Historical arbitrage opportunities study for US treasury futures

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Content

- Introduction
- Treasury futures position analysis
- Empirical study for historical arbitrage opportunities
- Reference

Introduction

- Treasury futures
  Treasury futures contract is an agreement of two parties to lock the price of an underlying treasury bond on a specific future time.

- History

  - Thirty-year Treasury bond futures were originally introduced on the Chicago Board of Trade in 1977.
  - After that there were many kind of treasury futures emerged: 10-year, 5-year, 2-year Treasury note and 30-year Ultra Treasury bond futures
  - This product line has experienced tremendous success with the increase of scale and global significance of U.S.Treasury investment over the years.

Introduction

Contract:

	2-Year T-Note Futures	3-Year T-Note Futures	5-Year T-Note Futures	10-Year T-Note Futures	Classic T-Bond Futures	Ultra T-Bond Futures
Contract Size	$200,000 face-value U.S. Treasury notes		$100,000 face-value U.S. Treasury notes		$100,000 face-value U.S. Treasury bonds	
Delivery Grade	T-notes with original maturity of not more than 5 years and 3 months and remaining maturity of not less than 1 year and 9 months from 1st day of delivery month but not more than 2 years from last day of delivery month	T-Notes with original maturity of not more than 5-1/4 years and a remaining maturity of not more than 3 years but not less than 2 years, 9 months from last day of delivery month	T-notes with original maturity of not more than 5 years and 3 months and remaining maturity of not less than 4 years and 2 months as of 1st day of delivery month.	T-notes maturing at least 6-½ years but not more than 10 years, from 1st day of delivery month.	T-bonds with remaining maturity of at least 15 years but no more than 25 years.	T-bonds with remaining maturity of at least 25 years but no more than 30 years
Invoice Price	Invoice price = settlement price x conversion factor (CF) + accrued interest, CF = price to yield 6%					
Delivery Method	Via Federal Reserve book-entry wire-transfer					
Contract Months	March quarterly cycle – March, June, September, December					
Trading Hours	Open Auction: 7:20 am-2:00 pm, Monday-Friday; Electronic: 6:00 pm - 4:00 pm, Sunday-Friday (Central Times)					
Last Trading & Delivery Day	Business day preceding last 7 business days of month; last delivery day is last business day of delivery month					
Price Quote	In percent of par to one-quarter of 1/32nd of 1% of par ($15.625 rounded up to nearest cent)		Quoted in percent of par to one-half of 1/32nd of 1% of par ($15.625 rounded up to nearest cent)		Quoted in percent of par to 1/32nd of 1% of par ($31.25)	

Introduction

- Underlying bond
  Each treasury futures contract has an underlying bond, the price of the futures is based on this underlying bond. For example, US 10 years treasury futures is based on the coupon rate 6% and 10 years US federal bond. In fact the underlying bond may not exist in the real world, it is only used as a standard to calculate the price of the futures.
- Eligible to delivery bond
  At the delivery date, the short position of the treasury futures has the right to buy the bond from market, there are some bonds in the market that meet the requirements for delivery, the pool of those bonds is called the eligible to delivery bond.
- Cheapest to delivery bond(CTD)
  Still at the delivery date, the shorter will choose the cheapest bond for deliverying, this bond is called cheapest to delivery bond(CTD). The right that the shorter can choose the bond for deliverying maybe the most important difference between the stock index futures and treasury futures.

Introduction

- Conversion factor

  Since the shorter has the right to choose CTD, we should find a standard to compare those bonds. It is the reason to introduce the conversion factor. The formula is as follows:

  $CF = \sum_{i=1}^{n} \frac{C_i}{(1+YTM)^{T_i}}$, where

  CF: conversion factor
  C_i: the i term cash value of the CTD
  YTM: the coupon rate of the underlying bond
  T_i: the payment time of cash value
  The meaning for conversion factor is that we discounted the future cash flow of the CTD with the coupon rate of the underlying bond. typically, we used the 6% for the YTM and if the coupon rate of CTD is bigger than 6%, the CF will be bigger than 1 and vice versa. It is because that the CTD is more valueable than the underlying, so the long position will pay more money at the delivery time.

Introduction

- Invoice price

  Invoice price is the real money that the long position pay to the short position at the delivery date.

  Invoice Price = Futures price * Conversion Factor(CF) + Accrued Interest

  From this formula, we can also see where to use the conversion factor.

