Indexing: ordered indexes, continued

Lecture 3
2ID35, Spring 2015

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Admin

Project part 2 will be posted later today. Once posted:

- find your team mates
- study your paper
- determine your research problem
- first report due on or before 10 May

Where we've been

Last time

- File structures

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- I/O model of computation

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- File structures
- I/O model of computation
- external sorting
- B+ trees

Today's agenda

Ordered indexes, continued

- R trees
- GiST: generalized search trees

Ordered indexing: spatial data

Why spatial/multi-dimensional data?

- GIS, CAD design, multimedia, ...
- composite search keys are not enough

Ordered indexing: R-trees

- height-balanced, dynamic search tree
- designed to minimize I/O

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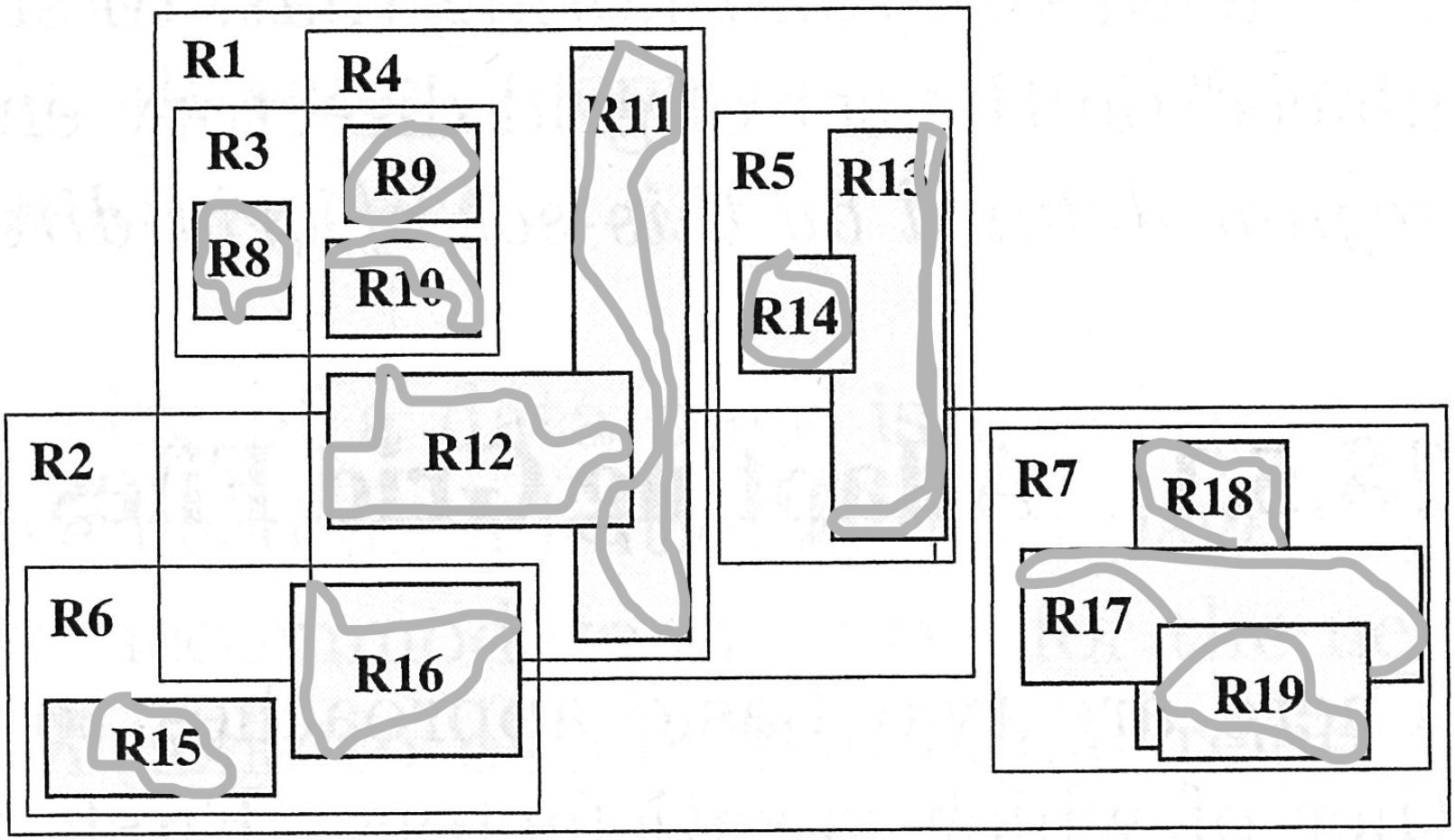
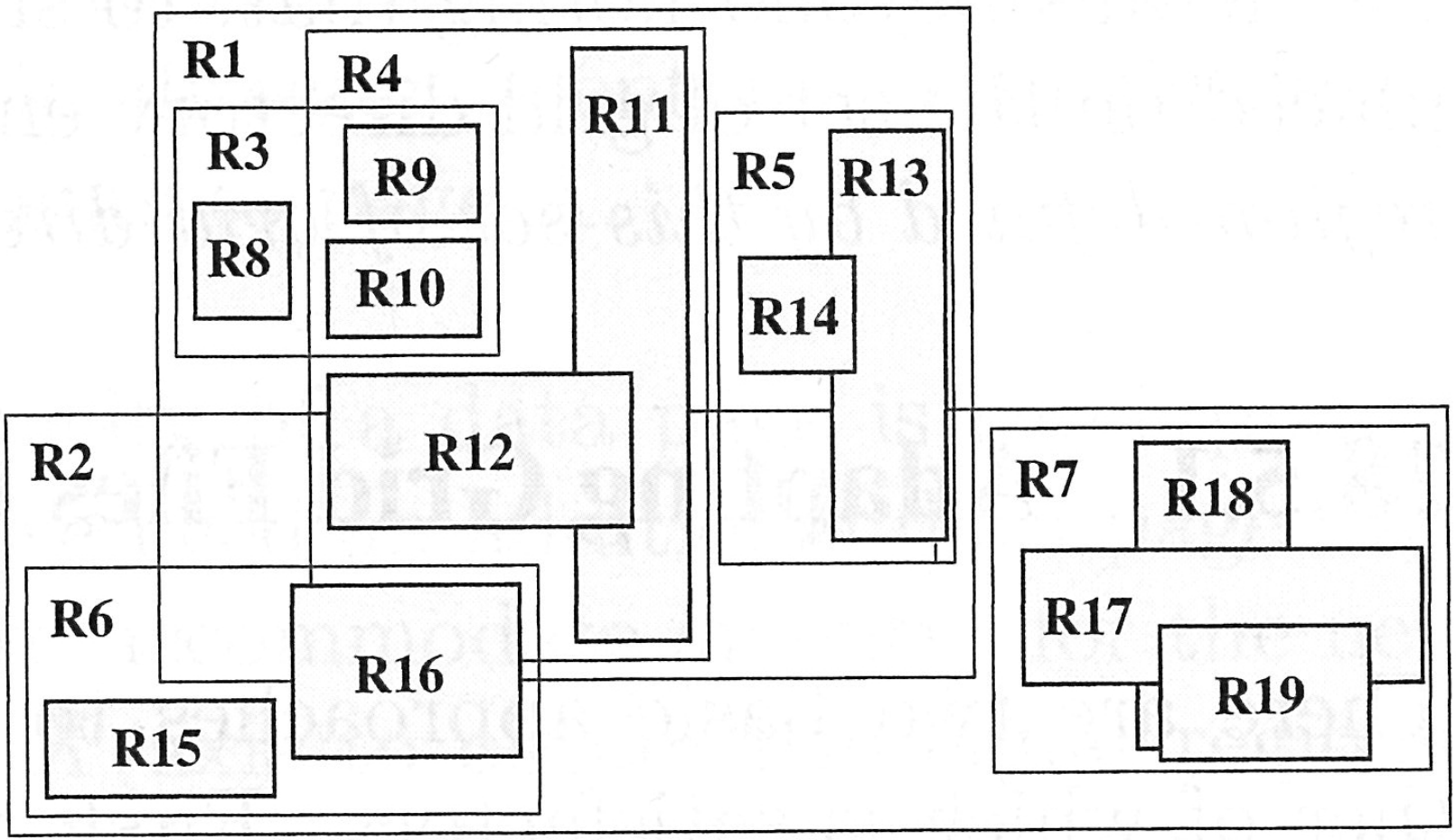
- height-balanced, dynamic search tree
- designed to minimize I/O
- instead of range of values, a rectangular bounding box is associated with each node
- efficient spatial search

