Bioinformatics

Discrete Mathematics and Optimization. Final Exam (Retake): 13.12.2023

Write problems in separate sheets.

Problem 1 (Combinatorics and counting, 3.5 pt)**.** Answer the following questions. Each question can be answered independently of the rest.

a) (1 pt) Count the number of ways to create a committee with a president, a secretary and a vocal from a group of 7 people, 4 of them men and 3 of them women, with the restriction that not all the members can be of the same sex.

b) (1.5 pt) Count the number of trees on n vertices with exactly $n-2$ leaves. (Hint: How is their Prüfer code?)

c) (1 pt) Prove by induction that for each integer n, $n(n+1)(n+2)$ is multiple of 6.

Problem 2 (Recurrences, 2.5 pt)**.** Answer the following questions. Each question can be answered independently of the rest.

a) (1.25 pt) Find a recurrence relation for the number of words over the alphabet $\{A, B, C, D\}$ without two consecutive equal letters.

b) (1.25 pt) Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$, $a_0 = 5, a_1 = 12$.

Problem 3 (Graphs, 4 pt)**.** Answer the following questions. Each question can be answered independently of the rest.

a) (1 pt) Among a group of 5 people, is it possible for everyone to be friends with exactly 2 of the people in the group? What about 3 of the people in the group? We consider that friendship is symmetric.

b) (1 pt) Find and draw the tree whose Prüfer code is $(1, 2, 3, 2, 1)$.

c) (1 pt) An spanning tree of G is a subgraph of G which is a tree. How many spanning trees does the complete graph K_n have? and the cycle of length n, C_n?

d) (1 pt) Explain Prim's algorithm and apply it to find the minimum spanning tree of the following graph starting at vertex A: