Chapter 4: Geocentric Models

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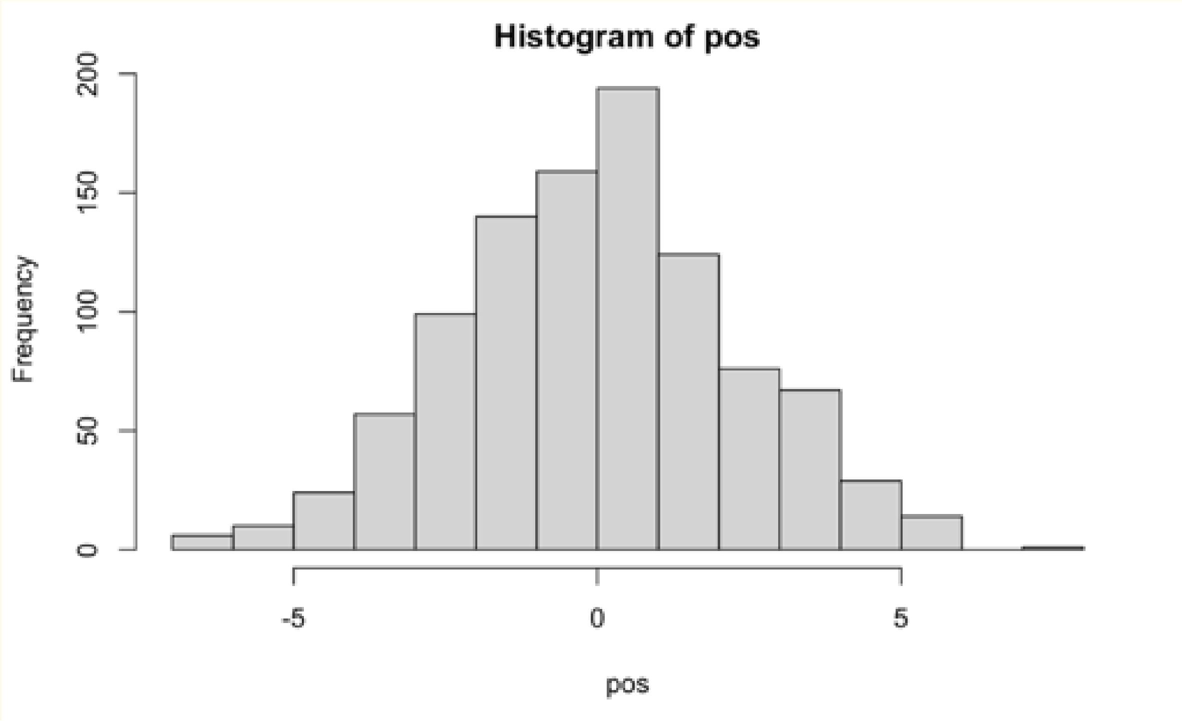
Why are normal distributions normal?

- Any process that adds together random values from the same distribution converges to normal

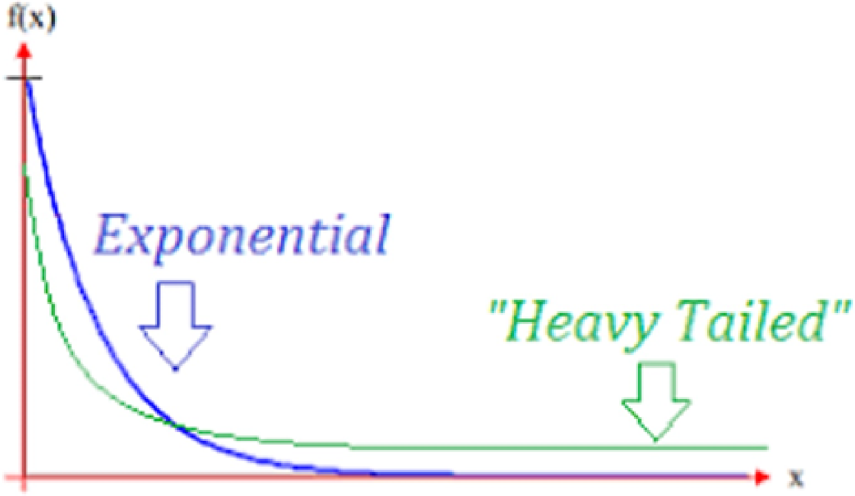
- Everything other than the mean is a fluctuation form the average value



