Risk Model

Group 3 – Risk Model - Operational Risk
Quantitative Risk Management – ECON 6295_10
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Business Case and Data Description

Our group is tasked with assisting our client, the operational risk department of a firm, in forecasting operational losses over the next 12 months. To achieve this, we leveraged their Internal loss database (ILD), which contains 213 unique events recorded over a 200-month period. The ILD documents operational loss events only when the loss amount exceeds a threshold of 1, meaning there are no records of events with losses below this value. Not every month has recorded loss events.

The ILD includes Period (Indicating the month of the event), EventID (a unique identifier for each event) and Amount (the monetary loss associated with each event).

Model Description

We employed LDA, one of various methods for calculating operational risk capital. This method applies a standard actuarial technique to model a firm's operational losses by combining a loss frequency distribution with a loss severity distribution. Using this model for both distributions, we then combine the results through Monte Carlo simulation to derive the probability distribution of the total operational loss.

Loss Frequency

It is usually difficult to select the distribution that best fits the observed data due to the scarcity of low-frequency events, and it generally is sound to use the Poisson distribution to model the number of events; we also found that it is relatively easy to estimate a Poisson variate due to the mean and variance both being the frequency parameter λ. Figures i and ii show the deviation of the actual distribution of operational risk events from the theoretical distribution which we observed in the S-shaped pattern of the Q-Q plot, that the actual distribution has heavier tails and more extreme values than a Poisson distribution.

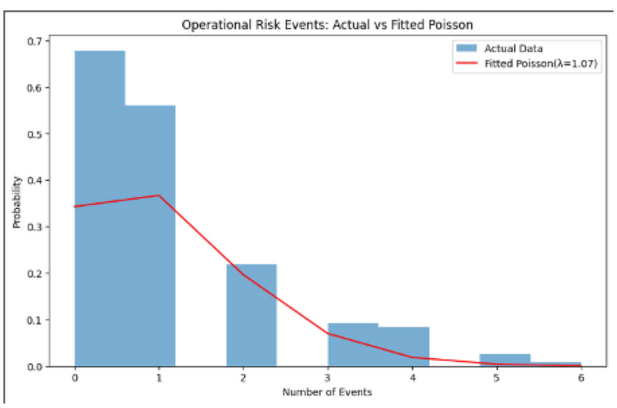

Fig i.

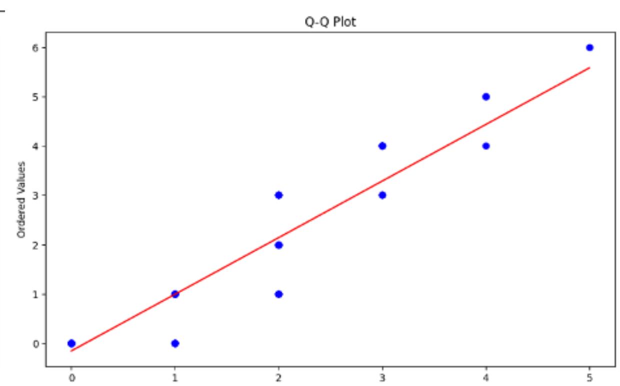

Fig ii.

Our data revealed varying patterns of event occurrence, with some periods experiencing multiple events while others show no activity (like Periods 4 and 5 with 0 events). Based on the standard deviation of 1.257332, we found considerable variation in event frequency, and although most periods had few events, there were periods with higher numbers of events as indicated in the positive skewness value of 1.421599. Although the fitted model may overestimate the probability of single events and underestimate zero events, we used the Poisson distribution to model frequency because it captures the general pattern of event frequencies.

The LDA curve relies on the compound distribution of loss frequency and amounts over time. To forecast operational losses using LDA, we calculated loss severity, which follows a Probability Density Function due to the continuous nature of loss distributions. Since the given data doesn't fit a normal distribution, we analyzed potential distribution transformations to best fit it. Initial analysis revealed the raw loss amount distribution is highly asymmetrical and skewed. We also found studies demonstrating suitable transformations and distributions for modeling operational loss severity.

