

Homework 2

Graphs and networks I

⑤ $m \geq 0$

$$G_m : \text{len } 2m+1$$

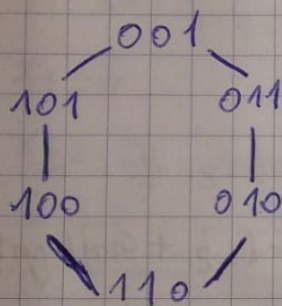
$$\{0,1\} \rightarrow \text{num ones} = m \text{ or } m+1$$

adjacent if different by 1 bit

a) $m=1$

$$\text{len } G_m = 2 \cdot 1 + 1 = 3$$

can have 1 or 2 ones $\{001, 011, 101, 110, 100, 101\}$



b) $m \geq 1$

for X_1 , with $m \geq 1$, has vertices $\text{length}(G_1) - 1$
 $\hookrightarrow m$, min number of 1s

$\rightarrow 2m+1 - m = m+1 \rightarrow$ number of zeros that can be flipped to obtain adjacent vertex \rightarrow degree of a vertex $= m+1$

c) $2 E(G_m) = \sum d(\text{vert}) \rightarrow E(G_m) = \frac{\text{num vectors} \cdot (m+1)}{2}$

num vectors = permutations of m in length, which is equal to those of $m+1$

in length (reversed starting positions, so we will multiply by 2)


$$V(G_m) = \binom{2m+1}{m} = \binom{2m+1}{m+1} \rightarrow V(G_m) = 2 \binom{2m+1}{m}$$

$$E(G_m) = \frac{2 \binom{2m+1}{m} (m+1)}{2} = \binom{2m+1}{m} (m+1)$$

d) Yes, because we can separate it in 2 subgraphs where each vertex is 1 vertex from the other subgraph away from another vertex of its own subgraph

⑤ e) its connected, as if we flip $0 \rightarrow 1$ in anythm of m or n 1 per flip we will construct a path from 0 to 1 in G_m , as all 1 's numbers on the graph will be created through this flips the graph will be connected

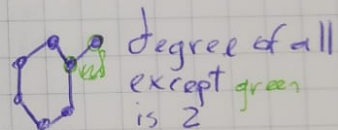
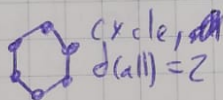
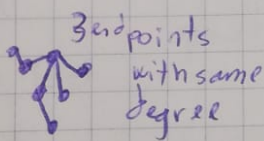
The diameter is equal to the path length, which is $2m+1$

⑥ $2 \leq \text{vertices}$  not counting loops or multiple edges

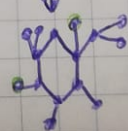
at least 2 vertices have the same degree

Because all graphs have endpoints, and there are at least 2, the exception is if the graph is cyclic, but in that

case at least 2 vertices of the cycle will have the same degree



↓ more than 1 vertex of the cycle with added vertices



there is more than one endpoint, which have degree 1 so at least 2 vertices have the same degree