ▷ black, and  $w$  has already finished

In a DAG, finish time determines order

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## Topological sorting based on DFS

▷ Push  $v$  onto stack as it finishes

PRESWEEP:  $slot \leftarrow |V|$

POSTEDGESCAN:  $order[--slot] \leftarrow v$

# Topological Sort Example (1)

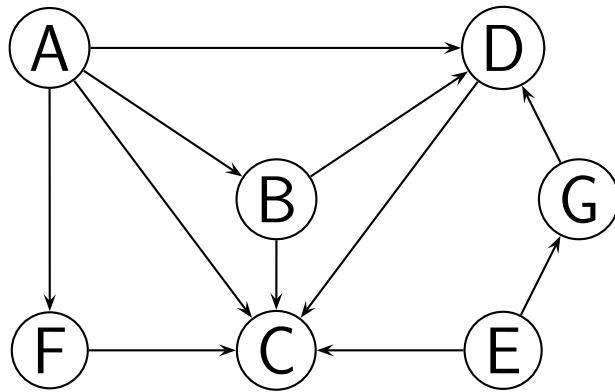


Figure 1a: Original graph

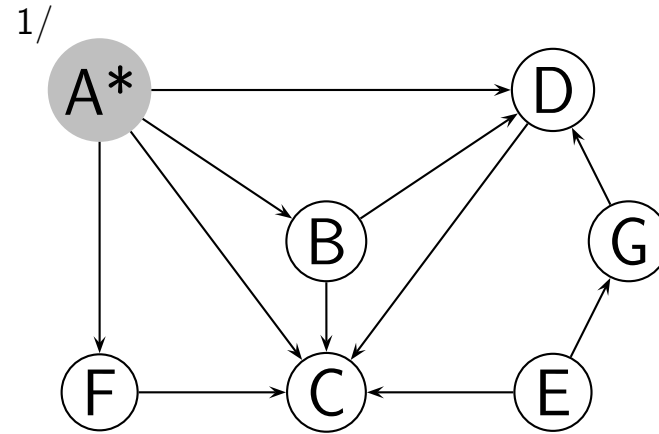


Figure 1b: Start at A

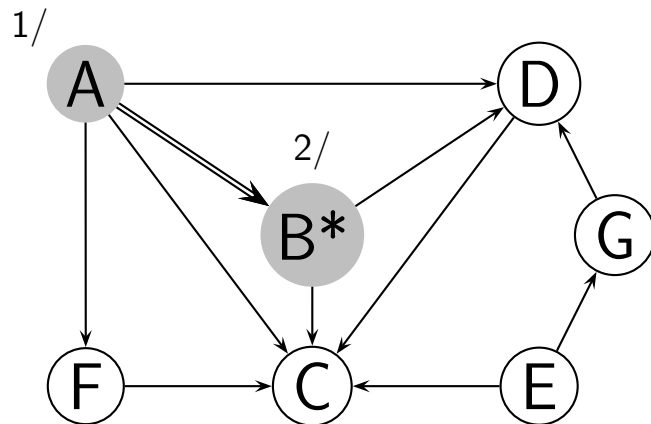


Figure 1c: Move to B

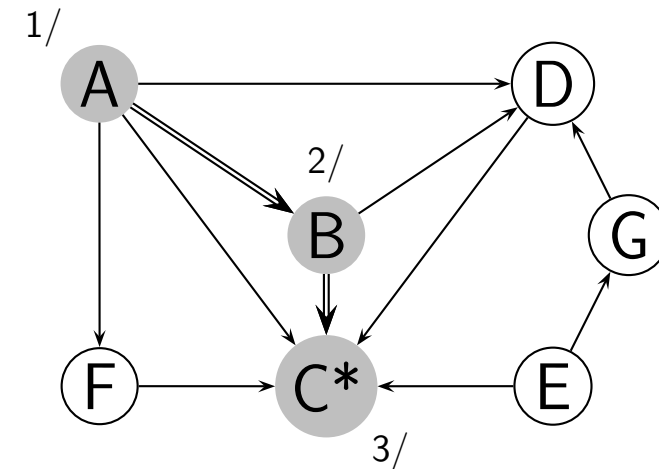


Figure 1d: Move to C

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## Topological Sort Example (2)