Loss Severity

Dutta & Perry analyze operational risk modeling, finding that Box-Cox transformation improves the fit of severity distributions to bank loss data, especially before parametric fittings, yielding better goodness-of-fit statistics than untransformed data. The transformed data effectively captures tail behavior for operational risk capital estimation. The lognormal distribution is widespread for modeling operational loss severity due to its fit for moderately heavy-tailed data (Peters, Dutta, K. C., & Perry, J. 2012). Flexible models like α-stable, g-and-h, and generalized beta (GB2) distributions are suggested for handling heavy tails and extreme losses (Peters, G. W., & Sisson, S. A. 2009). We investigate Log-normal, Gamma, Box-Cox, and Exponential transformations (Frachot, A., Georges, P., & Roncalli, T. 2001; Dutta K. and J. Perry 2006; El Adlouni et al. 2011; Kuketayev Lecture 2024). Histogram analyses show the original loss severity distribution is significantly left-skewed (see Appendix 1). Further, we evaluated histogram transformations, identifying Log-normal, Gamma, Box-Cox, and Exponential methods (Frachot et al. 2001; Dutta & Perry 2006; El Adlouni et al. 2011; Kuketayev Lecture 2024) and included t-distribution and truncated t-distribution transformations based on Group 2's model risk presentation.

In Appendix 1, the original distribution shows a long tail of extreme loss values. Log and Square Root of Log transformations best normalize the data, reducing skewness towards a normal distribution. T-distribution and Truncated T transformations improve upon the original but still show some skewness and heavier tails. Direct comparison indicates that the log-normal transformation offers a better fit with a more symmetric, bell-shaped curve. The T-Distribution transformation maintains slight positive skewness and heavier tails than log-normal. In summary, Log and Square Root of Log transformations effectively normalize the skewed original loss severity data, making them preferable for further statistical analysis and modeling.

Our Analysis loss severity transformations shows the Square Root of Log transformation is optimal for operational risk modeling for three reasons and it has strong goodness-of-fit metrics, including a favorable Shapiro-Wilk p-value (0.1263), the lowest Anderson-Darling statistic, and balanced distribution properties with low skewness and kurtosis. This approach addresses the original data's high skewness (3.95) and kurtosis (19.73), suitable for typical and extreme loss modeling in LDA. The logarithmic followed by square root transformation manages extreme values while maintaining interpretability for threshold data above 1. However, we found that it may risk overfitting the loss severity data. Despite good normality metrics, it might misleadingly normalize data, which may not suit operational risk modeling. Operational risk theory suggests loss severity often follows heavy-tailed distributions, with lognormal widely accepted. Thus, the simple log transformation is preferable as it is theoretically justified for financial losses, reduces overfitting risk, and allows easier interpretation, accurately representing important tail risks for operational risk assessment.

Choosing Log Normal Instead

Analysis of goodness-of-fit shows the Log transformation is most suitable, outperforming the square root log transformation. Strong statistical performance is indicated by the second-best Anderson-Darling statistic (3.918078), favorable Kolmogorov-Smirnov test results (KS normal: 0.113224, KS lognorm p-value:

0.304747), and the second-highest Shapiro-Wilk statistic (0.932828). The Log transformation is also theoretically sound for operational risk modeling as it avoids overfitting, preserves tail characteristics, and aligns with risk practices. In contrast to Box-Cox, Exponential, Cube Root, and T-distribution, which show poorer fits, the Log transformation balances fit, theory, and practicality for operational risk modeling.

In implementing our log-normal transformation for the LDA model, we assume loss events are independent, frequency and severity are uncorrelated, there is no temporal autocorrelation in loss amounts, and individual losses are independent. If these assumptions fail, we would consider copula methods for dependence modeling, time series analysis, or mixed models for frequency-severity correlation. Our log-normal transformation handles overdispersion by accounting for greater variance than the mean and managing extreme values. We could enhance this with a spliced distribution for body/tail modeling, mixed distributions, and adjustments for threshold effects to ensure accurate risk capturing. Furthermore, Peters, Dutta, K. C., & Perry, J. (2012) support using Log Normal for loss severity modeling, providing evidence for addressing overdispersion from the Log Normal distribution.