Heavy tails

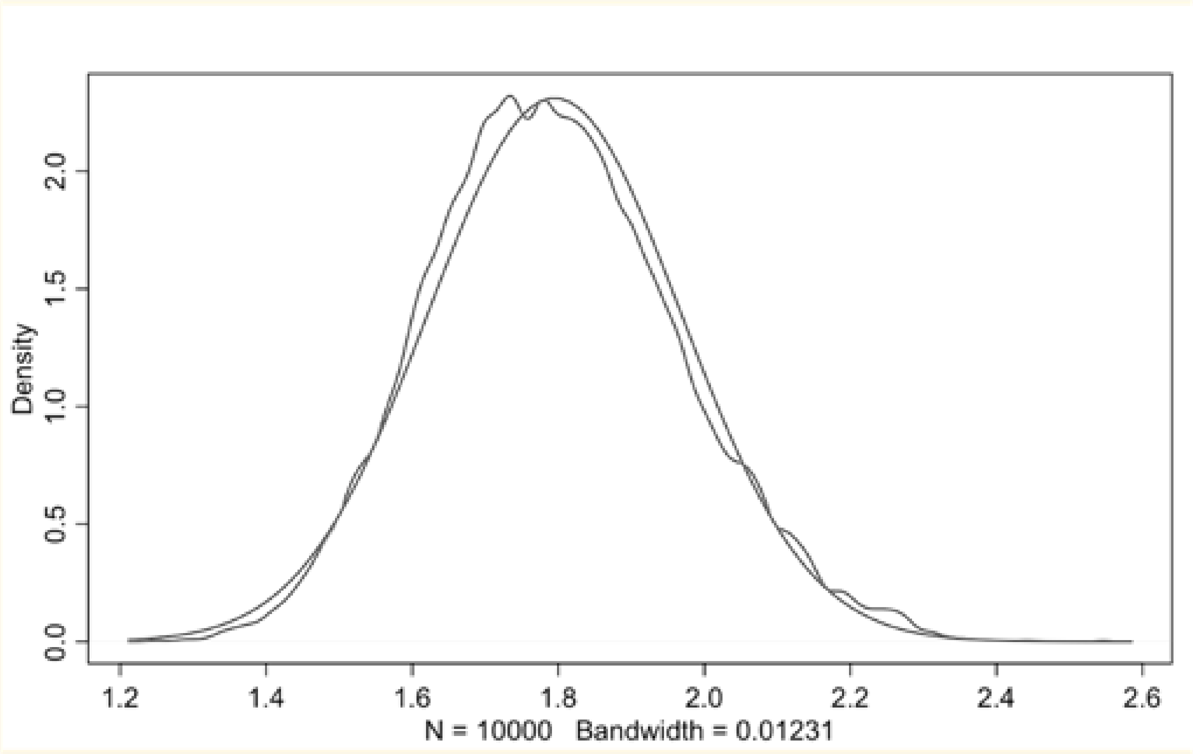
- There must be more uncertainty about how big the variance is than about how small it is

- If the variance is estimated to be near zero, then you know for sure that it can’t be much smaller. But it could be a lot bigger



Gaussian model of growth rate

- small effects that multiply together are approximately additive
- they also tend to stabilize on Gaussian distributions



A Language for Describing Models

1. Define variables: Data and Parameters

1. Define variables in terms of each other

1. Form a joint generative model by combining by combining variables to simulate hypothetical observations and analyze real ones

$$h_i \sim \text{Normal}(\mu, \sigma)$$
$$\mu \sim \text{Normal}(178, 20)$$
$$\sigma \sim \text{Uniform}(0, 50)$$

$$\mu_i = \alpha + \beta(x_i - \bar{x})$$
$$\alpha \sim \text{Normal}(178, 20)$$
$$\beta \sim \text{Normal}(0, 10)$$
$$\sigma \sim \text{Uniform}(0, 50)$$

Gaussian model of height

Generative model

How does **height** influence **weight**?

$$H \longrightarrow W \longleftarrow U$$

$$W = f(H, U)$$

"Weight is some function of height & unmeasured stuff"

Statistical model

How does average **weight** change with **height**?

$$W_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta H_i$$
$$\alpha \sim \text{Normal}(0, 10)$$
$$\beta \sim \text{Uniform}(0, 1)$$
$$\sigma \sim \text{Uniform}(0, 10)$$

Quadratic Approximation (QUAP)

- Approximate the posterior distribution as a multivariate Gaussian distribution
- Maximum A Posteriori (MAP) - a probabilistic framework for solving the problem of density estimation
- Provides approximations for each parameters marginal distribution

When $H = 0$, $W = 0$

Weight increases (on avg) with height

Weight (kg) is less than height (cm)

sigma must be positive

What the hell is going on

- Please help me

Linear Prediction

Regression - a general term, describes one or more predictor variables to model the distribution of one or more outcome variables.

Francis Galton - regress to the mean

Linear Prediction

Linear Model Strategy - make mu into a linear function of the predictor variable

Dear Golem: 'Consider all the lines that relate one variable to the other. Rank all of these lines by plausibility, given these data.'

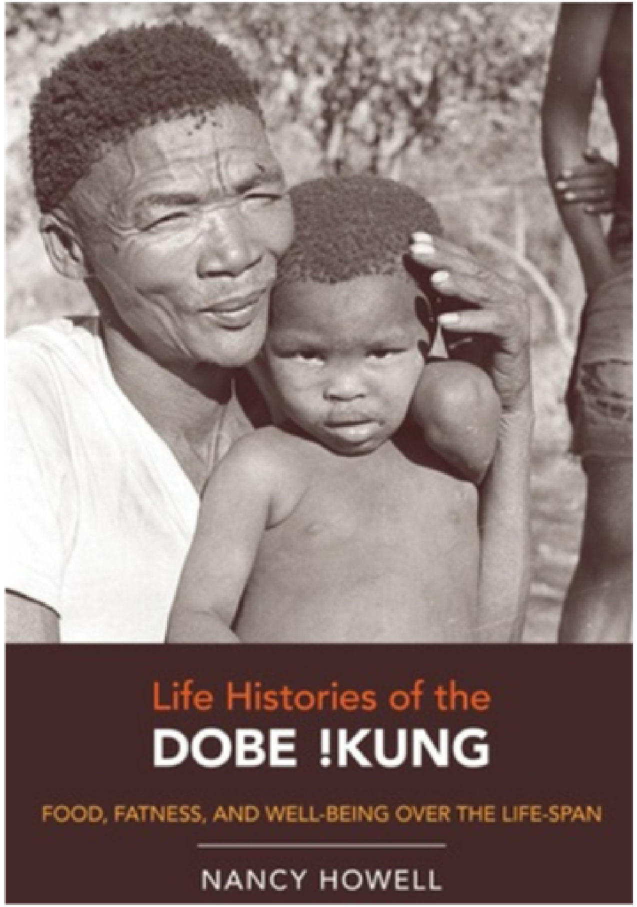
Golem: Here’s a posterior distribution.