Introduction

- Delivery process
  - Position day
    The day in which the CBOT is given notice by the futures short that he plans to make delivery of a certain amount of bonds in two business days.
  - Notice day
    The board of CBOT will inform the long position that she will be delivered to the next day.
  - Delivery day
    The short must have in his account by 10 A.M. CST the bonds he has specified he would deliver and actually must deliver them by 1 P.M. CST to the long party

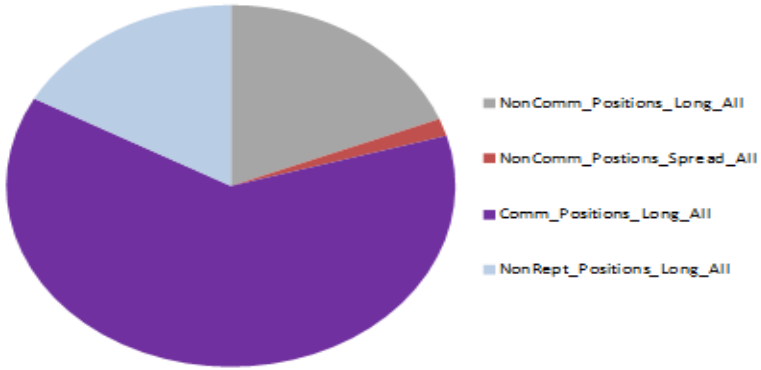
Position analysis

We can get the futures position report on the US Commodity Futures Trading Commission(CFTC). The report can be downloaded from the following website: http://www.cftc.gov/MarketReports/index.htm
In the following, I listed the meaning of the fields of this position report.

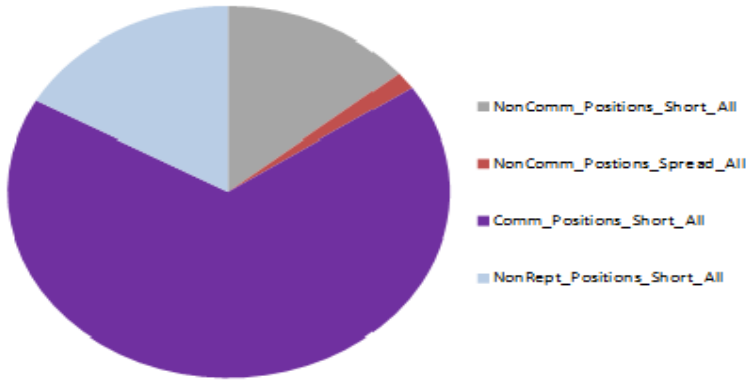
- Non-Commercial:The speculator positions, most of them are hedging funds.
- Commercial: The main purpose for this positions is to hedge risk.
- Total: Total positions.
- Non-Reportable position: Most of them are non institution investors and individual investor.
- Spreads: Hold both short and long position in the same time, most of them are arbitragers.

Position analysis

We got the CFTC report from 2000 to 2013 and drew the following charts:
Long position structure:



Short position structure:



Position analysis

We also studied the annual report for the company Pimco,which is the world largest bond investor. The website to download the annual report is as follows:
https://investments.pimco.com/Products/pages/283.aspx
The following chart listed the products in Pimco which included the US treasury futures.

