Bioinformatics

Discrete Mathematics and Optimisation

Problem Sheet Linear Programming

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1. Consider the linear program given by

$$\begin{aligned}
\text{maximise} \quad & 4x_1 + 3x_2 + 2x_3 \\
\text{subject to} \quad & x_1 + 2x_2 + 3x_3 \leq 6 \\
& 2x_1 + x_2 + x_3 \leq 3 \\
& x_1 + x_2 + x_3 \leq 2 \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}$$

Write it in equational standard form, find a basic feasible solution and apply the simplex method in order to find an optimal solution.

2. (a) Consider the linear program given by

$$\begin{aligned}
\text{maximise} \quad & 3x + 4y \\
\text{subject to} \quad & 3x + 2y \leq 12 \\
& 5x + 10y \leq 30 \\
& 2y \leq 5 \\
& x, y \geq 0.
\end{aligned}$$

Determine the feasible region and solve the problem graphically. Do the same for the new objective function $6x + 12y$.

(b) Rewrite both linear programs in standard equational form. Solve the first one using the simplex method, starting at the basic feasible solution suggested by the slack variables. Solve the second one using the simplex method, starting at the solution of the first.

3. Can you determine an optimal solution to the following linear program?

$$\begin{aligned}
\text{maximise} \quad & 11x + y \\
\text{subject to} \quad & 3x - 2y \leq 12 \\
& -5x \geq -31 \\
& x, y \geq 0.
\end{aligned}$$

4. Write each of the following two linear programs in the standard equational form with maximisation:

$$\begin{aligned}
(a) \text{ minimise} \quad & 2x_1 + 2x_2 - 4x_3 \\
\text{subject to} \quad & 2x_1 + 2x_2 + 2x_3 = 10 \\
& -2x_1 + 6x_2 - x_3 \leq -10 \\
& -x_1 + 3x_2 \geq 3 \\
& x_1 \leq 0 \text{ and } x_2, x_3 \geq 0
\end{aligned}$$

$$\begin{aligned}
(b) \text{ maximise} \quad & 3x_1 - 7x_2 + 5x_3 \\
\text{subject to} \quad & x_2 - x_3 \leq -9 \\
& -x_1 - 2x_3 \geq 5 \\
& 4x_1 - x_2 = 6 \\
& x_1 \leq 0 \text{ and } x_2 \geq 0.
\end{aligned}$$

5. Consider the linear program given by

$$\begin{aligned} \text{maximize} \quad & x_1 + 3x_2 + 2x_3 \\ \text{subject to} \quad & x_1 + 3x_2 + 3x_3 \leq 6 \\ & 2x_1 + x_2 + 2x_3 \leq 2 \\ & x_1 + 2x_2 + x_3 \leq 3 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) Write it in equational standard form and find a basic feasible solution.

(b) Apply the simplex method in order to find an optimal solution.

6. A cargo plane has three compartments for storing cargo: front, centre, rear. They have the following limits on both weight and space:

Compartment	Weight capacity (tonnes)	Space capacity (m^3)
Front	10	6800
Centre	16	8700
Rear	8	5300

The weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain balance of the plane. The following four cargos are available for shipment in the next flight:

Cargo	Weight (tonnes)	Volume (m^3/tonne)	Profit (€/tonne)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

Any proportion of these cargos is accepted. The objective is to determine how much of each cargo C1, C2, C3 and C4 should be accepted and how to distribute them among the compartments so that the total profit for the flight is maximized.

(a) Formulate the above problem as a linear program.

(b) What assumptions are made in formulating this problem as a linear program?

7. Due to a recent increase of the population in the area, a local hospital needs to build more recovery rooms to keep functioning decently. On average, different numbers of recovery rooms are required on different days of the week, as specified in the following table:

Day	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Demand	17	13	15	19	14	16	11

Sanitary rules state that if used, each recovery room must function five consecutive days and then be left unused for two consecutive days. The room management service of the hospital wants to devise an efficient schedule for the use of the recovery rooms.

Formulate a linear program to minimise the total number of recovery rooms.

8. We must run a program P and we have two available machines M_1 and M_2. Each execution of P spends $4s$ on M_1 and $1s$ on M_2. The cost per execution on M_1 is 1 cents and in M_2 is 4 cent. The energy consumption per execution on M_1 is $3mW$, while on M_2 it is $1mW$. We must run at least 10 executions in total, but we can not exceed $37s$ of running time nor 28 cents of cost. We want to minimize the energy consumption.

(a) Write a linear program to solve the problem.

(b) Draw the feasible region of the problem and identify a solution graphically.

(c) Write the linear program in equational form and run the simplex method on it.