Linear Prediction

Howell1 (adult) data

d <- Howell1

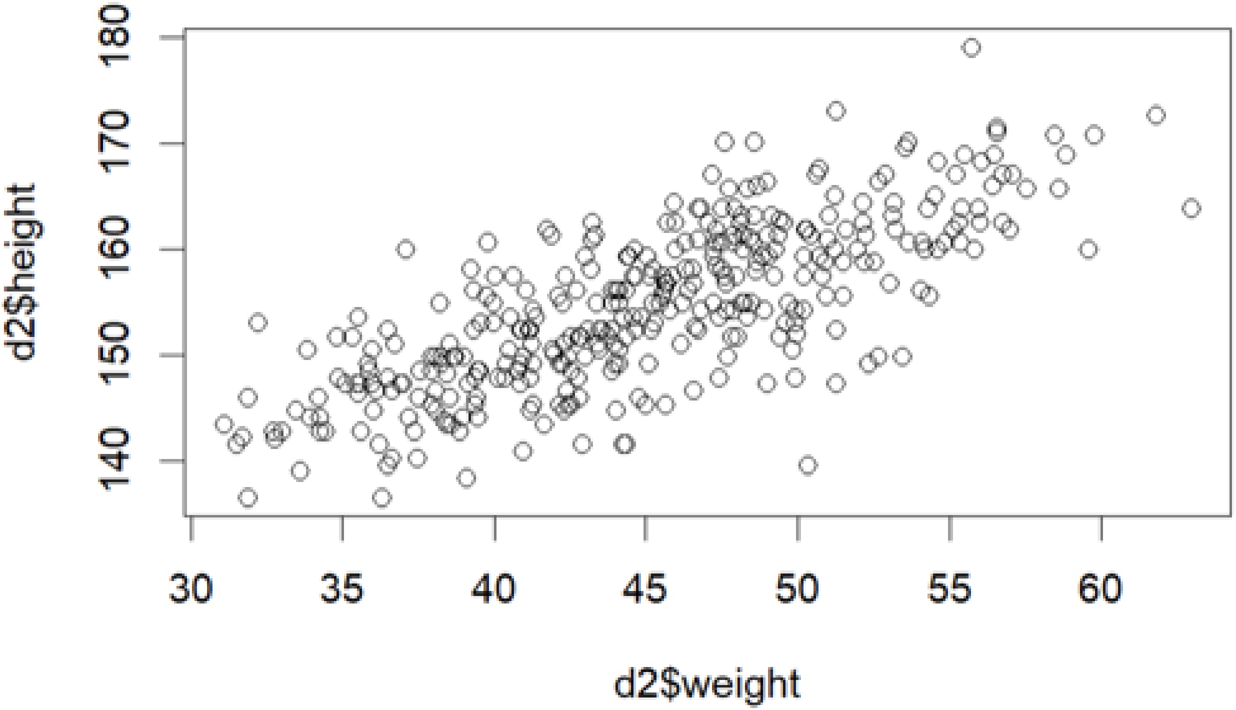
d2 <- d[d$age >= 18,]



Linear Prediction

plot(d2$height ~ d2$weight)

Height **covaries** with weight.



Linear Prediction - Posterior Distribution

$h_i \sim \text{Normal}(\mu, \sigma)$	`height ~ dnorm(mu,sigma)`
$\mu \sim \text{Normal}(178, 20)$	`mu ~ dnorm(178,20)`
$\sigma \sim \text{Uniform}(0, 50)$	`sigma ~ dunif(0,50)`

OK of Gaussian of height

$h_i \sim \text{Normal}(\mu_i, \sigma)$	`height ~ dnorm(mu,sigma)`
$\mu_i = \alpha + \beta(x_i - \bar{x})$	`mu <- a + b*(weight-xbar)`
$\alpha \sim \text{Normal}(178, 20)$	`a ~ dnorm(178,20)`
$\beta \sim \text{Log-Normal}(0, 1)$	`b ~ dlnorm(0,1)`
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Linear Prediction - Posterior Distribution