Ordered indexing: R-trees



Ordered indexing: R-trees



R-trees: structure

Properties

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  [*n-dimensional bbox, pointer to record*]

  bounding box is tightest bounding box for data object

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  the box covers all boxes in subtree rooted at child
- same minimal occupancy rules as for B+ trees

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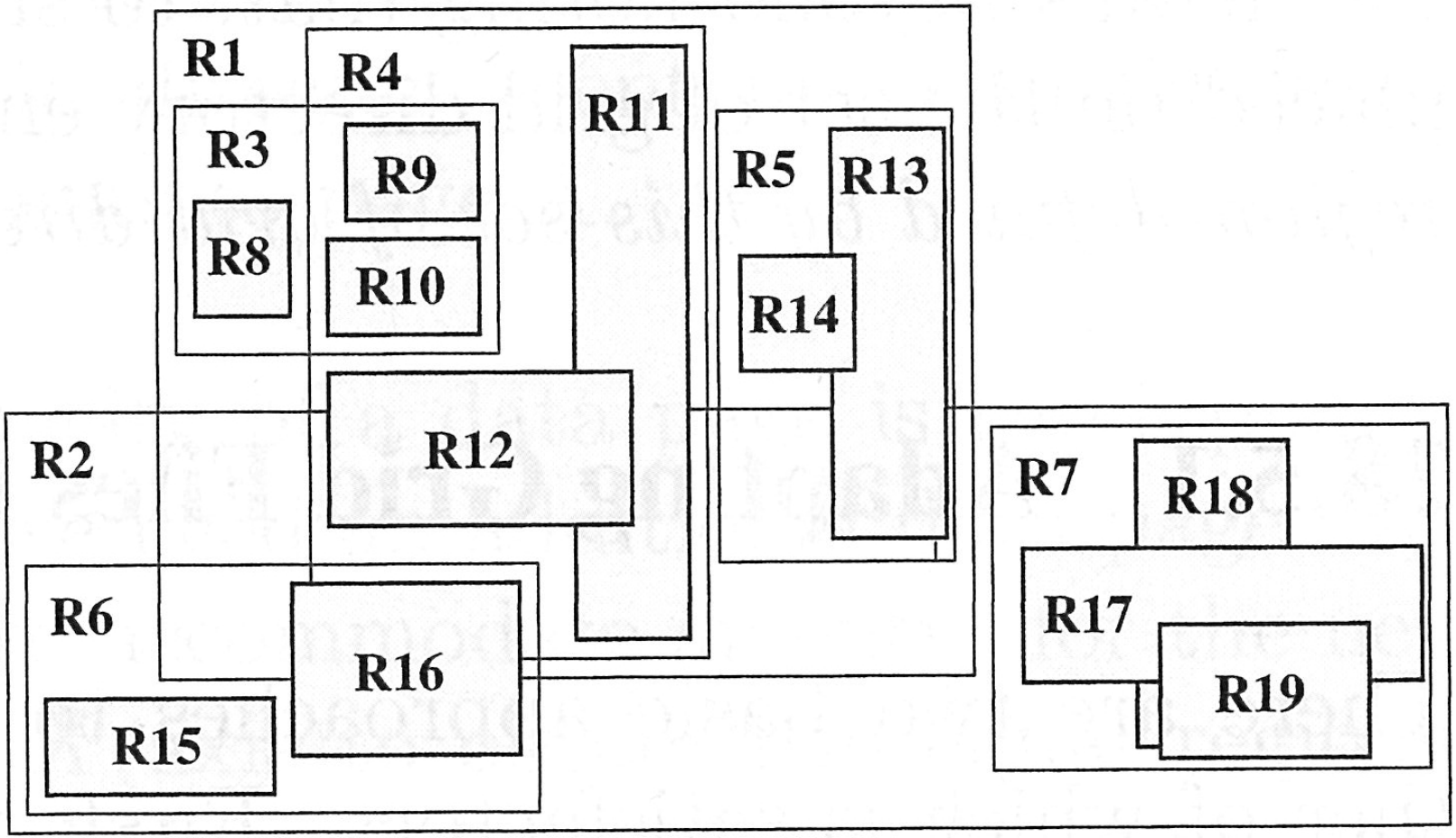

