

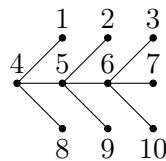
Bioinformatics

Discrete Mathematics and Optimisation

Problem Sheet Graphs and Networks I

1. Trees and Prüfer codes.

- (a) Compute the Prüfer code of the following tree.



- (b) Draw the tree with Prüfer code $(3, 1, 1, 2, 2, 2, 3, 3)$.

2. A leaf of a tree is a vertex of degree one.

- (a) Show that every tree T on $n \geq 2$ vertices has at least two leaves.
(b) If a tree T has k leaves and has no vertices of degree 2, what is the maximum number of vertices of T ?
(c) By using Prüfer codes, determine how many (labelled) trees there are with n vertices and $n - 2$ leaves. Find all such trees with $n = 5$ vertices.

3. Find the diameter of the following graphs. Justify your answers.

- (a) Complete graph K_n .
(b) Complete bipartite graph $K_{n,m}$.
(c) Path P_n .
(d) Cycle C_n .

4. The n -cube Q_n is a graph which has as vertex set all binary words of length n , and the edges are pairs of words which differ in precisely one entry.

- (a) How many vertices and edges does Q_n have?
(b) What is the diameter of Q_n ?
(c) Show by induction on n that Q_n has a cycle of length 2^n for all $n \geq 2$.

5. Let m be a positive integer and G_m be the graph whose vertices are the binary words $x_1x_2\cdots x_{2m+1}$, $x_i \in \{0,1\}$ with m or $m+1$ ones, and two words are adjacent in G_m if they differ in precisely one entry.
- (a) Draw G_1 , i.e. G_m when $m = 1$.

Let $m \geq 1$. The next questions concern G_m in general.

- (b) What is the degree of a vertex in G_m ?
- (c) How many edges are in G_m ?
- (d) Is G_m bipartite?
- (e) Is G_m connected? What is its diameter?
6. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.
7. A forest is an acyclic graph, that is, each connected component of a forest is a tree.
- (a) How many edges does a forest with n vertices and k connected components have?
- (b) How many (labelled) forests with two connected components are there with $n = 6$ vertices?
8. Let G be a graph with n vertices and m edges.
- (a) How many induced subgraphs does G have?
- (b) How many spanning subgraphs does G have?