Homework 2
Graphs and networks I

(5) $m \geq 0$

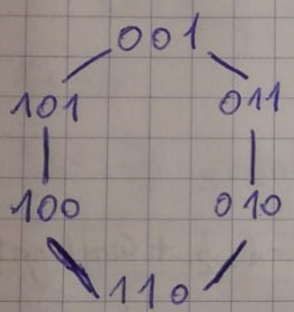
G_m : len $2m+1$

$\{0,1\}$ $\rightarrow$ num ones $= m$ or $m+1$

adjacent if different by 1 bit

a) $m=1$

len $G_m = 2\cdot1+1=3$
can have 1 or 2 ones $\Big\}$ $\{001, 011, 010, 110, 100, 101\}$



b) $m \geq 1$

for X_1, with $m \geq 1$, has vertices length$(G_1) - 1$ $\rightarrow$
↳ m, min number of 1s

$\rightarrow 2m+1-m = m+1 \rightarrow$ number of zeros that can be flipped to obtain adjacent vertex $\rightarrow$ degree of a vertex $= m+1$

c) $2E(G_m) = \sum d(\text{vect}) \rightarrow E(G_m) = \dfrac{\text{numvectors}\cdot(m+1)}{2}$

num vectors = permutations of m in length, which is equal to those of $m+1$ in length (reversed starting positions, so we will multiply by 2)

$V(G_m) = \binom{2m+1}{m} = \binom{2m+1}{m+1} \rightarrow V(G_m) = 2\binom{2m+1}{m}$

$E(G_m) = \dfrac{2\binom{2m+1}{m}(m+1)}{2} = \binom{2m+1}{m}(m+1)$

d) Yes, because we can separate it in 2 subgraphs where each vertex is 1 vertex from the other subgraph away from another vertex of its own subgraph

Homework 1

Counting

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$\{ATCG\} \to 4$

length n

no AA

$a_n = ?$

$a_0 = 1$

$a_1 = A, T, C$ or $G = 4$

$a_2 =$

/	TA	CA	GA	$= 15$
AT	TT	CT	GT	
AC	TC	CC	GC	
AG	TG	CG	GG	

$a_2 = 3 \cdot a_0 + 3 \cdot a_1 \to a_n = 3 \cdot a_{n-1} + 3a_{n-2}$

can be found as combinations ended in A $\cdot a_{n-2}$ + combinations that begin in A $\cdot a_{n-1}$ =

$= 3a_{n-2} + 3a_{n-1} = a_n$ both cases with $n \geq 2$ to avoid $n < 0$

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$a_n = 3a_{n-2} + 3a_{n-1} \to a_n - 3a_{n-2} - 3a_{n-1} = 0$

$\to x^2 - 3x - 3 = 0 \to \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \lambda_1 = \frac{3 + \sqrt{21}}{2}$

$\lambda_2 = \frac{3 - \sqrt{21}}{2}$

$a_n = A\lambda_1^n + B\lambda_2^n$

$a_0 = A\lambda_1^0 + B\lambda_2^0 = A + B = 1 \to A = 1 - B$

$a_1 = A\lambda_1^1 + B\lambda_2^1 = A\lambda_1 + B\lambda_2 = 4 \to 4 = (1-B)\lambda_1 + B\lambda_2 \to 4 = \lambda_1 - B\lambda_1 + B\lambda_2 \to$

$\to 4 = \frac{3+\sqrt{21}}{2} - \frac{B(3+\sqrt{21})}{2} + \frac{B(3-\sqrt{21})}{2} = 8 = 3+\sqrt{21} - B(3+\sqrt{21}) + B(3-\sqrt{21}) \to$

$\to 8 = 3 + \sqrt{21} - 3B - B\sqrt{21} + 3B - B\sqrt{21} \to 5 - \sqrt{21} = -B\sqrt{21} - B\sqrt{21} \to$

$\to 5 - \sqrt{21} = -B2\sqrt{21} \to B = \frac{5-\sqrt{21}}{-2\sqrt{21}} = \frac{-5+\sqrt{21}}{2\sqrt{21}} = \frac{\sqrt{21}-5}{2\sqrt{21}} = B$

$A = 1 - \frac{\sqrt{21}-5}{2\sqrt{21}} \to \frac{2\sqrt{21} - \sqrt{21} + 5}{2\sqrt{21}} = \frac{\sqrt{21}+5}{2\sqrt{21}} = A$

$a_n = A\lambda_1^n + B\lambda_2^n = \frac{\sqrt{21}+5}{2\sqrt{21}} \cdot \left(\frac{3+\sqrt{21}}{2}\right)^n + \frac{\sqrt{21}-5}{2\sqrt{21}} \cdot \left(\frac{3-\sqrt{21}}{2}\right)^n$