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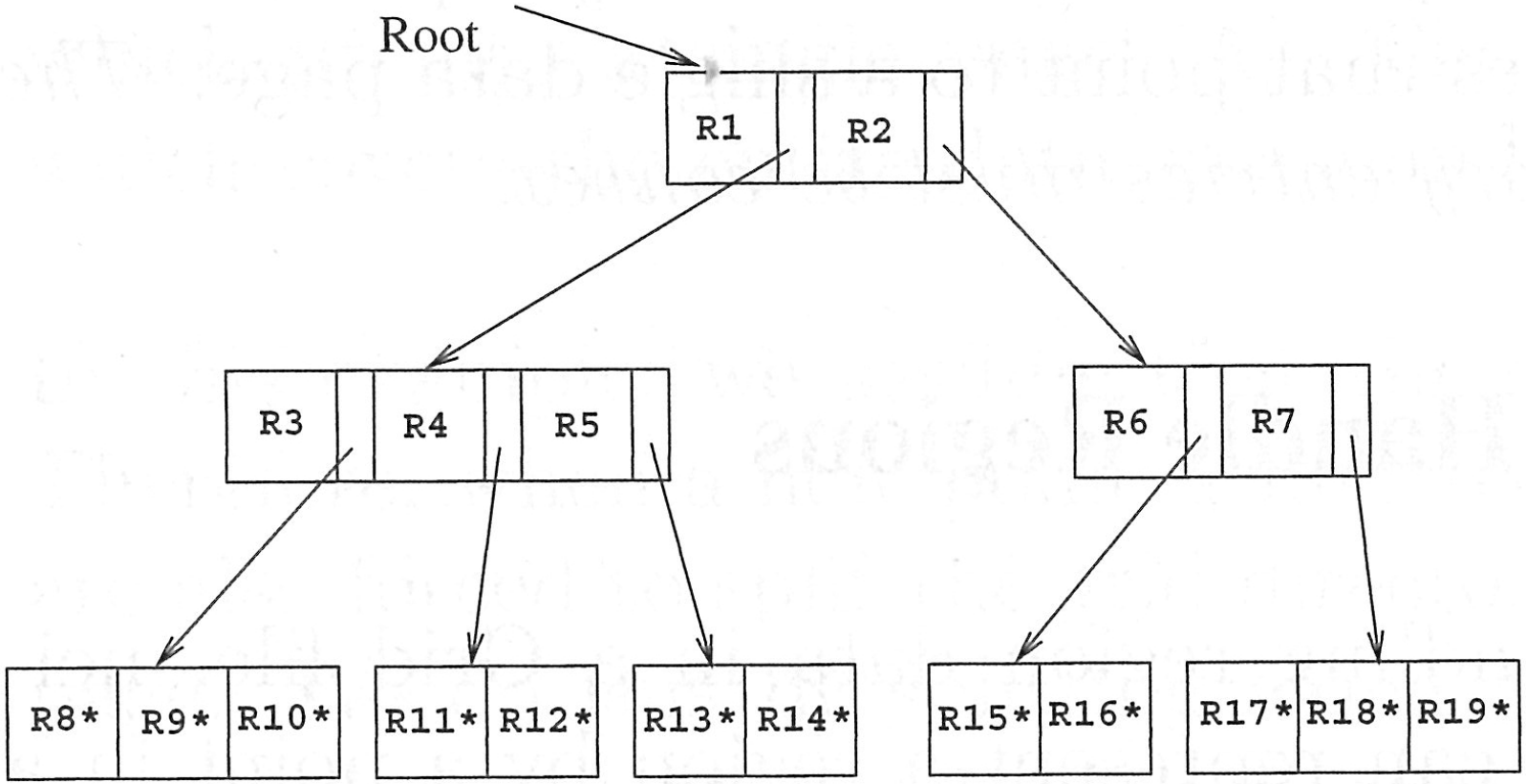

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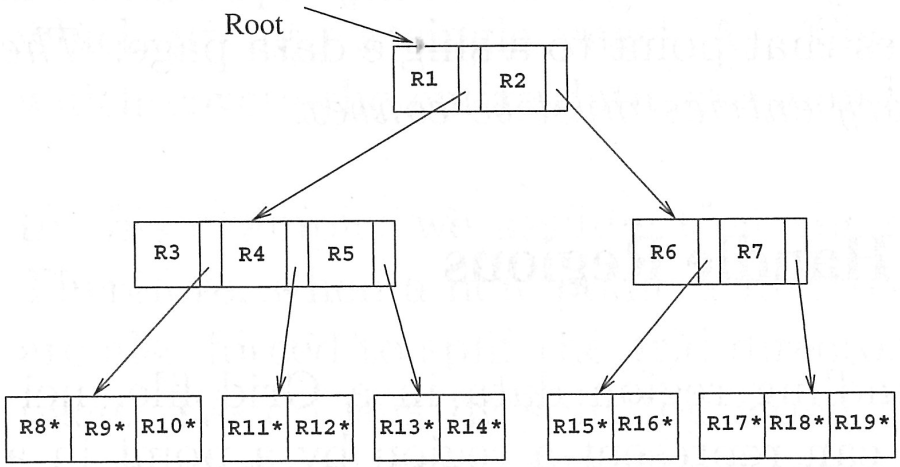

