Elektronik

Data Conversion

EITA10

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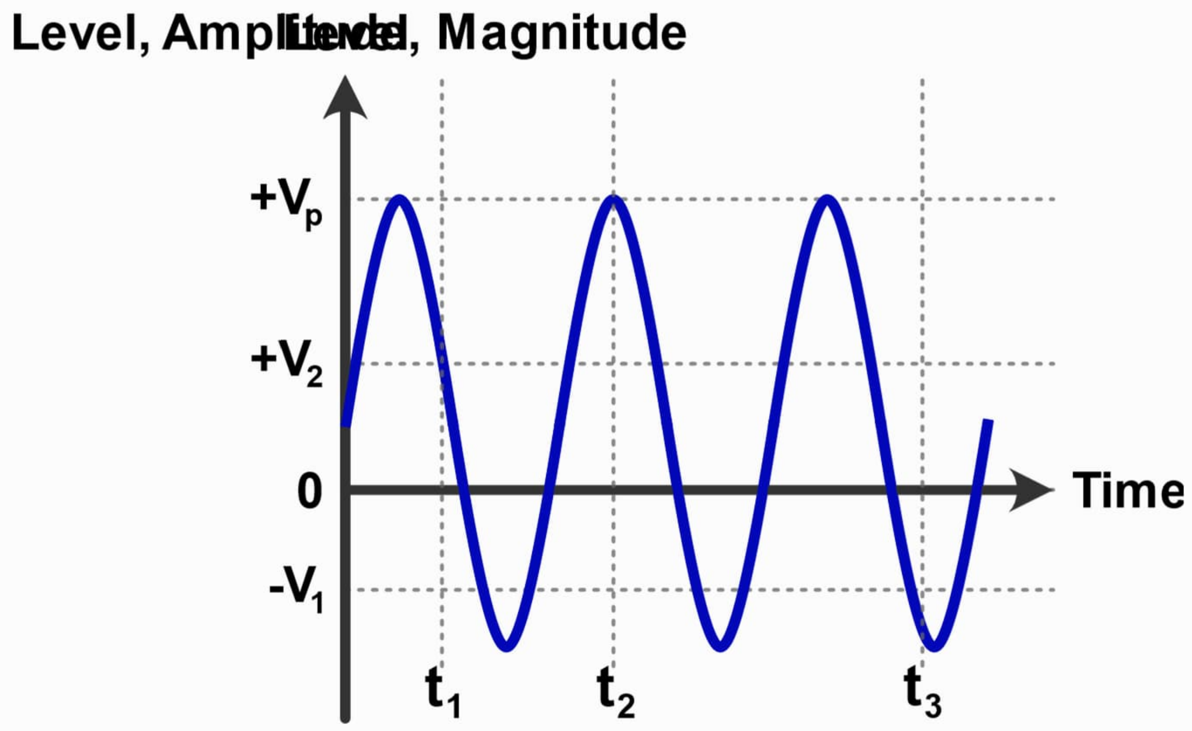
April 2025

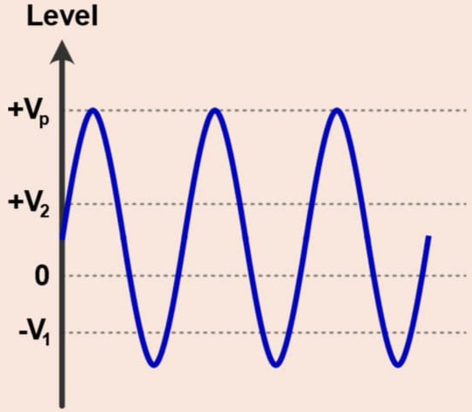
Electronic Signals

- Every signal is expressed in **two dimensions**:
  - **Level (amplitude)**
  - **Time**
- Could be a voltage, current, charge, etc.



Analog vs. Digital Signals

Analog (continuous-level)



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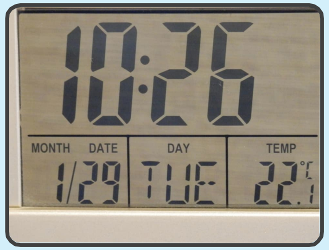
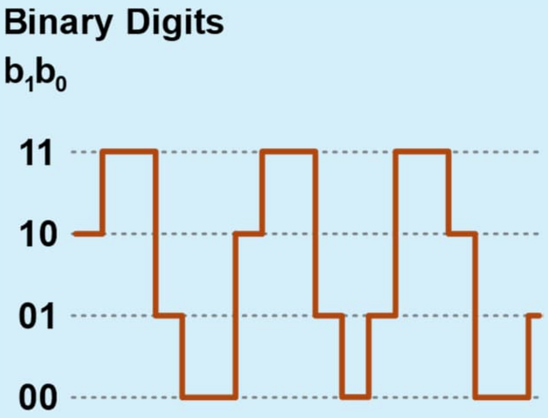
- Can take any value
- Real-world signals
    - Sound, light, image, biomedical, waves
    - Sensors and actuators

Digital (discrete-level)





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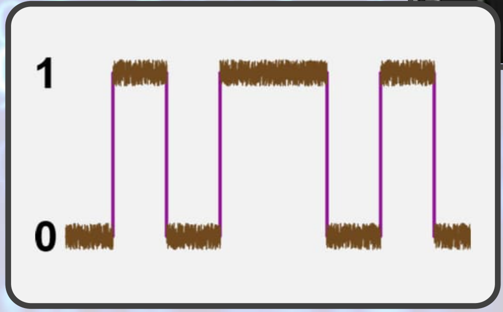
- Only takes specific levels
- Represented by binary digits (0 and 1)
- Boolean algebra
- Computational systems

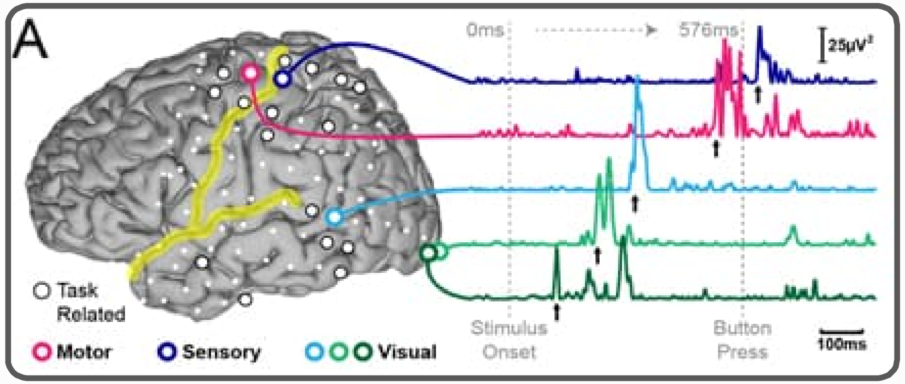
Computers are Digital

- Modular design
- Automated synthesis, hardware design language (HDL)
- Ease of data storage and transmission
- Noise and leakage immunity
- Integration with software
- Reconfigurability
- Scalability

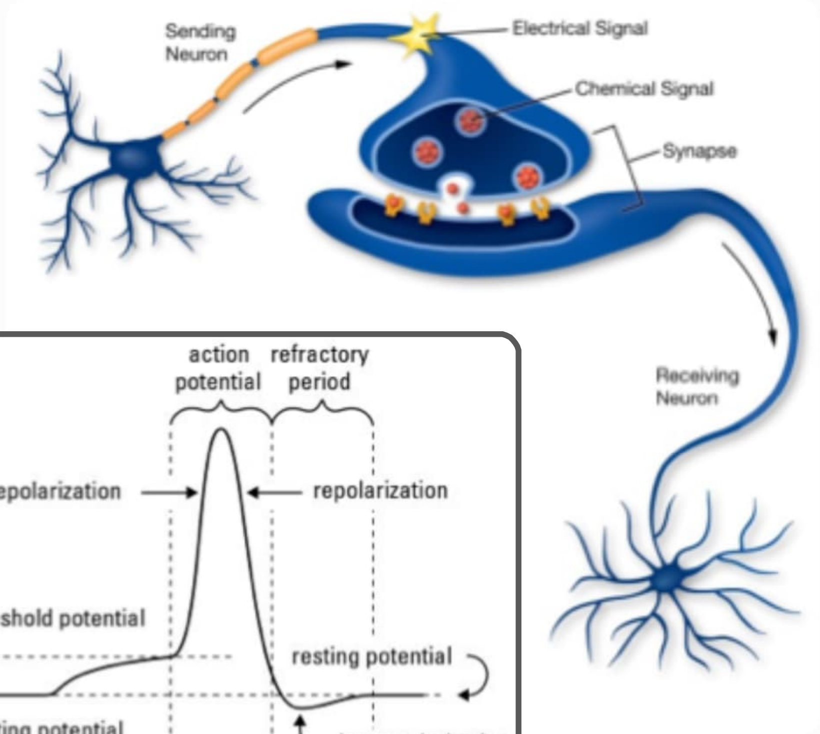


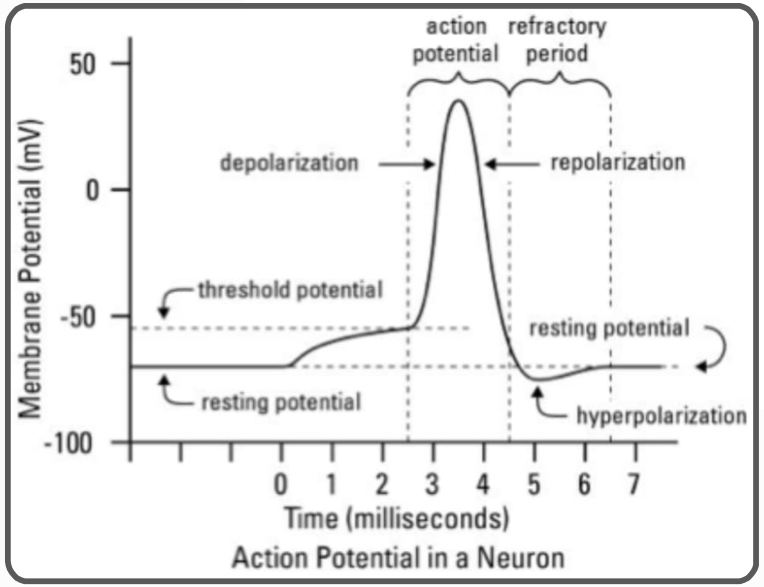
But the Real World is Analog

- **Biopotential signals**



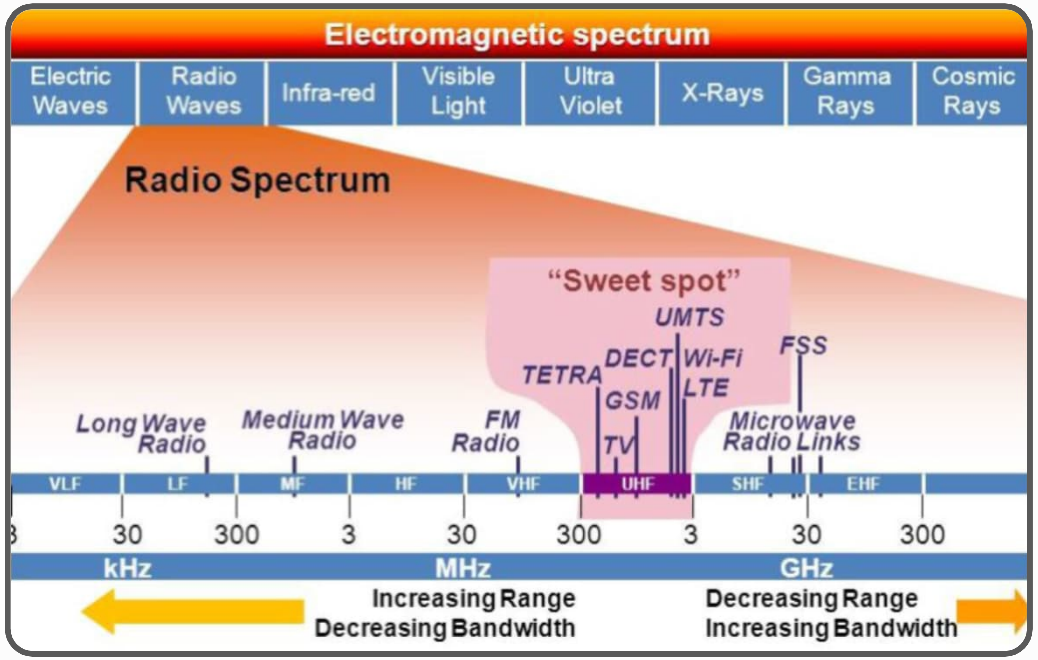
Signal	Frequency (Hz)	Dynamic Range
EEG	0.5 - 100	2 μV – 100 μV
ERG	0.2 – 200	0.5 μV – 1 mV
ECG	0.05 - 100	1 mV – 10 mV
EMG	2 – 500	50 μV – 5 mV

