How do we tell if a crease pattern can be folded?

Can we tell if a given crease pattern can be folded flat? Or more generally, if it can be folded at all? There are two important conditions that let us know when a crease pattern can be folded flat: Kawasaki's and Maekawa's theorems.

Note that these are necessary but not sufficient conditions for flat foldability; there are some crease patterns that satisfy these conditions but still cannot fold flat.

Map Foldability is a hard problem.

Unfortunately, studying maps is still difficult. It takes a computer a long time to figure out if a map crease pattern can be folded flat – in fact, the best way mathematicians know of is to simply try every possible way of folding the map and seeing if any way works.

Try it yourself!

Here are two map crease patterns. One of them can be folded flat, and the other can't. On the table to the left, you can try to find out which one can be folded. Remember that red lines are mountian folds and must point up; and blue lines are valley folds, and point down.

The Napkin Folding Problem

Can you fold a piece of paper (or a napkin) into a shape that is has a larger perimeter (distance around the outside) than the original square? Go ahead and to to do it at the table to the left. It seems impossible!

Answer

It might be surprising, but it turns out you can. Take a look at the model to the left—it has slightly greater perimeter than the starting square.

What is a Crease Pattern?

Crease patterns are the most important tool for studying origami. The lines specify where to fold the paper and the color indicates the direction of the fold, as shown to the right. We call the intersection of multiple creases at one point a *vertex*.