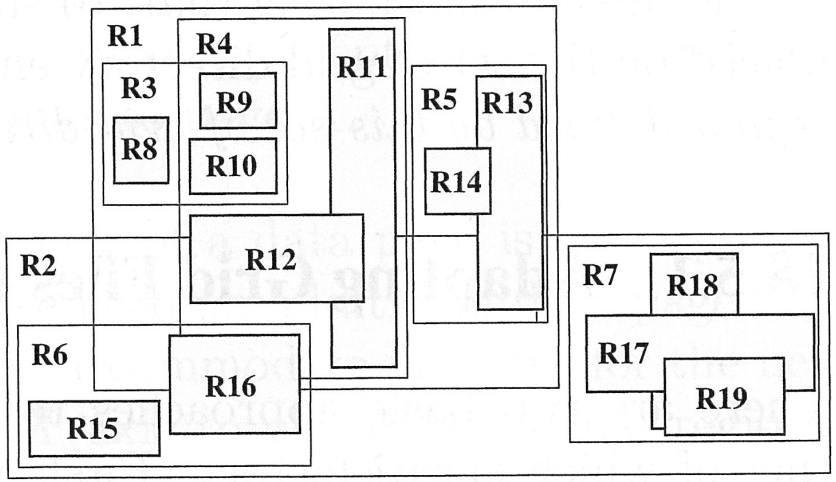

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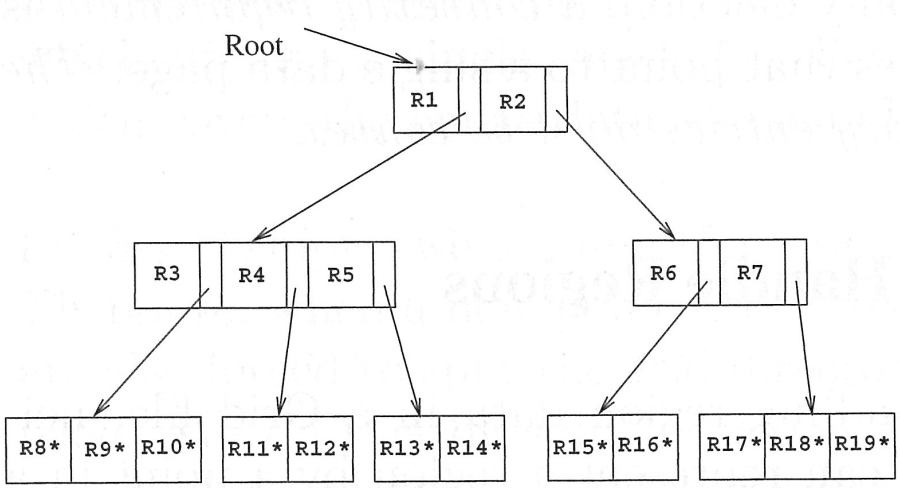

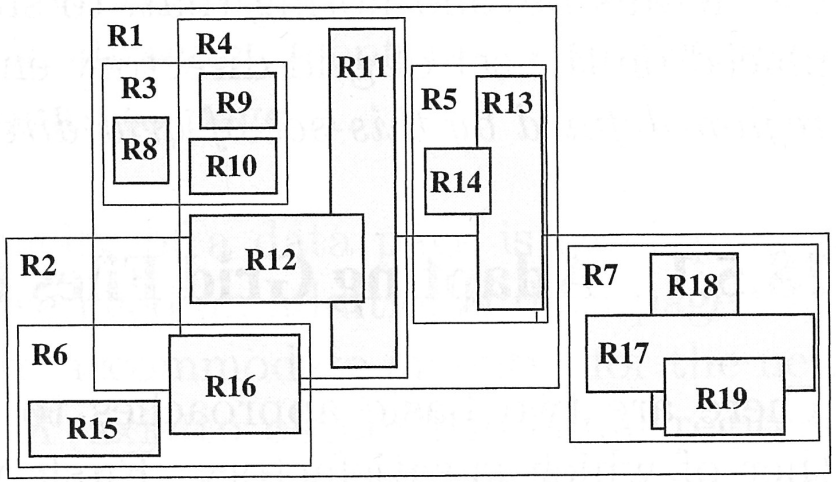

Note that, unlike B+ trees, we have no ordering on objects (and hence no ordering on tree nodes, siblings, etc.)

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Note that, unlike B+ tree, we may have to search multiple subtrees

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- if N is leaf
  - if N has space, insert
  - else, split N into N_1 and N_2, adjust N's parent to only cover new N_1, and add entry for N_2

R-trees: deletion

Delete is handled as expected, except that under-full nodes are just deleted, and the remaining entries re-inserted.

R-trees

R-trees

- Very popular spatial indexing mechanism (MySQL, Oracle, Postgres, ...)
- Intensively studied, leading to many variants such as R* and R+ trees
  - example: Hilbert R-trees

R-trees: Hilbert variant

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Hilbert R-trees

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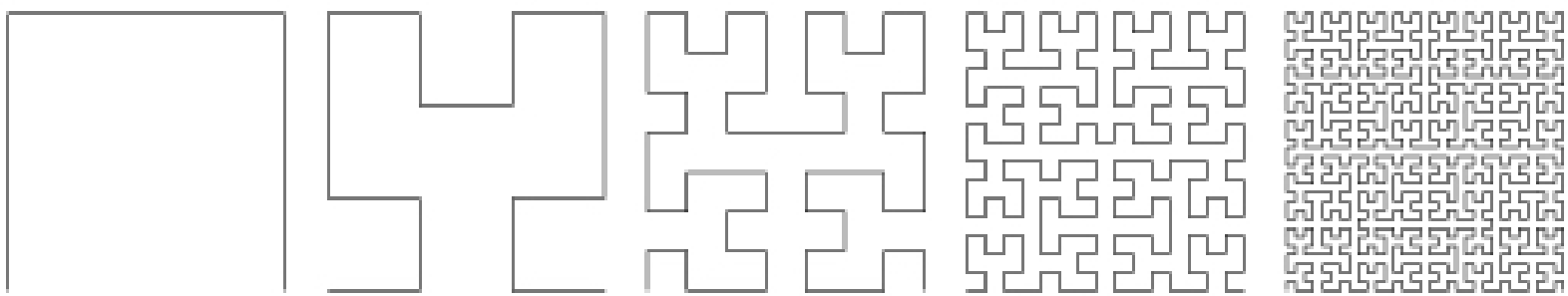
  - leads to poor space utilization (~70%)
- *key idea of Hilbert R-trees:* add an ordering to objects (and hence on nodes in the tree)
  - leads to excellent space utilization (~100%) and search/update costs (up to ~30% fewer page accesses)