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Priors

$h_i \sim \text{Normal}(\mu_i, \sigma)$	`height ~ dnorm(mu,sigma)`
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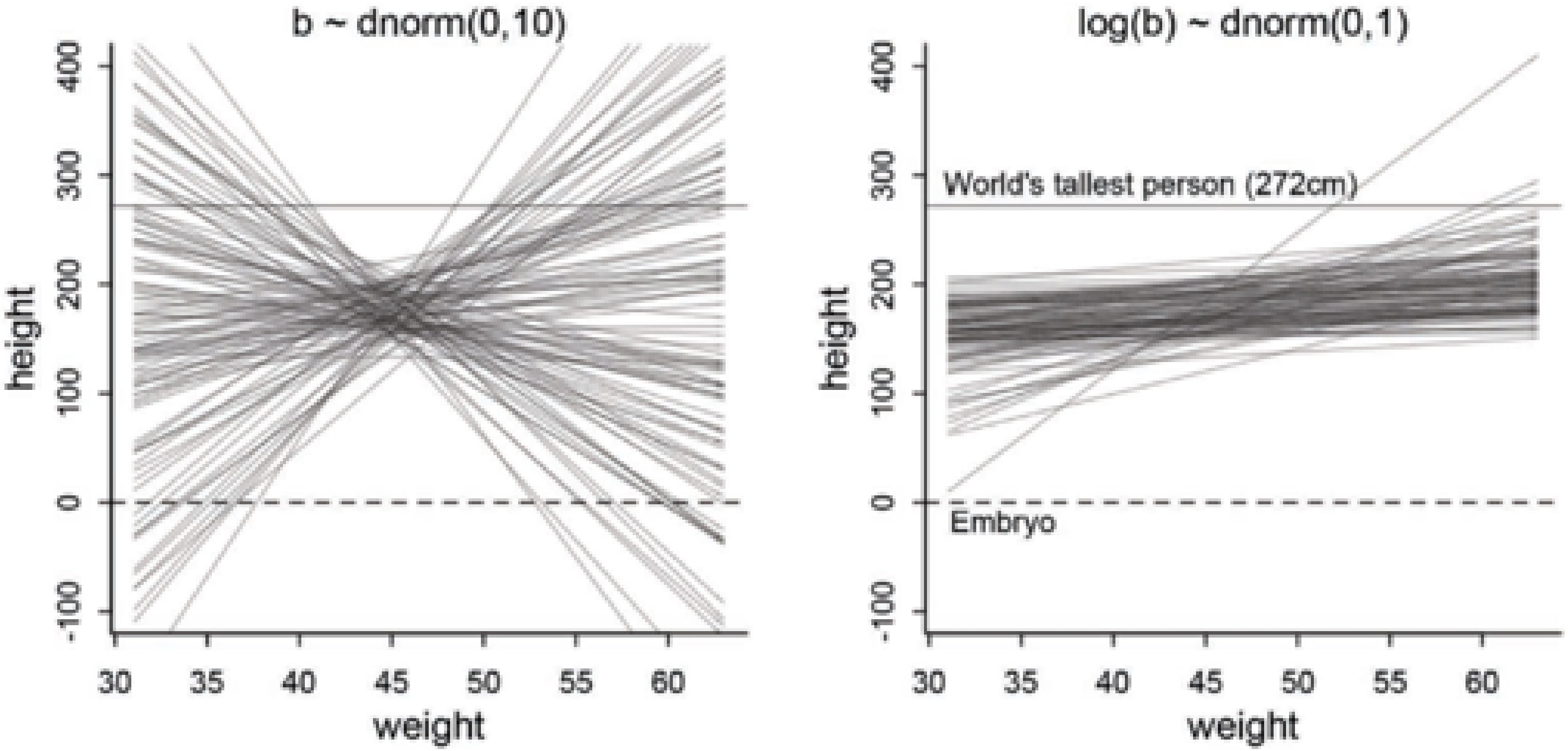
Linear Prediction - Priors



Linear Prediction - Posterior Distribution

$h_i \sim \text{Normal}(\mu, \sigma)$	`height ~ dnorm(mu,sigma)`
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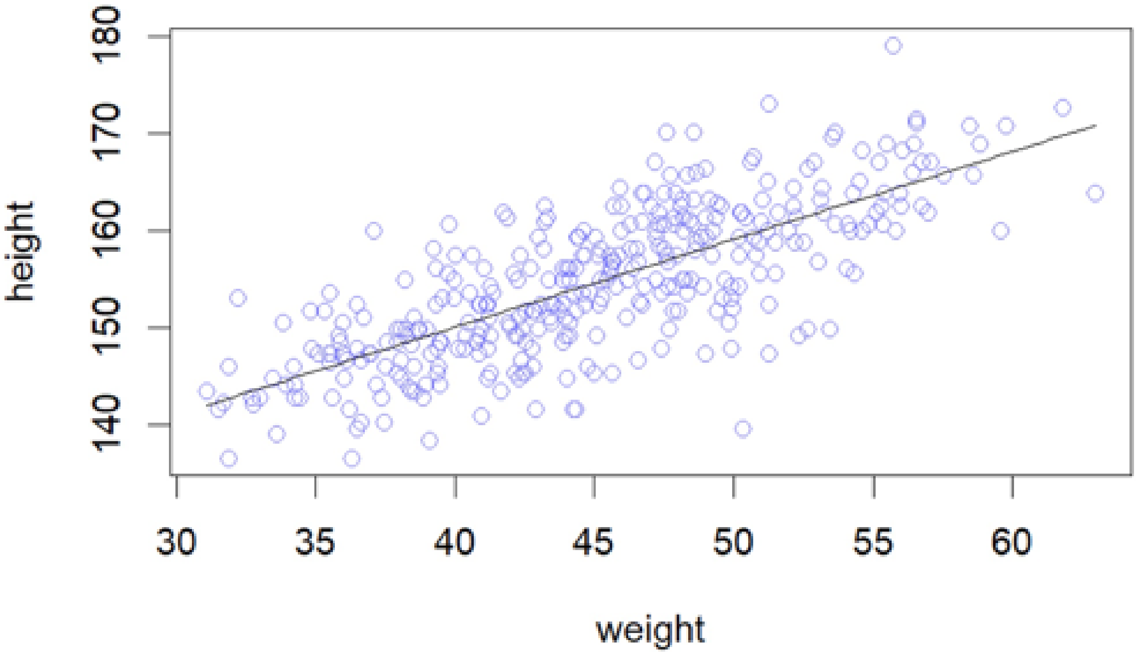
Mu function of weight