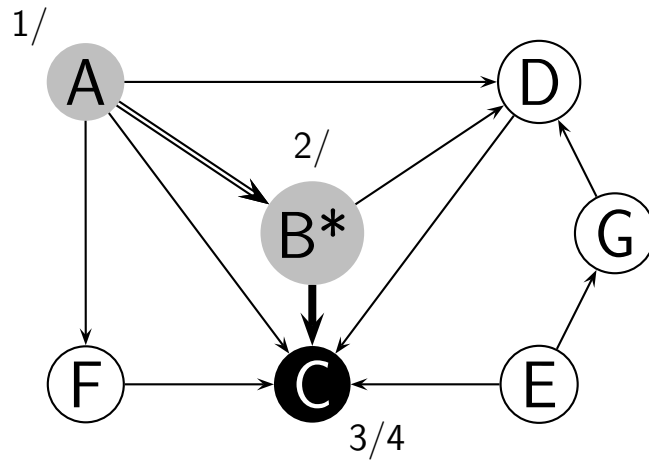


Figure 1e: Order = C

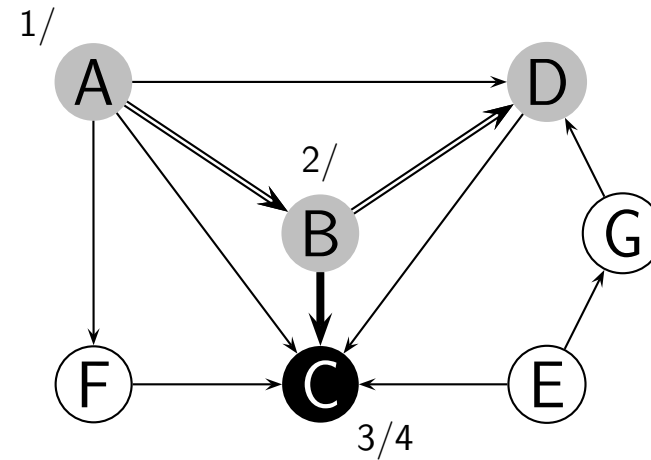


Figure 1f: Move to D

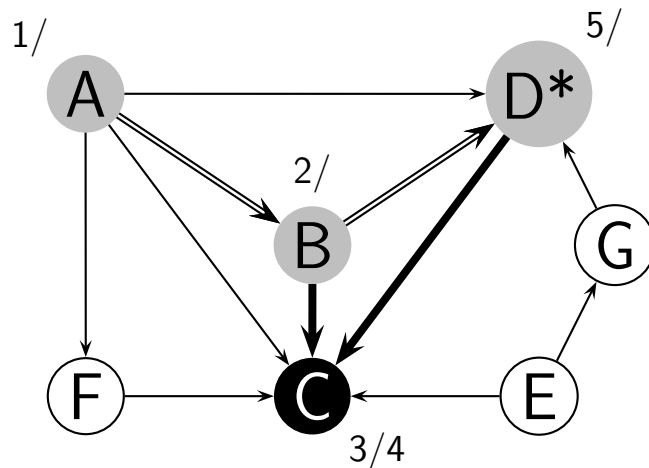


Figure 1g: Look at C, no move

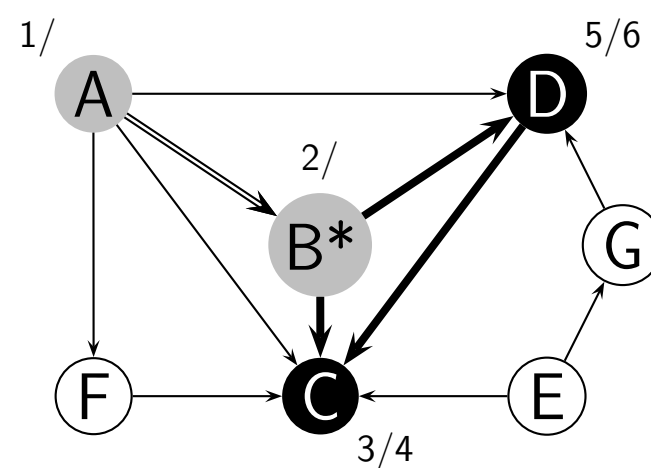


Figure 1h: Order = D, C

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# Topological Sort Example (3)