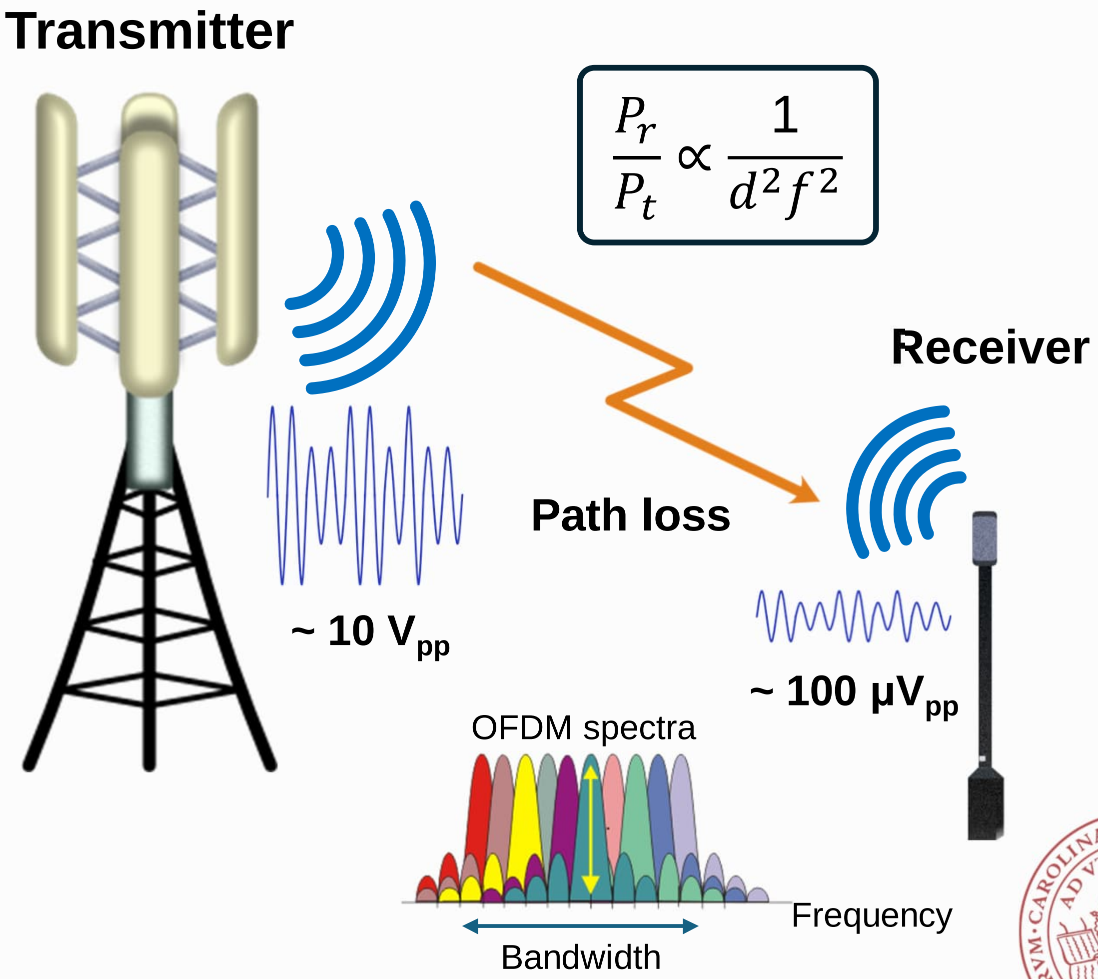




But the Real World is Analog

- Biopotential signals
- Telecommunication signals

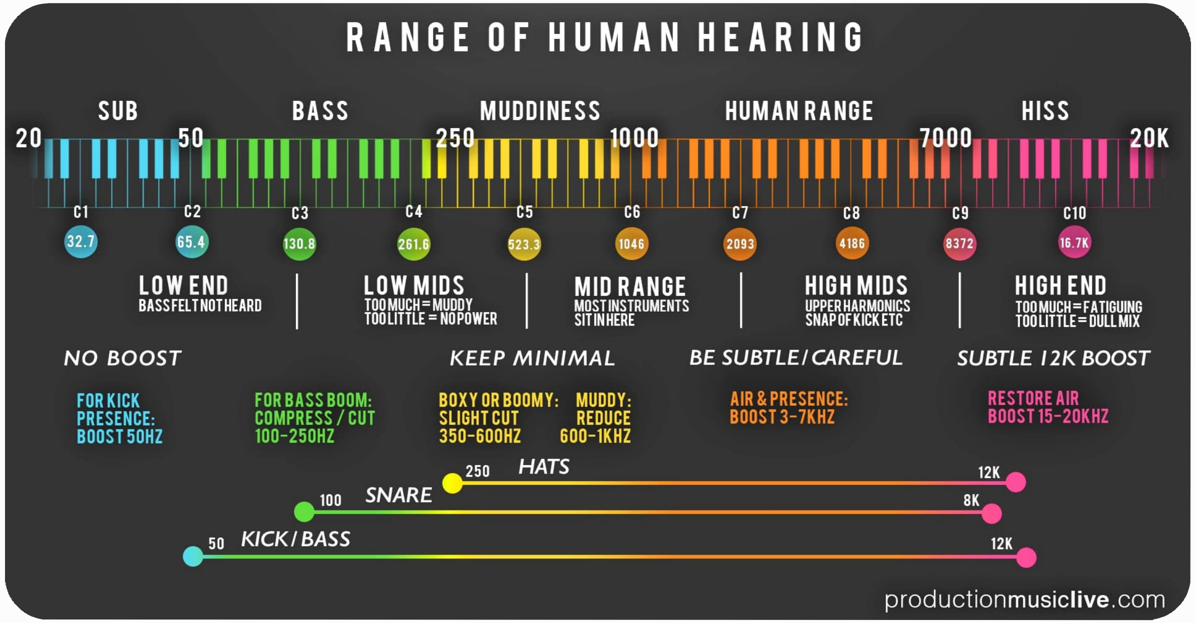


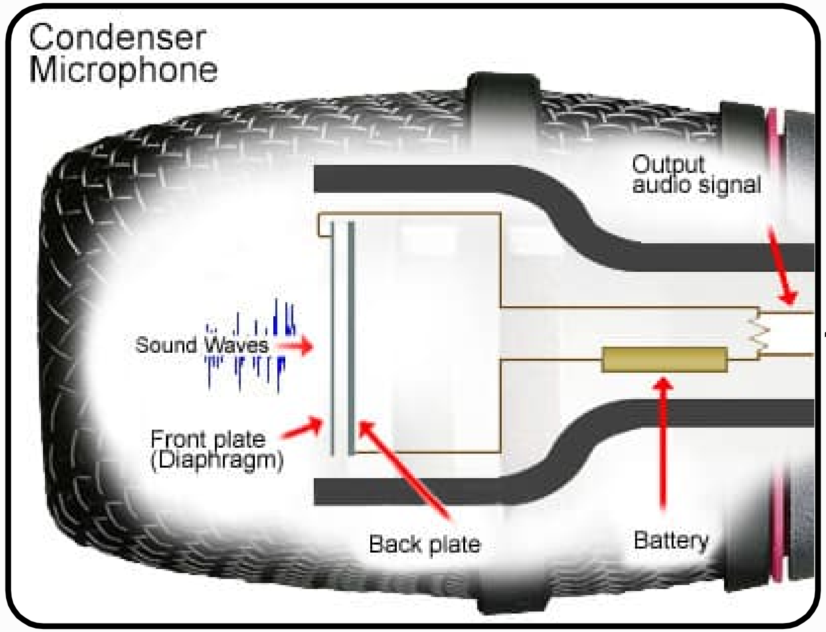


$$\frac{P_r}{P_t} \propto \frac{1}{d^2 f^2}$$

But the Real World is Analog

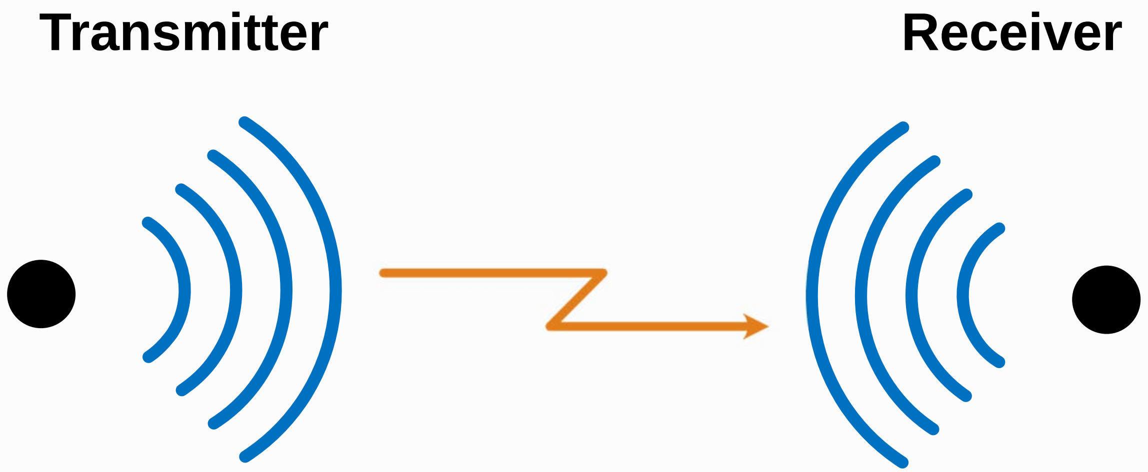
- Biopotential signals
- Telecommunication signals
- **Audio signals**





Real-World Analog Signals are Delicate!

- **Attenuation**
  - Path loss



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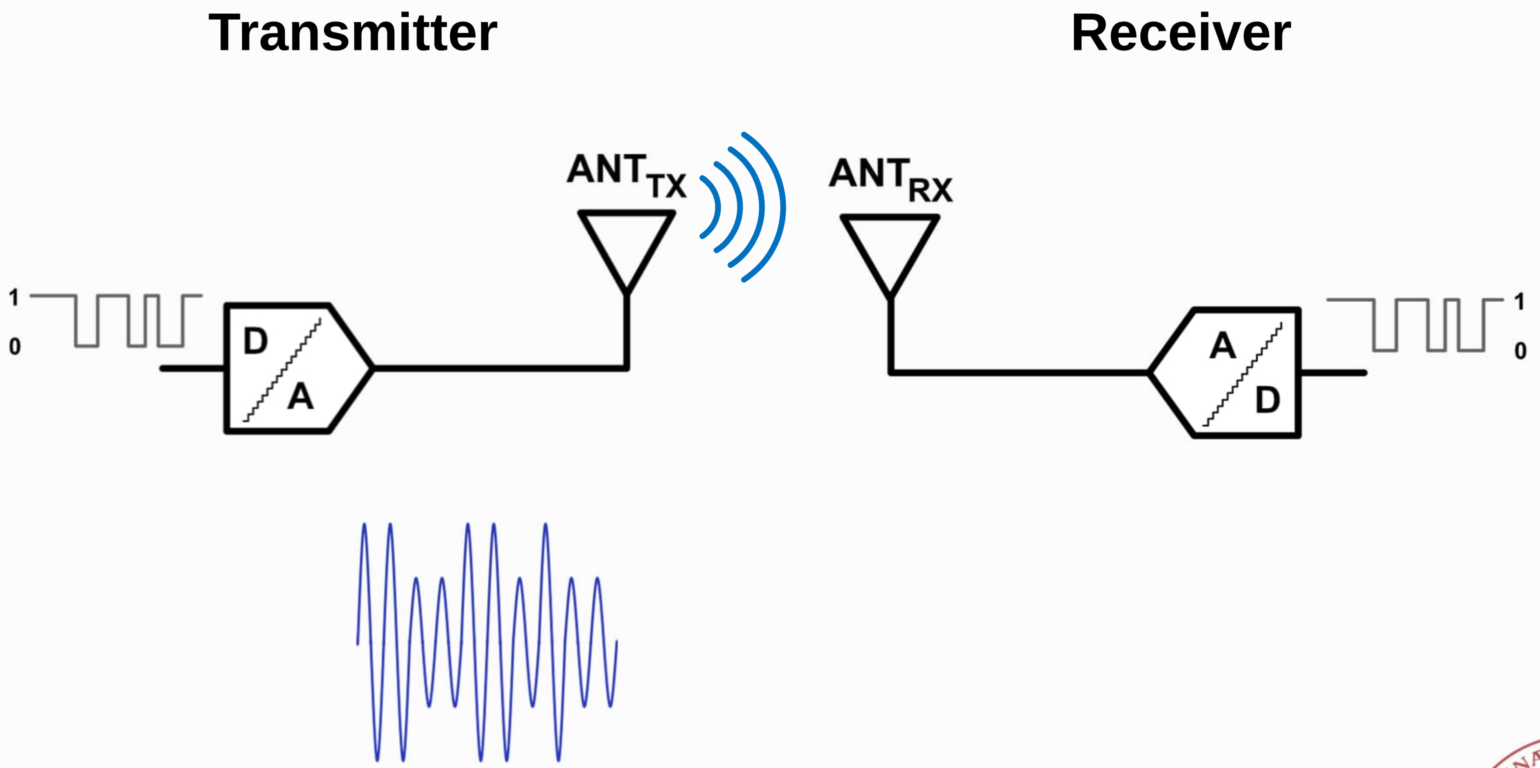

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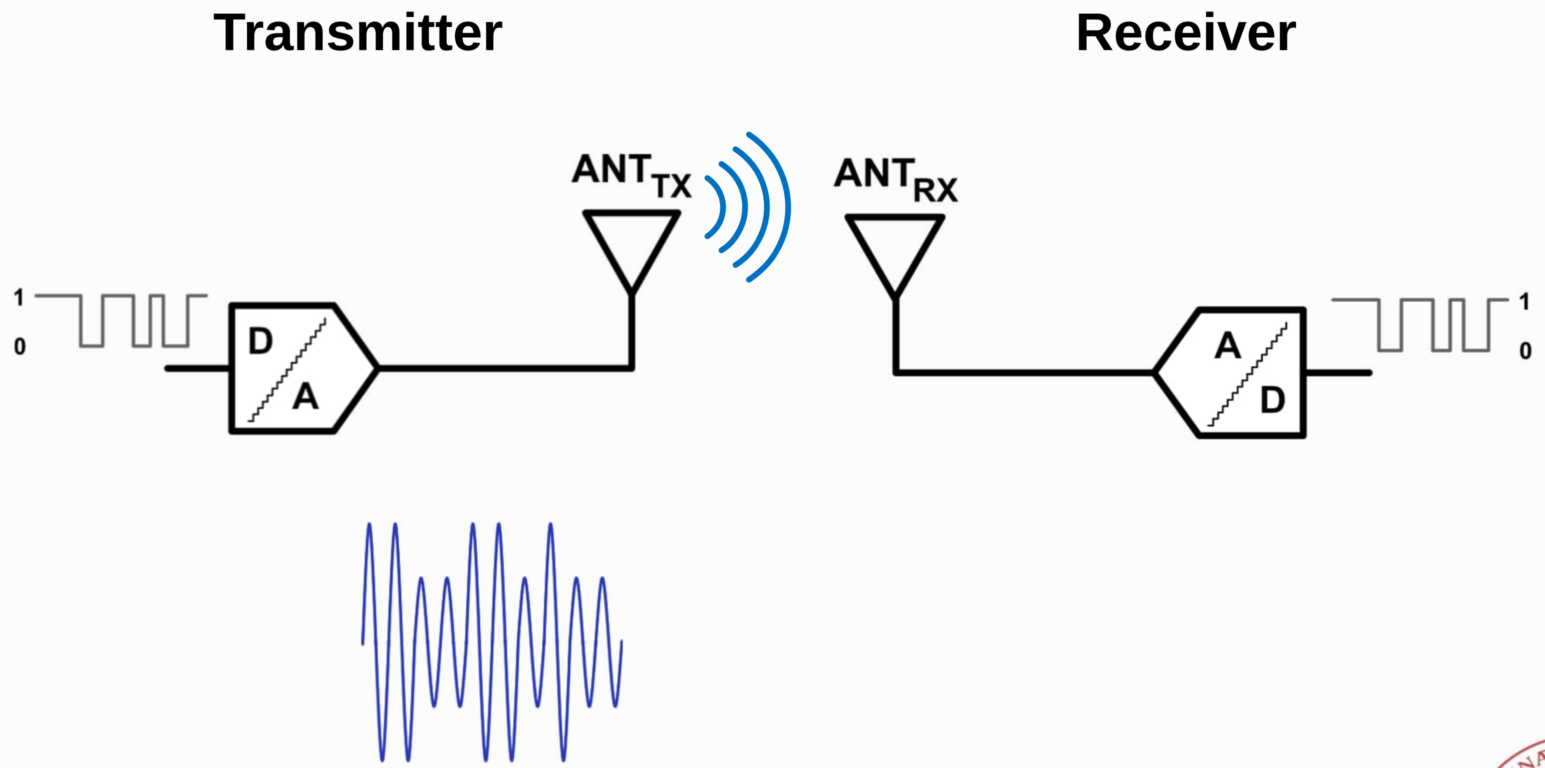

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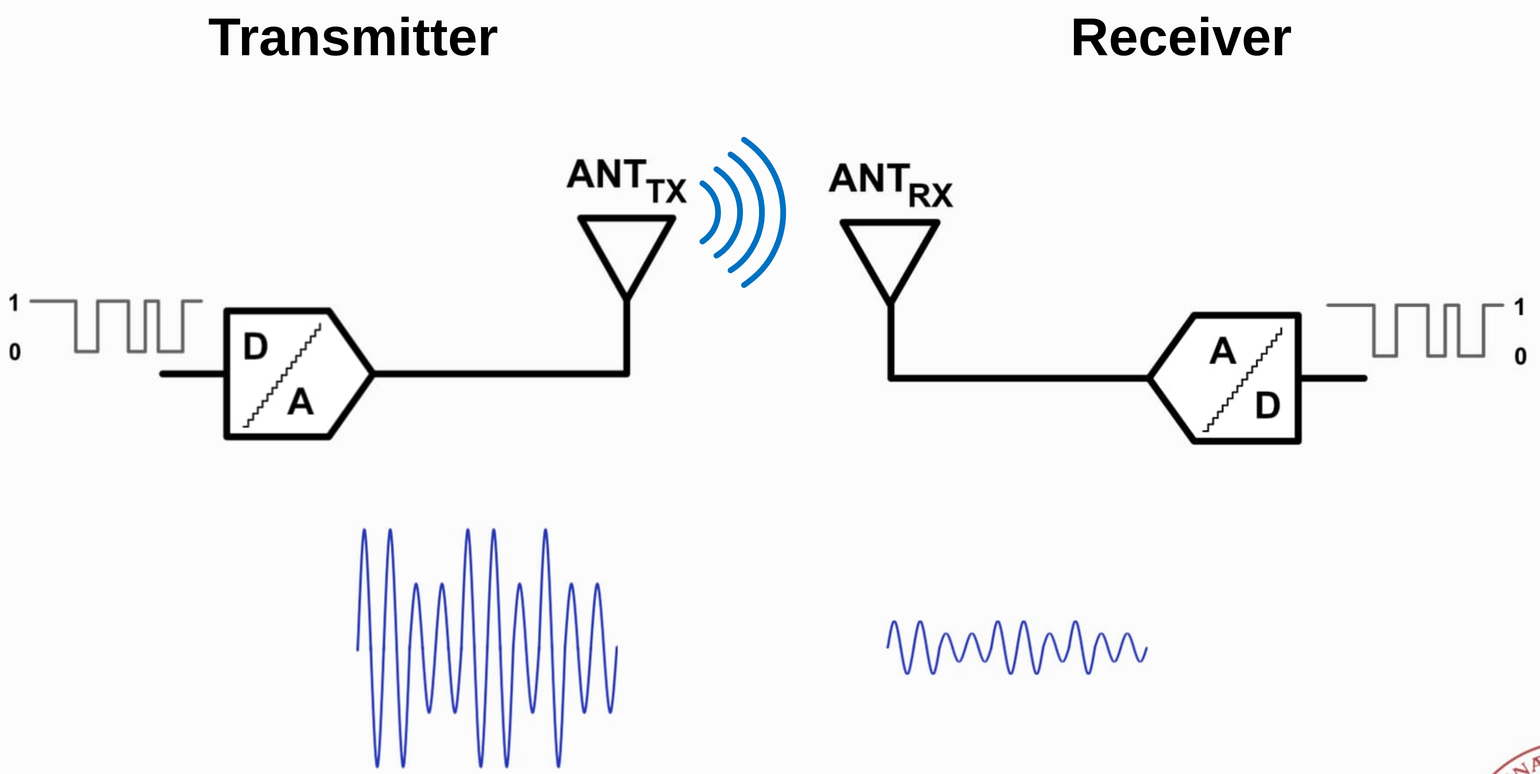
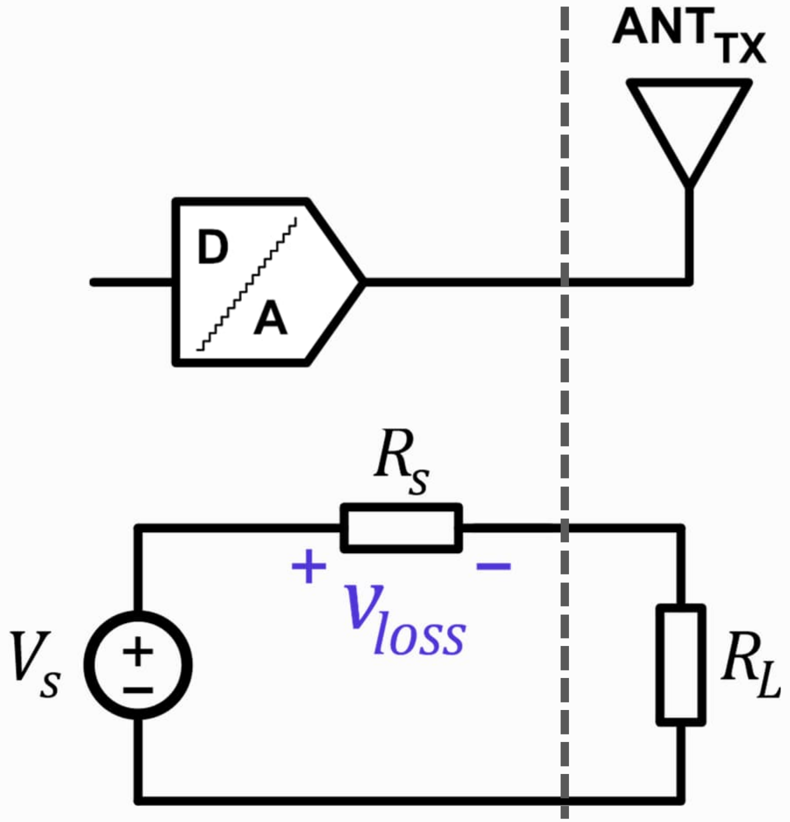


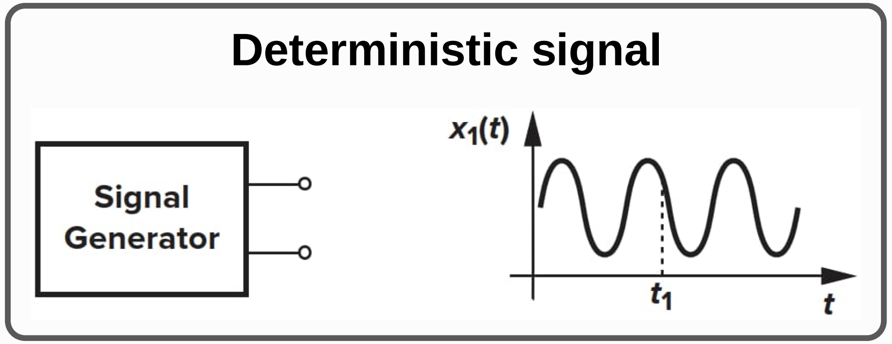
Real-World Analog Signals are Delicate!

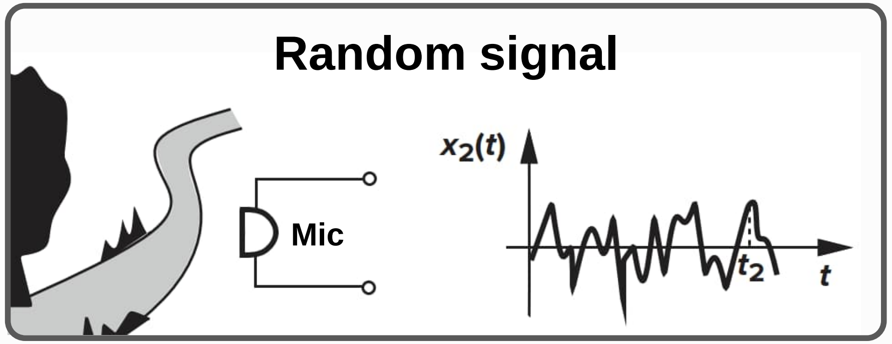
- **Attenuation**
  - Path loss
  - Power loss on the internal impedances



Real-World Analog Signals are Delicate!

- Attenuation
  - Path loss
  - Power loss on the internal impedances
- **Noise**
  - Random, stochastic process, unpredictable





Real-World Analog Signals are Delicate!