R-trees: Hilbert variant

Hilbert curves

- example of a "space filling" curve, proposed by Hilbert in 1891
- imposes a linear ordering on the points in the unit n-cube
- has very good spatial locality



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- extend each entry in non-leaf nodes to also include the largest Hilbert value (LHV) among the boxes contained in the subtree rooted at the entry

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  [*LHV, n-dimensional bbox, pointer to child node*]
- Search as usual; insertion/deletion are guided by Hilbert value of the inserted/deleted box
  - also, Hilbert ordering can be effectively used for bulk-loading/building the index

Search trees, in general

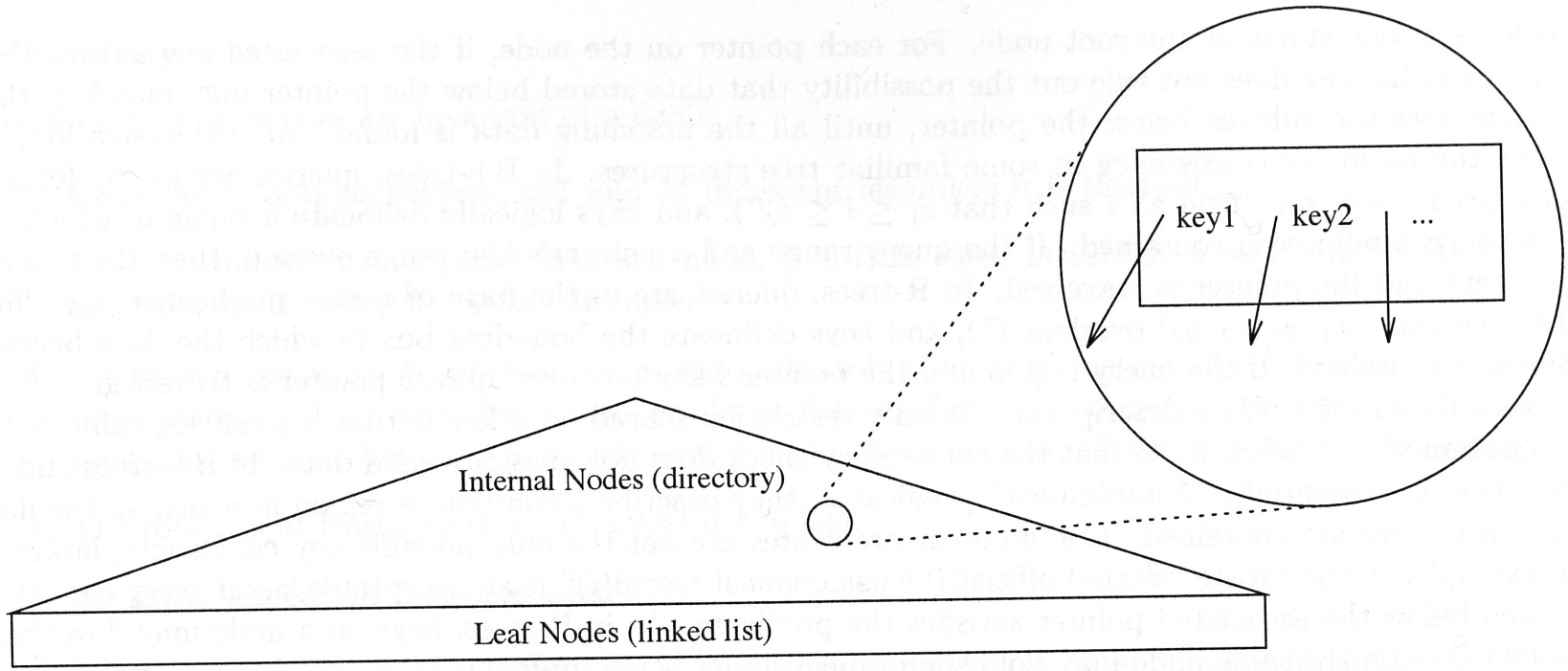
There has been an explosion in types of data: geographic data, multimedia, CAD, document libraries, sequence data, fingerprint data, biochemical and bioinformatics data, ...

Search trees, in general

Consequently, there has been an explosion of tree-based indexing techniques ...

often based on specialized search trees, or search trees over extensible data types.

