Homework 1

Counting

5

$\{ATCG\} \to 4$

length n

no AA

$a_n = ?$

$a_0 = 1$

$a_1 = A, T, C$ or $G = 4$

$a_2 =$

/	TA	CA	GA
AT	TT	CT	GT
AC	TC	GC	GC
AG	TG	CG	GG

$= 15$

$a_2 = 3 \cdot a_0 + 3 \cdot a_1 \to a_n = 3 \cdot a_{n-1} + 3a_{n-2}$

can be found as combinations ended in $A \cdot a_{n-2}$ + combinations that begin in $A \cdot a_{n-1} =$

$= 3a_{n-2} + 3a_{n-1} = a_n$ both cases with $n \geq 2$ to avoid $n < 0$

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$a_n = 3a_{n-2} + 3a_{n-1} \to a_n - 3a_{n-2} - 3a_{n-1} = 0$

$\to x^2 - 3x - 3 = 0 \to \dfrac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} : \quad \lambda_1 = \dfrac{3 + \sqrt{21}}{2}$

$\lambda_2 = \dfrac{3 - \sqrt{21}}{2}$

$a_n = A\lambda_1^n + B\lambda_2^n$

$a_0 = A\lambda_1^0 + B\lambda_2^0 = A + B = 1 \to A = 1 - B$

$a_1 = A\lambda_1^1 + B\lambda_2^1 = A\lambda_1 + B\lambda_2 = 4 \to 4 = (1-B)\lambda_1 + B\lambda_2 \to 4 = \lambda_1 - B\lambda_1 + B\lambda_2 \to$

$\to 4 = \dfrac{3+\sqrt{21}}{2} - \dfrac{B(3+\sqrt{21})}{2} + \dfrac{B(3-\sqrt{21})}{2} = 8 = 3 + \sqrt{21} - B(3+\sqrt{21}) + B(3-\sqrt{21}) \to$

$\to 8 = 3 + \sqrt{21} - 3B - B\sqrt{21} + 3B - B\sqrt{21} \to 5 - \sqrt{21} = -B\sqrt{21} - B\sqrt{21} \to$

$\to 5 - \sqrt{21} = -B2\sqrt{21} \to B = \dfrac{5 - \sqrt{21}}{-2\sqrt{21}} = \dfrac{-5 + \sqrt{21}}{2\sqrt{21}} = \dfrac{\sqrt{21} - 5}{2\sqrt{21}} = B$

$A = 1 - \dfrac{\sqrt{21} - 5}{2\sqrt{21}} \to \dfrac{2\sqrt{21} - \sqrt{21} + 5}{2\sqrt{21}} = \dfrac{\sqrt{21} + 5}{2\sqrt{21}} = A$

$a_n = A\lambda_1^n + B\lambda_2^n = \dfrac{\sqrt{21} + 5}{2\sqrt{21}} \cdot \left(\dfrac{3 + \sqrt{21}}{2}\right)^n + \dfrac{\sqrt{21} - 5}{2\sqrt{21}} \cdot \left(\dfrac{3 - \sqrt{21}}{2}\right)^n$