JR Concepcion
Independent Study
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A Comparative Analysis of Portfolio Optimization Techniques: Modern Portfolio Theory, Sharpe Ratio, and Kelly Criterion (Quadratic Utility)

I. Introduction.

In the ever-changing landscape of finance, portfolio optimization and management remain the most important parts of investment strategy. As mathematics and statistics continually play a more crucial role year by year, balancing the goal for high returns with the necessity of managing risk only continues to evolve. Over the past several decades, various quantitative strategies have emerged, each offering unique approaches to portfolio management. Among the most recognizable are Modern Portfolio Theory (MPT), the Sharpe optimization technique, and the Kelly Criterion (Quadratic Utility). We've seen numerous reworks of these methods and models such as the Black-Litterman Model, adjustments to the Fama-French Model(s), and a number of others. These strategies, grounded in robust mathematical frameworks discovered over 50 years ago, offer distinct methods of optimizing portfolios based on different risk-return paradigms.

This study aims to compare the performance specifically of these three quantitative strategies over the last five years, using a selection of four widely recognized index ETFs: the Russell 2000, Dow Jones, NASDAQ, and S&P 500. By focusing on these index ETFs, we minimize the external noise often introduced by individual companies or specific industries, leveraging the intrinsic diversification benefits these ETFs provide. However, this choice also introduces potential challenges, particularly the issues of multicollinearity and high correlations among the selected assets, which could influence the effectiveness of the optimization strategies.

The study will retrieve data from 2014 to 2018, reflecting a period that avoids the extreme volatility of the 2008 financial crisis while still encompassing the subsequent market recovery. While 10 years of historical data would be preferred, the volatility component can heavily skew our results. By evaluating the performance of these portfolios from 2019 to 2023, we aim to assess the real-world applicability of these strategies in managing investment portfolios in the modern era of markets. This period, although marked by the COVID-19 pandemic and other market fluctuations, provides a more stable environment for testing the resilience, effectiveness, and reliability of each strategy.

II. Literature Review

A. Modern Portfolio Theory (MPT)

Modern Portfolio Theory, introduced by Harry Markowitz in 1952, forever changed the way investors approached portfolio construction. At its core, MPT operates on the principle that a portfolio’s risk can be reduced, and potentially its returns increased by combining assets that have low correlations with one another. The theory introduces the concept of the Efficient Frontier, a set of optimal portfolios that offer the highest expected return for a given level of risk. The concept of the Efficient Frontier is still used heavily today by financial advisors and asset managers, as we are yet to find a more effective replacement.

Mathematical Foundation: MPT relies heavily on mean-variance optimization, where the expected return of a portfolio is calculated as a weighted sum of the expected returns of the individual assets. The variance of the portfolio is calculated using the covariance matrix of asset returns. The goal is to minimize the portfolio’s variance for a given expected return or for a given level of risk. This portfolio is often referred to as a “Tangency Portfolio”.

Critiques and Limitations: Despite how groundbreaking the discovery was for the industry, MPT has faced criticism over the years. One major critique is its reliance on historical data to predict future returns and covariances, which may not always hold true. Additionally, MPT assumes that investors are rational and markets are efficient, assumptions that have been challenged by behavioral finance and empirical evidence. With our analysis, our primary limitation is the high correlation of assets and the similar expected returns amongst the ETFs.

B. Sharpe Ratio Optimization

The Sharpe Ratio, developed by William F. Sharpe, is one of the most widely used metrics for evaluating the performance of an investment compared to a risk-free asset. It is calculated as the difference between the return of the portfolio and the risk-free rate, divided by the portfolio's standard deviation. This attempts to model the portfolio's total risk and is often referred to as a risk-to-reward metric.

Application in Portfolio Optimization: In portfolio optimization, the goal is to maximize the Sharpe Ratio. This approach balances return and risk more directly than MPT, as it normalizes returns by the risk taken. A portfolio with a higher Sharpe Ratio is considered better performing because it implies more return per unit of risk.

Critiques and Limitations: While the Sharpe Ratio is a powerful tool, it assumes that the returns of the portfolio are normally distributed and that investors are risk-averse in a linear manner. In reality, returns can be skewed, exhibit kurtosis, and contain a number of other factors that can heavily skew the metric, especially in Emerging Markets or Small-Caps. Investors also may have different degrees of risk tolerance depending on the context, which the Sharpe Ratio does not account for, such as diversification without extra parameters.

