

Bioinformatics

Discrete Mathematics and Optimisation

Problem Sheet Counting and Enumeration (Programming)

1. Recall exercise 6 from the first problem sheet. Write a **python** script that determines in how many ways some given word can be rearranged.
2. Write a **python** script that can both return you a specific Fibonacci number F_n as well as a list of all Fibonacci numbers smaller than some given m .
3. The *Catalan numbers*¹ C_n satisfy the non-linear recurrence equation

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

where $C_0 = 1$. One can prove (through various means) that

$$C_n = \binom{2n}{n} - \binom{2n}{n+1}.$$

Write a **python** script that determines the Catalan number for a given value of n . The script should contain three methods having the same output:

- (a) The first function should directly calculate C_n using the closed formula stated above.
- (b) The second function should determine C_n iteratively through a **for**-loop.
- (c) The third function should determine C_n through recursive function calls.

Compare the run-time of the three approaches for different (small and large) values of n using the module **timeit**.

¹Named after the 19th century Belgian mathematician Eugène Charles Catalan.