- **Attenuation**
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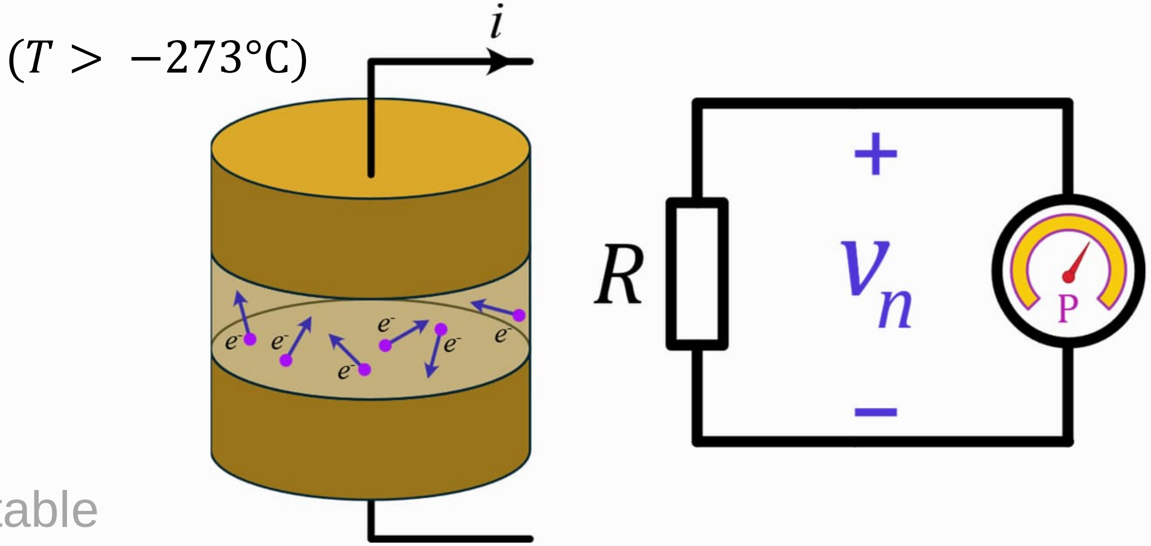
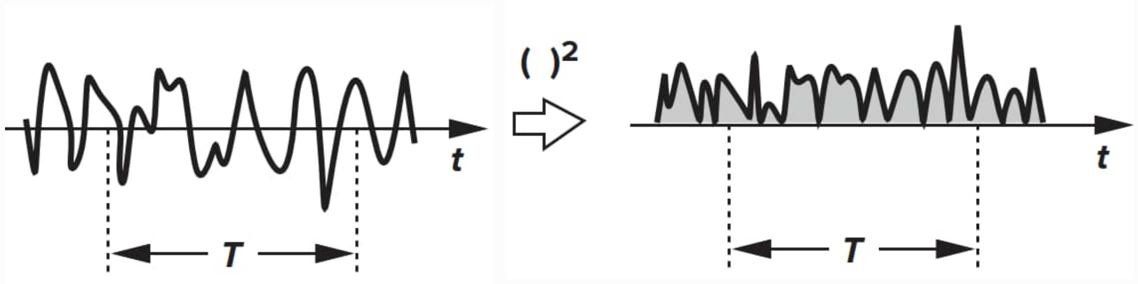
- **Noise**
  - Random, stochastic process, unpredictable
  - Johnson–Nyquist (thermal) noise

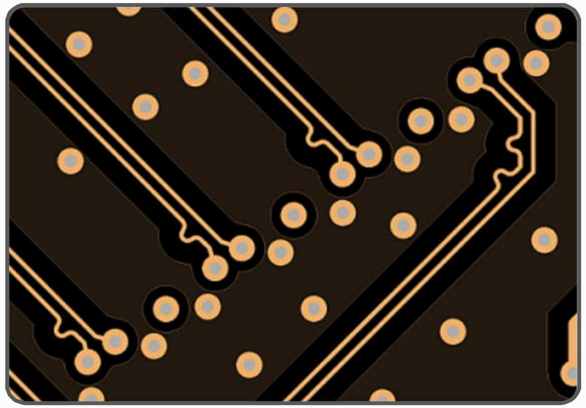


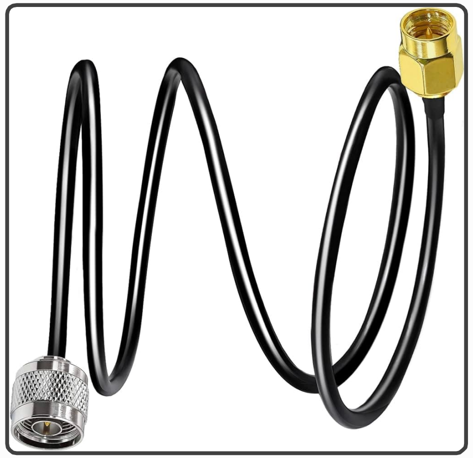


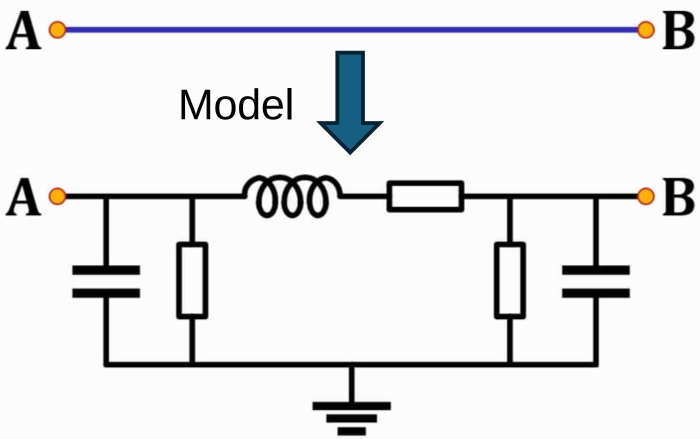
$v_{n,avg} = 0$ $P_{n,avg} \neq 0$

Real-World Analog Signals are Delicate!

- **Attenuation**
  - Path loss
  - Power loss on the internal impedances
- **Noise**
  - Random, stochastic process, unpredictable
  - Johnson–Nyquist (thermal) noise
- **Uncertainties and Variations**
  - Length of wires
  - Twist and bending in cables
  - Parasitic elements and mismatch
  - Model deviations







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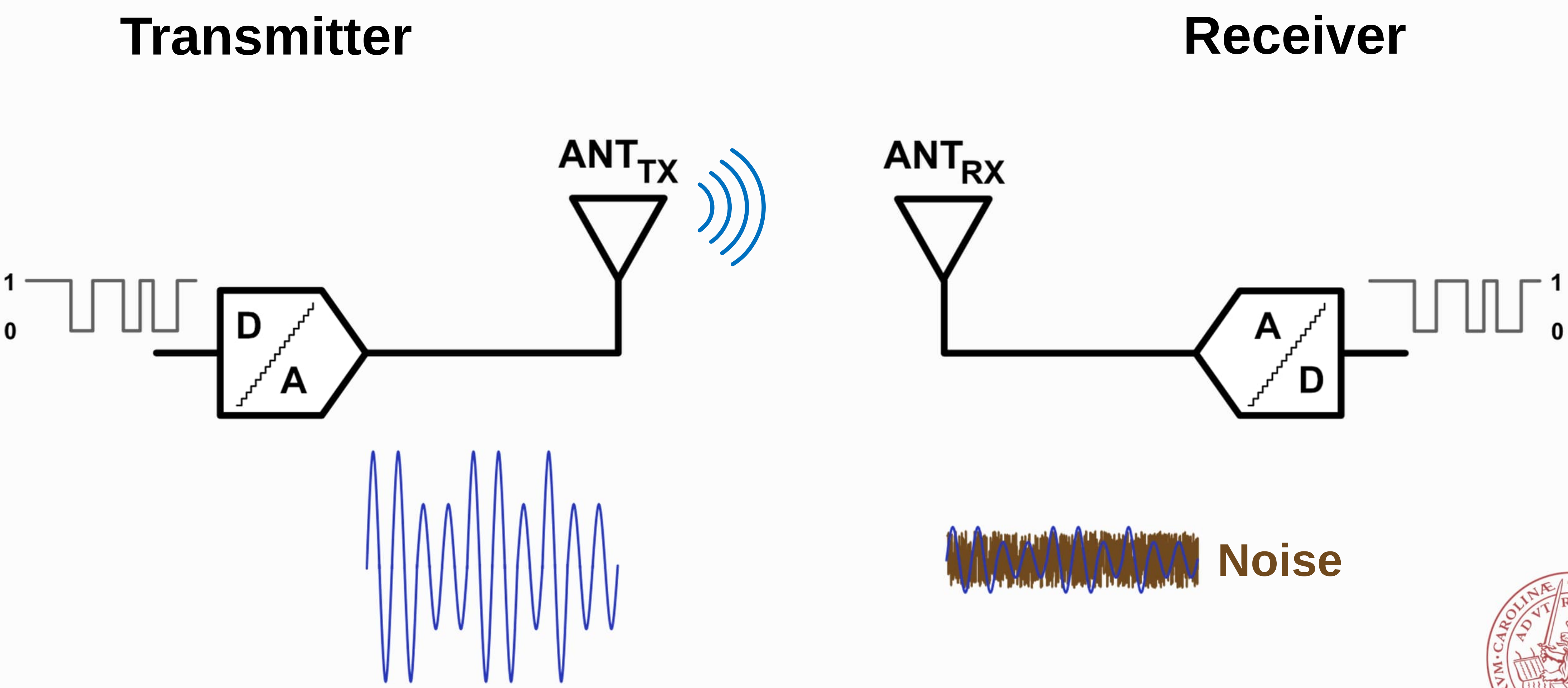
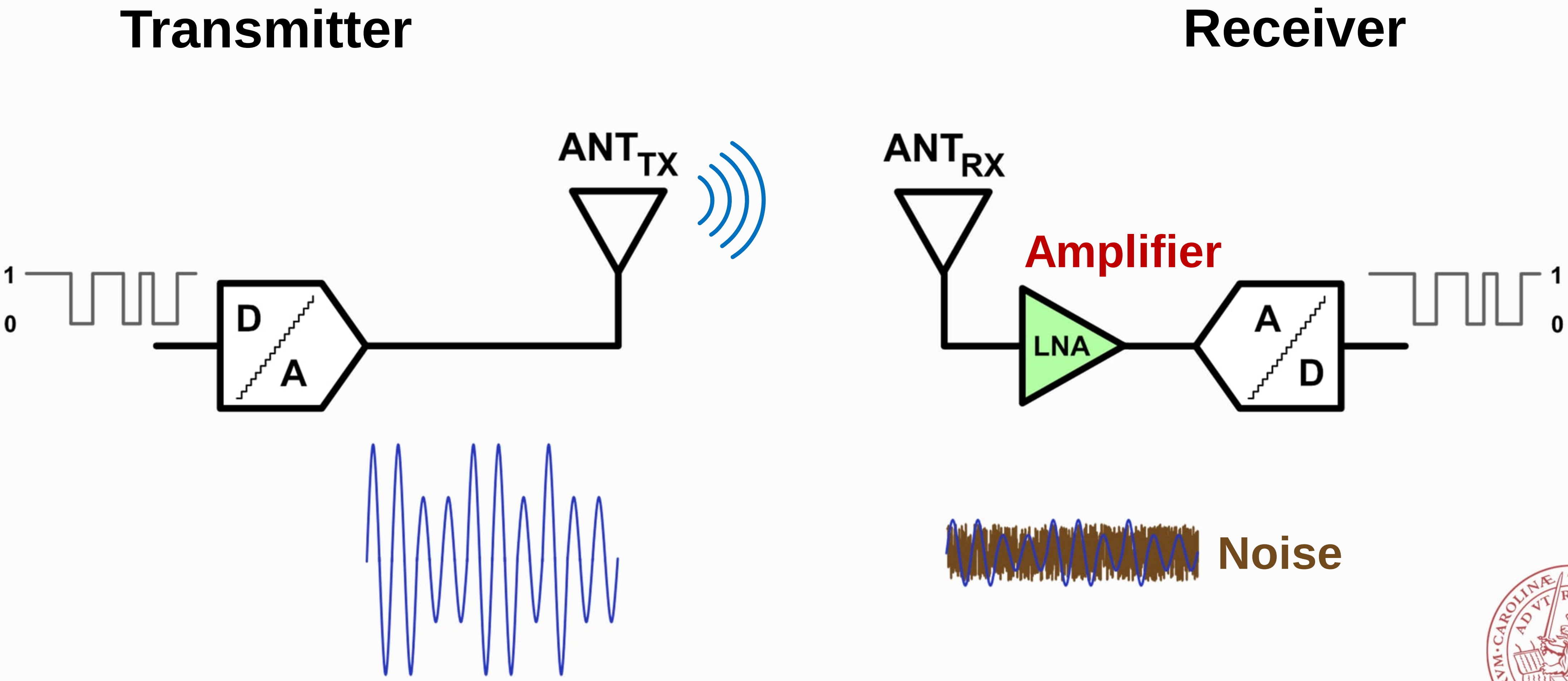


Amplification: Boosting the Signal’s Power



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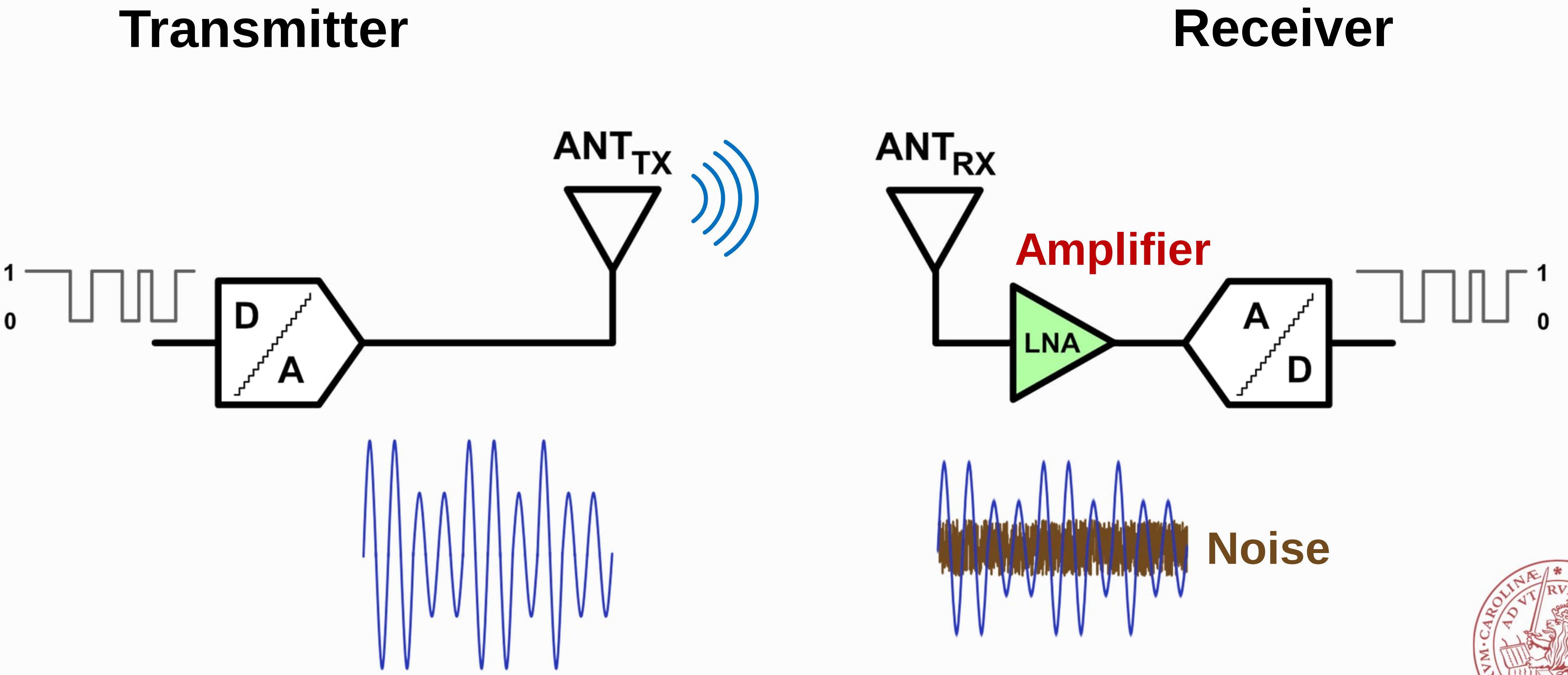
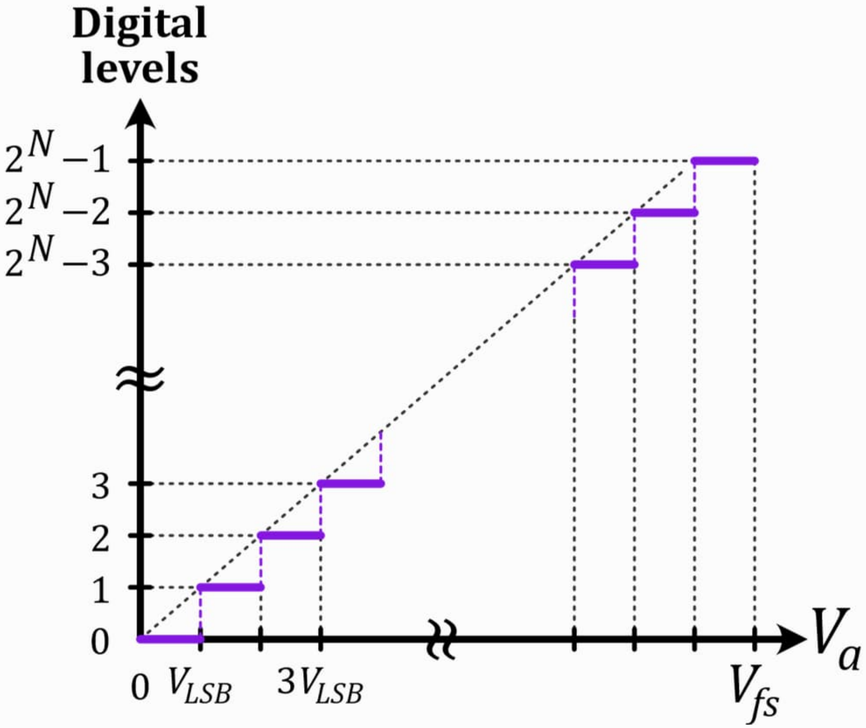


Quantization: Translation from Analog to Digital

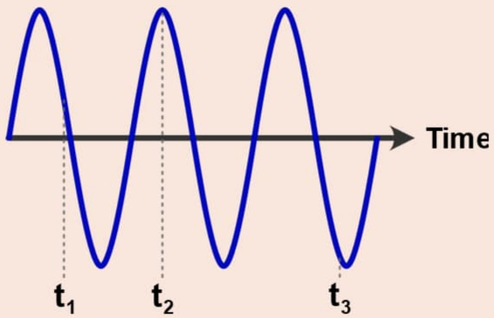
- Mapping analog intervals to digital levels
- Precision depends on the number of digits (resolution)
- In an N-bit binary quantizer:
  - Analog signal (V_a) varies from 0 to V_{fs}
  - For quantizers supporting negative values: $V_a \in \left[-V_{fs}/2\,,+V_{fs}/2\right]$
  - Number of digital levels: 2^N
  - Lowest significant bit corresponds to:

$$V_{LSB} = \frac{V_{fs}}{2^N - 1}$$



Continuous-Time (CT) vs. Discrete-Time (DT)

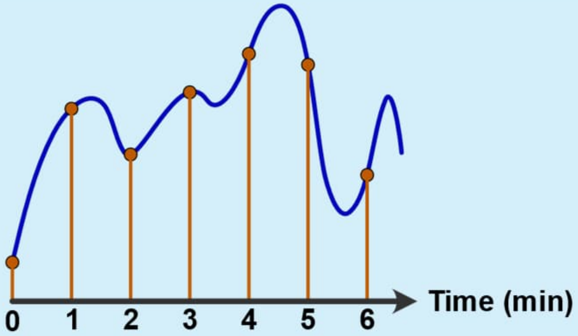
CT signals



- Change dynamically at any instant
- $x(t)$ can be defined for any $t \in \mathbb{R}$
- All real-world signals
- Too volatile to be processed!

DT signals

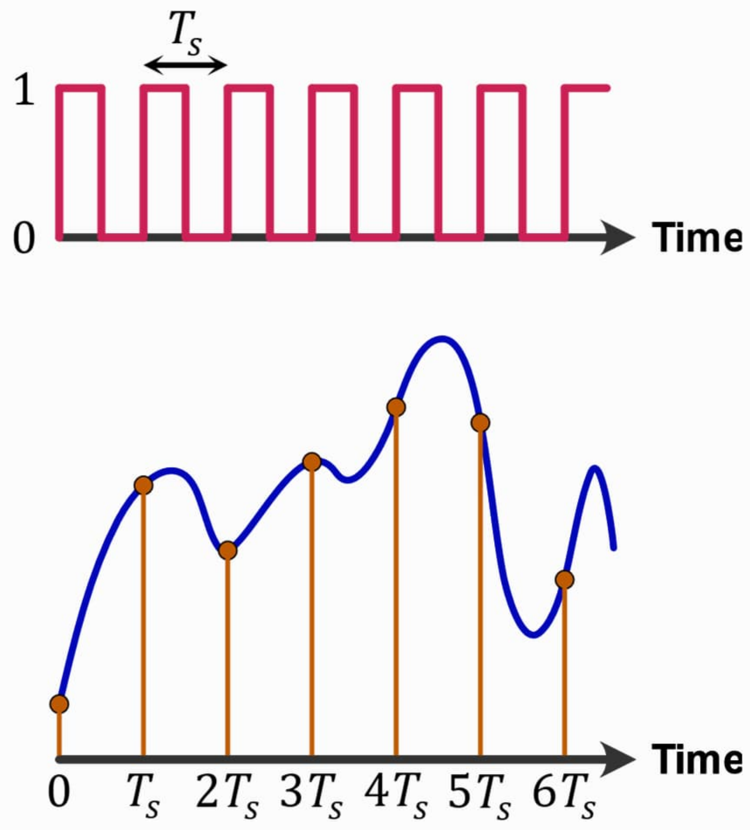
Refresh time: 1 min



- Change only at certain times
- $x[n]$ can be defined for any $n \in \mathbb{Z}$
- Sampled version of CT signals
- Commonly, equal sampling intervals

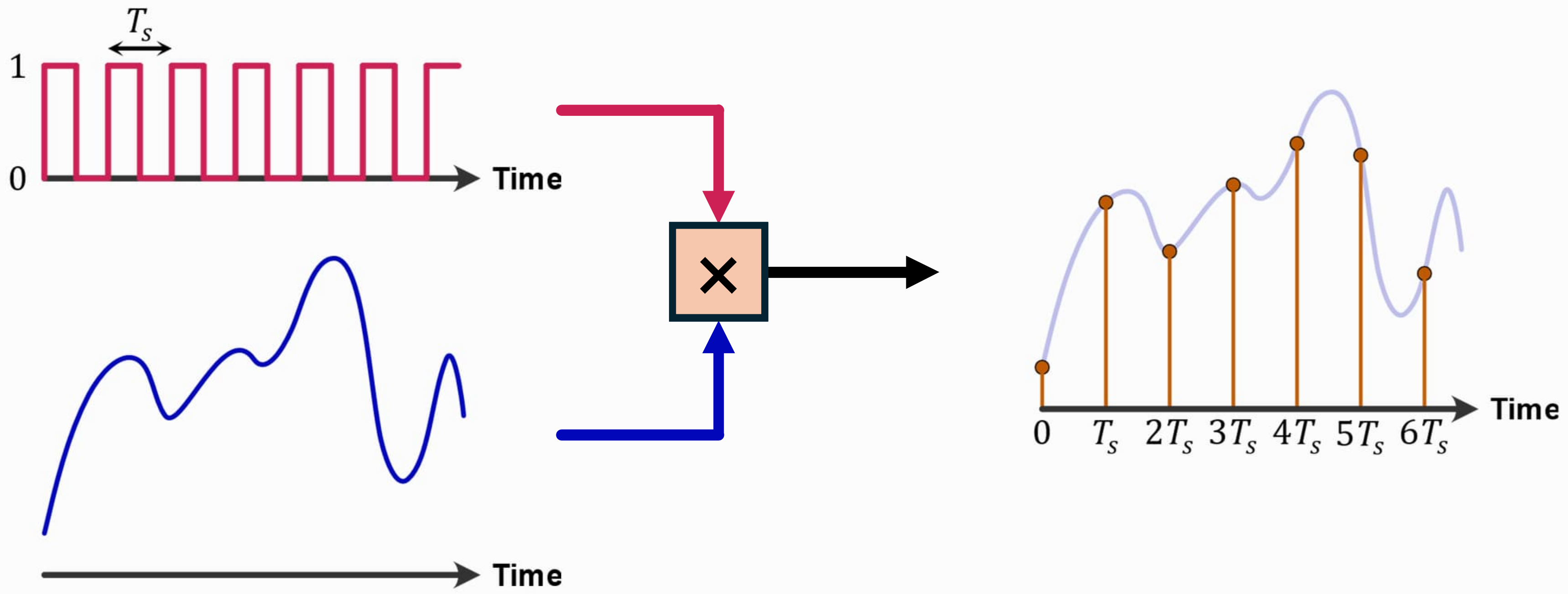
Sampling: Translation from CT to DT

- **Uniform sampling:** a periodic signal is used to sample CT signals and hold their corresponding DT signals.
- **Sampling period:** T_s
- **Sampling frequency:** $\boxed{f_s = 1/T_s}$
- Therefore, the k-th sample ($x[k]$) corresponds to the value of the CT signal at $t = kT_s$.



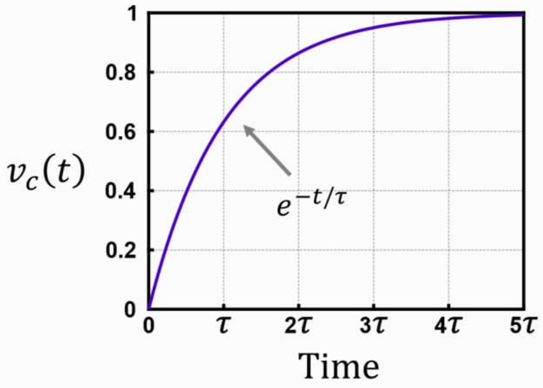
Pause and Ponder 1

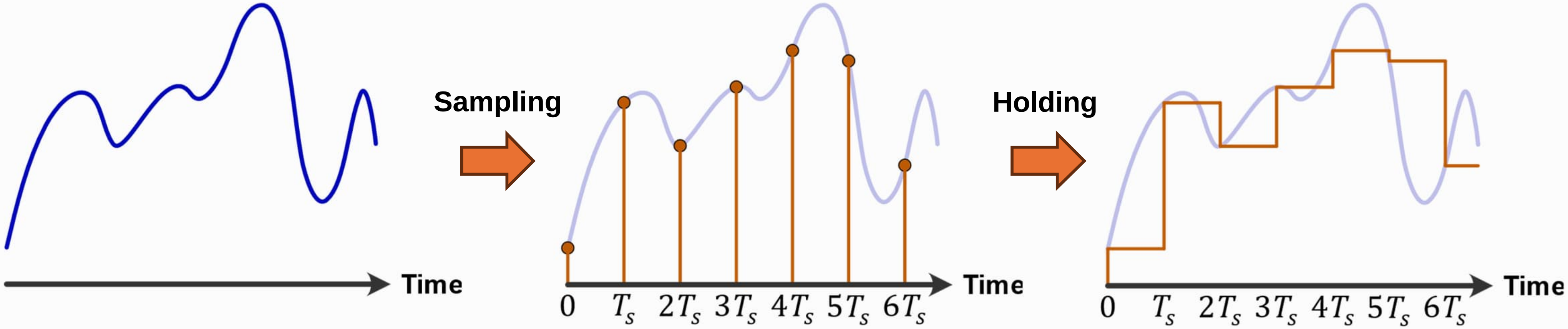
- What mathematical operation should be used to implement sampling?



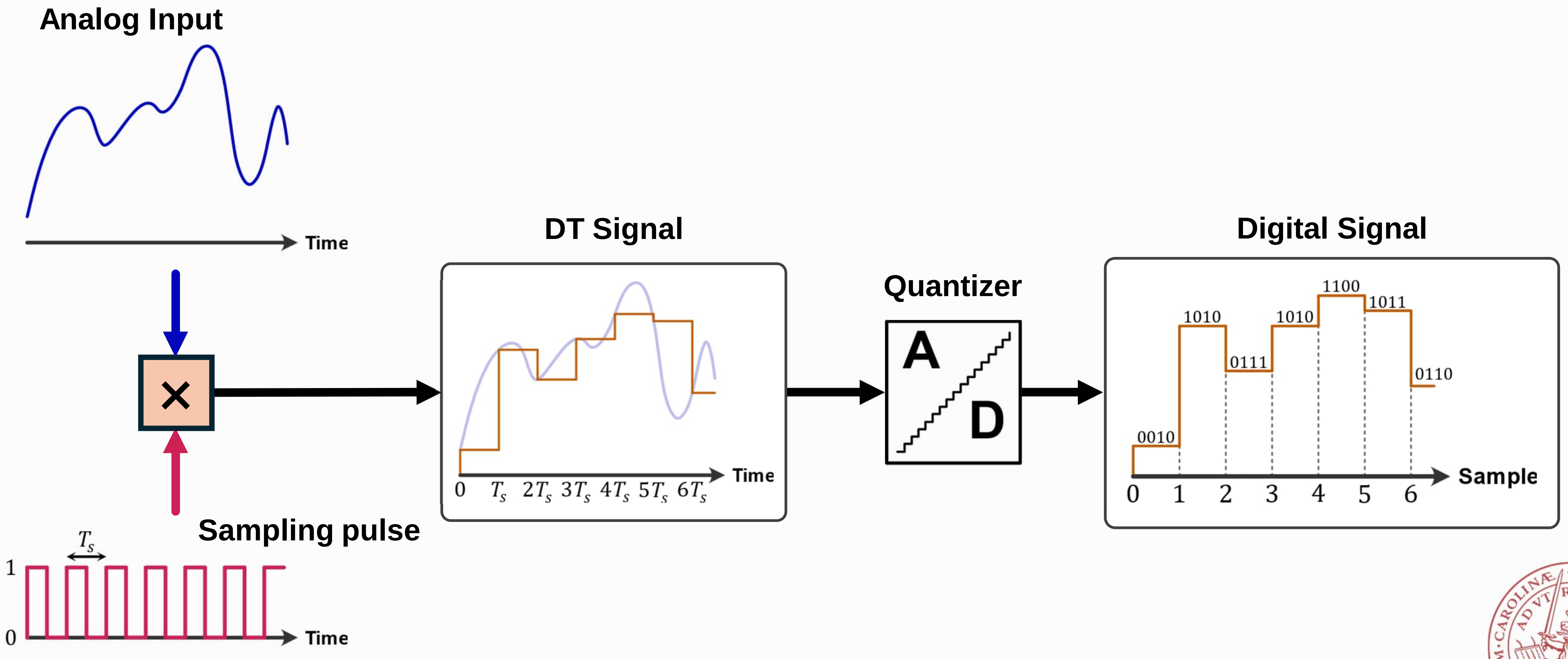
Pause and Ponder 2

- Considering the delay in signal processing systems, is it enough to sample signals at certain instants?

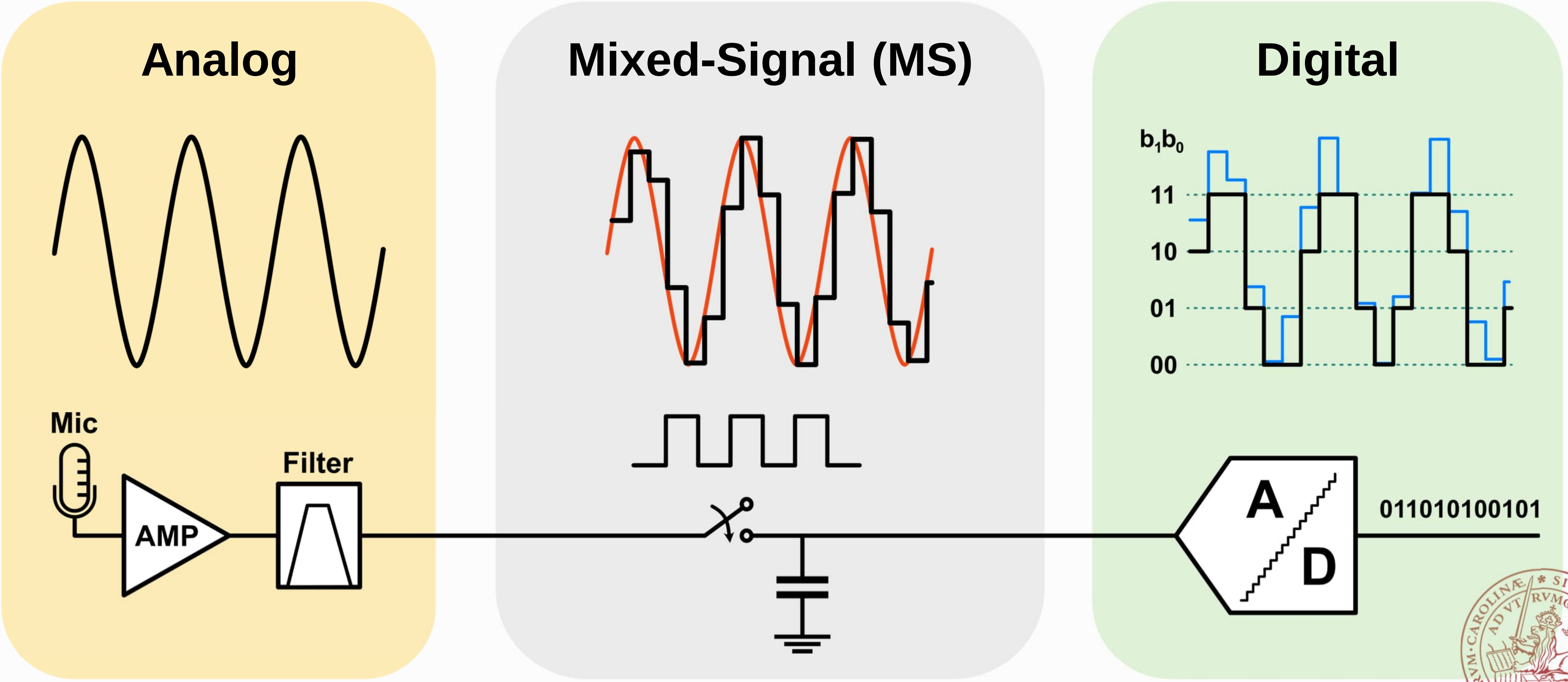




Data Conversion: Sampling + Quantization



Electronics: Amplify + Sample + Quantize + Process



Logarithmic Scale (Decibel)

- General definition: $|H(j\omega)|_{\mathrm{dB}} = 20\log_{10}|H(j\omega)|$
- $H(j\omega)$ can be a voltage gain, power gain, or any other type of transfer functions.
- Always represents a ratio → dB is unitless
- Useful for presenting wide-range quantities
- Can refer to a signal level in relation to a reference level:

dBV: $20\log_{10}\left(\frac{V_m}{1\ \mathrm{V}}\right)$ **dBm:** $10\log_{10}\left(\frac{P}{1\ \mathrm{m}W}\right)$

Voltage gain

$\lvert H(\mathrm{j}\omega)\rvert$	$\lvert H(\mathrm{j}\omega)\rvert_{\mathrm{dB}}$
10^n	$20n$ dB
10	20 dB
$\sqrt{2}$	≈ 3 dB
1	0 dB
$1/\sqrt{2}$	≈ -3 dB
0.1	-20 dB
10^{-n}	$-20n$ dB

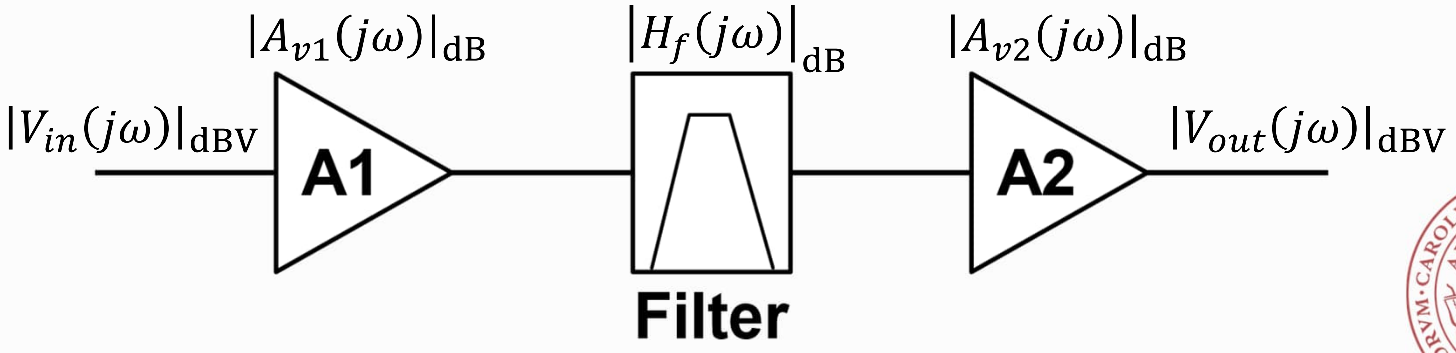
Why dB?