C. Quadratic Utility Portfolio

The Quadratic Utility Portfolio approach is another powerful method for optimizing portfolio allocation, particularly when an investor's preferences toward risk and return are quadratic. This method extends beyond the traditional mean-variance framework of Modern Portfolio Theory by incorporating the investor's risk aversion directly into the optimization process.

Application in Portfolio Optimization: The Quadratic Utility approach in portfolio optimization maximizes an investor's expected utility by balancing potential returns with associated risks. This is the flexibility that the Kelly Criterion often lacks in markets. While the Kelly Criterion focuses on maximizing long-term growth through optimal bet sizing, the Quadratic Utility method is better suited to the nature of financial markets, where investors are not bound by fixed intervals and face an infinite number of influencing factors. This approach allows for a more selective adjustment to varying levels of risk tolerance.

Critiques and Limitations: One limitation of the Quadratic Utility approach is its reliance on a risk aversion parameter, which can be difficult to estimate accurately as markets are always changing. Along with this, the quadratic utility function assumes that investors are only concerned with the mean and variance of returns, which can discount higher moments such as skewness and kurtosis, which might be relevant for some investors. We can see that these are quite similar to both the limitations of MPT and Sharpe.

III. Methodology

A. Data Collection

For this study, four major index ETFs were chosen to represent the U.S. equity market: the Russell 2000, Dow Jones, NASDAQ, and S&P 500. These ETFs were selected due to their broad market coverage, liquidity, and popularity among investors. The data spans from January 1, 2014, to December 31, 2018, providing a granular historical dataset to analyze. The performance of these ETFs from January 1, 2019, to December 31, 2023, will be used to evaluate the effectiveness of the portfolio strategies.

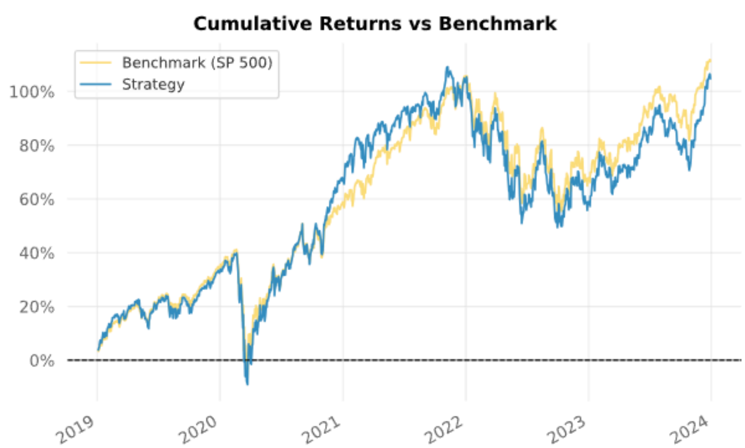
Data Sources: Historical price data for the ETFs will be sourced from Yahoo Finance (yfinance package), which provides all types of returns data, but here we will focus on adjusted close. This data will be used to calculate daily returns, which are then compounded to derive monthly and annual returns for portfolio analysis.

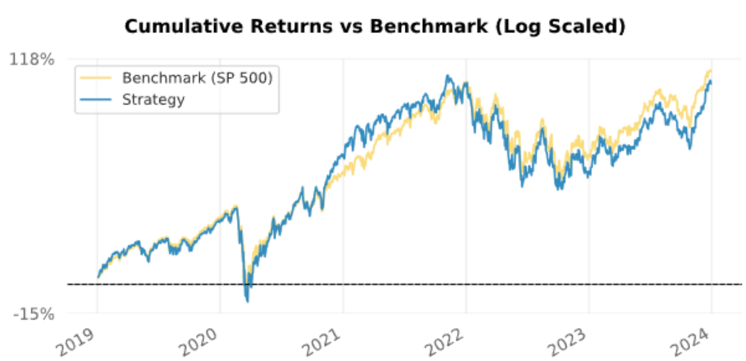
B. Portfolio Construction

1. **Equally Weighted Portfolio:** Each ETF is assigned an equal weight (25%), providing a straightforward benchmark for comparison.
2. **Modern Portfolio Theory (MPT) Portfolio:** Constructed using mean-variance optimization, with a focus on maximizing return for a given level of risk.
3. **Sharpe Ratio Portfolio:** Optimized to achieve the highest possible Sharpe Ratio, balancing return and risk.
4. **Kelly Criterion Portfolio:** Allocated according to the Kelly Criterion, aiming to maximize the long-term growth rate.

