Jessica Page

Homework 4

Problem 1]

(a) To obtain numerical results similar to the analytic answer

$$y = -\sin(\pi\cos(x))$$

it was necessary to set the tolerable error to 0.5, and the step size h equal to 0.01 using RK45 and allowing the first iteration of y_{i+1} and z_{i+1} calculations to be calculated with h. Proper data fitting was not implemented, but the comparison shows the code was working properly. (fig. 1)

(b) Comparing RK4 and RK5 methods individually added assurance that the process was working correctly as they are indistinguishable. (fig. 2)

(c) Using a variable step size required to meet a lower tolerance of 5×10^{-3}, the step size varied as a function of x as shown below in figure 3.

(d) The minimum step size h_{min} was found to be 1.7×10^{-6}. Using constant h_{min} as the step size for comparison, using the variable step size method was found to be 39ms in real time and 47ms in user time faster than a constant step size. The output is shown below.

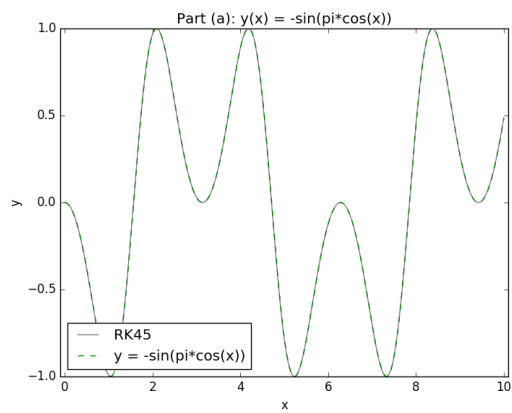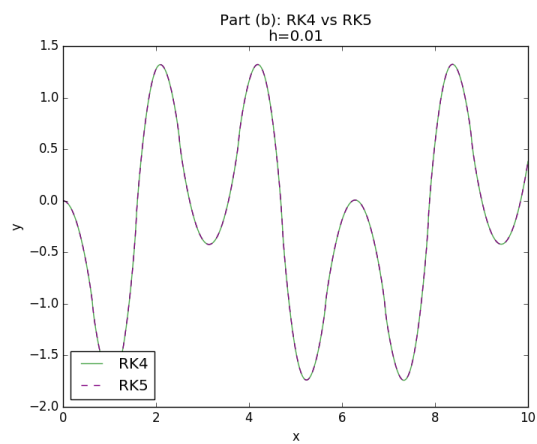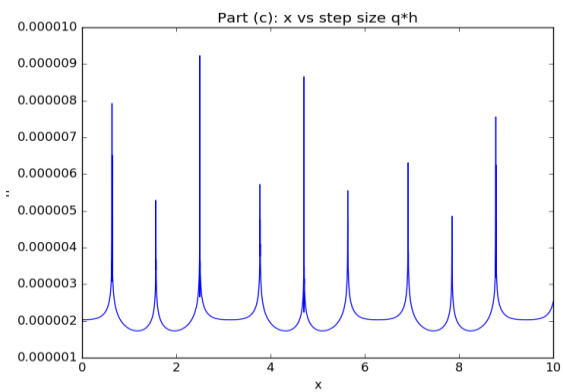| Constant h_{min}	variable h
real 0m0.776s	real 0m0.737s
user 0m0.702s	user 0m0.655s
sys 0m0.062s	sys 0m0.077s

Figure 1

Figure 2

Figure 3

Problem 2]

(a) Using the RK45 method to solve for y(t) yielded an exponential decay in height with time with a $y_{max} \sim 5$. Tolerance was set to 0.001 and dt was set to 0.1. (fig. 4)

(b) The results were not the accurate physical trend. The computational results are due to a variable q which can increase rapidly. The function was not analyzed well in the time for $y < y_{high}$ so it appears to automatically begin decreasing. The small amount of difference between y_{i+1} and z_{i+1} leads to a high q since they were identical during times when y is negative due to setting $Q_{out} = 0$ and the differences in RK4 and RK5 are mostly contained in the upper k values.

