Bioinformatics

Discrete Mathematics and Optimisation

Problem Sheet Iterative Methods

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1. Let $f(x) = x^2 + \sqrt{x} - 15$. Find an approximation of a zero of $f(x)$ using the Newton method with three iterations:
   (a) starting at $x_0 = 4$,
   (b) starting at $x_0 = 1$.

2. Let $f(x, y) = x^2 - 4xy + 5y^2 - \ln(xy)$.
   (a) Show that $f(x, y)$ is strictly convex in $C = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$.
   (b) Perform the first two iterations of the Newton method on $f(x, y)$ starting at $(1, 1)$.

3. Let $f(x, y) = x^2 + e^{y^2}$ and $g(x, y) = x^4 + 2y^2$.
   (a) Argue that f has a unique global minimum in $\mathbb{R}^2$. Perform the first iteration of the Newton method on f with initial point $(1, 1)$.
   (b) Argue that g has a unique global minimum in $\mathbb{R}^2$. Perform the first iteration of the steepest descent method on g with initial point $(1, 2)$.

4. Let $f(x, y) = 2x^2 + y^2 - xy + 2x + y + 4$.
   (a) Write $f(x, y)$ as a quadratic function
   $$f(x, y) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + c.$$
   (b) Explain why $f(x, y)$ has a global minimum, and find it by the Newton method with initial point $(-1, -1)$.
   (c) Find the first iteration of the steepest descent method with initial point $(-1, -1)$.

5. Let $f(x, y) = (x^2 + y^2) + \big((x - 4)^2 + (y - 2)^2\big) + \big((x - 1)^2 + (y - 4)^2\big)$, which gives the sum of the squares of the distances from a point (x, y) in the plane to the points $\mathbf{a} = (0, 0)$, $\mathbf{b} = (4, 2)$ and $\mathbf{c} = (1, 4)$.
   (a) Show that $f(x, y)$ is a strictly convex function in $\mathbb{R}^2$.
   (b) Find the point $\mathbf{d} = (d_1, d_2)$ which minimizes the sum of the squares of the distances to $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.
   (c) Starting at the point $(1, 1)$, find the point of the first iteration of the steepest descent method applied to the function $f(x, y)$.
   (d) Consider the function $g(x, y) = f(x, y) + x^3y^3$. Starting at the point $(1, 1)$, find the point of the first iteration of the Newton method applied to the function $g(x, y)$.