Cournot Model with Symmetric Costs

Jikhan Jeong

April 12, 2022

This tutorial is just for my students; therefore, please do not upload on the web

- Consider an industry with N = 2 firms selling a homogeneous product (duopoly).
- Every firm independently and simultaneously chooses its profit maximizing output (q_1 for firm 1 and q_2 for firm 2).
- The inverse demand function: $p(q_1, q_2) = a - b(q_1 + q_2)$, where a, b > 0
- Firm 1's total cost function is $TC_1(q_1) = cq_1$, where c >0
- Firm 2's total cost function is $TC_2(q_2) = cq_2$, where c >0.
- Here, marginal cost for Firms 1 and 2 are the same (Yes, this problem is a sysmetrics cost case).
- **Firm1**'s profit function is: max $\pi_1 = TR_1 - TC_1 = p(q_1, q_2)q_1 - cq_1 = [a - b(q_1 + q_2)]q_1 - cq_1$
- To maximize the Firm 1's profits, Firm 1 differentiate its profit function with repect to q_1,
- First order condition (FOC) respect to $q_1 : \frac{\partial \pi_1}{\partial q_1} = \frac{[a-b(q_1+q_2)]q_1 - cq_1}{\partial q_1} = a - 2bq_1 - bq_2 - c = 0$
- Rearranging and solving for q_1, then we can get a Firm 1's "best response function (BRF)" as follow:
- $\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c = 0$
- $a - 2bq_1 - bq_2 - c = 0$
- $a - bq_2 - c = 2bq_1$, both sides are divided by 2b, then
- $q_1(q_2) = \frac{a-bq_2-c}{2b} = \frac{a-c}{2b} - \frac{q_2}{2} \leftarrow$ Best response function of Firm 1 (BRF1)
- **Firm2**'s profit function is: max $\pi_2 = TR_2 - TC_2 = p(q_1, q_2)q_2 - cq_2 = [a - b(q_1 + q_2)]q_2 - cq_2$
- Firms 1 and 2 have the same marginal cost (i.e., c); therefore, it is a symmetric problem. BRF and the optimal quantity must be the same between Firms 1 and 2.
- $q_2(q_1) = \frac{a-bq_1-c}{2b} = \frac{a-c}{2b} - \frac{q_1}{2} \leftarrow$ Best response function of Firm 2 (BRF2).
- Plug BRF2 into BRF1 to get the optimal quanity 1, q_1^*
- $q_1(q_2) = q_1(q_2(q_1)) = \frac{a-bq_2(q_1)-c}{2b} = \frac{a-c}{2b} - \frac{q_2(q_1)}{2} = \frac{a-c}{2b} - \frac{1}{2}[\frac{a-c}{2b} - \frac{q_1}{2}]$, then
- $q_1(q_2) = \frac{a-c}{2b} - \frac{1}{2}[\frac{a-c}{2b} - \frac{q_1}{2}]$, the function contains only one unknown variable q_1^*
- $q_1 = \frac{a-c}{2b} - \frac{1}{2}[\frac{a-c}{2b} - \frac{q_1}{2}]$, to simplify the equestion, multiply both sides with 4b (because 4b is the maximum denominators among variables)
- $4bq_1 = 2(a - c) - (a - c) + bq_1 = (a - c) + bq_1$
- $4bq_1 - bq_1 = 3bq_1 = (a - c)$
- $q_1^* = \frac{(a-c)}{3b}$, as it is a symmetric problem, $q_1^* = q_2^*$. Heavy math? Yes, but we are almost done!!!!!
- Inverse demand curve is $p(q_1, q_2) = a - b(q_1 + q_2)$, we can get the optimal price by pluging the optimal quantities for Firms 1 and 2.
- $p(q_1^*, q_2^*) = a - b(q_1^* + q_2^*) = a - b[\frac{2(a-c)}{3b}] = \frac{3a}{3} - \frac{(2a-2c)}{3} = \frac{a+2c}{3} = p^*$
- $p^* = \frac{a+2c}{3}$
- Now we can get optimal profits for each Firm. It is a symmetric problem so that the maximum profit for Firms 1 and 2 must be the same.
- Plug optimal quanity and price for the profit function for the Firm 1
- $\pi_1 = TR_1 - TC_1 = p(q_1, q_2)q_1 - cq_1 = [a - b(q_1 + q_2)]q_1 - cq_1$

- $\pi_1^* = \pi_2^* = p(q_1, q_2)q_1 - cq_1 = (p^* - c)q_1^* = (\frac{a+2c}{3} - c)\frac{(a-c)}{3b} = \frac{a+2c-3c}{3}\frac{(a-c)}{3b} = \frac{(a-c)(a-c)}{9b} = \frac{(a-c)^2}{9b}$
- Summary of results
-

	Firm 1	Firm 2
Profit Function	$\pi_1 = p(q_1, q_2)q_1 - cq_1 = [a - b(q_1 + q_2)]q_1 - cq_1$	$\pi_2 = p(q_1, q_2)q_2 - cq_2 = [a - b(q_1 + q_2)]q_2 - cq_2$
BRF	$q_1(q_2) = \frac{a-bq_2-c}{2b} = \frac{a-c}{2b} - \frac{q_2}{2}$	$q_2(q_1) = \frac{a-bq_1-c}{2b} = \frac{a-c}{2b} - \frac{q_1}{2}$
Optimal q	$q_1^* = \frac{(a-c)}{3b}$	$q_2^* = \frac{(a-c)}{3b}$
Optimal p	$p^* = \frac{a+2c}{3}$	
Max profit	$\pi_1^* = \frac{(a-c)^2}{9b}$	$\pi_2^* = \frac{(a-c)^2}{9b}$

- Sympolic Python Duopoly Example with Python: `https://github.com/Jikhan-Jeong/Symbolic-Python/blob/master/4_2_2022_Symbolic_Python_for_Economic_Analysis.ipynb`