Search trees, in general



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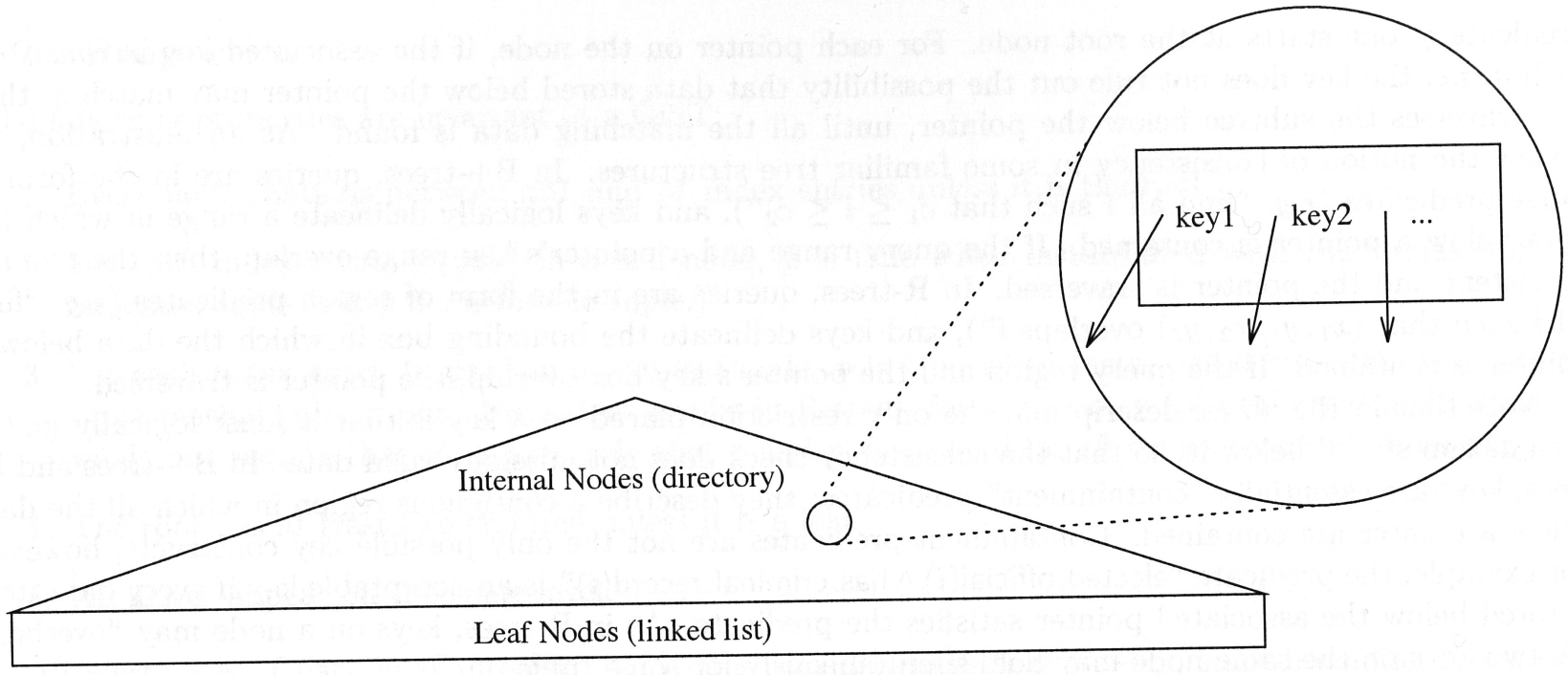

What can we say in general about tree-based ordered indexes? How far can we generalize this structure?

GiST: generalized search trees

Proposal for generic tree-index infrastructure

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Available as part of PostgreSQL

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- for each index entry (p, ptr) in an internal node, p is true when instantiated with the values from any tuple reachable from ptr
- the root has at least two children unless it is a leaf
- all leaves appear on the same level

GiST: generalized search trees

Key designer must define the following methods

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- Penalty(E_1, E_2), for inserting E_2 into subtree at E_1
- PickSplit(P), into two new lists P_1 and P_2, maintaining occupancy requirements

GiST: generalized search trees

Basic infrastructure:

- Search
- Search on linearly ordered domains
  - using FindMin and Next

GiST: generalized search trees

Basic infrastructure:

- Search
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  - using FindMin and Next
- Insert
  - using ChooseSubTree, Split, and AdjustKeys

GiST: generalized search trees

Basic infrastructure:

- Search
- Search on linearly ordered domains
  - using FindMin and Next
- Insert
  - using ChooseSubTree, Split, and AdjustKeys
- Delete

GiST: search

Algorithm Search(R, q)

Input: GiST rooted at R, predicate q

Output: all tuples that satisfy q

Sketch: Recursively descend all paths in tree whose keys are consistent with q.

S1: [Search subtrees] If R is not a leaf, check each entry E on R to determine whether Consistent(E, q). For all entries that are Consistent, invoke Search on the subtree whose root node is referenced by E.`ptr`.

S2: [Search leaf node] If R is a leaf, check each entry E on R to determine whether Consistent(E, q). If E is Consistent, it is a qualifying entry. At this point E.`ptr` could be fetched to check q accurately, or this check could be left to the calling process.