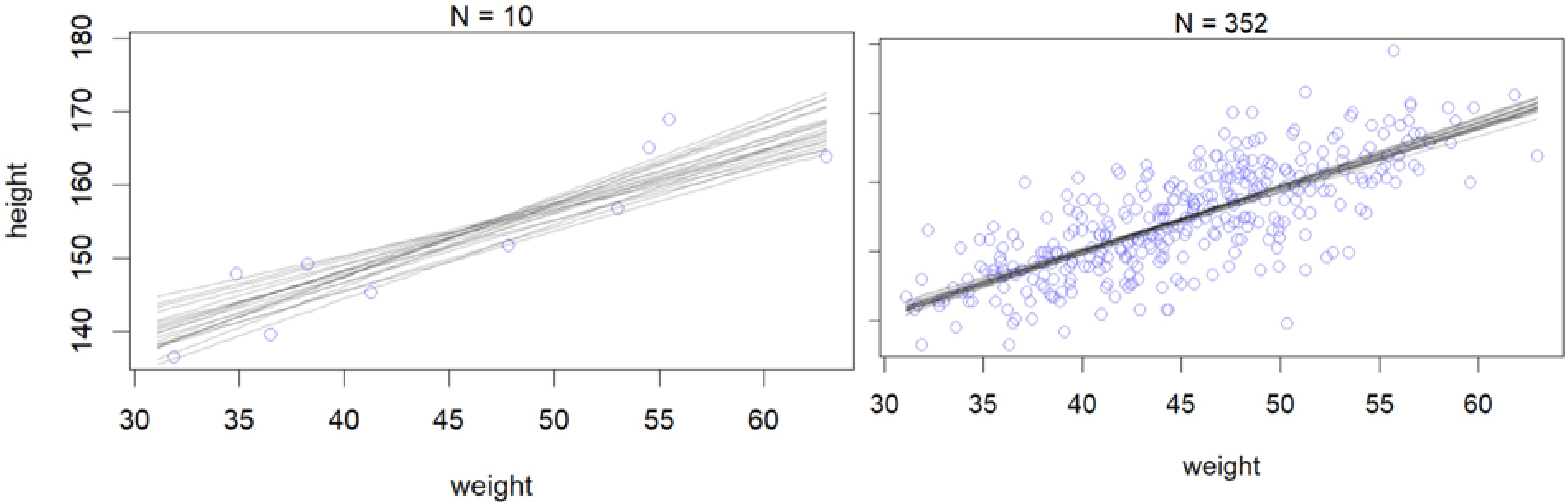
Linear Prediction - Uncertainty

Mu - made of parameters with distributions.

Posterior distribution considers every regression line connecting height and weight and assigns a relative plausibility to each.



Linear Prediction - Uncertainty



Curves from Lines

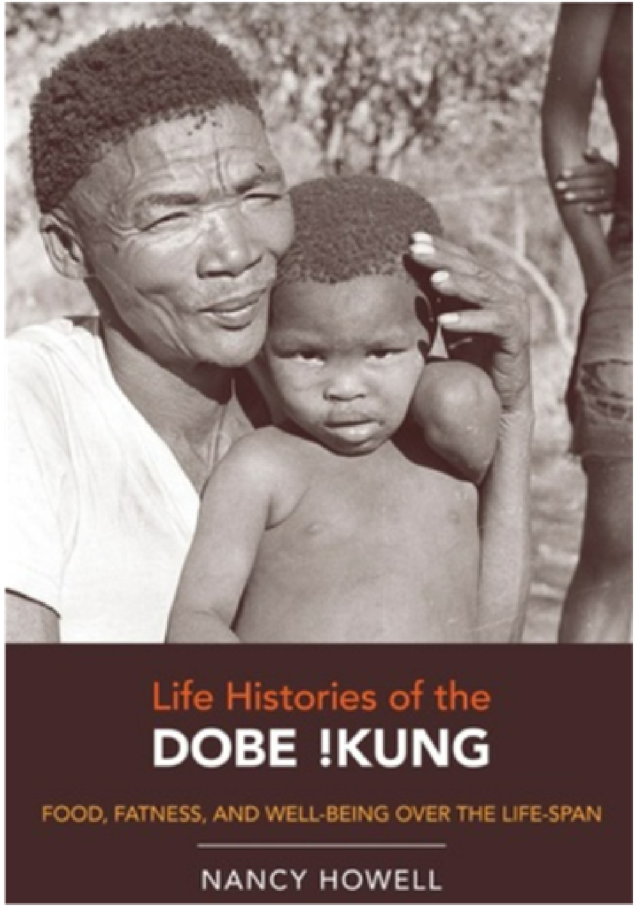
2 approaches in Chapter 4: **Polynomial** and **B-spline.**

Polynomial - add extra predictors that are powers of a variable.

Polynomial Regression

Howell1 (adult) data

```
d <- Howell1
d2 <- d[d$age >= 18, ]
```



Polynomial Regression

Linear

$$h_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta(x_i - \bar{x})$$
$$\alpha \sim \text{Normal}(178, 20)$$
$$\beta \sim \text{Log-Normal}(0, 1)$$
$$\sigma \sim \text{Uniform}(0, 50)$$

Polynomial

$$h_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_1 x_i + \beta_2 x_i^2$$
$$\alpha \sim \text{Normal}(178, 20)$$
$$\beta_1 \sim \text{Log-Normal}(0, 1)$$
$$\beta_2 \sim \text{Normal}(0, 1)$$
$$\sigma \sim \text{Uniform}(0, 50)$$

Polynomial Regression

Why is Beta1 dlnorm and Beta 2 dnorm?