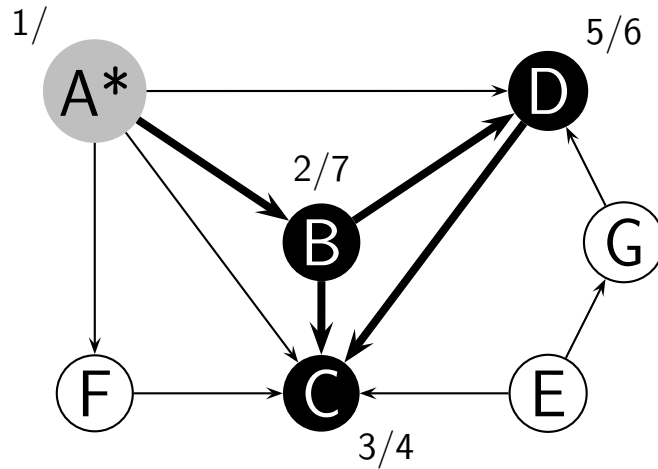


Figure 1i: Order = B,D,C

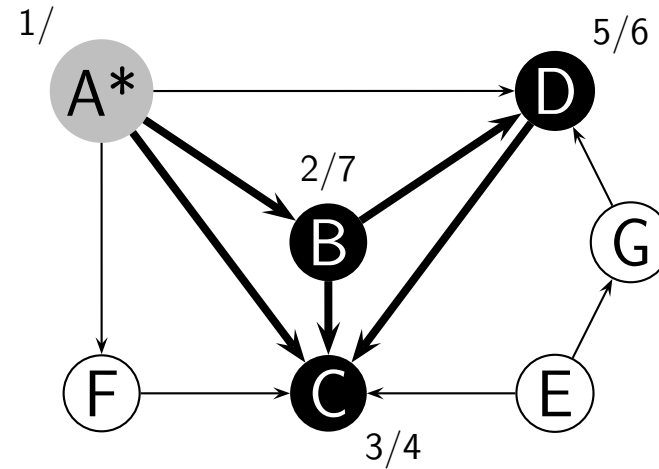


Figure 1j: Look at C, no move

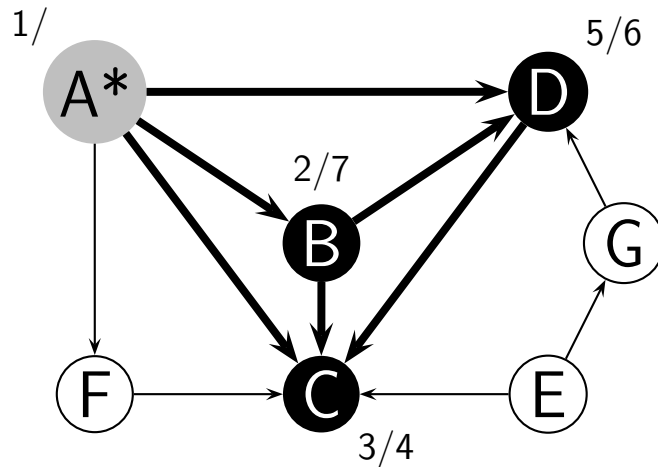


Figure 1k: Look at D, no move

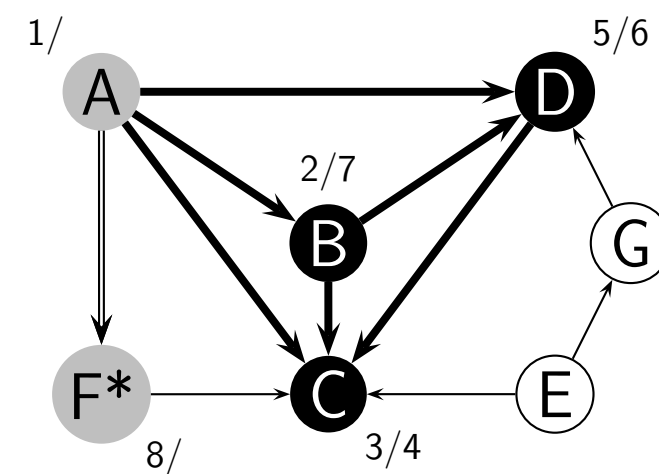


Figure 1l: Move to F

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# Topological Sort Example (4)