Based on research papers supporting Log Normal transformation, we assumed Log-normal distribution would be a good fit. These assumptions highlight the need for careful consideration when choosing a loss distribution for operational risk modeling. While the lognormal distribution offers a starting point, it's important to acknowledge its limitations and explore alternative models that better capture the full spectrum of potential losses, especially for extreme events.

Issues addressed in the Model Risk Team (Group 4s) evaluation of the Risk Model

We recommended log-normal transformation for loss severity modeling, but Group 4 argued that the log-normal, exponential, and Gamma distributions fail to capture the heavy tails of the data. While presenting our model's risk, they criticized our Exponential Distribution plot despite our detailed and clear explanation and selection of our log-normal choice, based on goodness of fit test and other tests.The Group 4 team reviewed our presentation slides but did not delve into evaluating our methods or offering recommendations on the transformations we selected for loss severity modeling. When we inquired about their thoughts on our approach during the presentation, they indicated that they did not see a need for considering our methods. While their feedback was brief, it presents an opportunity for us to refine our approach and ensure alignment with their expectations in the future.

We found that Group 4's suggestion of the T-distribution at the 25th percentile has several flaws which were not addressed properly .There were significant limitations of their method as we overlooked during their presentation. They Compared the truncated t-distribution at the 25th percentile with the log-normal showing significant differences in statistical fits for operational risk modeling. The log-normal performed better overall, with lower Anderson-Darling stats (3.918078 vs 15.909088), improved Kolmogorov-Smirnov results (0.113224 vs 0.202404), and superior Shapiro-Wilk stats (0.932828 vs 0.798098). Its properties also examined moderate skewness (0.822397 vs 1.326878) and lower kurtosis (0.139431 vs 0.693440). Despite the truncated t-distribution has advantages: it manages the threshold effect, fits tail behavior better as seen in the KDE plot, provides theoretical support for threshold data, and conservatively estimates extreme losses. While the log-normal has a more balanced spread, the truncated t-distribution indicates a sharper decline at lower values, possibly aligning more closely with actual operational losses. Despite the log-normal's superior fit and simpler industry usage, the truncated t-distribution's capacity to handle heavy tails, conservative estimates, and compliance with regulatory standards could make it a better option for modeling extreme loss events.

Model Estimation

Our model's structure presented in Table I shows a constant mean with GARCH volatility using a Standardized Student's t distribution, fitted using Maximum Likelihood estimation. The influence of past squared residuals on current volatility (ARCH effect) seems to be statistically significant ($p < 0.05$), suggesting that past shocks have an impact on current volatility; the p-value of beta[1] however mentions that volatility is not persistent over time.