(c) Increasing the tolerance to 0.5 and setting q < 2 yielded a more accurate solution. (fig. 5)

(d) The reasoning behind a fixed time step is the same argument made in (b). If q can vary unchecked its possible to increase rapidly making it possible to skip over important points in the domain. (fig. 6)
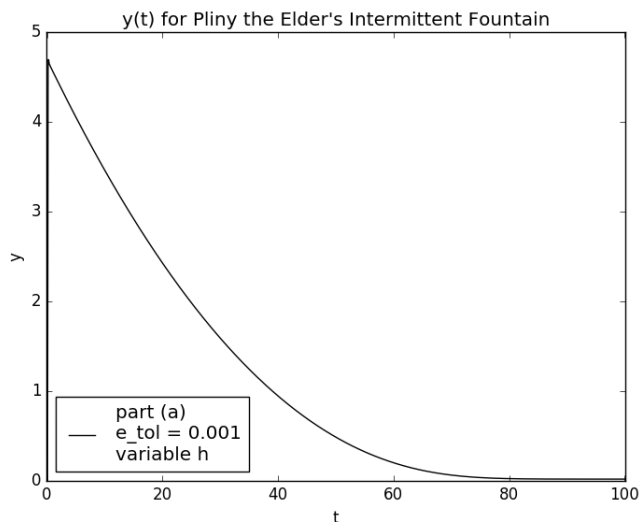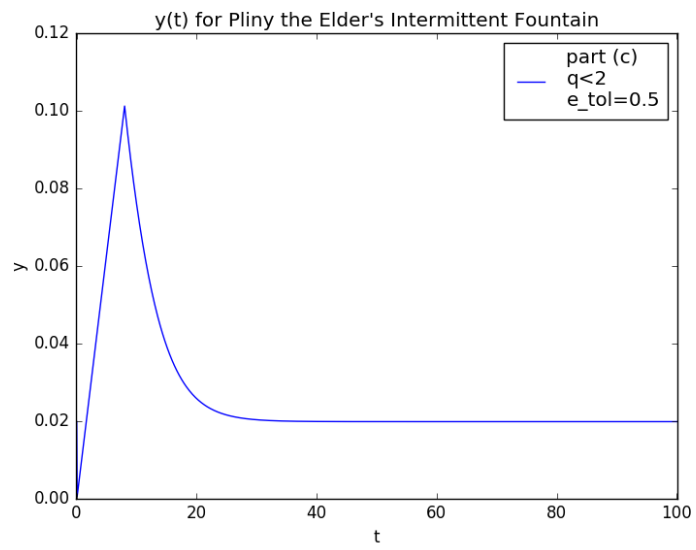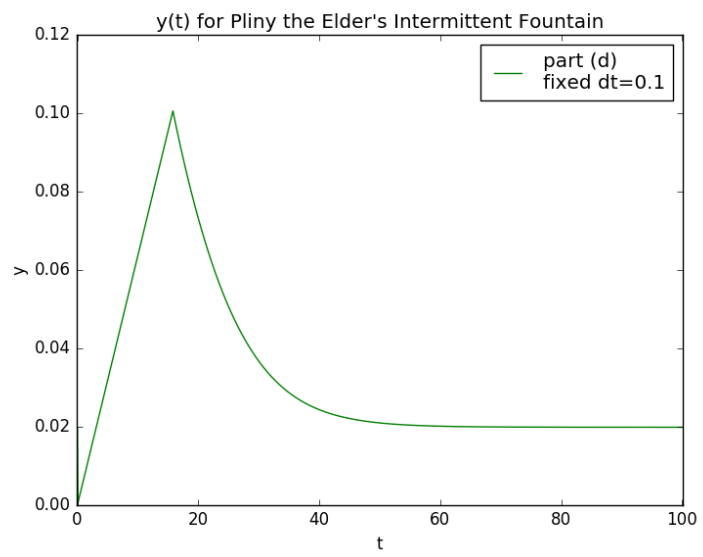

Figure 4

Figure 5

Figure 6