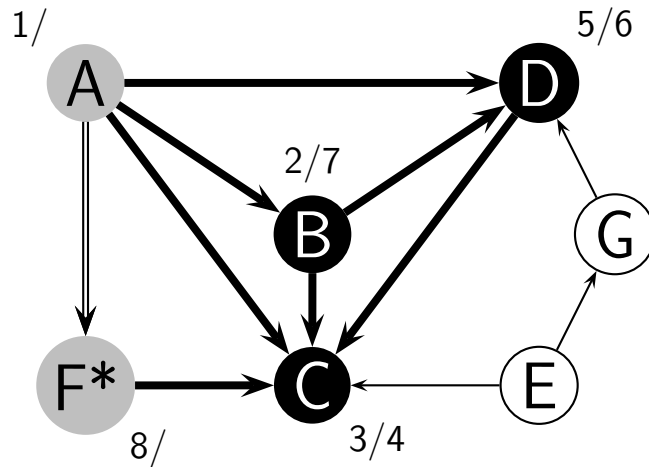


Figure 1m: Look at C, no move

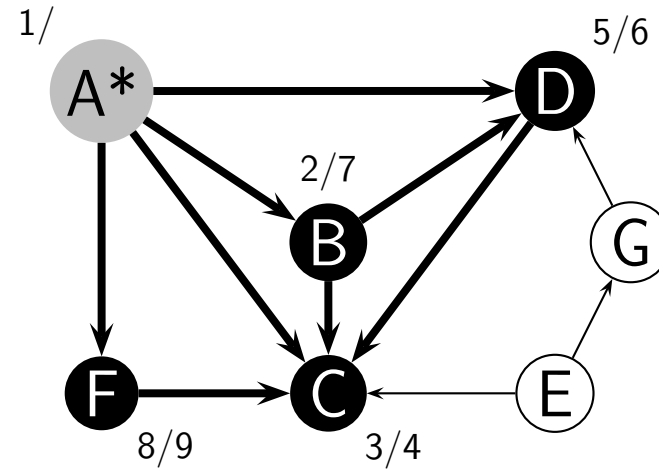


Figure 1n: Order = F,B,D,C

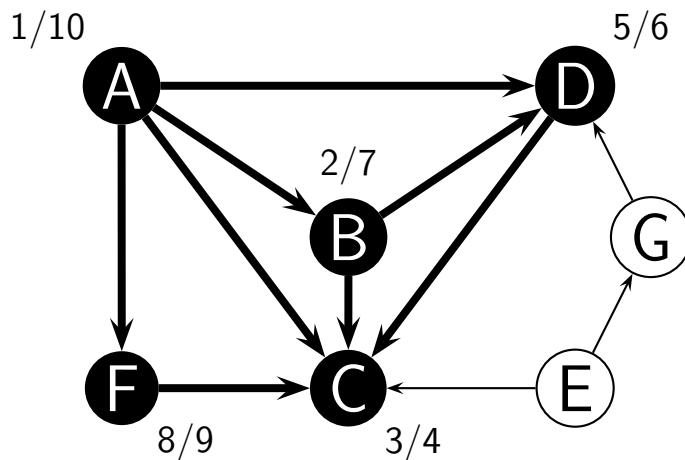


Figure 1o: Order = A,F,B,D,C

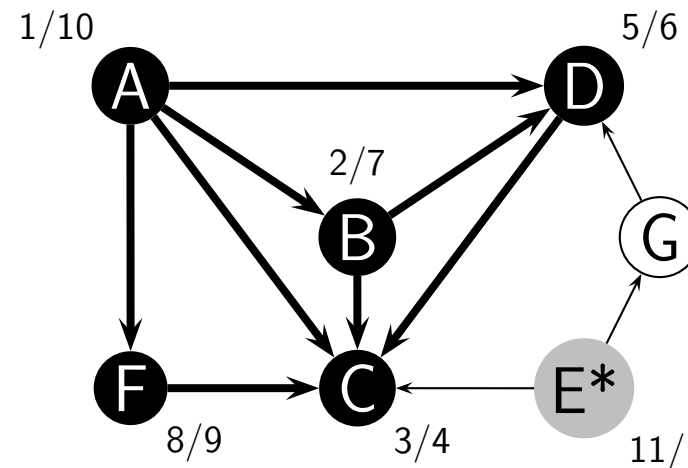


Figure 1p: Start new DFS at E

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# Topological Sort Example (5)