IV. Interpretation of Results

1. Modern Portfolio Theory (MPT):





Key Performance Metrics

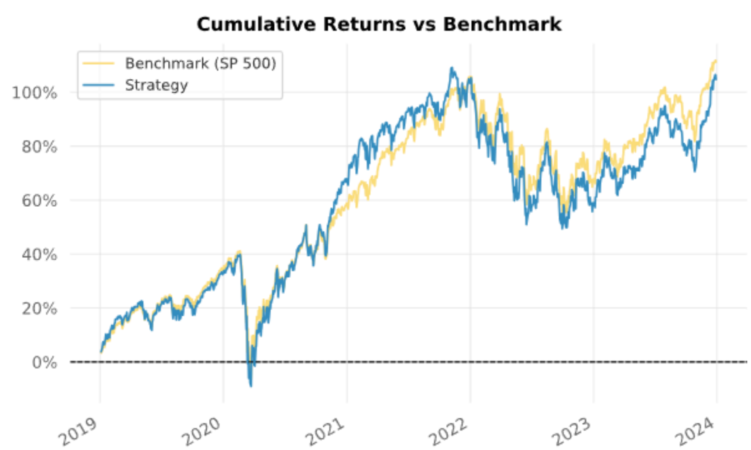
Metric	Benchmark (SP 500)	Strategy
Risk-Free Rate	0.0%	0.0%
Time in Market	100.0%	100.0%
Cumulative Return	111.2%	104.69%
CAGR﹪	10.91%	10.43%
Sharpe	0.82	0.74
Prob. Sharpe Ratio	96.4%	94.77%
Smart Sharpe	0.7	0.63
Sortino	1.15	1.03
Smart Sortino	0.98	0.88
Sortino/√2	0.81	0.73
Smart Sortino/√2	0.7	0.62
Omega	1.15	1.15
Max Drawdown	-33.72%	-35.03%
Longest DD Days	708	781
Volatility (ann.)	20.98%	22.99%
R^2	0.96	0.96
Information Ratio	-0.0	-0.0
Calmar	0.32	0.3
Skew	-0.55	-0.7
Kurtosis	11.61	10.64
Expected Daily	0.06%	0.06%
Expected Monthly	1.25%	1.2%
Expected Yearly	16.13%	15.4%

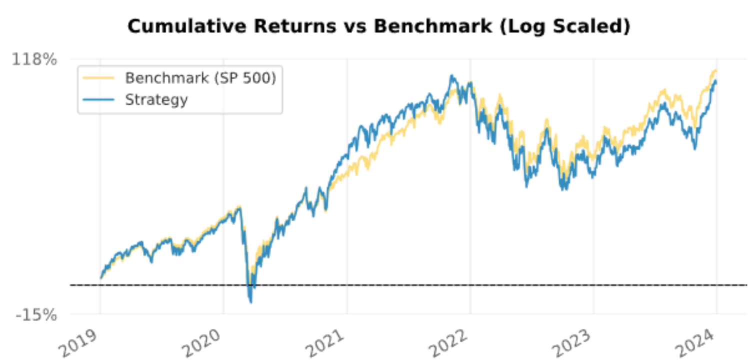
The MPT portfolio delivered a balanced approach by maximizing returns while controlling for risk. However, the results show that the MPT portfolio effectively reduced volatility, but not to the scale of the benchmark. It also did not surpass the benchmark in terms of cumulative return or risk-adjusted performance. The MPT portfolio achieved a cumulative return of 104.69%, slightly below the S&P 500’s 111.2%, and had a Sharpe Ratio of 0.74 compared to the benchmark’s 0.82.

Despite this, the MPT portfolio offered a degree of downside protection, as reflected in its maximum drawdown of -35.03%, close to the S&P 500's -33.72%. However, the strategy’s risk-adjusted metrics, such as the Sortino Ratio (1.03 to the benchmark's 1.15), indicate that it might not have managed downside risk as efficiently.

Overall, while MPT remains a foundational approach in portfolio optimization, this analysis proves its reliability in modern markets. While it did not outperform our benchmarks, it proved to still be a viable tool for asset managers.