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GARCH Model Summary:
--------------------------------------------------
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -378.085
Distribution:  Standardized Student's t  AIC:                          766.171
Method:            Maximum Likelihood   BIC:                           776.559
                                        No. Observations:                   59
Date:              Wed, Nov 13 2024   Df Residuals:                       58
Time:                      21:37:26   Df Model:                            1
                               Mean Model
=========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
-------------------------------------------------------------------------
mu             7.8373     14.821      0.529      0.597 [-21.211, 36.886]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega       7299.3554   4782.337      1.526      0.127 [-2.074e+03,1.667e+04]
alpha[1]       0.3716      0.174      2.131  3.309e-02     [2.982e-02,  0.713]
beta[1]        0.3259      0.284      1.147      0.252      [ -0.231,  0.883]
                              Distribution
===========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
---------------------------------------------------------------------------
nu           189.5979     99.358      1.908  5.636e-02 [ -5.140,3.843e+02]
===========================================================================
```

Table 1: *Garch model summary*

Month	Forecasted VaR	Volatility
1	0.00	105.88
2	0.00	122.96
3	0.00	133.58
4	0.00	140.52
5	0.00	145.16
6	0.00	148.31
7	0.00	150.47
8	0.00	151.96
9	0.00	152.99
10	0.00	153.71
11	0.00	154.20
12	0.00	154.55

Table 2: *Operational Risk Capital Analysis*

In Table 2, the consistently zero Value at Risk (VaR) forecasts highlight a fundamental mismatch between the GARCH framework and the characteristics of operational loss data. One may be tempted to assume that consistently zero Value at Risk (VaR) is due the GARCH's model characteristic of conditional variance exhibiting a high degree of persistence. Our careful analysis however revealed that the mismatch may be due to

GARCH's unsuitability for the heavy-tailed nature of operational losses. To find potential improvements to our model, 12-month operational losses were forecasted using ARIMA.

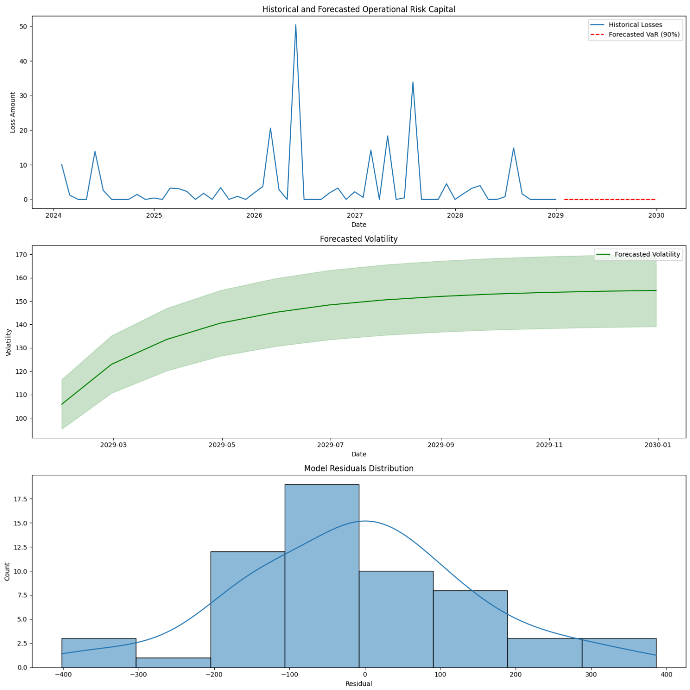

Fig iii.

Our GARCH (1,1) model demonstrated significant limitations as shown in Fig iii. in its application to forecasting operational losses. With an adjusted R-squared of 0.000, the model showed poor explanatory power, failing to capture meaningful variance in the data. Moreover, the consistently zero Value at Risk (VaR) forecasts highlight a fundamental mismatch between the GARCH framework and the characteristics of operational loss data, which typically lack volatility clustering. While GARCH is effective for modeling time series with