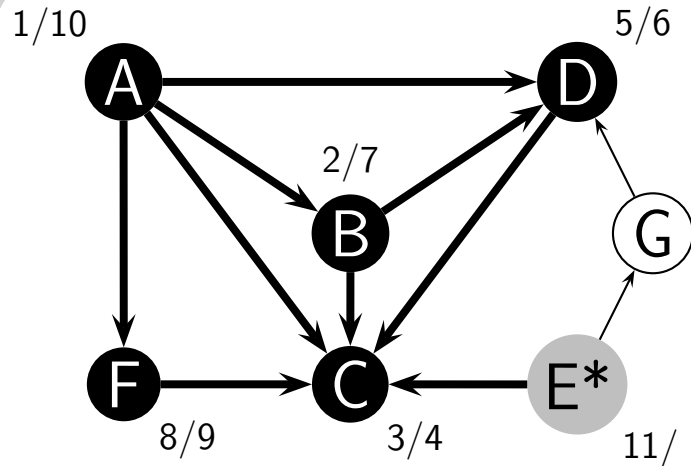


Figure 1q: Look at C, no move

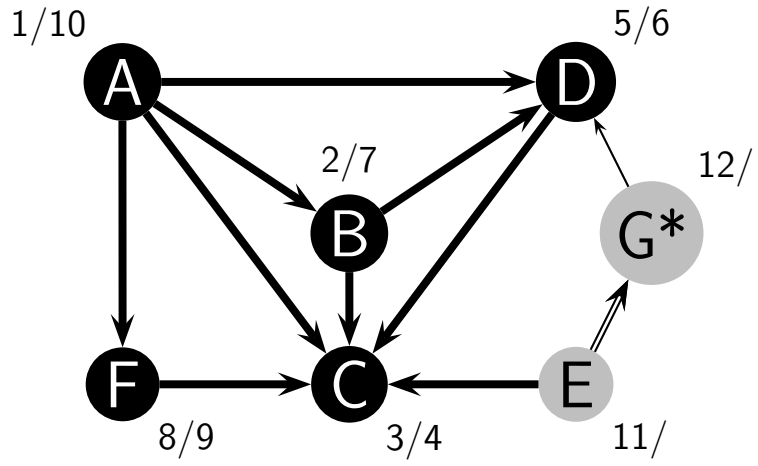


Figure 1r: Move to G

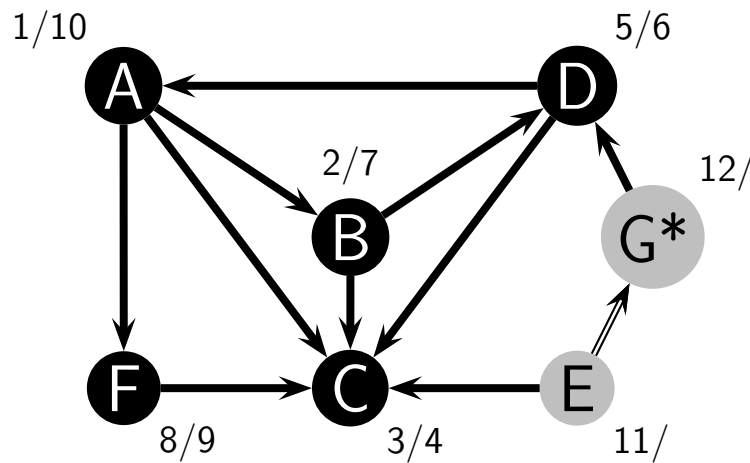


Figure 1s: Look at D, no move