2. Sharpe Ratio Optimization:





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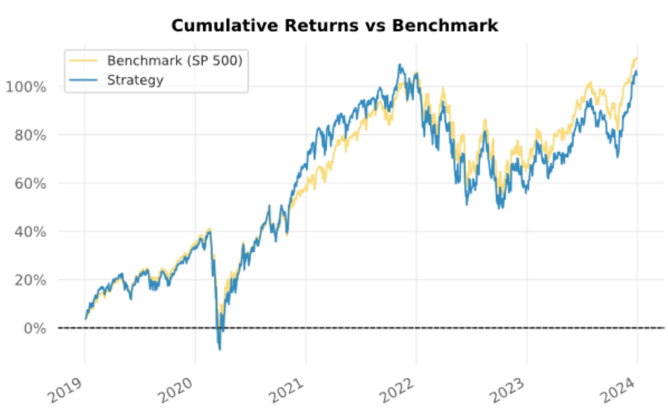
Expected Yearly	16.13%	15.41%

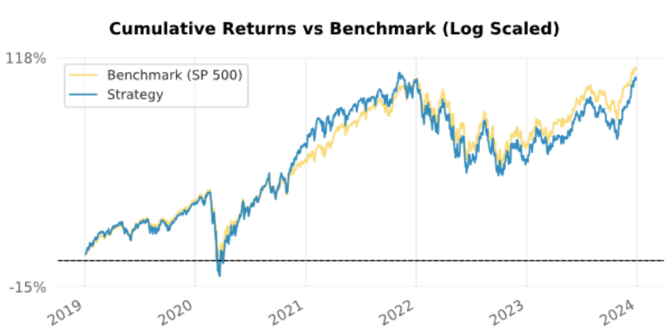
The portfolio optimized for the Sharpe Ratio demonstrated solid performance in terms of risk-adjusted returns, as anticipated. The Sharpe Ratio for this strategy came in at 0.74, slightly lower than the S&P 500's 0.82. This was identical to our previous portfolio. This indicates that while the strategy was effective in managing risk, it did not fully outperform the benchmark in terms of converting risk into return. The strategy's cumulative return was 104.73%, which is close to the S&P 500’s 111.2%, reflecting a competitive performance despite the lower Sharpe Ratio. However, this was slightly higher than the MPT Portfolio.

These results were slightly disappointing as despite optimizing for Sharpe, we still could not seem to beat the S&P’s ratio. This can be directly related to our weight parameter as allowing it to optimize with no parameters left us with a portfolio with a 0.86 Sharpe, distributing the portfolio mainly to the NASDAQ, with some allocation in the S&P.

In conclusion, the Sharpe Ratio portfolio remains a compelling option for investors who prioritize risk management, particularly those who are risk-averse and seek to balance return with volatility. Given our parameters, we lacked the heavy upside returns due to our weight parameters as previously mentioned.

3. Quadratic Utility Portfolio:





Key Performance Metrics

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Expected Daily	0.06%	0.06%
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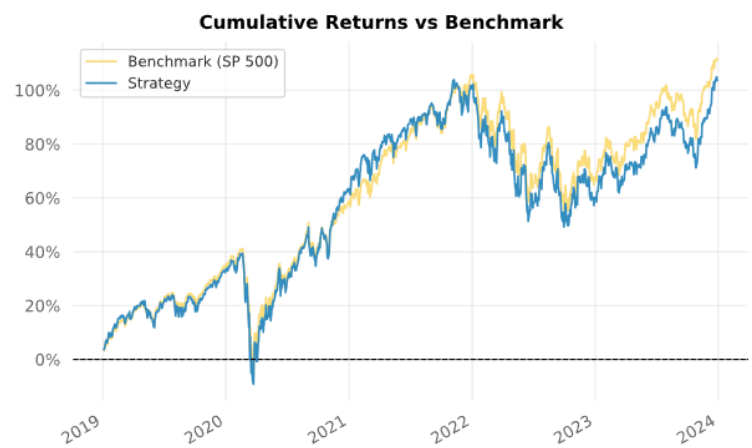
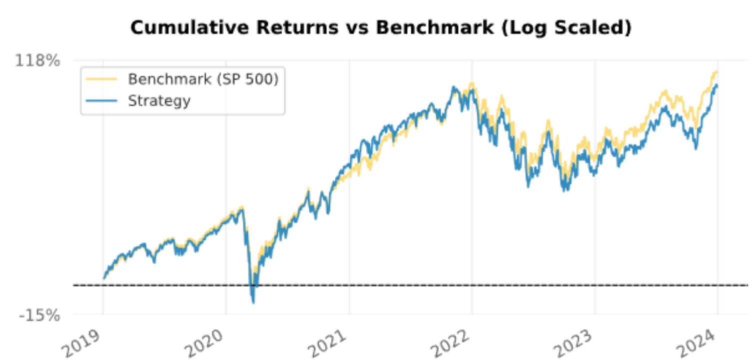
The Quadratic Utility Portfolio, designed to balance expected returns with risk according to an investor's risk aversion, performed quite well over the analysis period. The results show that this portfolio achieved a cumulative return of 104.73%, which is slightly lower than the S&P 500's 111.2%. As you may notice, this return is identical to the return of the Sharpe Portfolio (due to them being weighted equally). The Sharpe ratio of 0.74 is somewhat lower than the benchmark's 0.82, indicating a slightly less efficient conversion of risk into return as we've previously seen.

