Homework 5

Problem sheet extrema with several variables

⑥ $f(x,y) = x^3 - 12xy + 8y^3$

a) $\nabla f = \left(\frac{df}{dx}, \frac{df}{dy}\right) = (3x^2 - 12y, -12x + 24y^2)$

$\nabla f = (0,0) \rightarrow \frac{df}{dx} = 0 \quad \frac{df}{dy} = 0 \rightarrow 3x^2 - 12y = 0, \; -12x + 24y^2 = 0$

$3x^2 - 12y = 0 \rightarrow x^2 = \frac{12y}{3} \rightarrow x^2 = 4y \rightarrow x = 0 \rightarrow y = 0 \; \hat{} \; (x,y) = (0,0)$

$-12x + 24y^2 = 0 \rightarrow \frac{24y^2}{12} = x \rightarrow 2y^2 = x \rightarrow y = 1 \rightarrow x = 2, \; (x,y) = (2,1)$

Critical points $(0,0)(2,1)$

b) $\frac{df''}{dx^2} = 6x$

$\frac{df''}{dxy} = -12 \qquad \frac{df''}{dyx} = -12$

$\frac{df''}{dy^2} = 48y$

Hessian matrix

$\begin{pmatrix} \frac{df''}{dx^2} & \frac{df''}{dxy} \\ \frac{df''}{dyx} & \frac{df''}{dy^2} \end{pmatrix} \rightarrow He = \begin{pmatrix} 6x & -12 \\ -12 & 48y \end{pmatrix}$

For critical point $(0,0) \rightarrow \begin{pmatrix} 0 & -12 \\ -12 & 0 \end{pmatrix} = 0 - (-12 \cdot -12) = -144 < 0 \rightarrow$ Saddle point

For critical point $(2,1) \; \begin{pmatrix} 6 \cdot 2 & -12 \\ -12 & 48 \cdot 1 \end{pmatrix} = \begin{pmatrix} 12 & -12 \\ -12 & 48 \end{pmatrix} = 576 - (-12 \cdot -12) =$

$= 576 - 144 = 432 > 0 \rightarrow$ Minima $\rightarrow$ Relative minima

Critical point $(0,0)$ is a saddlepoint, critical point $(2,1)$ is a relative minima