time-varying volatility, its assumptions are unsuitable for the sporadic and heavy-tailed nature of operational losses. Additionally, the residual analysis further emphasizes the model's inadequacy, as it struggles to account for the event-driven dynamics of operational losses. Given these limitations, the GARCH model does not provide actionable insights for operational risk forecasting. Instead, focusing on the Loss Distribution Approach (LDA) with Monte Carlo simulations and employing heavy-tailed severity distributions such as lognormal or α-stable would be more appropriate for capturing the complexities of operational losses. so we decided to test with Arima.

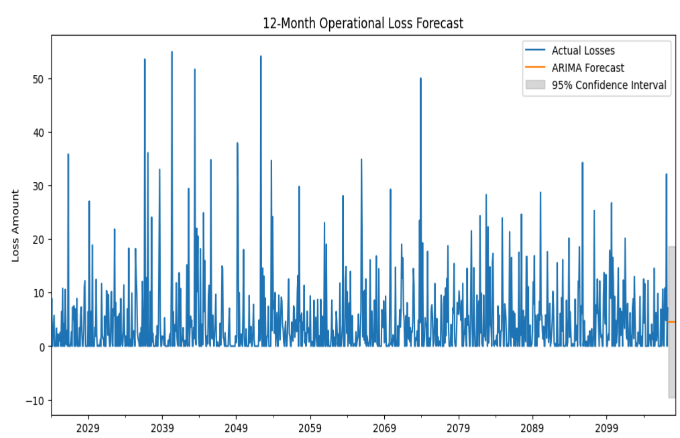

Fig iv. 12 months operational loss forecast

The ARIMA (1,1,1) as shown in table 3 and Fig iv ,model also presented challenges in forecasting operational losses. While it produced a relatively stable forecast trend of around 5 units, this conservative estimate significantly underestimated the volatility and extreme spikes evident in the historical data. Operational loss data is characterized by irregular and sporadic extreme events, which ARIMA is ill-suited to capture due to its assumption of predictable patterns and stationarity. The increasing uncertainty in longer-term forecasts further underscores the model's limitations, as it struggles to reflect the tail risk critical for operational risk management. Although ARIMA may serve as a baseline forecasting tool, its inability to handle the heavy-tailed and event-driven nature of operational losses makes it inadequate for this context.

Date	Value
2107-05-31	4.523606
2107-06-30	4.518208
2107-07-31	4.518197
2107-08-31	4.518197
2107-09-30	4.518197
2107-10-31	4.518197
2107-11-30	4.518197
2107-12-31	4.518197
2108-01-31	4.518197
2108-02-29	4.518197
2108-03-31	4.518197
2108-04-30	4.518197

Table 3:Arima Forecasting

Monte Carlo

The loss distribution as shown in figure v and summary of Annual losses on Table 4 was computed based on 1000 simulations using the Monte Carlo method. The simulation reveals a highly skewed distribution of operational losses with a mean of 4.52 and a standard deviation of 7.18, indicating substantial variability and uncertainty in operational loss predictions. The 90th percentile of the loss distribution, which is also the tail loss, is $12.44 million, implying that 90% of the time, portfolio losses will not exceed $12.44 million over a one-year period.

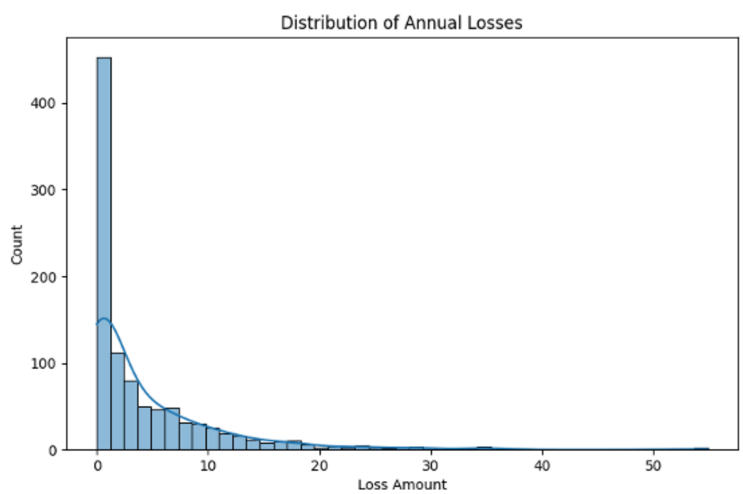

Fig v. *Annual Losses*

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Annual Loss Analysis:
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Number of simulations: 1000

Basic Statistics:
Mean: 4.52
Standard Deviation: 7.18
Median: 1.78
Skewness: 3.02
Kurtosis: 12.63

Percentiles:
50th percentile: 1.78
75th percentile: 6.23
90th percentile: 12.44
95th percentile: 17.85
99th percentile: 34.62
```