The Quadratic Utility Portfolio's ability to maintain a balance between return and risk is further supported by its volatility, which was 22.99% annually, slightly higher than the S&P 500's 20.98%. This indicates that while the portfolio took on more risk, it did so in a controlled manner, aligning with the investor's risk tolerance parameters. With a different pool of assets, specifically individual equities, we'd be able to truly test the ability of the technique.

In conclusion, the Quadratic Utility Portfolio proved to be a well-performing strategy, compared to our S&P benchmark. It is also very unique that all three portfolios contain an identical Sharpe, yet can be justified by our lack of diversification and parameters.

4. Benchmark Portfolio (Equally Weighted):





Key Performance Metrics

Metric	Benchmark (SP 500)	Strategy
Risk-Free Rate	0.0%	0.0%
Time in Market	100.0%	100.0%
Cumulative Return	111.2%	103.55%
CAGR﹪	10.91%	10.34%
Sharpe	0.82	0.75
Prob. Sharpe Ratio	96.4%	95.01%
Smart Sharpe	0.7	0.64
Sortino	1.15	1.05
Smart Sortino	0.97	0.89
Sortino/√2	0.81	0.74
Smart Sortino/√2	0.69	0.63
Omega	1.15	1.15
Max Drawdown	-33.72%	-34.92%
Longest DD Days	708	774
Volatility (ann.)	20.98%	22.32%
R^2	0.98	0.98
Information Ratio	-0.01	-0.01
Calmar	0.32	0.3
Skew	-0.55	-0.67
Kurtosis	11.61	11.87
Expected Daily	0.06%	0.06%
Expected Monthly	1.25%	1.19%
Expected Yearly	16.13%	15.27%

The equally weighted benchmark portfolio had a solid performance over the five-year period, closely mirroring the S&P 500 in terms of cumulative returns. The benchmark achieved a cumulative return of 103.55%, slightly lower than the S&P 500's 111.2%. This can be due to several factors, but can certainly be pinned to the fact of holding a less optimal asset equally to more optimal assets.

In terms of risk-adjusted metrics, the benchmark portfolio had a Sharpe ratio of 0.75, which is somewhat lower than the S&P 500's 0.82. This indicates that while the equally weighted portfolio offered solid returns, it did so with slightly less efficiency in terms of risk. Additionally, the portfolio's maximum drawdown of -34.92% was slightly higher than the S&P 500's -33.72%, reflecting a slightly higher exposure to market downturns. This was our top performer in all statistics listed within our test portfolios.

Despite these slight underperformances to the benchmark, the equally weighted portfolio demonstrated lower skewness (-0.67 compared to the S&P 500’s -0.52), implying it might have experienced fewer extreme negative returns. The Sortino ratio, which focuses on downside risk, was also very close to that of the S&P 500, further underscoring the portfolio’s resilience. This was one of the most unique findings of this portfolio as this difference is quite stark and could have an impact over the long term.

Overall, while the benchmark portfolio did not outperform the S&P 500 in terms of raw returns or risk-adjusted metrics, its performance was very competitive. The results suggest that for investors seeking a straightforward, diversified investment approach without the need for complex optimization, an equally weighted portfolio is a viable option.

B. Potential Limitations

1. Dependence on Historical Data:
All the optimization strategies employed in this study rely on historical data to estimate expected returns, volatilities, and covariances. However, as the saying goes, past performance is not always indicative of future results. As stated in the initial document, historical data may not capture the evolving dynamics of financial markets, with examples like large market corrections and or crashes. This limitation is particularly relevant for MPT, which heavily depends on the accuracy of the covariance matrix.

2. Sensitivity to Input Parameters:
Both the Sharpe Ratio and Quadratic Utility optimizations are sensitive to the input parameters, such as expected returns, volatilities, and the risk-free rate. Small changes in these parameters can significantly impact the resulting portfolio allocations, potentially leading to different outcomes in practice. In our study, we normalized the risk-free rate to 0% so it would not be a

factor. Considering risk-free rates were significantly higher when the methods were designed, focusing primarily on the market factors was the approach.