Polynomial

$$h_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_1 x_i + \beta_2 x_i^2$$
$$\alpha \sim \text{Normal}(178, 20)$$
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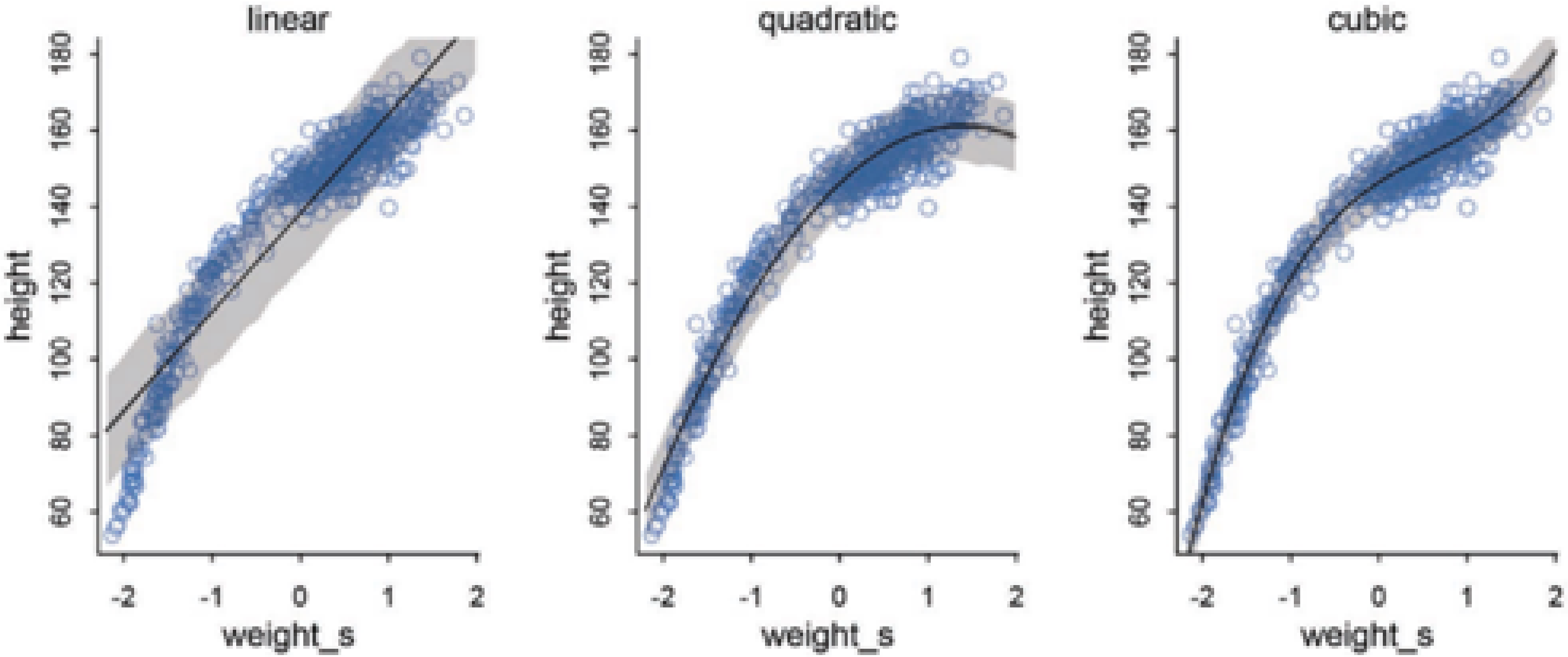
Polynomial Regression



B (Basis)-Splines

B-spline - generate synthetic predictor variables that turn off specific a parameters within a range of the real predictor variables.

Basis Function - each synthetic variable (many Betas)

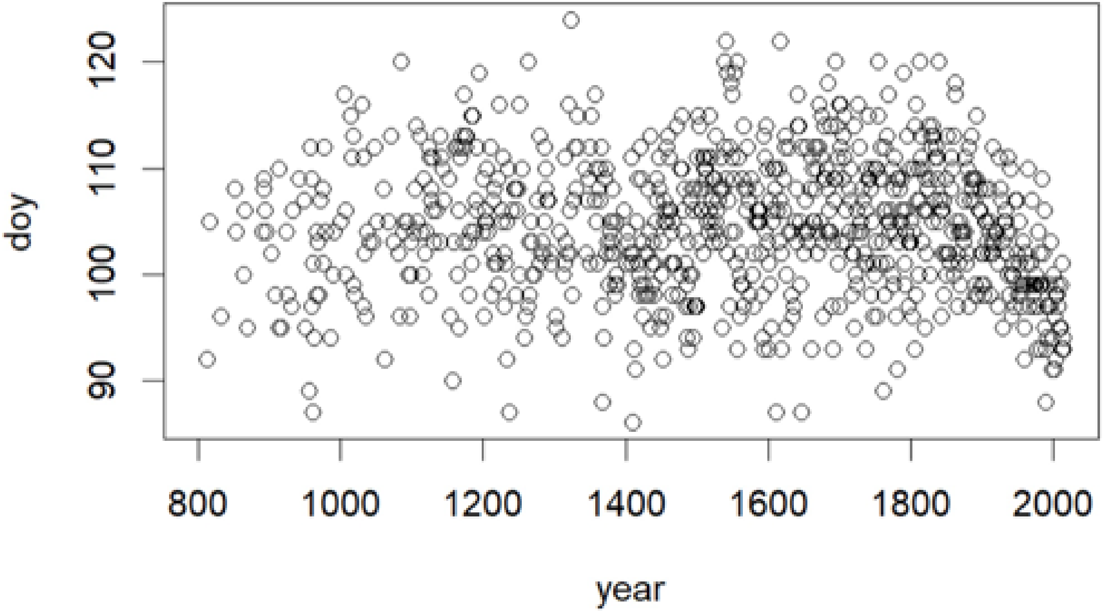
Knots - pivot points used to divide the horizontal axis.