PIMCO Inflation Response Multi-Asset Fund	U.S. Treasury 10-Year Note June Futures		Long	240
	U.S. Treasury 10-Year Note May Futures	$129.00	short	3
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	3
PIMCO Investment Grade Corporate Bond Fund	U.S. Treasury 10-Year Note June Futures		Long	877
	U.S. Treasury 10-Year Note May Futures	$ 129.000	short	147
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	147
PIMCO Moderate Duration Fund	U.S. Treasury 10-Year Note June Futures		Long	116
PIMCO Unconstrained Tax Managed Bond Fund	U.S. Treasury 2-Year Note June Futures		Short	55
	U.S. Treasury 5-Year Note June Futures		Short	163
	U.S. Treasury 5-Year Note June Futures		Long	105
	U.S. Treasury Ultra Long-Term Bond June Futures		Long	1
PIMCO High Yield Portfolio	Euro-OAT France Government 10-Year Bond June Futures		Short	60
	U.S. Treasury 10-Year Note June Futures		Long	950
PIMCO Investment Grade Corporate Portfolio	U.S. Treasury 10-Year Note May Futures	$129.00	Short	56
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	56
PIMCO Long Duration Corporate Bond Portfolio	U.S. Treasury 10-Year Note May Futures	$ 129.000	Short	188
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	188
PIMCO Low Duration Portfolio	U.S. Treasury 5-Year Note June Futures		Short	105
PIMCO Moderate Duration Portfolio	U.S. Treasury 5-Year Note June Futures		Short	46
PIMCO Real Return Portfolio	U.S. Treasury 10-Year Note May Futures	$129.00	Short	49
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	49
PIMCO U.S. Government Sector Portfolio	U.S. Treasury 10-Year Note June Futures		Long	12,239
PIMCO International Portfolio	Euro-Bund 10-Year Bond June Futures		Short	893
PIMCO Total Return Fund	Euro-Bund 10-Year Bond June Futures		Short	7,243
	U.S. Treasury 10-Year Note June Futures		Long	69,160
	U.S. Treasury 10-Year Note May Futures	$129.00	Short	3,975
	U.S. Treasury 10-Year Note May Futures	$130.00	Short	1,400
	U.S. Treasury 10-Year Note May Futures	$130.00	Short	1,400
	U.S. Treasury 10-Year Note May Futures	$132.50	Long	3,975
	U.S. Treasury 10-Year Note May Futures	$133.00	Long	1,400

Basis

Although in US market, the treasury futures is typically used for hedging risk, it is still very meaningful that we do some researches for the arbitrage opportunities. The most common used method is to find the non zero basis.

Basis=(bond price - future price * conversion factor)

If the basis is not equal to zero, it means that there are some difference between the spot and futures, then the arbitrage opportunities may exist. our strategy is to calculate the theorem price of the futures and compare with the real future price to see if the non zero basis exists.

Basis

Theorem price of treasury futures:

$$F_t = \frac{(P_t + AI_t) * (1 + r * \frac{(T-t)}{365}) - \sum_{i=1}^{n} \frac{C_i}{f} * (1 + \frac{\bar{r}}{365} * (T - S_i)) - AI_T}{CF}$$

Here:

P_t : clean price of bond

AI_t: interest at time t

r: loan rate

$\bar{r}$:re-investment rate

f: is the coupon payment frequency

s_i: coupon payment time

AI_T: interest at time T

Arbitrage strategy

If we found the theorem price is lower than the real price, we can use the following method to realize arbitrage:

At time t	loan money $P_t + AI_t$ from bank with the loan rate r, buy a bond and sell short treasury futures.
At time T	Delivery the bond to the long side of treasury futures and get the money F*CF, Return the loan to the bank.
Realized profit	$F*CF + \sum_{i=1}^{n} \frac{C_i}{f}(1 + \frac{\bar{r}}{365} * (T - S_i)) + AI_T - (P_t + AI_t) * (1 + r * \frac{T-t}{365})$

Arbitrage strategy

Next, we apply this strategy into our real world to find the historical arbitrage opportunities for 5 years treasury futures(ZFU3) and 10 years treasury futures(TYU3). The following chart listed the basic information of their eligible to delivery bonds (data from Bloomberg):
For ZFU3, there are 8 eligible for delivery bonds:

Bond code	Conversion factor	Coupon rate	Bond ISSUE Date	Bond maturity date
US912828UA67 Govt	0.8044	0.625	2012/11/30	2017/11/30
US912828UZ19 Govt	0.7874	0.625	2013/4/30	2018/4/30
US912828UE89 Govt	0.8056	0.75	2012/12/31	2017/12/31
US912828UR92 Govt	0.7989	0.75	2013/2/28	2018/2/28
US912828UU22 Govt	0.7956	0.75	2013/4/1	2018/3/31
US912828UJ76 Govt	0.8069	0.875	2013/1/31	2018/1/31
US912828VE70 Govt	0.7991	1	2013/5/31	2018/5/31
US912828VK31 Govt	0.8113	1.375	2013/7/1	2018/6/30

Arbitrage strategy

For TYU3, there are 17 eligible for delivery bonds:

