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Homework 4

Linear programming

⑤

$$\max\ x_1+3x_2+2x_3$$

subject to $x_1+3x_2+3x_3 \le 6$

$2x_1+x_2+2x_3 \le 2$

$x_1+2x_2+x_3 \le 3$

$x_1, x_2, x_3 \ge 0$

a) → equational standard form

$x_1+3x_2+3x_3+s_1=6$ ⟹ $s_1 = 6-x_1-3x_2-3x_3$

$2x_1+x_2+2x_3+s_2=2$ ⟹ $s_2 = 2-2x_1-x_2-2x_3$

$x_1+2x_2+x_3+s_3=3$ ⟹ $s_3 = 3-x_1-2x_2-x_3$

$z = x_1+3x_2+2x_3$

Basic feasible solution:

$(x_1, x_2, x_3, s_1, s_2, s_3) = (0,0,0,6,2,3)$

b) $s_1 = 6-x_1-3x_2-3x_3$ → x with biggest coefficient → $x_1 = 6-3x_2-3x_3-s_1$

$s_2 = 2-(6-3x_2-3x_3-s_1)-x_2-2x_3$ →

→ $s_2 = 2-12+6x_2+6x_3+2s_1-x_2-2x_3$ → $s_2 = -10+5x_2+4x_3+2s_1$

$s_3 = 3-(6-3x_2-3x_3-s_1)-2x_2-x_3 = s_3 = 3-6+3x_2+3x_3+s_1-2x_2$

→ $s_3 = -3+x_2+2x_3+s_1$

$z = (6-3x_2-3x_3-s_1)+3x_2+2x_3 = 6-x_3-s_1 = z$

$x_1 = 6-3x_2-3x_3-s_1$

$s_2 = -10+x_2+4x_3+2s_1$

$s_3 = -3+x_2+2x_3+s_1$

$z = 6-x_3-s_1$

$(6,0,0,0,-10,-3)$

$x_1 = 0$ ⟹ $x_2 \le 2$

$s_2 = 0$ ⟹ $x_2 \le 2$

$s_3 = 0$ ⟹ $x_2 \le 3$

b) From a)

if $s_1 = 0 \rightarrow x_2 \leq 2$

if $s_2 = 0 \rightarrow x_2 \leq 2$ $\quad (0, 0, 6, 2, 3)$

if $s_3 = 0 \rightarrow x_2 \leq 3/2$

$x_2 \rightarrow s_3$

$s_3 = 3 - x_1 - 2x_2 - x_3 \rightarrow 2x_2 = 3 - x_1 - x_3 - s_3 \rightarrow x_2 = \frac{3}{2} - \frac{x_1}{2} - \frac{x_3}{2} - \frac{s_3}{2}$

$$x_2 = \frac{3}{2} - \frac{x_1}{2} - \frac{x_3}{2} - \frac{s_3}{2}$$

$$s_1 = 6 - x_1 - 3\left(\frac{3}{2} - \frac{x_1}{2} - \frac{x_3}{2} - \frac{s_3}{2}\right) - 3x_3 = \frac{12}{2} - \frac{2x_1}{2} - \frac{9}{2} + \frac{3x_1}{2} + \frac{3x_3}{2} + \frac{3s_3}{2} - \frac{6x_3}{2} =$$

$$s_1 = \frac{3}{2} + \frac{x_1}{2} - \frac{3x_3}{2} + \frac{3s_3}{2}$$

$$s_2 = 2 - 2x_1 - \left(\frac{3}{2} - \frac{x_1}{2} - \frac{x_3}{2} - \frac{s_3}{2}\right) - 2x_3 = \frac{4}{2} - \frac{3}{2} - \frac{4x_1}{2} + \frac{x_1}{2} + \frac{x_3}{2} - \frac{4x_3}{2} + \frac{s_3}{2} =$$

$$s_2 = \frac{1}{2} - \frac{3x_1}{2} - \frac{3x_3}{2} + \frac{s_3}{2}$$

$$z = x_1 + 3\left(\frac{3}{2} - \frac{x_1}{2} - \frac{x_3}{2} - \frac{s_3}{2}\right) + 2x_3 = \frac{2x_1}{2} + \frac{9}{2} - \frac{3x_1}{2} - \frac{3x_3}{2} - \frac{3s_3}{2} + \frac{4x_3}{2} =$$