Compact representation of wide-range signals and physical quantities

Quantity	Linear scale	dB scale
The sun's radiated power	4×10^{27} W	306 dBm
Typical FM transmitter output power	100 kW	80 dBm
Typical mobile phone output power	0.5 W	27 dBm
Received power from GPS satellite	2×10^{-16} W	-127 dBm
Cosmic background radiation in 1 kHz window	4×10^{-21} W	-174 dBm

$$|V_{out}(j\omega)|_{dBV} = |V_{in}(j\omega)|_{dBV} + |A_{v1}(j\omega)|_{dB} + |H_f(j\omega)|_{dB} + |A_{v2}(j\omega)|_{dB}$$

Easier gain calculation in multiple stage cascade systems



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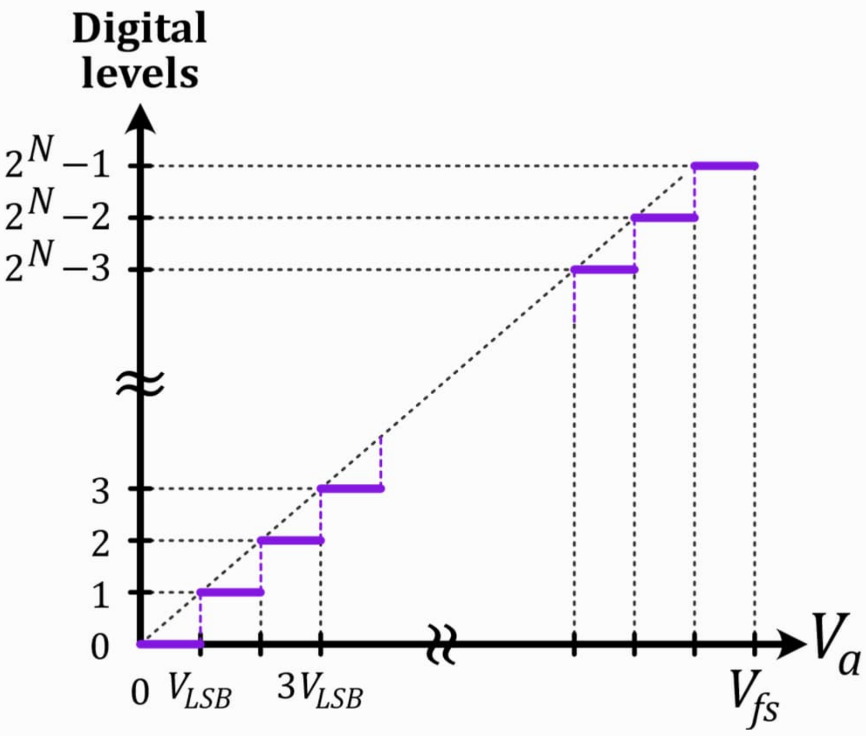
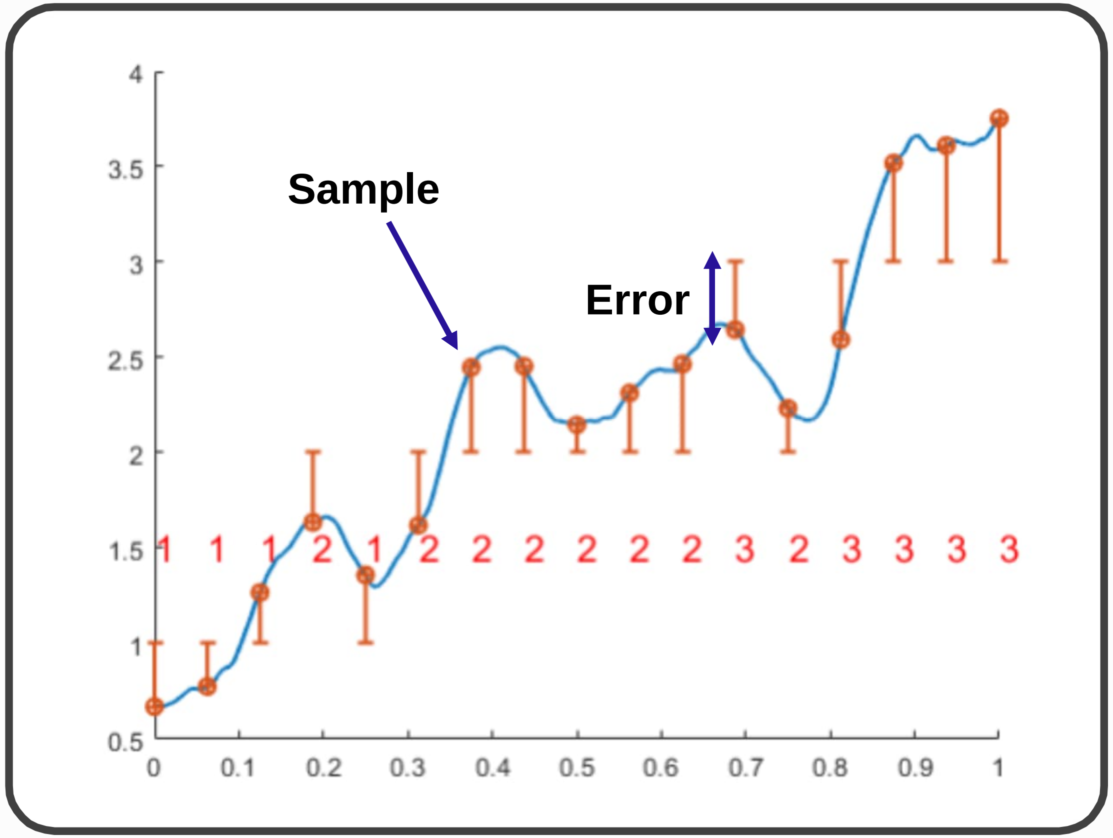


$$\Delta = V_{LSB} = \frac{V_{fs}}{2^N - 1}$$

Quantization: Example 1

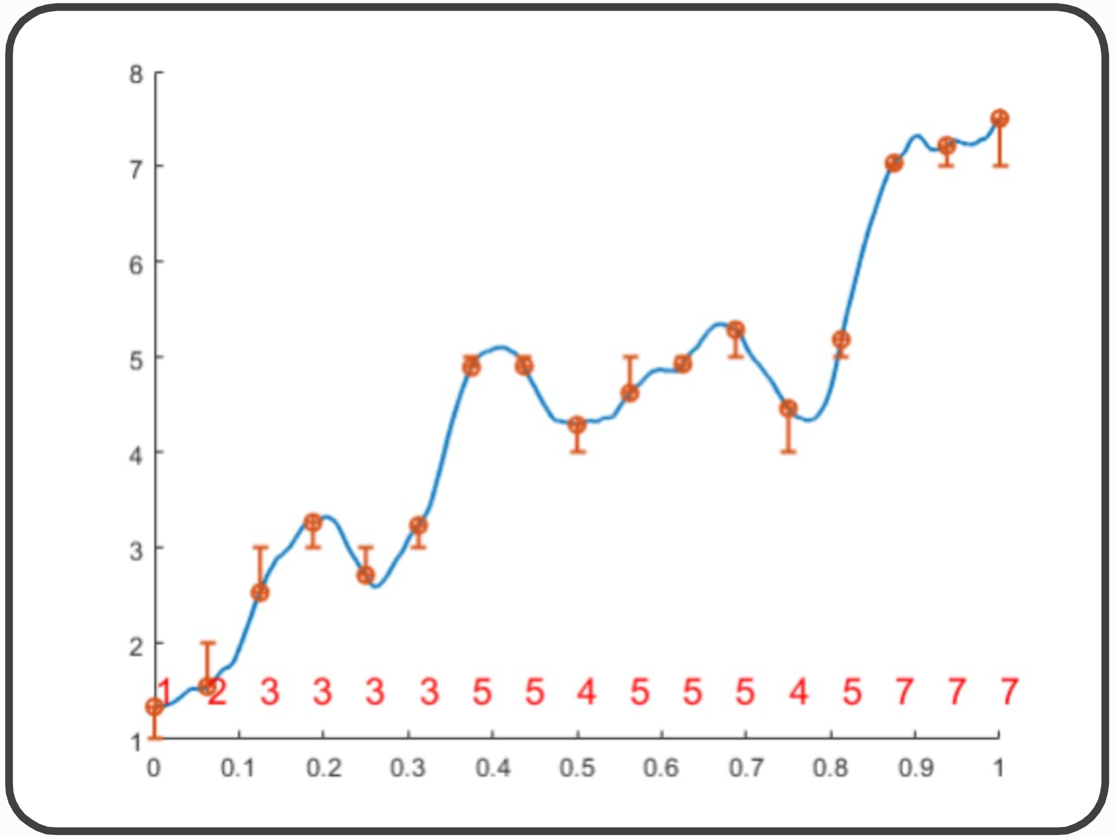
- 2-bit quantizer ($N = 2$)
- Number of discrete levels: $2^2 = 4$
- $\Delta = V_{LSB} = \frac{V_{fs}}{2^2-1} = \frac{V_{fs}}{3}$
- each sample Rounded to the closest integer
- Loss of information



Quantization: Example 2

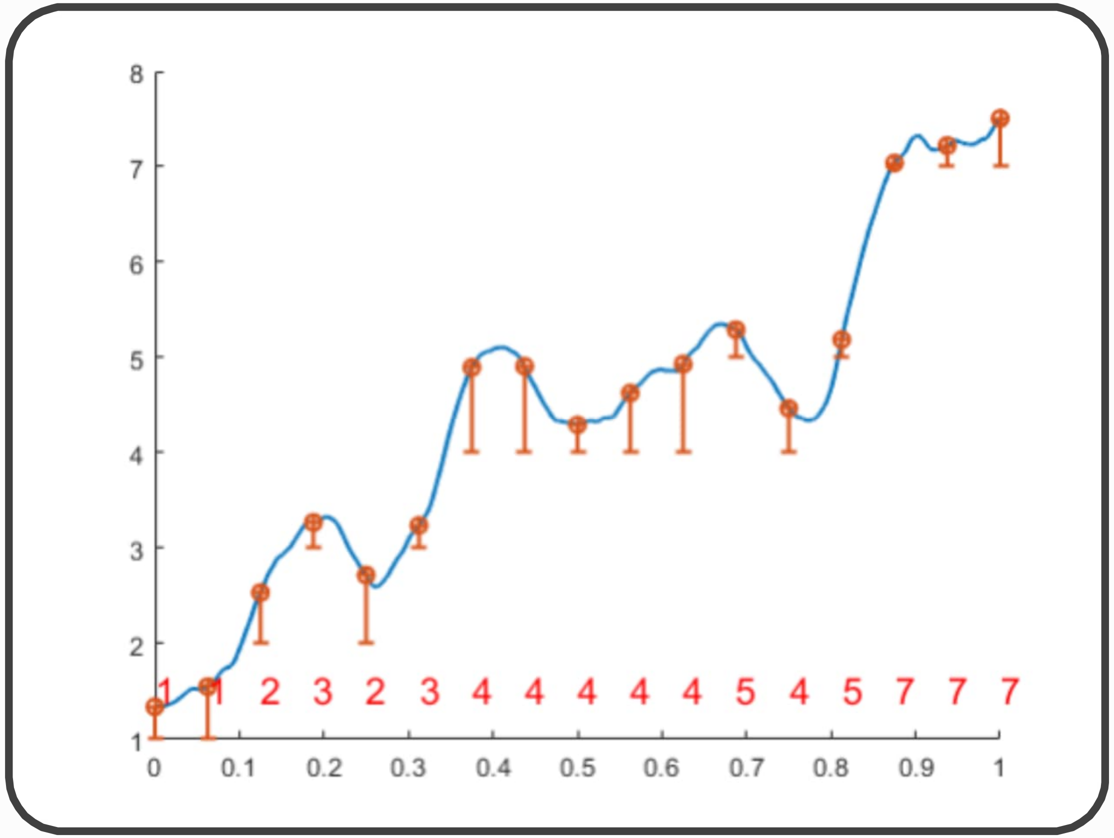
- 3-bit quantizer ($N = 3$)
- Number of discrete levels: $2^3 = 8$
- $\Delta = V_{LSB} = \frac{V_{fs}}{2^3-1} = \frac{V_{fs}}{7}$
- Rounds each sample to the closest integer
- Lower loss of information

    - Smaller quantization error (V_{err})



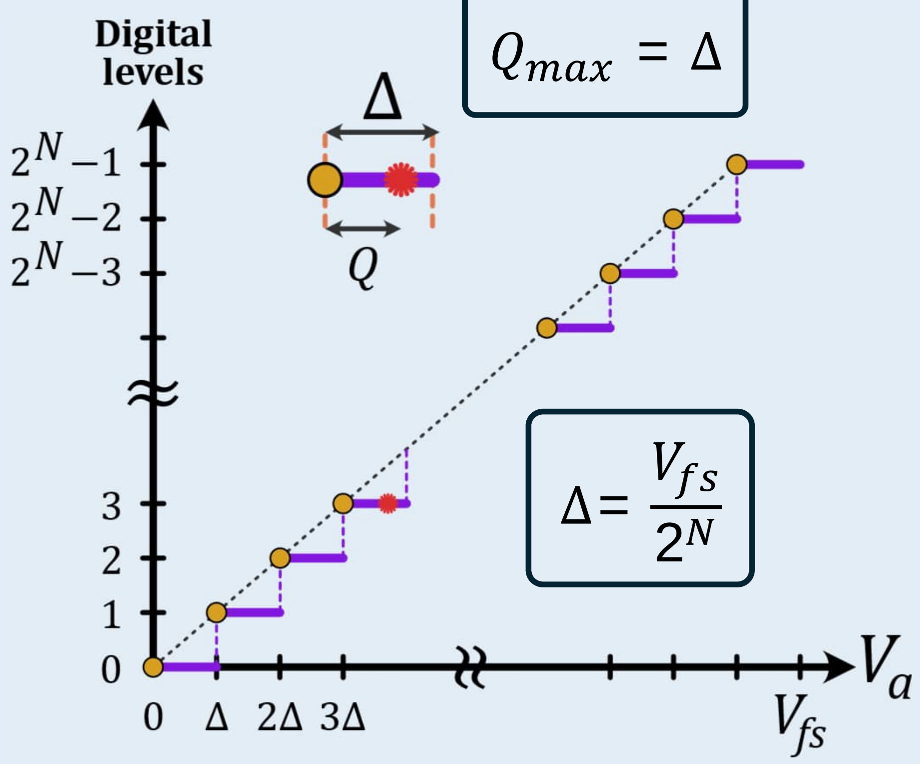
Truncating or Rounding?

- Truncating 3-bit quantizer
- Degradation of signal quality
- Quantization error depends on:
  - Number of bits (V_{LSB})
  - Rounding method
- **Truncation:**
  - Remove the decimals
  - $|V_{err}| \leq V_{LSB}$
- **Rounding** (previous slide):
  - $|V_{err}| \leq V_{LSB}/2$



Truncating or Rounding?

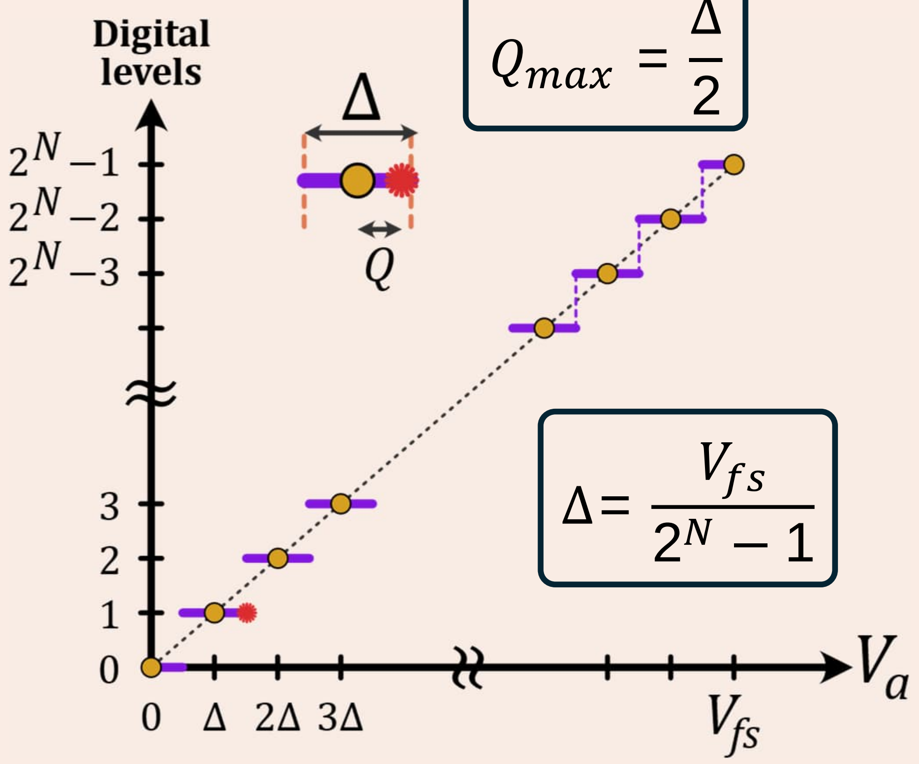
Truncating



$$Q_{max} = \Delta$$

$$\Delta = \frac{V_{fs}}{2^N}$$

Rounding



$$Q_{max} = \frac{\Delta}{2}$$

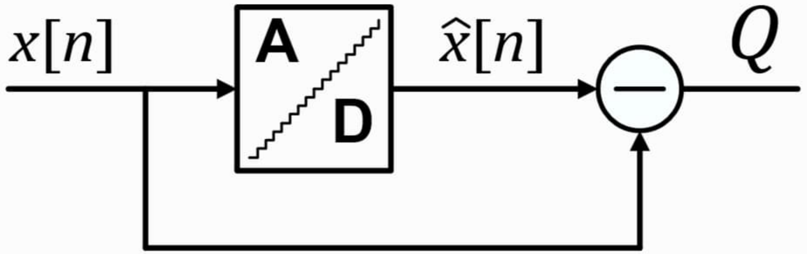
$$\Delta = \frac{V_{fs}}{2^N - 1}$$

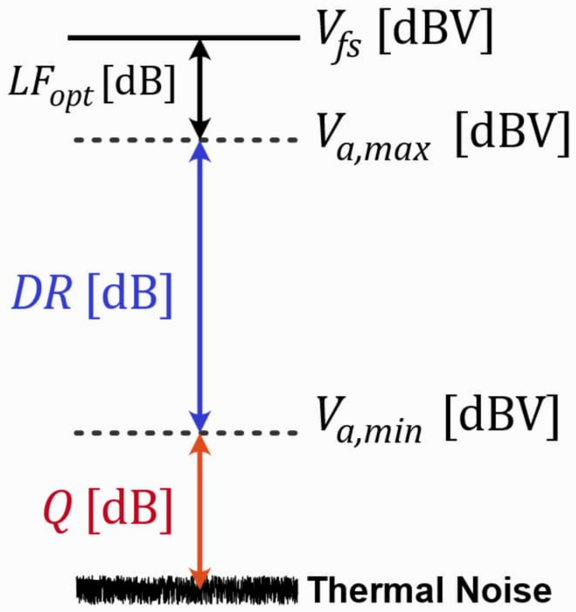
Quantization Noise (Error), Q

- This is not a real noise!
- However, has an effect like noise
  - Limits the dynamic range (DR) of data converters
- **DR:** Maximum range of analog signal that can be converted precisely.