3. Exclusion of Transaction Costs and Taxes:
The study does not account for transaction costs, taxes, or other frictions that could affect the portfolio's performance in a real-world setting. Frequent rebalancing, as simulated in the study, could add significant costs depending on portfolio size. This can drastically weigh on returns which can erode the returns of the optimized portfolios.

4. Optimization Weight Parameters:
The parameters that were set and used for weighting our portfolios (5% minimum, 35% maximum) introduce their limitations. These constraints were chosen to ensure diversification and to prevent any single asset from overweighting the portfolio. However, they may also limit the ability of the optimization algorithm to fully capitalize on the strengths of high-performing assets, especially during periods of strong market trends. By capping the maximum weight at 35%, the portfolios may not reflect the true optimal allocation. This could lead to underperformance compared to a fully unconstrained optimization with the best-expected return and risk. The minimum weight constraint of 5% ensures exposure to each asset, which also may prevent the portfolio from completely excluding assets. While these constraints are practical in application, they do introduce a trade-off between diversification and potential returns.

VI. Conclusion

A. Summary of Findings

The study results satisfy a large portion of our initial curiosity, while also helping generate even more questions. The top-performing portfolio within the study turned out to be the equal-weighted portfolio, which was the only non-optimized portfolio. We also saw extremely poor performance within the reweighted portfolios, leaving them to be fully excluded from the study to provide more focus on these portfolios. Performance amongst the optimized portfolios was nearly identical, or within a 1% to 3% margin of one another.

The final results indicate that all portfolio optimization methods perform relatively the same with the basket of assets. The results do not indicate a clear winner but do show that they are still relevant in modern markets. Intuitively the approach of taking a return and dividing it by its volatility (standard deviation or variance) would make sense, but what makes it truly amazing is the period in which these strategies were discovered, and can still be applicable.

Weighting constraints within each portfolio simulation also played a large factor in our study, as we saw two different methods both became weighted equally to one another, along with the majority of portfolios having assets containing both the minimum and maximum values. Testing was also done with no constraints leading to a wild array of results, but seemingly greater Sharpe's and returns. Allowing these algorithms to run without constraints comes with benefits and drawdowns, but it is not a reliable strategy when considering any variation of practical application.

B. Future Research

1. Testing Additional Strategies:
Incorporating other optimization methods, such as the Black-Litterman model which better incorporates future returns, or risk-parity methods may allow for better and more diversified results. These strategies also differ in mathematical approach, potentially leading to significantly more unique results.

2. Expanding the Asset Selection:
Expanding the study to include different asset classes, such as individual equities, emerging markets, or fixed-income assets could enhance the understanding of how these strategies perform across a more diverse set of investments. It would also test the common optimization limitations as there it adds a variety of other uncorrelated factors. These factors may be a better stress test for the relevance of these optimization methods, as they have proven their relevance within the US markets.

3. Incorporating Real-World Constraints:
Future studies could account for transaction costs, taxes, and liquidity constraints to provide a more realistic assessment of each strategy's performance in practice. These costs can add up to be quite significant depending on portfolio size and the quantities of tradings occurring within the portfolio

4. Market Cycle Analysis:
Extending the evaluation period to include different market cycles, including both bull and bear markets, would provide a more comprehensive view of each strategy's long-term effectiveness. As computational abilities continue to develop, the possibilities of this grow.

Tools and Sources:

-Python:

Pandas: Used for data manipulation and analysis, particularly in handling time series data, calculating returns, and managing large datasets.

Numpy: Essential for numerical computations, including portfolio weight adjustments, covariance calculations, and optimization tasks.

YFinance: A Python library that simplifies the process of downloading historical market data, such as prices and returns for the selected ETFs.

PyPortfolioOpt: A library that provides tools for portfolio optimization, including mean-variance optimization (MPT), Sharpe Ratio maximization, and other advanced portfolio construction methods.

QuantStats: A Python library that offers detailed performance reports for portfolios, including various risk and return metrics, and comparisons against benchmarks like the S&P 500.

Empyrical: Utilized to calculate performance metrics and perform other statistical analyses related to finance and investments.

Google Colab:

Jupyter Notebooks provided by Colab were a versatile tool for conducting exploratory data analysis, running simulations, and documenting the steps taken during the portfolio optimization process. Using Google's Environment over my own allowed for faster computing times, especially with large optimizations.

Resources:

Quantitative Asset Management: Michael Robbins