$$z = \frac{9}{2} - \frac{x_1}{2} + \frac{x_3}{2} - \frac{3s_3}{2}$$

if $x_2 = 0 \rightarrow x_3 \leq 3$

if $s_1 = 0 \rightarrow x_3 \leq 1$ $\quad (0, \frac{3}{2}, 0, \frac{3}{2}, \frac{1}{2}, 0)$

if $s_2 = 0 \rightarrow x_3 \leq 1/3$

$x_3 \rightarrow x_5$

$s_2 = \frac{1}{2} - \frac{3x_1}{2} - \frac{3x_3}{2} + \frac{s_3}{2} \rightarrow 3x_3 = 1 - 3x_1 + s_3 - 2s_2 \rightarrow x_3 = \frac{1}{3} - \frac{3x_1}{3} + \frac{s_3}{3} - \frac{2}{3}s_2 \rightarrow$

$$x_3 = \frac{1}{3} - x_1 - \frac{2}{3}s_2 + \frac{s_3}{3}$$

$$x_2 = \frac{3}{2} - \frac{x_1}{2} - \left(\frac{1}{3} - x_1 - \frac{2}{3}s_2 + \frac{s_3}{3}\right)\frac{1}{2} - \frac{s_3}{2} \rightarrow x_2 = \frac{3}{2} - \frac{x_1}{2} - \frac{1}{6} + \frac{x_1}{2} + \frac{2}{6}s_2 - \frac{s_3}{6} - \frac{s_3}{2}$$

$$= \frac{9}{6} - \frac{3x_1}{6} - \frac{1}{6} + \frac{3x_1}{6} + \frac{2}{6}s_2 - \frac{s_3}{6} - \frac{3s_3}{6} = x_2$$

$$x_2 = \frac{4}{3} + \frac{s_2}{3} - \frac{2s_3}{3}$$

b) $s_1 = \frac{3}{2} + \frac{x_1}{2} - \frac{3}{2}\left(\frac{1}{3} - x_1 - \frac{2}{3}s_2 + \frac{s_3}{3}\right) + \frac{3s_3}{2} = \frac{3}{2} + \frac{x_1}{2} - \frac{3}{6} + \frac{3x_1}{2} + \frac{6s_2}{6} - \frac{3s_3}{6}$

$+ \frac{3s_3}{2} \rightarrow \frac{9}{6} + \frac{3x_1}{6} - \frac{3}{6} + \frac{9x_1}{6} + \frac{6s_2}{6} - \frac{3s_3}{6} + \frac{9s_3}{6} = s_1$

$s_1 = 1 + 2x_1 + s_2 + s_3$

$z = \frac{9}{2} - \frac{x_1}{2} + \frac{1}{2}\left(\frac{1}{3} - x_1 - \frac{2}{3}s_2 + \frac{s_3}{3}\right) - \frac{3s_3}{2} \rightarrow z = \frac{9}{2} - \frac{x_1}{2} + \frac{1}{6} - \frac{x_1}{2} - \frac{2s_2}{6} + \frac{s_3}{6} - \frac{3s_3}{2}$

$\rightarrow z = \frac{27}{6} - \frac{3x_1}{6} + \frac{1}{6} - \frac{3x_1}{6} - \frac{2s_2}{6} + \frac{s_3}{6} - \frac{9s_3}{6}$

$z = \frac{14}{3} - x_1 - \frac{s_2}{3} - \frac{4s_3}{3}$

$(0, \frac{4}{3}, \frac{1}{3}, 1, 0, 0)$

$x_1 = 0$

$x_2 = \frac{4}{3}$ $\quad max \rightarrow x_1 + 3x_2 + 2x_3 = 0 + 3\cdot\frac{4}{3} + 2\cdot\frac{1}{3} = \frac{12}{3} + \frac{2}{3} = \boxed{\frac{14}{3}} = max$

$x_3 = \frac{1}{3}$