$$\boxed{DR = V_{a,max}[dBV] - V_{a,min}[dBV]}$$

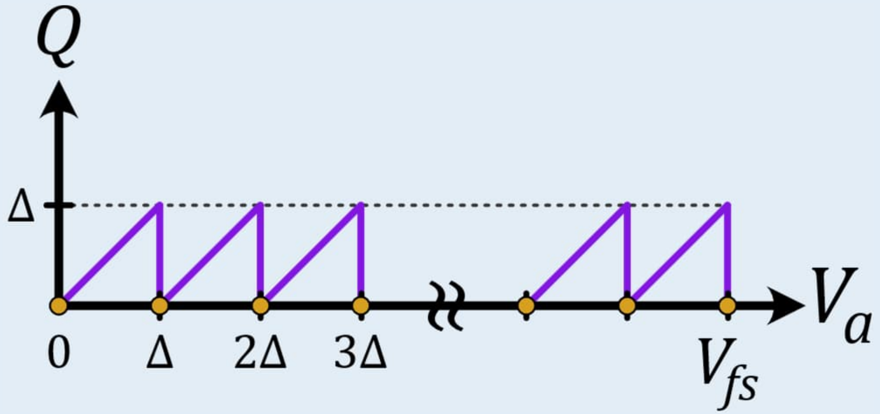
- $V_{a,max}$: Limited by nonlinearities
- $V_{a,min}$: Limited by quantization error and thermal noise





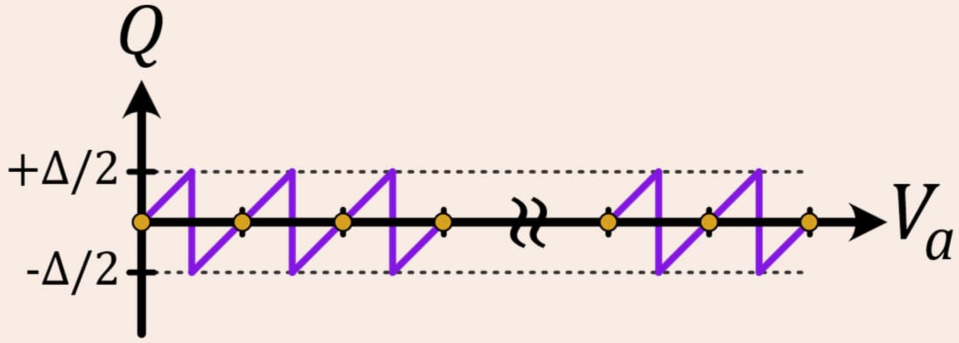
Quantization Noise

Truncating



Average Power: $\boxed{P_Q = \frac{1}{\Delta}\int_0^{\Delta} Q^2 dV_a = \frac{\Delta^2}{3}}$

Rounding



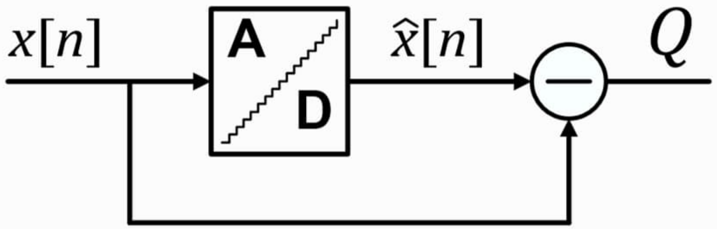
Average Power: $\boxed{P_Q = \frac{1}{\Delta}\int_0^{\Delta} Q^2 dV_a = \frac{\Delta^2}{12}}$

Rounding results in **6 dB** lower quantization noise.

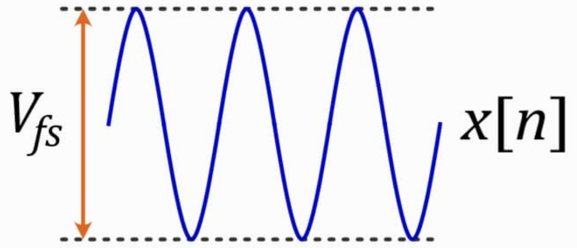
Signal-to-Noise Ratio (SNR)

- The ratio of signal power to noise power

$$\boxed{SNR = \frac{P_{sig}}{P_N}} \qquad \boxed{SNR[\text{dB}] = 10\log_{10} SNR}$$



- A measure of conversion accuracy



- In an ideal N-bit quantizer:
  - Full-scale sinusoidal input signal: $V_{sig,pp} = V_{fs}$
  - Signal power: $P_{sig} = \frac{\left(V_{sig,pp}/2\right)^2}{2} = \frac{V_{fs}^2}{8} = \frac{\left(2^N-1\right)^2\Delta^2}{8}$
  - Quantization noise power: $P_Q = \frac{\Delta^2}{12}$

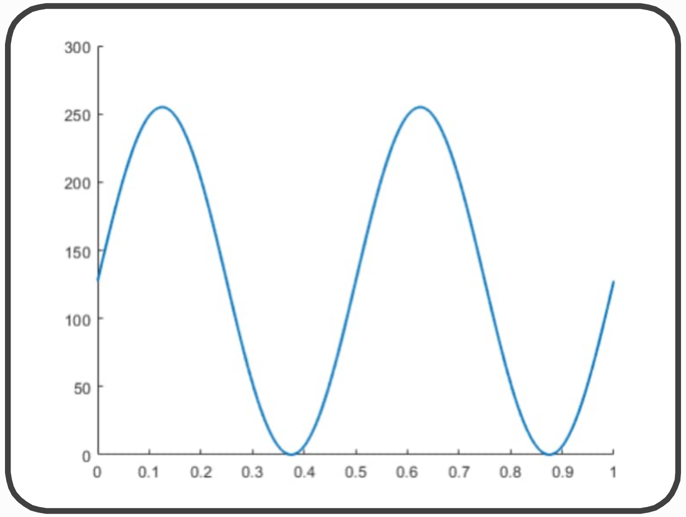
$$SNR = \frac{P_{sig}}{P_Q} = \frac{3}{2}(2^N-1)^2$$

$$\boxed{SNR[\text{dB}] \cong 6.02 \times N + 1.76\ \text{dB}}$$

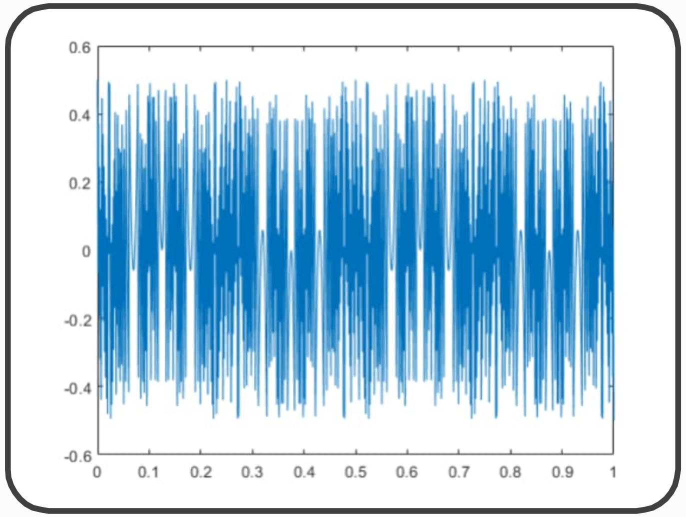
Signal-to-Noise Ratio (SNR)

- Each additional bit of resolution → **6 dB higher SNR**
- Example:
  - $N = 8$
  - $\Delta = 1$
  - $V_{fs} = 2^8 - 1 = 255$
  - $P_{sig} = 8128$
  - $P_Q = 0.083$
  - $SNR = \frac{P_{sig}}{P_Q} \cong 9.8 \times 10^4$
  - $SNR[dB] \cong 50$ dB

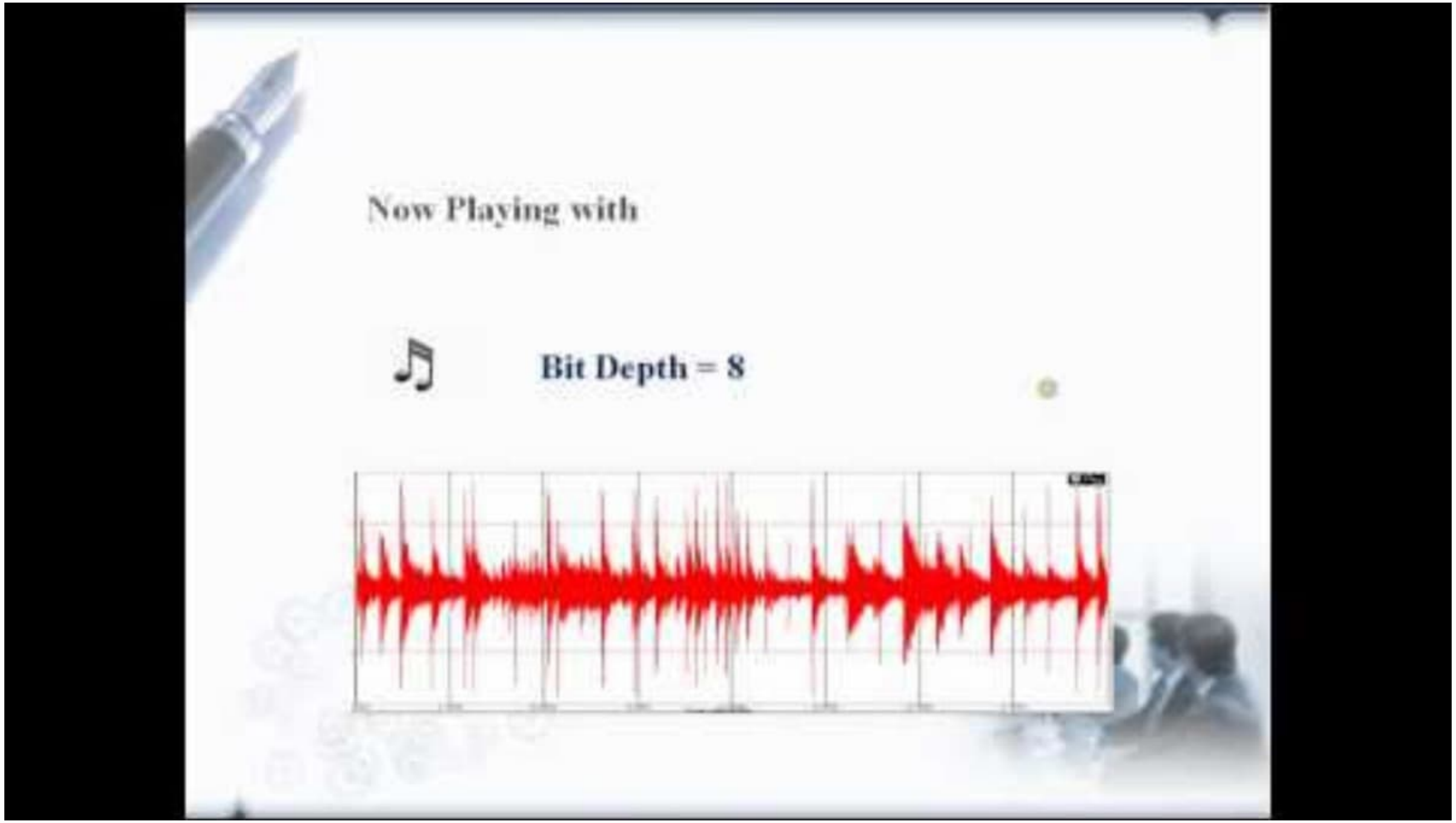
Input signal



Quantization noise



Audio Demonstration of Quantization Noise



Effective Number of Bits (ENOB)

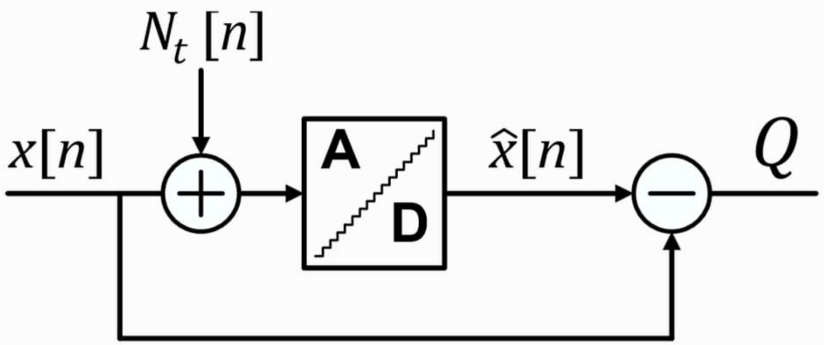
- In practice, electronic quantizer adds **thermal noise** (N_t) to analog samples.
- Thermal noise is added to quantization noise.

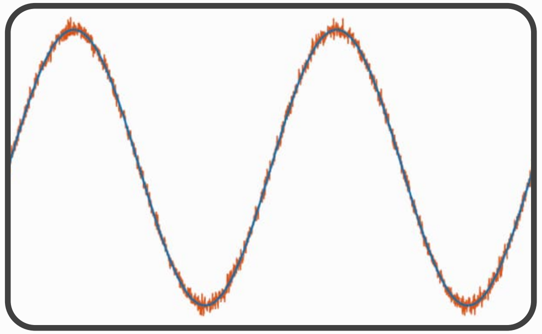
$$N_{tot}[n] = N_t[n] + Q[n]$$

- **Degraded SNR** compared to ideal quantizer

$$SNR = \frac{P_{sig}}{P_{N,t} + P_Q}$$

- **ENOB:** practical achievable resolution of a quantizer including thermal noise and other imperfections:





$$ENOB \triangleq \frac{SNR[\text{dB}] - 1.76\text{ dB}}{6.02}$$

Transfer function of an Ideal Quantizer

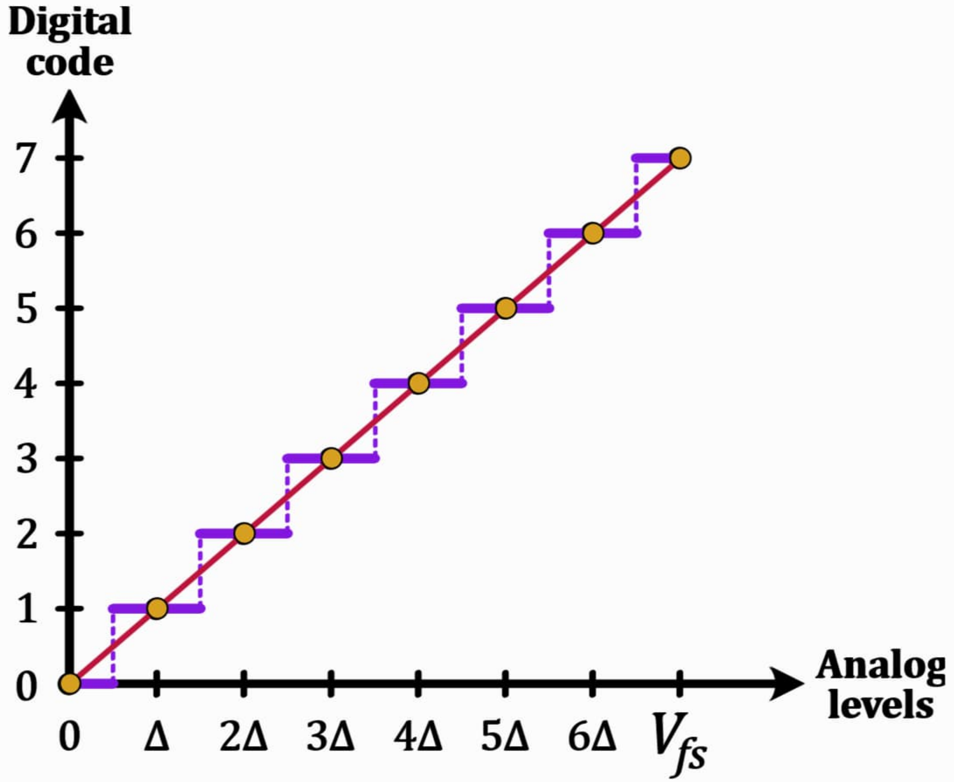
- A linear function that relates analog levels (V_a) to digital codes (D)

$$\boxed{D = G(V_a - V_{a0})}$$

- In an ideal quantizer:

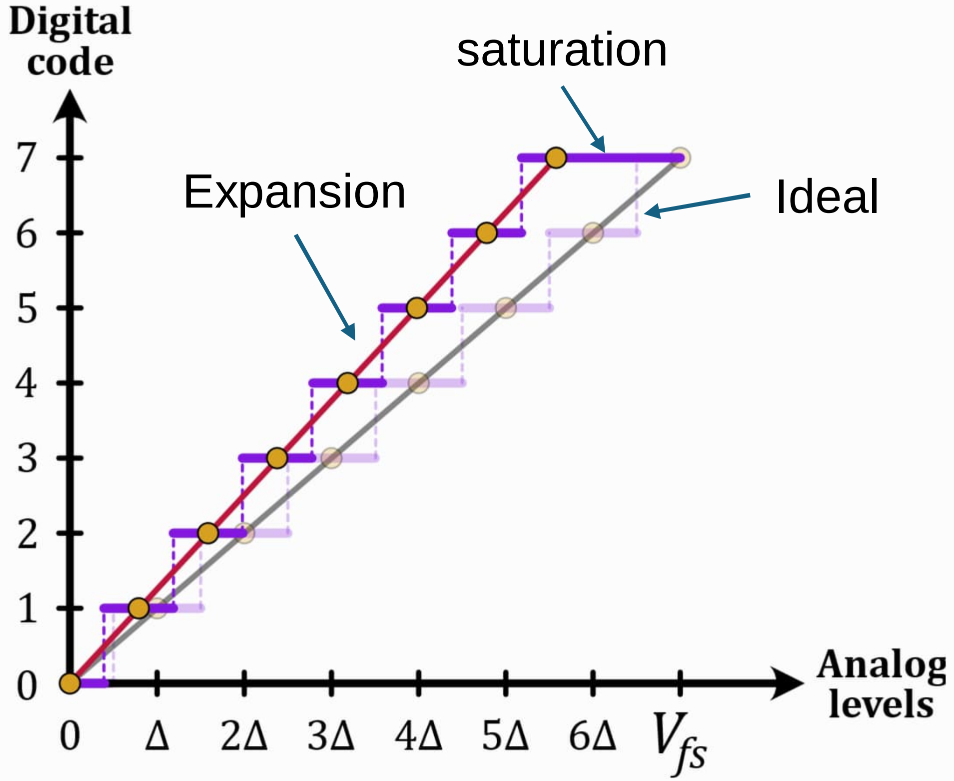
$$\boxed{G = \frac{1}{\Delta}} \qquad \boxed{V_{a0} = 0}$$