Table 4: Summary Annual Loss

Garch Models

Assumptions

The GARCH model operates under several key assumptions to ensure accurate and meaningful results. First, stationarity assumes that the time series statistical properties, such as mean, variance, and autocorrelation, remain constant over time. This stability is crucial for generating reliable forecasts. Second, the normality of innovations presumes that error terms are normally distributed, facilitating straightforward estimation and inference. Finally, the model relies on constant parameters, assuming that the coefficients in the variance equation do not change over the sample period, ensuring the consistency of volatility forecasts.

Limitations

Despite its usefulness, the GARCH model has notable limitations. Model misspecification can occur if the correct order of the model (p, q) is not selected, leading to biased results. Parameter instability may arise during periods of market turbulence or structural changes, reducing the model's accuracy. Additionally, the non-normality of innovations can undermine parameter estimates if returns exhibit skewness or kurtosis. Another challenge is model order selection, as choosing the appropriate lag structure can be complex. Lastly, conditional heteroskedasticity, while modeled explicitly, may not capture all real-world volatility patterns, limiting the model's applicability in specific financial environments.

ARIMA MODELS

Assumptions

ARIMA models are built on several fundamental assumptions. Stationarity is essential, meaning the time series should have constant statistical properties like mean and variance over time. The model also assumes linearity, where the relationship between the dependent variable and its lagged values is linear. Another critical assumption is constant variance, where the variance of the error terms remains stable throughout the time series. Additionally, independence of errors requires that the residuals from the model are uncorrelated, ensuring unbiased and efficient estimates.

Limitations

Despite the model's relevance, ARIMA models have several limitations. Model order selection can be complex, requiring careful evaluation of lag orders. Non-stationarity in data can lead to inaccurate and inconsistent results if not adequately addressed through differencing. Structural breaks, such as changes in market conditions, can disrupt model accuracy. Outliers and anomalies pose another challenge by distorting the estimation process. Lastly, the constant volatility assumption may not fit well with financial time series that often show time-varying volatility.

In context, GARCH models generally outperformed ARIMA models in forecasting volatility, especially when dealing with time series exhibiting time-varying volatility, which is a common characteristic of financial data. It is important to note that these are general assumptions and limitations. Specific models and applications may have additional considerations. Model validation and diagnostic checks are essential to assess the adequacy of the model and the reliability of the results.

Monte Carlo Simulation

Monte Carlo simulation is a powerful statistical technique used to model uncertainty and assess risk by running many simulations. Its application spans finance, engineering, and scientific research. However, its effectiveness depends on key assumptions and faces certain limitations.

Assumptions

Monte Carlo simulations assume that the underlying model is accurate and well-calibrated. The random number generator used must produce truly random samples to avoid biases in the simulation results. Additionally, each simulation must be independent to ensure unbiased estimates, allowing for a realistic representation of possible outcomes.

Limitations

Despite its advantages, Monte Carlo simulation has notable limitations. Model risk arises if the underlying model does not accurately represent real-world dynamics, leading to unreliable predictions. The computational cost can also be significant, especially when dealing with complex models or many simulations. Finally, uncertainty in parameter estimates can propagate through the simulation, reducing the reliability of the results and requiring careful interpretation.

It's important to note that these assumptions and limitations can impact the accuracy and reliability of forecasts. Model selection, parameter estimation, and diagnostic checking are crucial steps in ensuring the validity of the models. By considering these factors, users can better understand the strengths and weaknesses of Monte Carlo simulations in decision-making processes.

Conclusion

With Alternative methods, including ARIMA and GARCH models, we evaluated the data but found it inadequate for the sporadic and heavy-tailed characteristics of operational loss data. ARIMA struggled with extreme events, while GARCH failed to address the absence of volatility clustering inherent in operational losses. Monte Carlo simulations proved instrumental in capturing the variability and tail risk, providing a robust framework for risk assessment.

While the log-normal distribution demonstrated strong fit and interpretability, we acknowledged the merits of alternative models, such as the truncated t-distribution, for handling heavy tails and threshold effects. However, the log-normal model remains our recommended approach due to its balanced performance and alignment with industry practices.

Our study underscores the importance of selecting appropriate models for operational risk management, emphasizing the need for flexibility and adaptability in addressing unique data characteristics. Future research should explore hybrid approaches, including spliced distributions and copula methods, to enhance model accuracy and capture dependencies between frequency and severity. By refining these models, we can provide more reliable tools for operational risk forecasting and decision-making.

References

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Appendix

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=== Summary of All Goodness-of-Fit Tests ===
                 AD_normal     AD_exp  KS_normal_stat  KS_normal_pvalue  \
Original         25.104254   8.776876        0.268881      4.332925e-14
Log               3.918078   4.826520        0.113224      7.799170e-03
Sqrt_Log          0.662068  32.509180        0.055421      5.119880e-01
Cube_Root         8.132941  52.144026        0.139172      4.626812e-04
BoxCox           13.257090  17.450092        0.185543      6.834732e-07
Exponential       6.951069  24.482559        0.143950      2.579943e-04
Sqrt_Exponential 10.658855  46.705854        0.164421      1.683740e-05
Square_Root      11.294471  34.824237        0.166999      1.163107e-05
T_dist           15.909088  43.264286        0.202404      3.979985e-08
T_transform       7.081077  14.013921        0.138531      4.996176e-04

