Old final exam

(1) Consider the following conjunctive query Q:

$$result(A, B, C, D, E, F) \quad \leftarrow \quad r(A, B, C), s(A, F, E), t(E, D, C), u(A, C, E)$$

1. Give the hypergraph representation of Q.
   - Solution. The node set of this hypergraph is $\{A, B, C, D, E, F\}$. The edge set is $\{\{A, B, C\}, \{A, F, E\}, \{C, D, E\}, \{A, C, E\}\}$.
2. Is Q acyclic? If so, can it be made cyclic by removing one hyperedge? Otherwise, can it be made acyclic by removing one hyperedge?
   - Solution. Yes, Q is acyclic. A tree decomposition can be formed by taking $\{A, C, E\}$ as root and all other edges as children of the root. If we were to remove the edge $\{A, C, E\}$, then a tree decomposition of Q would not be possible and hence Q would be cyclic.

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(2) Given a schedule S over transactions $\{T_1, \ldots, T_n\}$, the "strong graph" associated with S is the directed graph sg(S) having exactly one node for each transaction of S and an edge from T_i to T_j (for $i \neq j$) if and only if in S, for some object X there is an action $\alpha_i(X)$ of T_i on X which appears before an action $\alpha_j(X)$ of T_j on X.

Prove or disprove the following claims.

1. If sg(S) is acyclic, then S is conflict serializable.
   - Solution. This claim is true. First, we note that pg(S), the precedence graph of S, is a subgraph of sg(S). Hence, if sg(S) is acyclic, then so is pg(S). By the Serializability Theorem we then have that S is conflict serializable.
2. If S is conflict serializable, then sg(S) is acyclic.
   - Solution. This claim is false. For example, the schedule $\langle R_1(A), R_2(A), R_1(A), C_1, C_2 \rangle$ is clearly conflict serializable, yet has a cycle in its strong graph.

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(3) Consider the following conjunctive queries.

$$\begin{array}{llll} Q_1: & result(A) & \leftarrow & r(A,B), r(A,C), s(B,D,E), s(B,F,F) \\ Q_2: & result(X) & \leftarrow & r(X,Y), r(X,W), s(Y,W,W), t(X) \end{array}$$

Is it the case that $Q_2 \subseteq Q_1$? Prove your answer.

Solution. Yes. The following variable mapping is a homomorphism from Q_1 to Q_2:

$A \rightarrow X, B \rightarrow Y, C \rightarrow Y, D \rightarrow W, E \rightarrow W, F \rightarrow W$.

By the Homomorphism Theorem we then have that $Q_2 \subseteq Q_1$.

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(4) Consider the "semi-difference" relational algebra operator, defined as

$$\begin{aligned} R \triangleright S &= \{r \in R \mid \neg\exists s \in S(r \bowtie s \in R \bowtie S)\} \\ &= R - (R \ltimes S). \end{aligned}$$

Formally prove or disprove the following proposals for relational algebra equivalences.

1. $\sigma_\theta(R \triangleright S) = \sigma_\theta(R) \triangleright S$, where θ is a standard single-table selection condition which mentions only attributes in R (i.e., $atts(\theta) \subseteq atts(R) - atts(S)$).

Solution. This proposal is true. Let $t \in \sigma_\theta(R \triangleright S)$. Then, (1) $\theta(t)$ is true; (2) $t \in R$; and, (3) there is no $s \in S$ such that $t \bowtie s \in R \bowtie S$. By (1) and (2), we have that (4) $t \in \sigma_\theta(R)$. By (3) and (4), we have that $t \in \sigma_\theta(R) \triangleright S$. Hence $\sigma_\theta(R \triangleright S) \subseteq \sigma_\theta(R) \triangleright S$.

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Going in the other directtion, suppose now that $t \in \sigma_\theta(R) \triangleright S$. Then, (1) $t \in \sigma_\theta(R)$ and (2) there is no $s \in S$ such that $t \bowtie s \in \sigma_\theta(R) \bowtie S$. By (1) we have that (3) $t \in R$ and (4) $\theta(t)$ is true. By (2) and (3) we have that (5) $t \in R \triangleright S$. By (4) and (5) we have that $t \in \sigma_\theta(R \triangleright S)$. Hence $\sigma_\theta(R \triangleright S) \supseteq \sigma_\theta(R) \triangleright S$.

Since $\sigma_\theta(R \triangleright S) \subseteq \sigma_\theta(R) \triangleright S$ and $\sigma_\theta(R \triangleright S) \supseteq \sigma_\theta(R) \triangleright S$, we conclude that the equality holds.

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2. $R \ltimes S = R \rhd (R \rhd S)$.

Solution. This proposal is true.

$$\begin{aligned} R \rhd (R \rhd S) &= R \rhd (R - (R \ltimes S)) \\ &= R - (R \ltimes (R - (R \ltimes S))) \\ &= R - (R - (R \ltimes S)) \\ &= R \ltimes S. \end{aligned}$$

The first two equalities hold by definition of $\rhd$. The third holds since $R \ltimes R' = R'$ for any $R' \subseteq R$, by definition of $\ltimes$. The final holds since $R - (R - R') = R'$ for any $R' \subseteq R$, by basic set theory.