GiST: search on linearly ordered domains

Algorithm FindMin(R, q)

Input: GiST rooted at R, predicate q

Output: minimum tuple in linear order that satisfies q

Sketch: descend leftmost branch of tree whose keys are Consistent with q. When a leaf node is reached, return the first key that is Consistent with q.

FM1: [Search subtrees] If R is not a leaf, find the first entry E in order such that Consistent(E, q)[1]. If such an E can be found, invoke FindMin on the subtree whose root node is referenced by E.`ptr`. If no such entry is found, return NULL.

FM2: [Search leaf node] If R is a leaf, find the first entry E on R such that Consistent(E, q), and return E. If no such entry exists, return NULL.

GiST: search on linearly ordered domains

Algorithm Next(R, q, E)

Input: GiST rooted at R, predicate q, current entry E

Output: next entry in linear order that satisfies q

Sketch: return next entry on the same level of the tree if it satisfies q. Else return NULL.

N1: [next on node] If E is not the rightmost entry on its node, and N is the next entry to the right of E in order, and Consistent(N, q), then return N. If $\neg$Consistent(N, q), return NULL.

N2: [next on neighboring node] If E is the rightmost entry on its node, let P be the next node to the right of R on the same level of the tree (this can be found via tree traversal, or via sideways pointers in the tree, when available [LY81].) If P is non-existent, return NULL. Otherwise, let N be the leftmost entry on P. If Consistent(N, q), then return N, else return NULL.

GiST: insertion

Algorithm Insert(R, E, l)

Input: GiST rooted at R, entry $E = (p, \texttt{ptr})$, and level l, where p is a predicate such that p holds for all tuples reachable from `ptr`.

Output: new GiST resulting from insert of E at level l.

Sketch: find where E should go, and add it there, splitting if necessary to make room.

I1. [invoke ChooseSubtree to find where E should go] Let L = ChooseSubtree(R, E, l)

I2. If there is room for E on L, install E on L (in order according to Compare, if IsOrdered.) Otherwise invoke Split(R, L, E).

I3. [propagate changes upward] AdjustKeys(R, L).

GiST: insertion

Algorithm ChooseSubtree(R, E, l)

Input: subtree rooted at R, entry $E = (p, \texttt{ptr})$, level l

Output: node at level l best suited to hold entry with characteristic predicate $E.p$

Sketch: Recursively descend tree minimizing Penalty

CS1. If R is at level l, return R;

CS2. Else among all entries $F = (q, \texttt{ptr}')$ on R find the one such that Penalty(F, E) is minimal. Return ChooseSubtree($F.\texttt{ptr}', E, l$).

GiST: insertion

Algorithm Split(R, N, E)

Input: GiST R with node N, and a new entry $E = (p, \texttt{ptr})$.

Output: the GiST with N split in two and E inserted.

Sketch: split keys of N along with E into two groups according to PickSplit. Put one group onto a new node, and Insert the new node into the parent of N.

SP1: Invoke PickSplit on the union of the elements of N and $\{E\}$, put one of the two partitions on node N, and put the remaining partition on a new node N'.

SP2: [Insert entry for N' in parent] Let $E_{N'} = (q, \texttt{ptr}')$, where q is the Union of all entries on N', and $\texttt{ptr}'$ is a pointer to N'. If there is room for $E_{N'}$ on Parent(N), install $E_{N'}$ on Parent(N) (in order if IsOrdered.) Otherwise invoke Split(R, Parent(N), $E_{N'}$)[2].

SP3: Modify the entry F which points to N, so that $F.p$ is the Union of all entries on N.

GiST: insertion

Algorithm AdjustKeys(R, N)

Input: GiST rooted at R, tree node N

Output: the GiST with ancestors of N containing correct and specific keys

Sketch: ascend parents from N in the tree, making the predicates be accurate characterizations of the subtrees. Stop after root, or when a predicate is found that is already accurate.

PR1: If N is the root, or the entry which points to N has an already-accurate representation of the Union of the entries on N, then return.

PR2: Otherwise, modify the entry E which points to N so that $E.p$ is the Union of all entries on N. Then AdjustKeys(R, Parent(N).)

GiST: deletion

Deletion (1) maintains balance of tree, and (2) keeps keys as specific as possible.

Follows B+tree style on linearly ordered domains, and otherwise R-tree style.

GiST

Generalizes a wide variety of indexing proposals.

GiST

Generalizes a wide variety of indexing proposals.

Permits DB engine designers to focus on core extensible infrastructure.

Promotes deeper study of indexing

Wrap up

- Indexing, part 1: ordered indexes
  - B+ trees
  - R trees
  - GiST: generalized search trees

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- Indexing, part 1: ordered indexes
  - B+ trees
  - R trees
  - GiST: generalized search trees
- next time: Indexing, part 2: hash indexes and indexability

Credits

- Our textbook (Silberschatz *et al.*, 2011)
- Ramakrishnan & Gehrke, 2003
- Hellerstein *et al.*, 1995