Bond code	Conversion factor	Coupon rate	Bond ISSUE Date	Bond maturity date
US912828UV05 Govt	0.7408	1.125	2013/4/1	2020/3/31
US912828VA58 Govt	0.7408	1.125	2013/4/30	2020/4/30
US912828VF46 Govt	0.7541	1.375	2013/5/31	2020/5/31
US912828TJ95 Govt	0.7055	1.625	2012/8/15	2022/8/15
US912828TY62 Govt	0.6991	1.625	2012/11/15	2022/11/15
US912828SV33 Govt	0.7202	1.75	2012/5/15	2022/5/15
US912828VB32 Govt	0.6956	1.75	2013/5/15	2023/5/15
US912828VJ67 Govt	0.7738	1.875	2013/7/1	2020/6/30
US912828RR30 Govt	0.7488	2	2011/11/15	2021/11/15
US912828SF82 Govt	0.7426	2	2012/2/15	2022/2/15
US912828UN88 Govt	0.7191	2	2013/2/15	2023/2/15
US912828RC60 Govt	0.7626	2.125	2011/8/15	2021/8/15
US912828NT32 Govt	0.8149	2.625	2010/8/16	2020/8/15
US912828PC88 Govt	0.8094	2.625	2010/11/15	2020/11/15
US912828QN35 Govt	0.8284	3.125	2011/5/16	2021/5/15
US912828ND89 Govt	0.8671	3.5	2010/5/17	2020/5/15
US912828PX26 Govt	0.862	3.625	2011/2/15	2021/2/15

Results

We test the results under different interest adjustment:
Data for ZFU3 is from 20130102 to 20130715, totally 134 trading days, for TYU3 is from 20120620 to 20130701, totally 279 trading days. The r we used is LIBOR rate and $\bar{r}$ we used is saving rate(Data From BB). I listed the results under different interest rate scenarios

interest adjustment=0:

Treasury futures name	Observation numbers	Arbitrage numbers	Average profit
ZFU3(5 years)	134	11	0.032%
TYU3(10 years)	279	64	0.1405%

Results

interest adjustment=0.0005:

Treasury futures name	Observation numbers	Arbitrage numbers	Average profit
ZFU3(5 years)	134	1	0.00838%
TYU3(10 years)	279	12	0.12%