Where should the knots go? You decide!

B-Splines

Cherry Blossoms in Japan from 812 CE - 2015 CE.

More “wiggly.”



B-Splines

$$\mu_i = \alpha + w_1 B_{i,1} + w_2 B_{i,2} + w_3 B_{i,3} + \ldots$$

mean

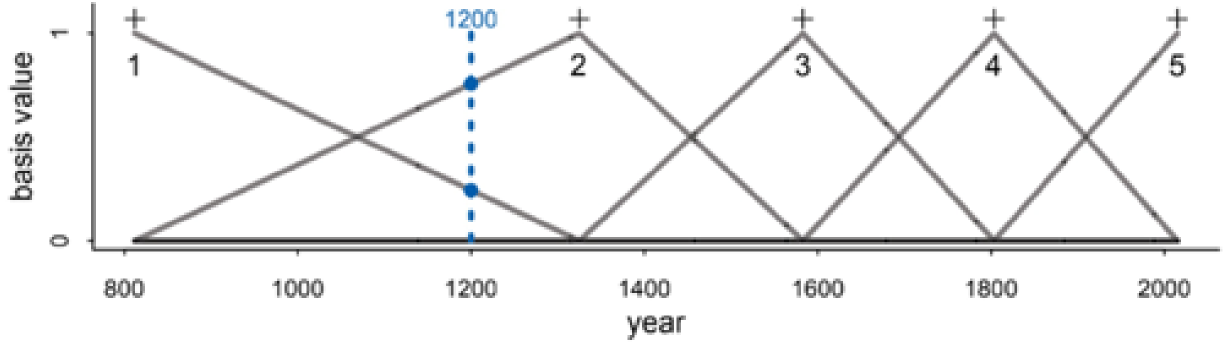
Intercept

Weight of basis function (influence wiggle)

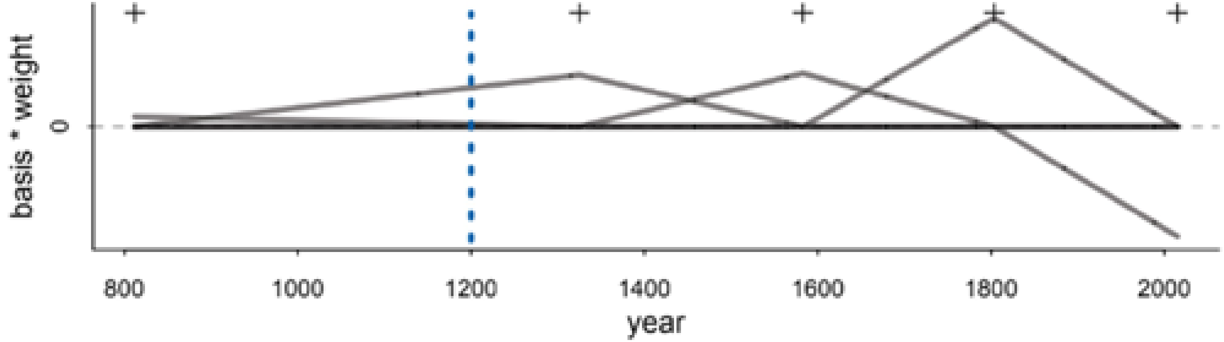
Basis function

B-Splines

Knots evenly spaced.



Knots weighted.



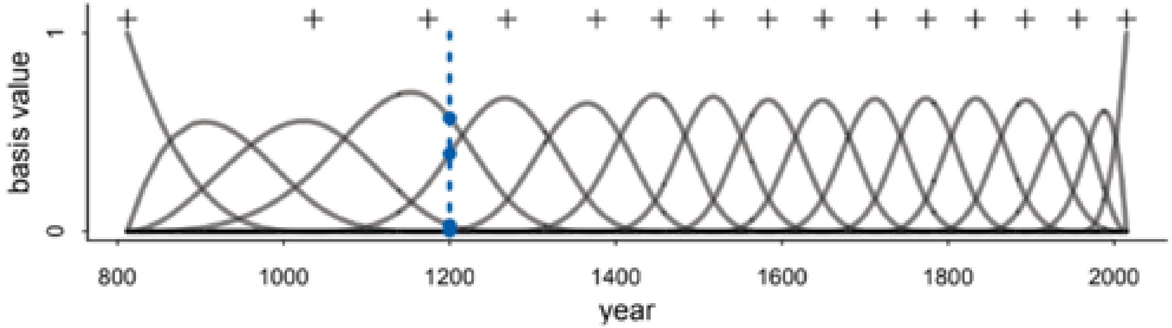
(Both Degree 1)