Gain **Offset**



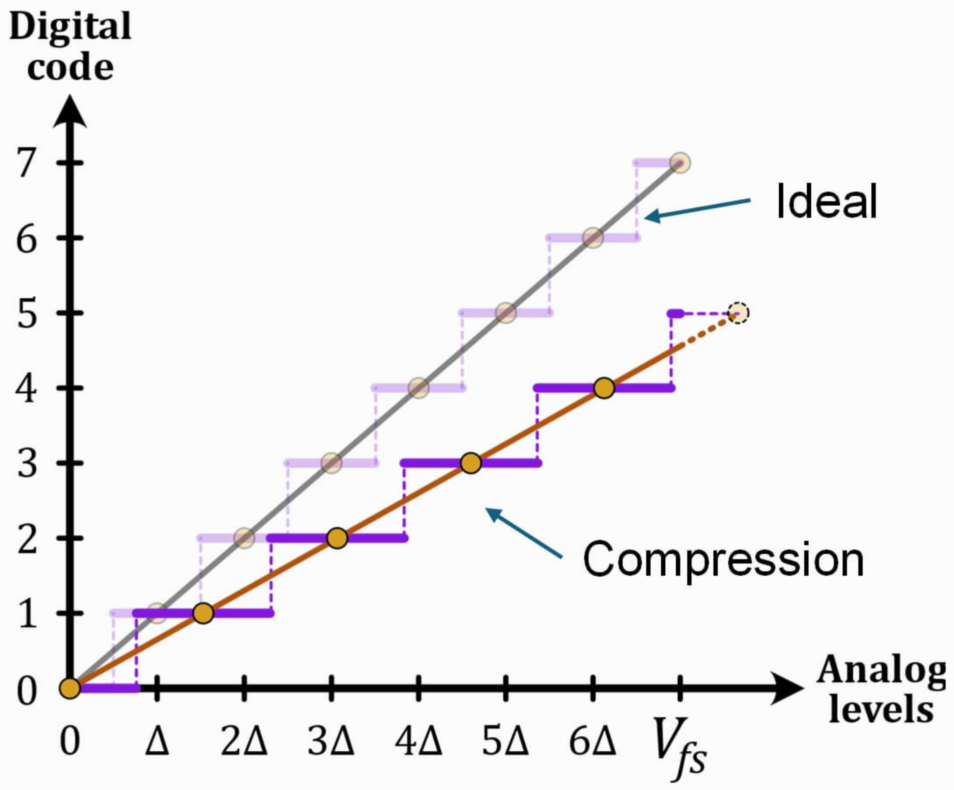
Quantization Non-idealities – Gain Error

- Smaller or larger effective V_{LSB} = Δ
- **Gain Expansion ($G_{meas} > G_{ideal}$):**
    - Saturation in the end
    - Part of full-scale range is lost
    - Limits the dynamic range



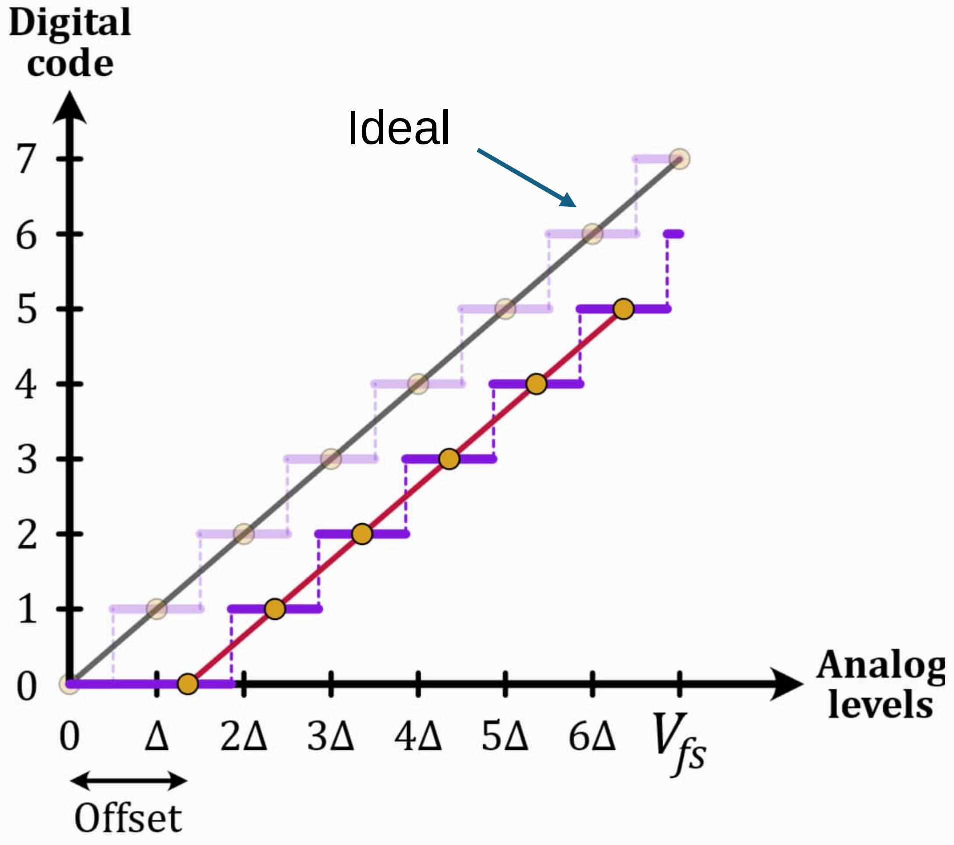
Quantization Non-idealities – Gain Error

- Smaller or larger effective V_{LSB} = Δ
- **Gain Expansion ($G_{meas} > G_{ideal}$):**
  - Saturation in the end
  - Part of full-scale range is lost
  - Limits the dynamic range
- **Gain Compression ($G_{meas} < G_{ideal}$):**
  - Unused digital code (6 and 7 here)
  - Lower effective bits → larger Q
  - Lower SNR



Quantization Non-idealities – Offset Error

- Interval for the first digital code (0) is stretched.
- $\Delta_0 > V_{LSB}$
- Normally can be corrected in digital domain.
- Requires rigorous measurement and modeling



Pause and Ponder 3

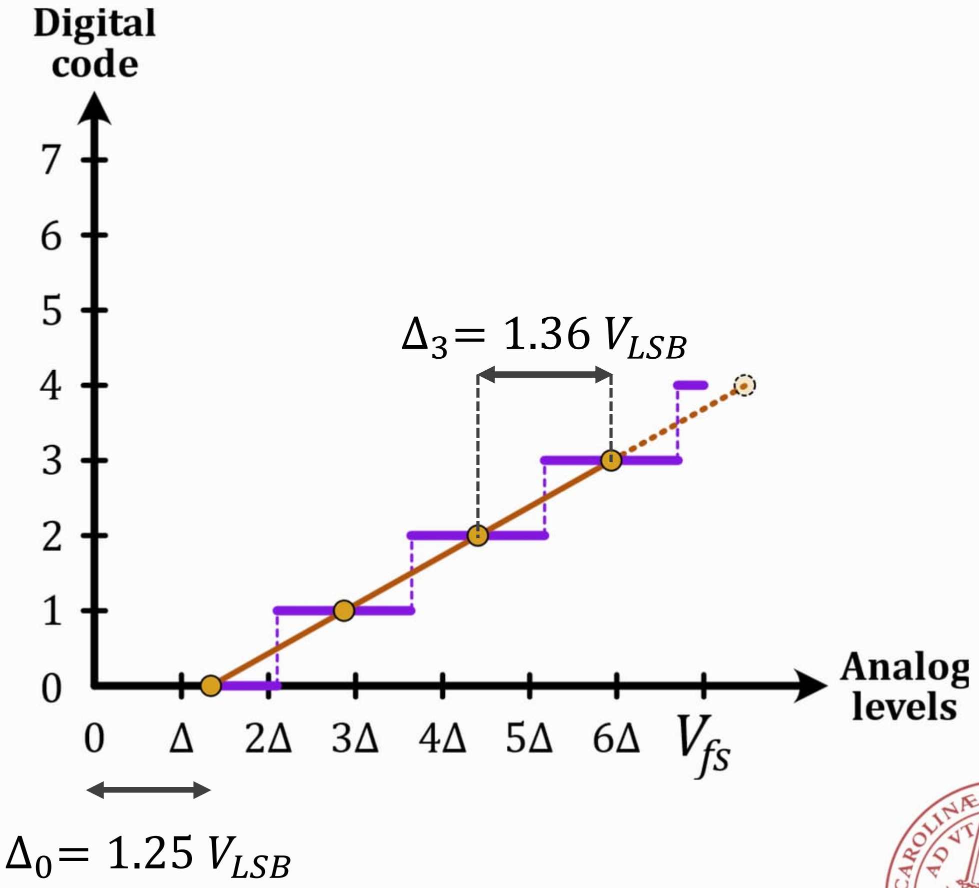
- After 100 rounds of measurements, we have plotted this transfer function for an ADC. Find offset and gain error.

$$D = G(V_a - V_{a0})$$

$$V_{a0} = \Delta_0 = 1.25\, V_{LSB}$$

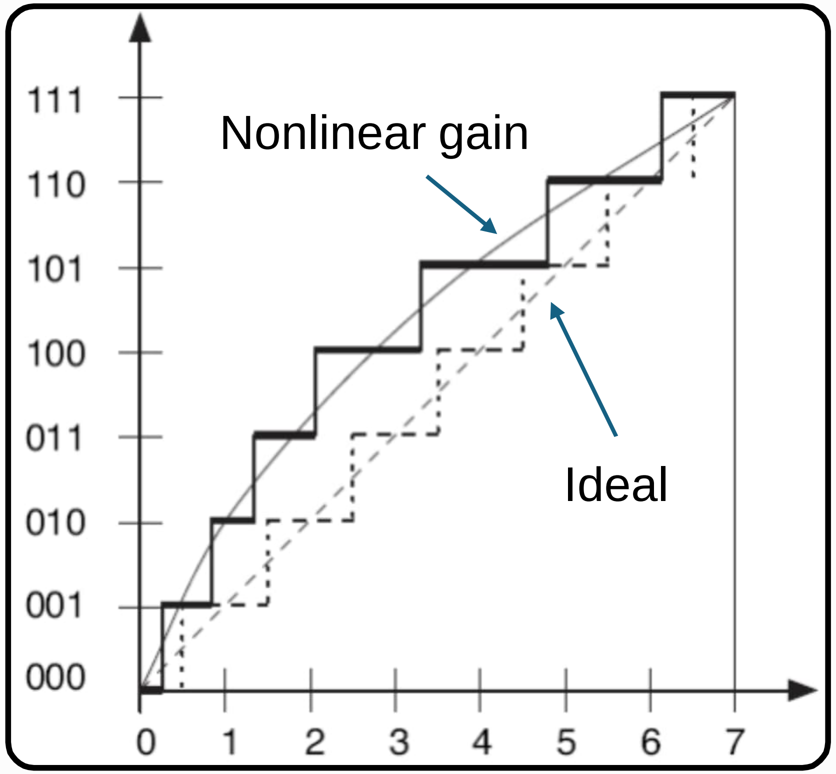
$$G = \frac{1}{\Delta_3} = 0.73\, \frac{1}{V_{LSB}}$$

$$G < 1 \;\rightarrow \text{Compression}$$



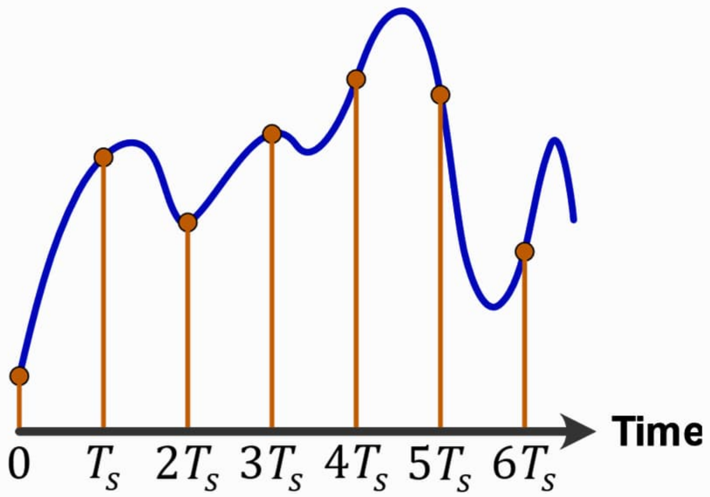
Quantization Non-idealities – Nonlinearity

- Nonlinearities of switches, capacitors, and resistors.
- Uneven intervals ($\Delta_i \neq \Delta_j$)
- Generates distortion in digital spectrum
    - Harmonics
    - Inter-mixing of signals
- Degrades DR

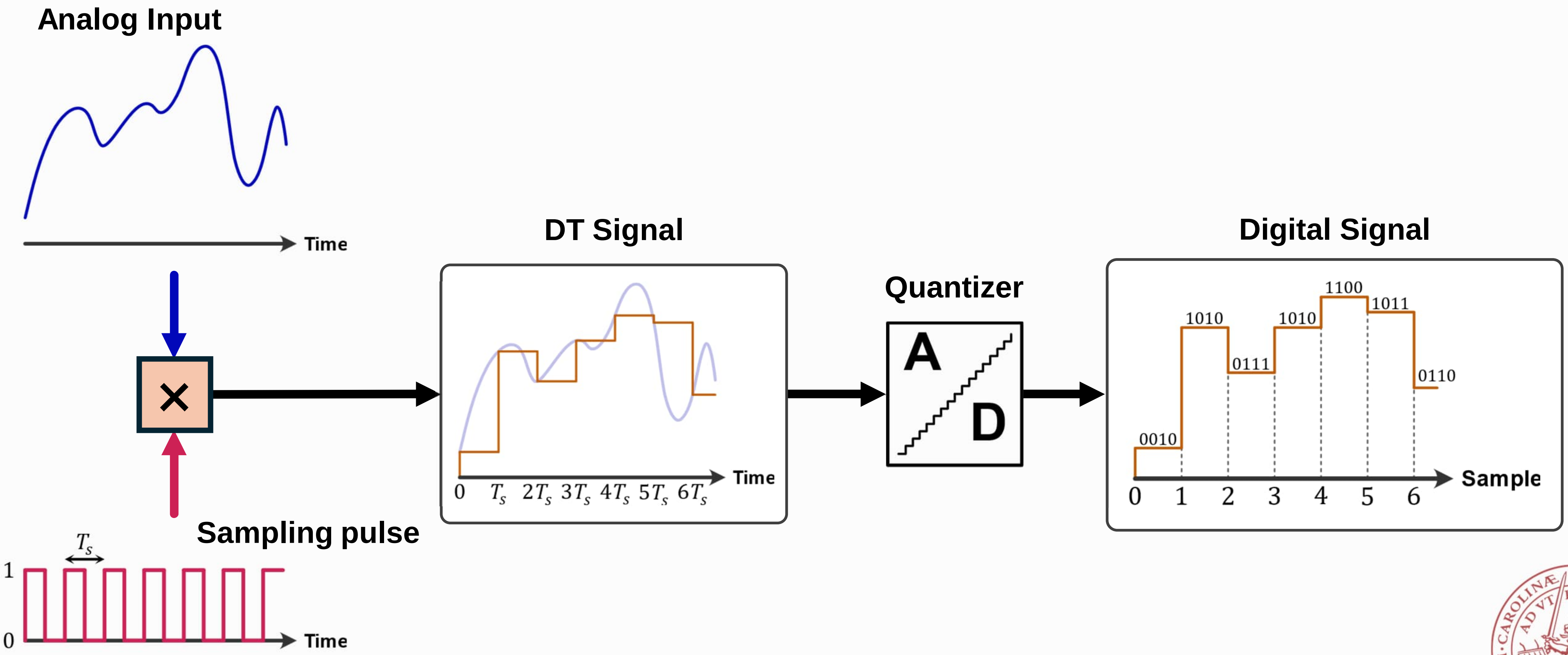


Sampling: Translation from CT to DT

- **Uniform sampling:** a periodic signal is used to sample CT signals and hold their corresponding DT signals.
- **Sampling period:** T_s
- **Sampling frequency:** $\boxed{f_s = 1/T_s}$
- Therefore, the k-th sample ($x[k]$) corresponds to the value of the CT signal at $t = kT_s$.

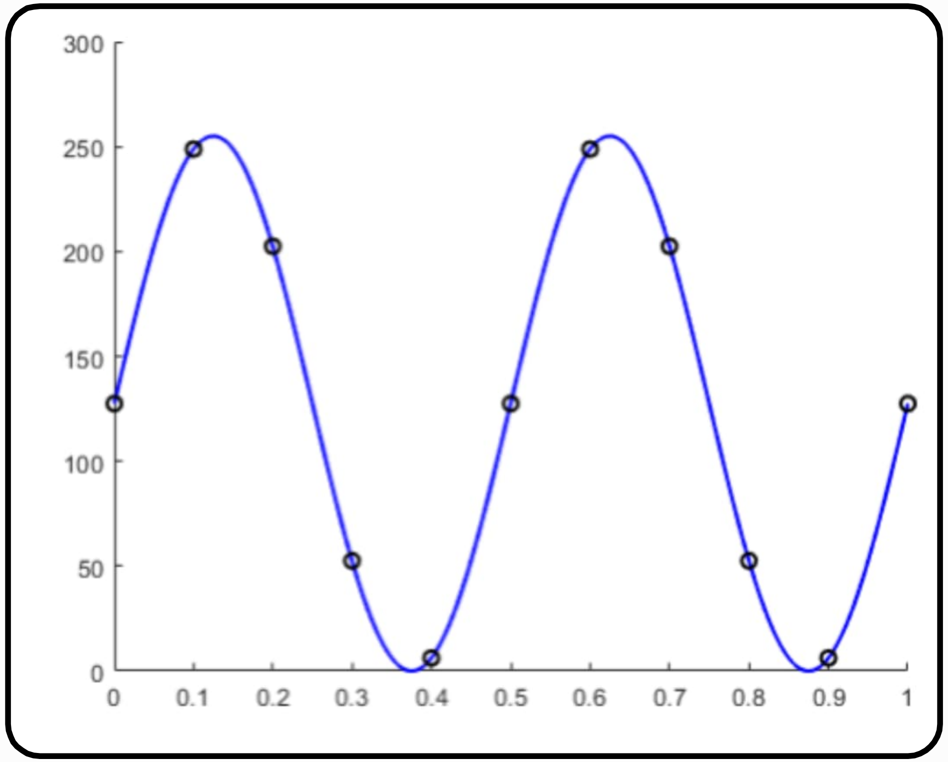


Data Conversion: Sampling + Quantization



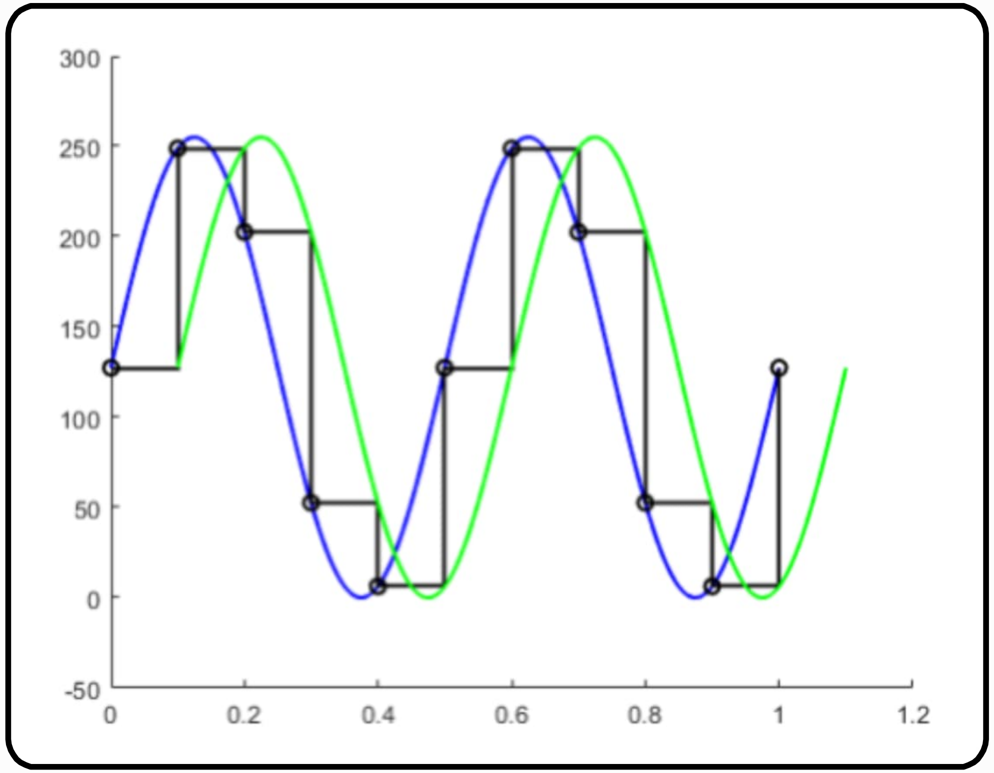
How Fast Should We Sample a Signal?

