

Angle Trisection with Origami

It is indeed possible to trisect any angle with folds! Here are the step by step instructions for doing so, making use of the axioms of origami pictured above. This technique was found in the 1970s by Hisashi Abe. What do you think makes origami more powerful than compass and straightedge?

Types of Foldability

Sometimes, a model can't be folded flat, but it can be folded in a 3D form. This flower is an example – you can see that if you tried to press it in a book, it would get crushed. Unfortunately, it's much more difficult to study 3D foldability. It isn't clear how to tell from the crease pattern that the flower won't fold flat.



Kawasaki's Theorem

A single vertex can be folded flat if and only if the sum of every other angle around the vertex is 180 degrees.



Maekawa's Theorem

A single vertex can be folded flat if and only if the difference between the number of mountain folds and the number of valley folds is plus or minus 2.

Some crease patterns are not foldable.

Here's an example of a crease pattern that can't be folded flat at all. In fact, it can't even be folded in a 3D form without curving the paper (unlike the flower above).

Notice that this crease pattern satisfies Maekawa's and Kawasaki's theorems--the fact that it still can't be folded flat demonstrates why these theorems are not sufficient.