Assignment 4 of Algorithm

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1. Linear-inequality feasibility

Proof:

suppose that we have an algorithm for linear programming, that means we can solve

$$\begin{aligned} \max \quad & \boldsymbol{c}^T \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}$$

given a linear inequality feasibility problem , our goal is to check whether there exist $\boldsymbol{x}^*$,

$$\boldsymbol{A}'\boldsymbol{x}^* \leq \boldsymbol{b}', \boldsymbol{x}^* \geq \boldsymbol{0}$$

which equals to

$$\begin{aligned} \max \quad & \boldsymbol{0} \cdot \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}'\boldsymbol{x} \leq \boldsymbol{b}' \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}$$

this problem can be solved using the same algorithm.

2. Airplane Landing Problem

Let $x_1, x_2, ..., x_n$ be the exact landing time of each airplane respectively, the problem can be written as

$$\begin{aligned} \max \quad & \min(x_2 - x_1, x_3 - x_2, ..., x_n - x_{n-1}) \\ \text{s.t.} \quad & s_1 \leq x_1 \leq t_1 \\ & s_2 \leq x_2 \leq t_2 \\ & \cdots \\ & s_n \leq x_n \leq t_n \end{aligned}$$

for instance, we have $n = 4, [10, 20], [40, 60], [75, 80], [100, 120]$ (here the minute is the metric of time), using tool cvxpy we can obtain the optimal solution 35 with optimal variables $x_1 = 10, 45, 80, 116$.

```python
#!/usr/bin/env python

import cvxpy as cvx
import numpy as np
import ecos

x = cvx.Variable(4,integer=True)
left = np.array([10,40,75,100])
right =np.array([20,60,80,120])

objective = cvx.Maximize(cvx.min(x[1:] - x[:3]))

constraints = [ x >= left, x <= right]

prob = cvx.Problem(objective,constraints)

prob.solve(solver = cvx.ECOS_BB)
print 'using solver ECOS_BB...'
print 'status:', prob.status
print 'optimal value:', prob.value
print 'optimal var:', x.value
```

```
joseph_chen@JosephChen-Desktop: ~/Documents/algorithms_course/python
joseph_chen@JosephChen-Desktop:~/Documents/algorithms_course/python$ ./landing.p
y
using solver ECOS_BB...
status: optimal
optimal value: 34.9999999999
optimal var: [  10.   45.   80.  116.]
joseph_chen@JosephChen-Desktop:~/Documents/algorithms_course/python$
```

3. Dual Simplex Algorithm

the result is showed below.

```
directly solving...
problem has 3 equalities and 0 inequalities
starting optimization...
status: feasible!
optimal value: -16.5
optimal x: [  0.    0.    0.    1.5   2.5  16.5]
solving dual problem...
problem has 0 equalities and 6 inequalities
starting optimization...
status: feasible!
optimal value: -16.5
optimal y: [ 2.5  6.   0. ]
using tool cvxpy
optimal value: -16.5
optimal x: [ -5.45737158e-22   6.11583402e-22  -4.40096362e-22   1.50000000e+00
   2.50000000e+00   1.65000000e+01]
```