- Depends on the maximum frequency of interest in the signal spectrum (f_{sig})
- Fourier transform of a signal determines f_{sig}
- In this example:
  - f_s = 10
  - f_{sig} = 2
- $f_s > 2 \times f_{sig}$, seems OK!



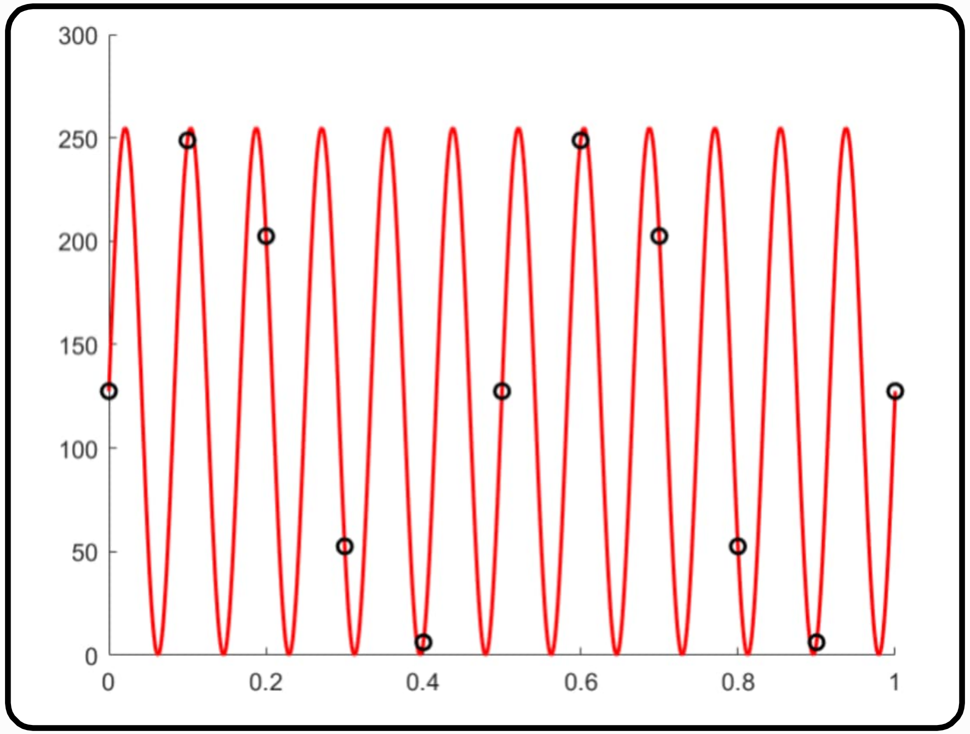
Signal Reconstruction

- The sampled signal is restored by interpolation.
- A filter takes average of samples and fill time intervals.
- Requires enough samples per second to reconstruct the signal with acceptable accuracy.



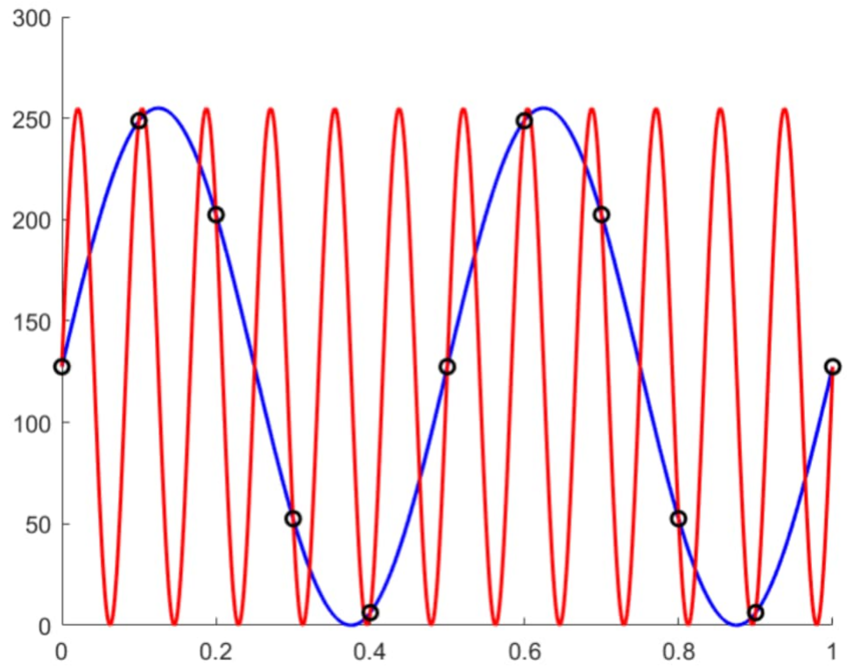
Undersampling

- $f_s < 2 \times f_{sig}$
- Misses rapid changes
- The signal might have several extreme values between two sample points
- Cannot be reconstructed



Aliasing

- Signals at different frequencies sampled by the same f_s, lead to the same sample points.
- We cannot differentiate them after sampling.
- Example:
  - f_s = 10
  - f_{blue} = 2, f_{red} = 12



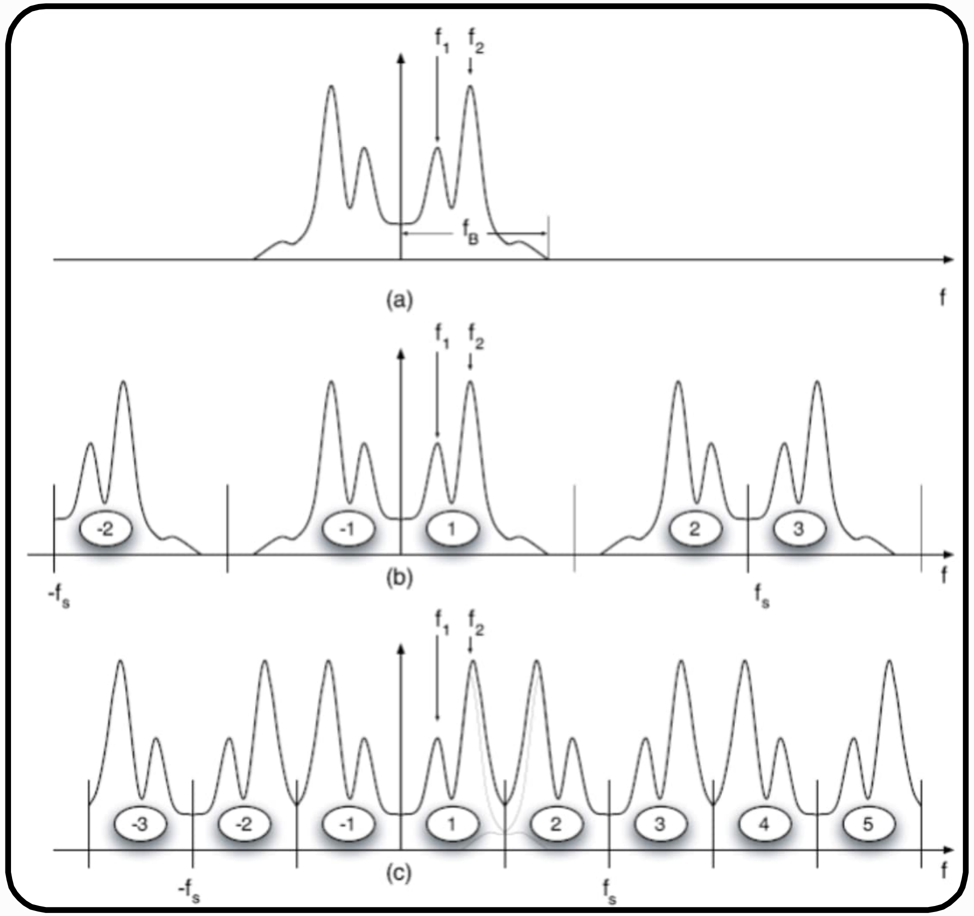
Aliasing

- In general aliased signals of a specific signal located at f_{sig}, exist at:

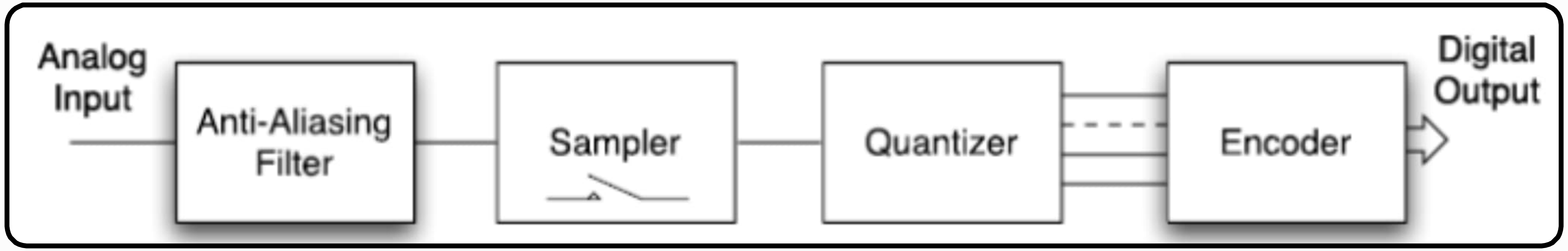
$$\boxed{f_n = n \times f_s \pm f_{sig}}$$

- Meaning, each signal itself repeats in spectrum as an image (copy) at:

$$\boxed{f_{im} = n \times f_s \pm f_{sig}}$$



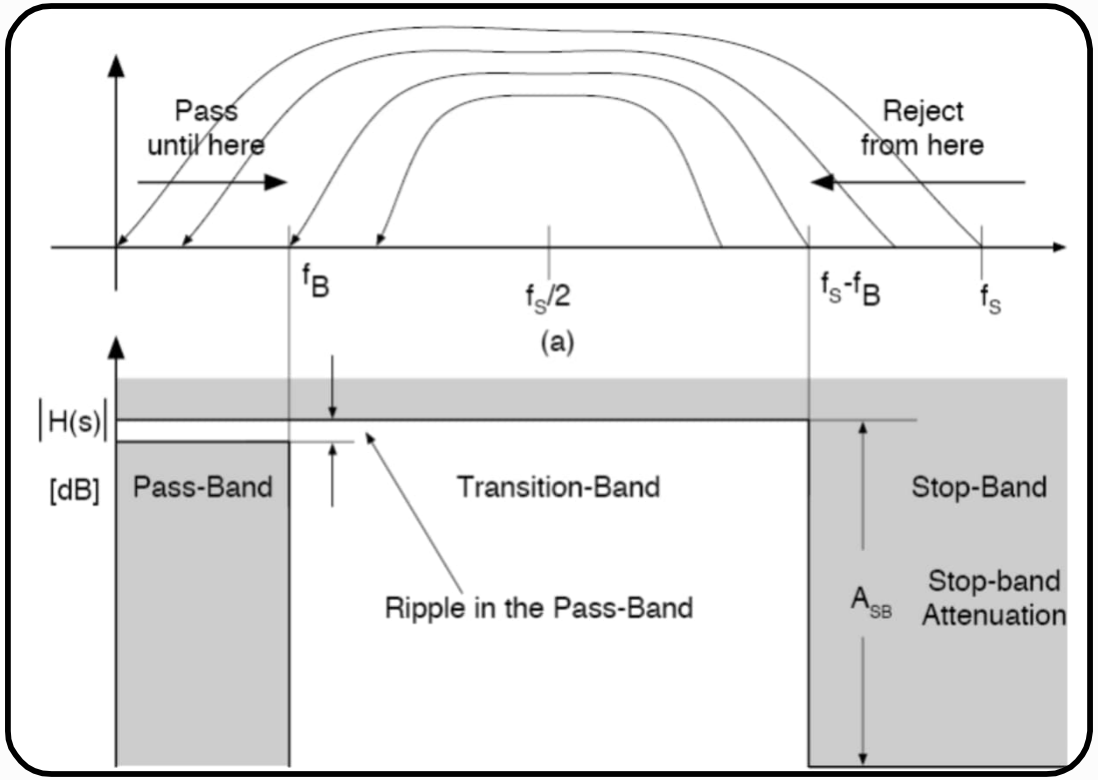
Anti-aliasing Filter



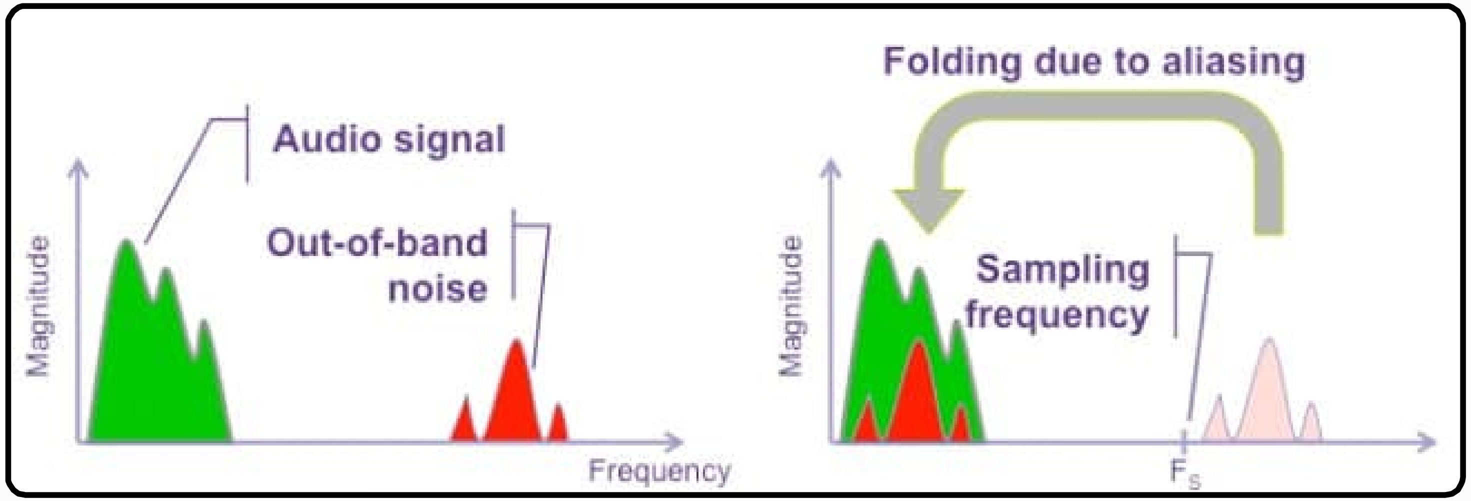
- Must reject interference signals at $f > f_s - f_{sig}$
  - Folded down into the spectrum of the useful signal
  - Cannot be performed digitally after sampling → analog filter is necessary
  - Disturbances with $f_B < f < f_s - f_B$ can be digitally filtered

Anti-aliasing Filter

- Desired signal
  - Weak, low-power
  - $f < f_B$
- Disorders that folds down
  - For frequencies $f_{\text{mirror}} < f_B$
  - $f \geq f_s - f_B$
  - Attenuated at least by A_{SB}
- Steep filter
  - Higher order



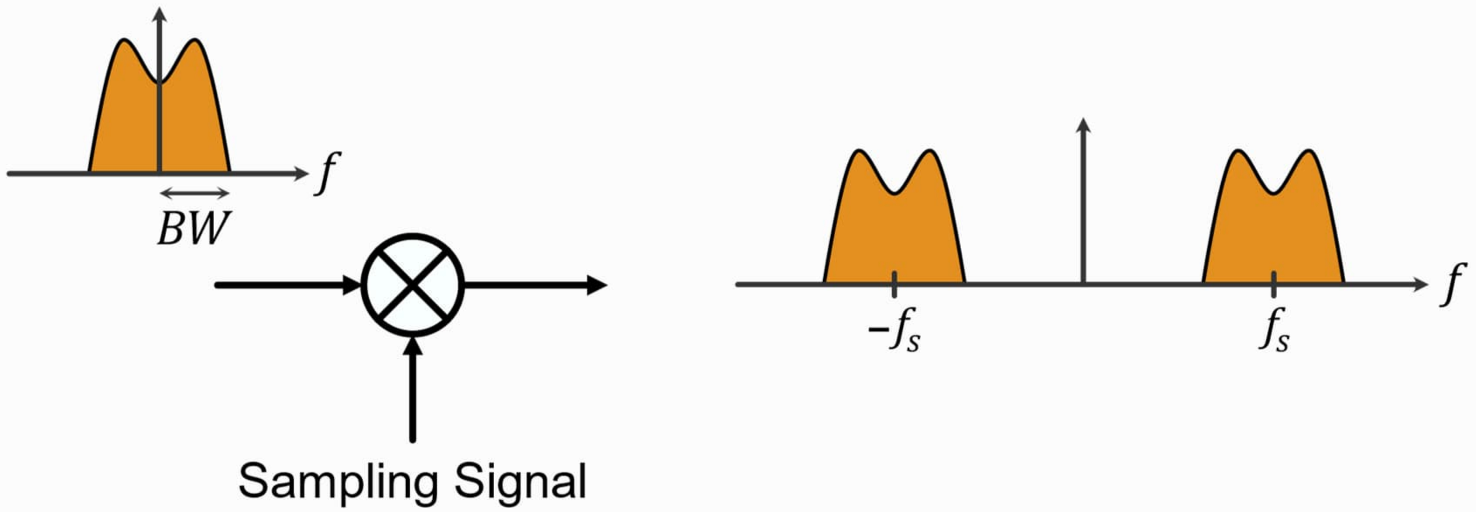
Noise Folding and Oversampling



- Noise also folded back
- Sampling at higher frequency leads to less folding into the band of interest.

Frequency Representation of Sampling

- In Fourier domain, multiplication is turned into convolution.
- Simply, shift by $\pm k f_s$ in frequency ($k \in \mathbb{Z}$)
- Copies of signal spectrum is shifted both upwards and downwards.



How Fast Should We Sample a Signal?

- Collision between copies of the signal spectrum is not allowed.
- The sampled signal would be undetectable if the copies overlap.
- Nyquist rate (minimum required f_s):

$$\boxed{f_{Nyquist} = f_{s,min} = 2 \times BW}$$