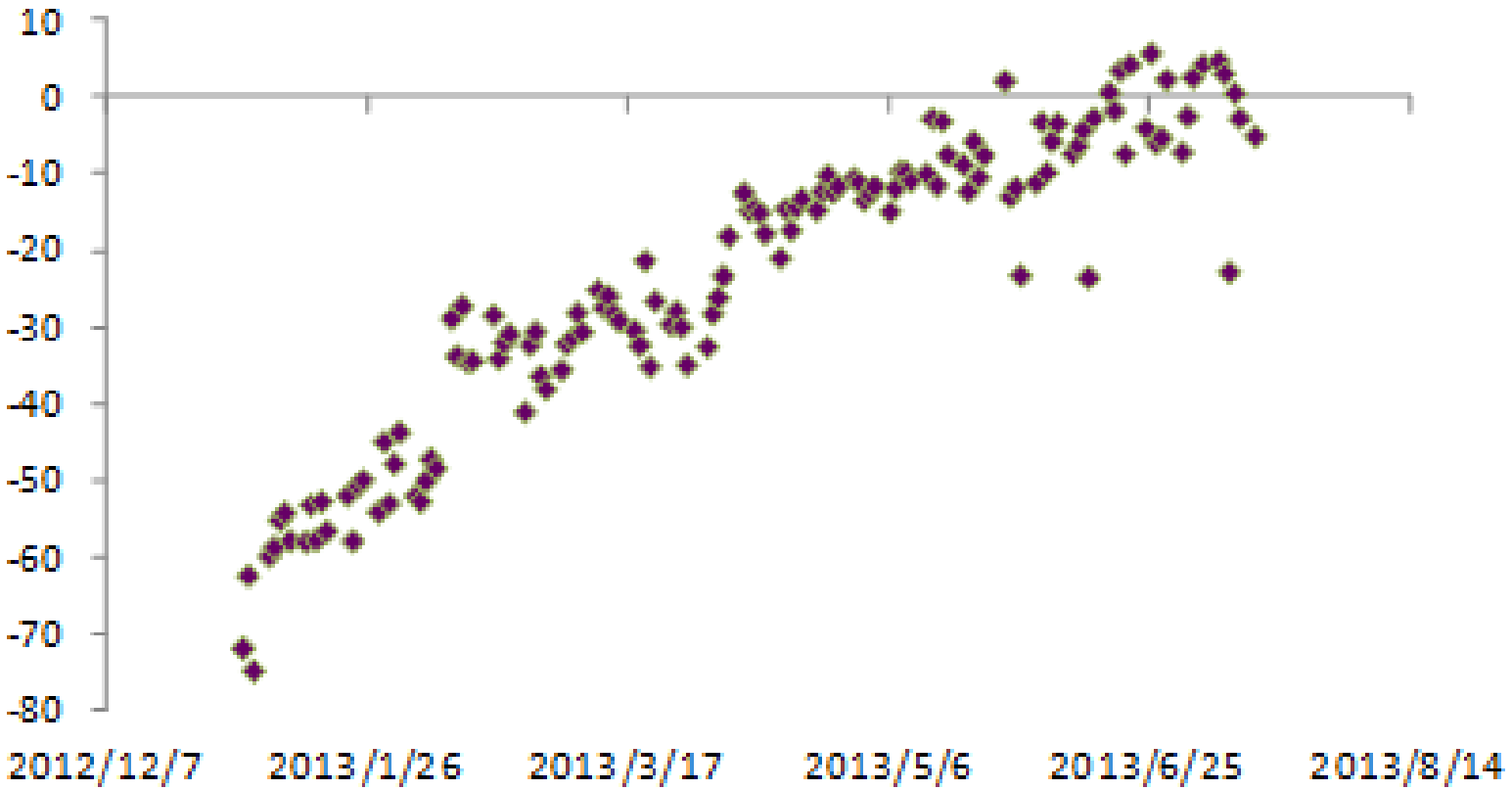
Results

interest adjustment=-0.0005:

Treasury futures name	Observation numbers	Arbitrage numbers	Average profit
ZFU3(5 years)	134	22	0.0505%
TYU3(10 years)	279	89	0.182%