B-Splines - Degree

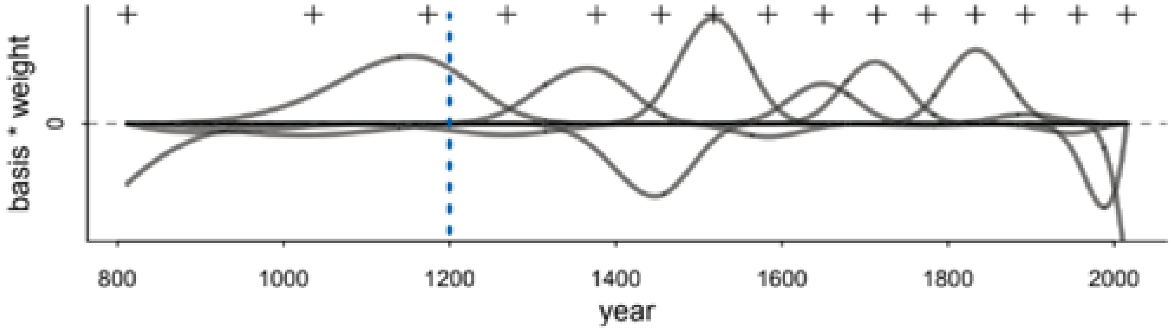
Degrees determine how many basis functions overlap at each knot.

B-Splines

Knots evenly spaced by year.



Knots weighted.



Degree 3 - 4 functions combine (cubic spline)

B-Splines - Priors

Probability of data

$$D_i \sim \text{Normal}(\mu_i, \sigma)$$

Probability of model

$$\mu_i = \alpha + \sum_{k=1}^{K} w_k B_{k,i}$$

Priors

$$\alpha \sim \text{Normal}(100, 10)$$

$$w_j \sim \text{Normal}(0, 10)$$

$$\sigma \sim \text{Exponential}(1)$$

Exponential distribution of sigma - contains no more than mean deviation

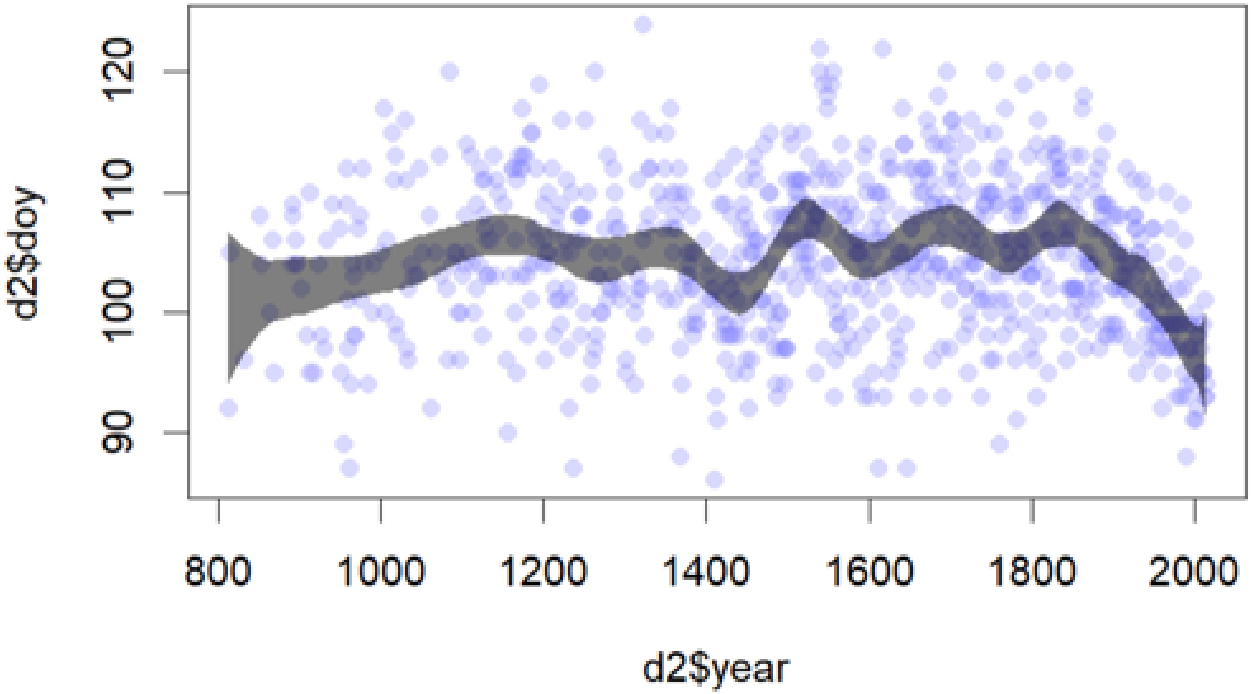
B-Splines - Posterior

Construct Posterior

```
m4.7 <- quap(
    alist(
        D ~ dnorm( mu , sigma ) ,
        mu <- a + B %*% w ,
        a ~ dnorm(100,10),
        w ~ dnorm(0,10),
        sigma ~ dexp(1)
    ), data=list( D=d2$doy , B=B ) ,
    start=list( w=rep( 0 , ncol(B) ) ) )
```

B-Splines - Posterior

Plot Posterior Predictions



B-Splines

How are this and this related?

$$\mu_i = \alpha + w_1 B_{i,1} + w_2 B_{i,2} + w_3 B_{i,3} + \ldots$$

$$D_i \sim \text{Normal}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \sum_{k=1}^{K} w_k B_{k,i}$$

$$\alpha \sim \text{Normal}(100, 10)$$

$$w_j \sim \text{Normal}(0, 10)$$

$$\sigma \sim \text{Exponential}(1)$$