                 KS_lognorm_stat  KS_lognorm_pvalue  KS_t_stat   KS_t_pvalue  \
Original                0.045613           0.749565   0.240183  2.677553e-11
Log                     0.065619           0.304747   0.108959  1.173291e-02
Sqrt_Log                0.056884           0.478537   0.055421  5.119892e-01
Cube_Root               0.058052           0.452614   0.142028  3.271077e-04
BoxCox                  0.055072           0.520112   0.191727  2.481450e-07
Exponential             0.143951           0.000258   0.143946  2.581279e-04
Sqrt_Exponential        0.164421           0.000017   0.174648  3.749447e-06
Square_Root             0.054523           0.532994   0.171222  6.265401e-06
T_dist                  0.072969           0.196908   0.246053  7.654257e-12
T_transform             0.138532           0.000500   0.138531  4.996330e-04

                  SW_stat     SW_pvalue     K2_stat      K2_pvalue
Original         0.569224  1.061952e-22  208.483121   5.351376e-46
Log              0.932828  2.570577e-08   20.635432   3.304249e-05
Sqrt_Log         0.989584  1.262900e-01    7.839278   1.984826e-02
Cube_Root        0.852722  1.958447e-13   72.077682   2.231158e-16
BoxCox           0.763357  3.327362e-17  119.201762   1.305167e-26
Exponential      0.901794  1.282553e-10   37.356304   7.730040e-09
Sqrt_Exponential 0.850237  1.471042e-13   40.437793   1.655941e-09
Square_Root      0.793147  4.402901e-16  106.632390   6.999560e-24
T_dist           0.798098  6.938955e-16   45.102829   1.607109e-10
T_transform      0.893560  3.786388e-11  944.063700  9.980957e-206
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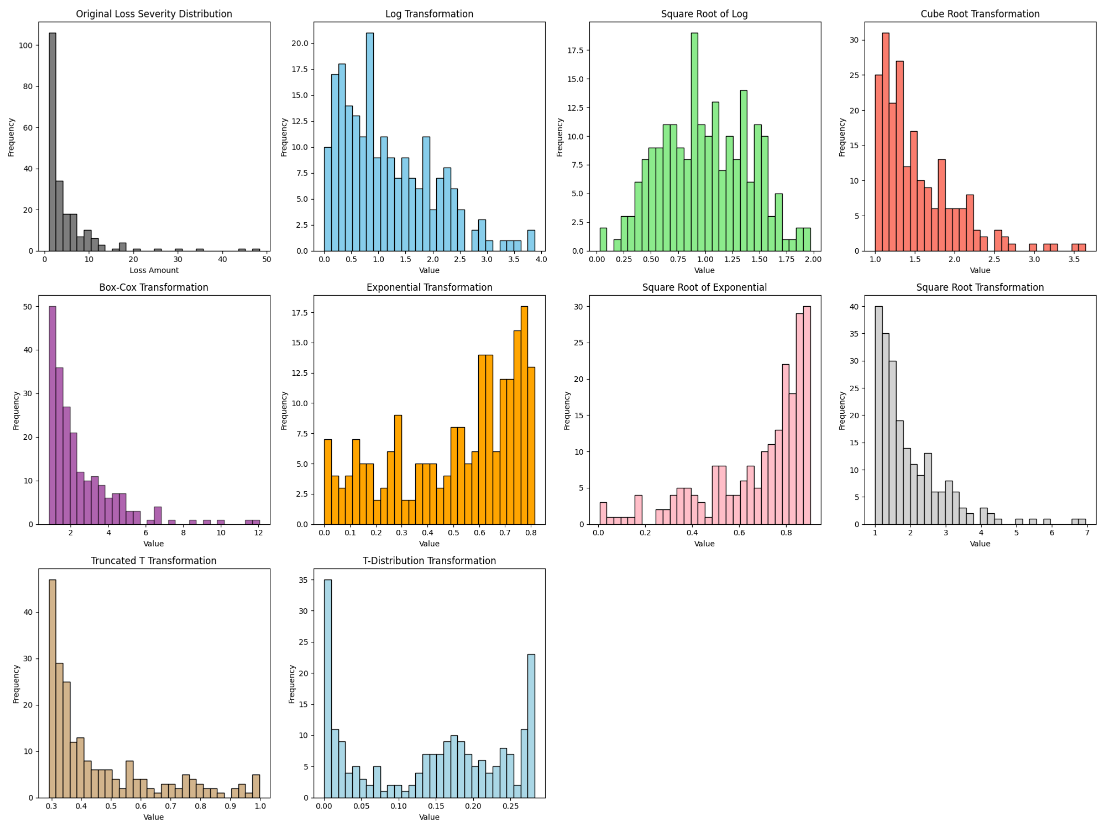
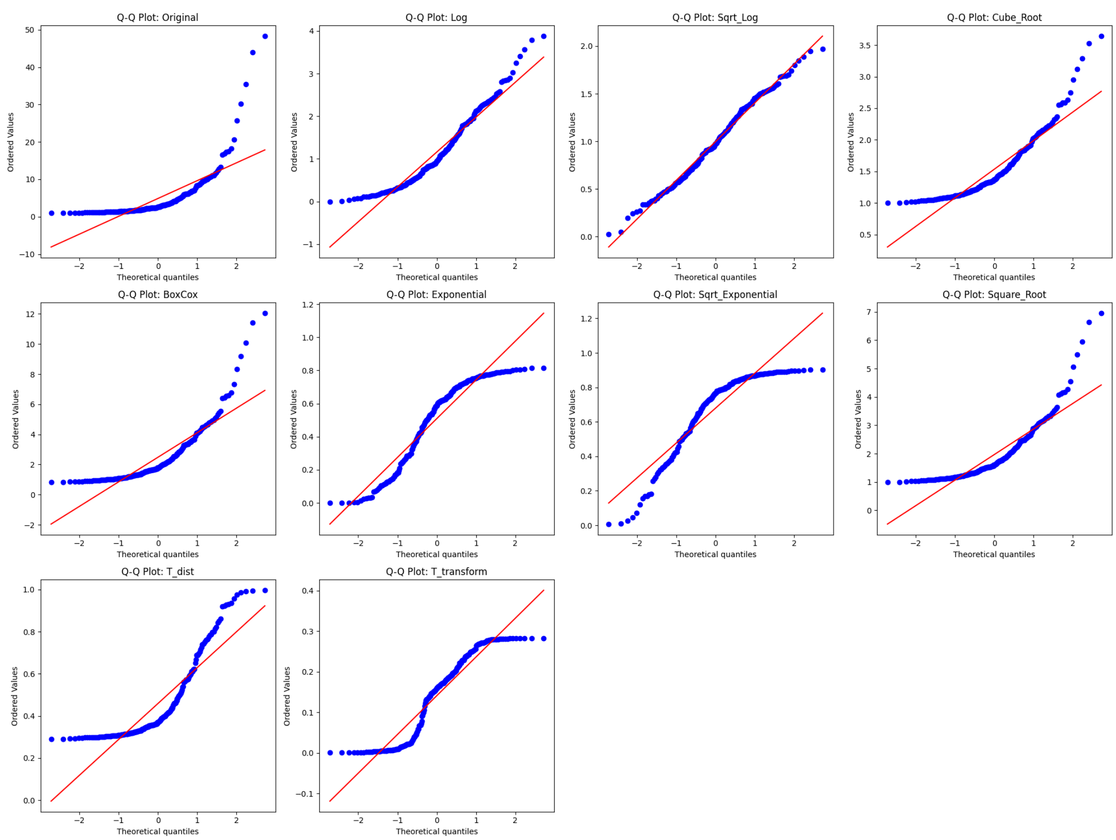

Original Loss Severity Distribution
Log Transformation
Square Root of Log
Cube Root Transformation
Box-Cox Transformation
Exponential Transformation
Square Root of Exponential
Square Root Transformation
Truncated T Transformation
T-Distribution Transformation
Frequency
Loss Amount
Value


Q-Q Plot: Original
Q-Q Plot: Log
Q-Q Plot: Sqrt_Log
Q-Q Plot: Cube_Root
Q-Q Plot: BoxCox
Q-Q Plot: Exponential
Q-Q Plot: Sqrt_Exponential
Q-Q Plot: Square_Root
Q-Q Plot: T_dist
Q-Q Plot: T_transform
Ordered Values
Theoretical quantiles