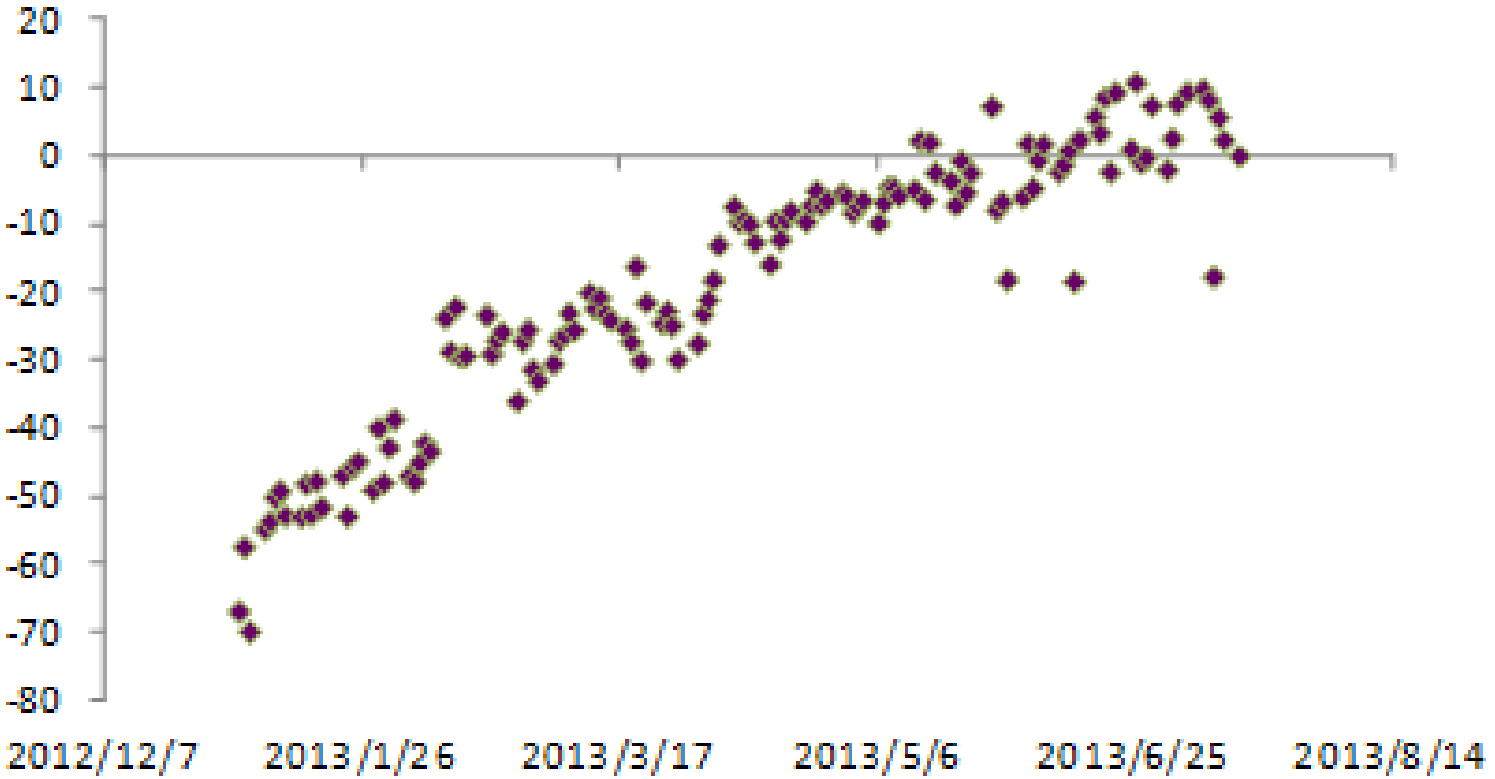
Results

The difference between True and theorem price for ZFU3
Under interest adjustment 0.



Results

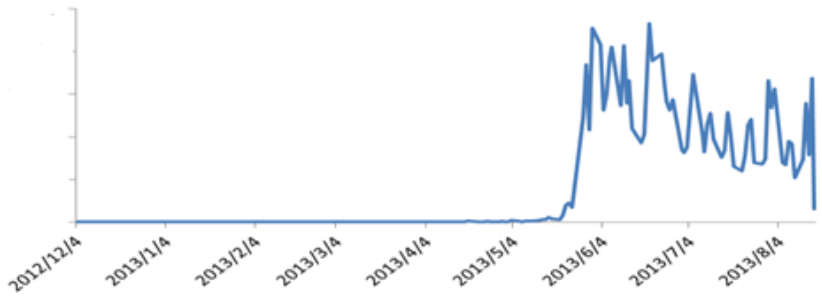
The difference between True and theorem price for ZFU3
Under interest adjustment -0.0005



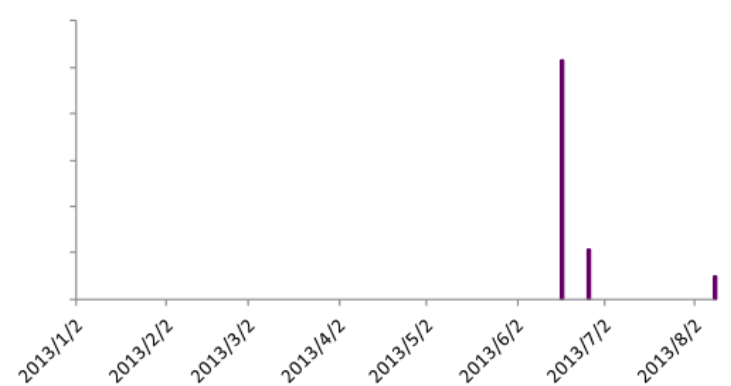
Conclusion

- Most treasury futures in US market are used to hedge risk. Reason: Mature market
- The arbitrage opportunities for long term treasury futures are more than short term treasury futures. Reason: The long term treasury futures are more sensitive to the economy environment change.
- Most arbitrage opportunities occurred closed to maturity. Reason: The trading volume of treasury futures increased dramatically when the time is close to maturity.

For TYU3:



For ZFU3:



Conclusion

- Although there are some arbitrage opportunities, the profit of the arbitrage is very low
- When the interest rate increases, the arbitrage opportunities decreases and vice versa. Reason: the influence of interest rate on the loan money is bigger than on the coupon reinvestment.
- We also did the same work for high frequency data from BB these days, details can be talked in the future if this topic can be published.

1)The handbook of fixed income security, 6 edition,Frank J. Fabozzi

2)Price Discovery In the Treasury Futures Market, MICHAEL W.BRANDT

3)Are There Arbitrage Opportunities in the Treasury -Bond Futures Market? ,Robert W.Kolb

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