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Second iteration

$$\nabla f(\vec{x_1}) = \left(4\cdot\left(\tfrac{-5}{7}\right) - \left(\tfrac{-6}{7}\right)+2 \;,\; 2\left(\tfrac{-6}{7}\right) - \left(\tfrac{-5}{7}\right)+1\right) = (0,0)$$

$$\vec{X_2} = \begin{pmatrix} \frac{-5}{7} \\ \frac{-6}{7} \end{pmatrix} - \frac{1}{7}\begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-5}{7} \\ \frac{-6}{7} \end{pmatrix}$$

$(-5/7, -6/7)$ is a global minimum → If we keep iterating the point will always be the same.

c) Find the first iteration of the steepest descent method with initial point $(-1,-1)$

$x_0 = (-1,-1)$

$x_1 = x_0 - t\nabla f(x_0)$

$\nabla f(-1,-1) = (-1,0)$

$\phi_{x_0}(t) = f(x_0 - t\nabla f(x_0));\ f((-1,-1) - t(-1,0));$

$f(-1+t,-1) = f(t-1,-1)$

$\phi_{x_0}(t) = 2(t-1)^2 + (-1)^2 - (t-1)(-1) + 2(t-1) + (-1) + 4 =$

$= 2(t^2-2t+1) + \cancel{1} + t - \cancel{1} + 2t - 2 - 1 + 4 = 2t^2 - t + 3$

$$\left.\begin{array}{l} \phi'_{x_0}(t) = 4t - 1 \\ \phi'_{x_0}(t) = 0 \end{array}\right\} \quad 4t - 1 = 0;\ t = \frac{1}{4}$$

$x_1 = (-1,-1) - \frac{1}{4}(-1,0) = \boxed{\left(\frac{-3}{4}, -1\right)}$

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4) Let $f(x,y) = 2x^2 + y^2 - xy + 2x + y + 4$

a) Write $f(x,y)$ as a quadratic function

$$f(x,y) = (x \;\; y) \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + (x \;\; y) \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + C =$$

$$= (q_1 x + q_3 x \quad q_2 x + q_4 x) \begin{pmatrix} x \\ y \end{pmatrix} + (b_1 x + b_2 y) + C =$$

$$= q_1 x^2 + q_3 xy + q_2 xy + q_4 y^2 + b_1 x + b_2 y + C =$$

(let $q_2 = q_3$)

$$= q_1 x^2 + 2 q_2 xy + q_4 y^2 + b_1 x + b_2 y + C$$

$$f(x,y) = 2x^2 + y^2 - xy + 2x + y + 4$$

$$\boxed{f(x,y) = (x \;\; y) \begin{pmatrix} 2 & -1/2 \\ -1/2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + (x \;\; y) \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 4}$$

b) Explain why $f(x,y)$ has a global minimum, and find it by the Newton method with initial point $(-1,-1)$.

$$\vec{X}_{k+1} = \vec{X}_k - H_f^{-1} \cdot \nabla f$$

First iteration

$$\nabla f = (f_x, f_y) = (4x - y + 2, 2y - x + 1)$$

$$\nabla f(\vec{X}_k) = (-4 + 1 + 2, -2 + 1 + 1) = (-1, 0)$$

$$H_f(x,y) = \begin{pmatrix} 4 & -1 \\ -1 & 2 \end{pmatrix} \longrightarrow H^{-1}_{f(x,y)} = \frac{1}{4 \cdot 2 - 1} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

$$\vec{X}_0 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \vec{X}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \frac{1}{7} \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} - \frac{1}{7} \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -5/7 \\ -6/7 \end{pmatrix}$$