Assignment 6 of Algorithm

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1. Integer Programming

Proof:

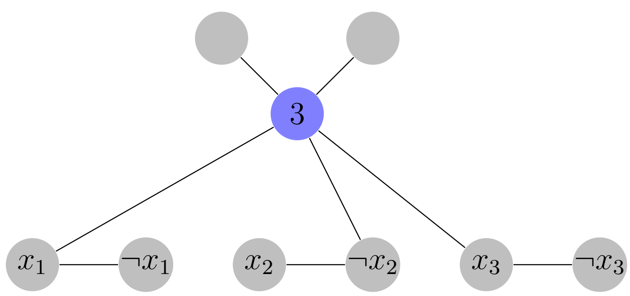
- Certificate: we can verify the constraints $Ax \geq b$ in polynomial time, thus it is a NP problem;
- NP-hard: Now we show that 3-SAT $\leq_P$ Integer Programming. Let $C_i(i = 1, 2, ..., m)$ be the clauses of 3-SAT problem $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$, $x = (x_1, x_2, ..., x_n)$.Once we construct a matrix A and b that each clause C_i can be True or False corresponding to the True or False of the constraint $a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n \geq b_i$, then ϕ is satisfied iff $Ax \geq b$ holds.
  For instance, $m = 3$, $n = 4$, $x_i = \{0, 1\}$, $\phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_2 \vee x_3 \vee x_4)$. The corresponding constraints are $x_1 + x_2 + (1 - x_3) \geq 1$, $(1 - x_1) + x_2 + x_4 \geq 1$ and $x_2 + x_3 + x_4 \geq 1$, we can easily construct the matrix A and b by rewriting the constraints.Thus 3-SAT is polynomially reducible to the Integer programming.

Hence , the Integer Programming problem is in NP-complete.

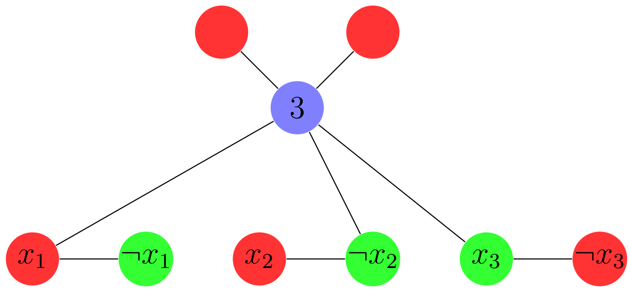
2. Mine-sweeper

Proof:

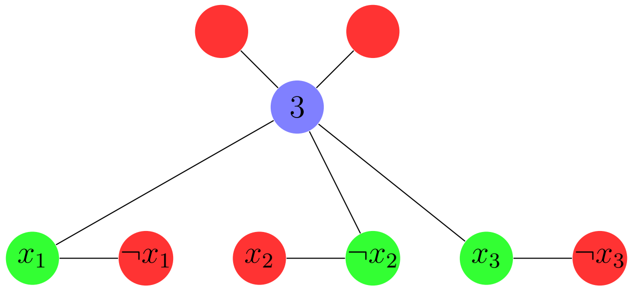
- Certificate: Given such a graph, we can verify whether a node v that is labeled m has exactly m neighboring nodes containing mines in polynomial time, thus it is a NP problem;
- NP-hard: Now we prove that $3 - SAT \leq_p Mine - consistency$.
  For instance,if a clause of $3 - SAT$ is $x_1 \vee \neg x_2 \vee x_3$ then we can construct a graph



If node v has a mine ,we color it *red*, otherwise we color it *green*.If $3-SAT$ is satisfied, then we can color the neighboring nodes of blue node 3 to make sure that blue node 3 has exactly 3 neighboring nodes containing mines. For instance, if $x_1 = 1, x_2 = 1, x_3 = 0$,then we color the graph



if the clause $x_1 \vee \neg x_2 \vee x_3$ is *False*, which means that $x_1 = 0, x_2 = 1, x_3 = 0$, then



we can not color the nodes to make sure that blue node 3 has exactly 3 neighboring nodes containing mines. Thus $3-SAT \leq_p mine-consistency$

Hence $mine-consistency$ is in NP-complete.