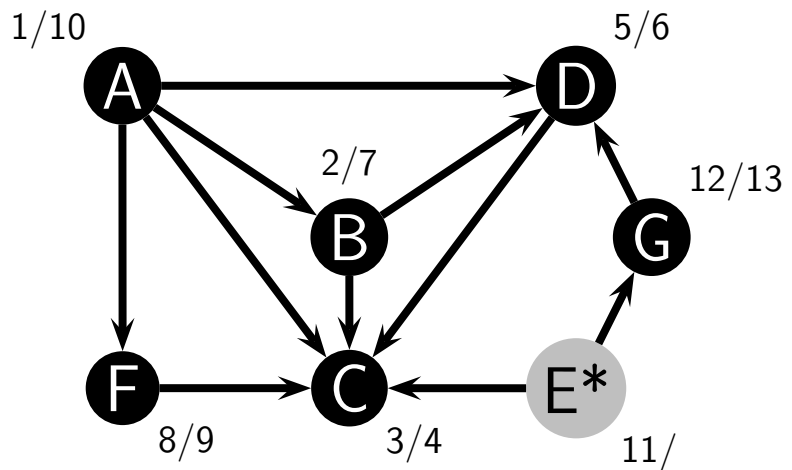


Figure 1t: Order = G, A, F, B, D, C

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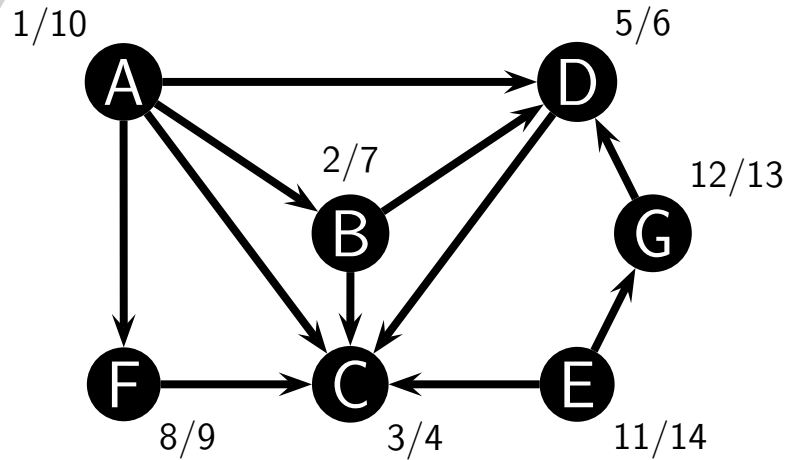


Figure 1u: Order = E,G,A,F,B,D,C