Elektronik

Basic Components and Circuit Theory

EITA10

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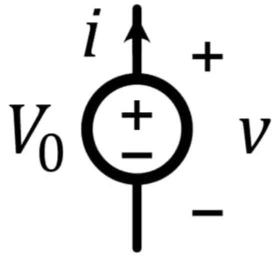
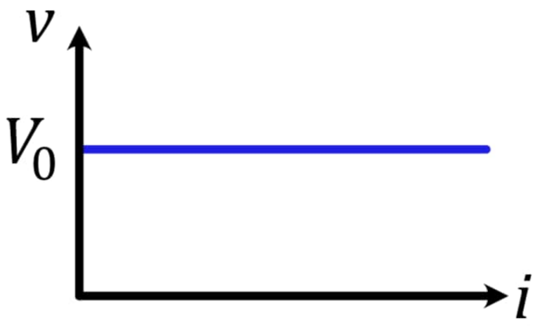
Mars 2025

We learned

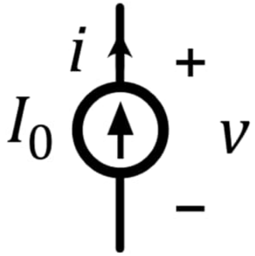
Voltage Source



$$v_{BA} = -\int_A^B \vec{E}\,.\,\overrightarrow{dl}$$

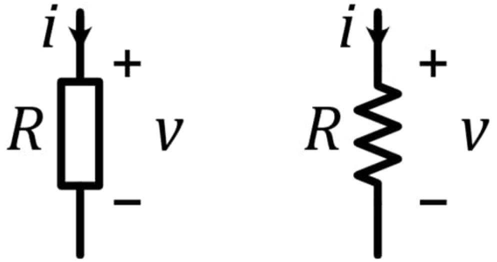


Current Source

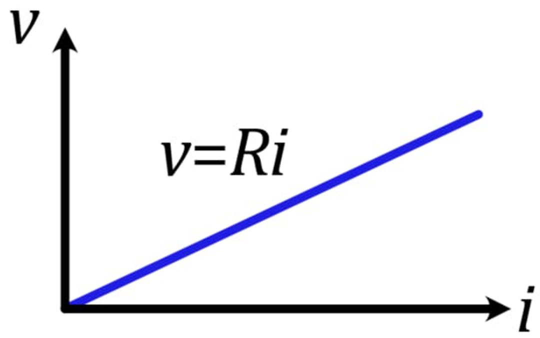


$$i = \frac{dq}{dt}$$

Resistor



$$v = Ri \quad i = Gv$$

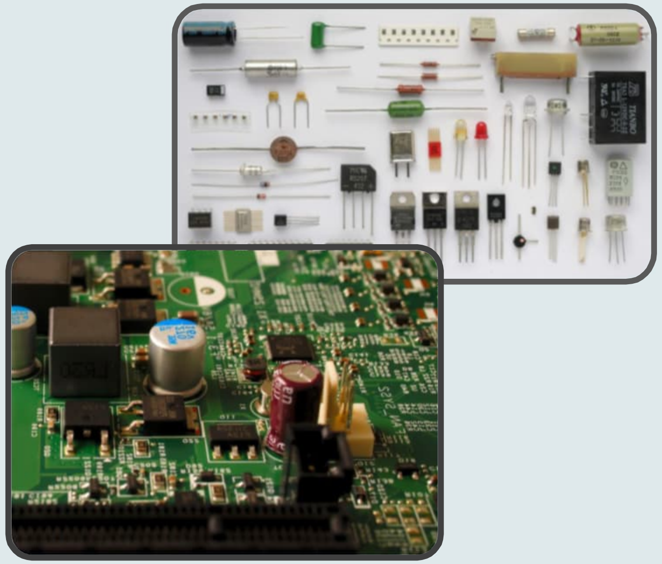


Today we learn

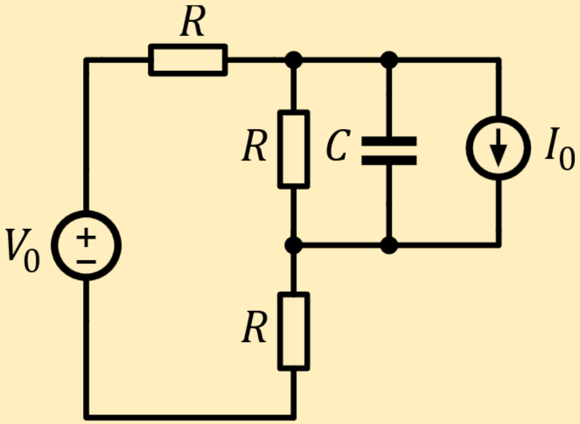
- Resistive circuits
- Kirchhoff’s voltage and current laws (KVL and KCL)
- Parallel and Series combination
- Voltage and current division
- Superposition
- Thevenin’s and Norton’s theorems
- Nodal analysis

Circuit Theory

Physical Reality
- Fields
- Flows
- Charges
- Material
- Particles

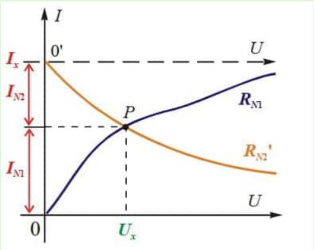


Circuit Models
- Abstracts
- Behavioral models
- Linear/non-linear
- Sources
- Passive/active components



Mathematical Calculations
- Matrices
- Algebraic equations
- Partial differential equations
- System of linear/non-linear equations
- Graphical methods
- Numerical methods

$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}$$

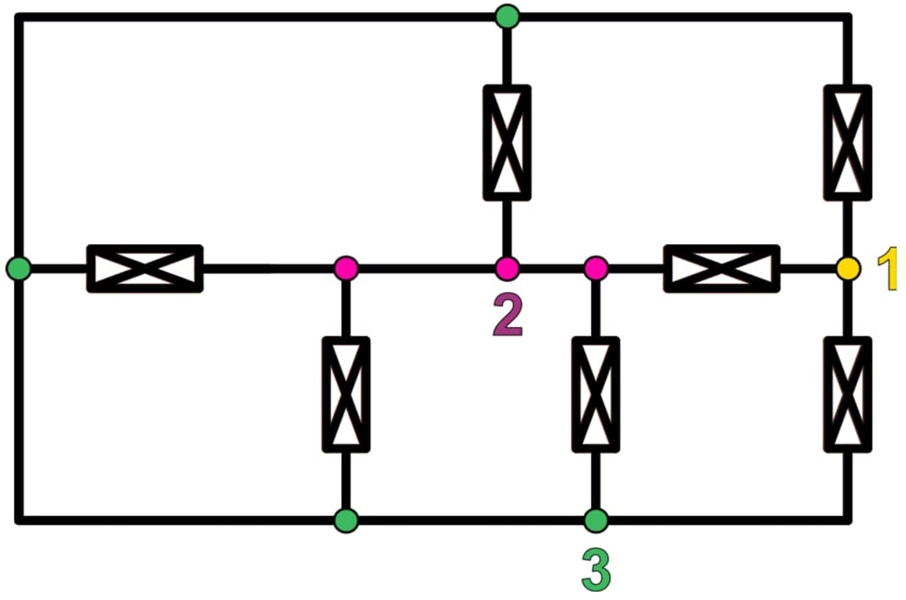


$$\boldsymbol{V} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \quad \boldsymbol{I} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$$\boldsymbol{Z} = \begin{pmatrix} Z_{11} & 0 & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix}$$

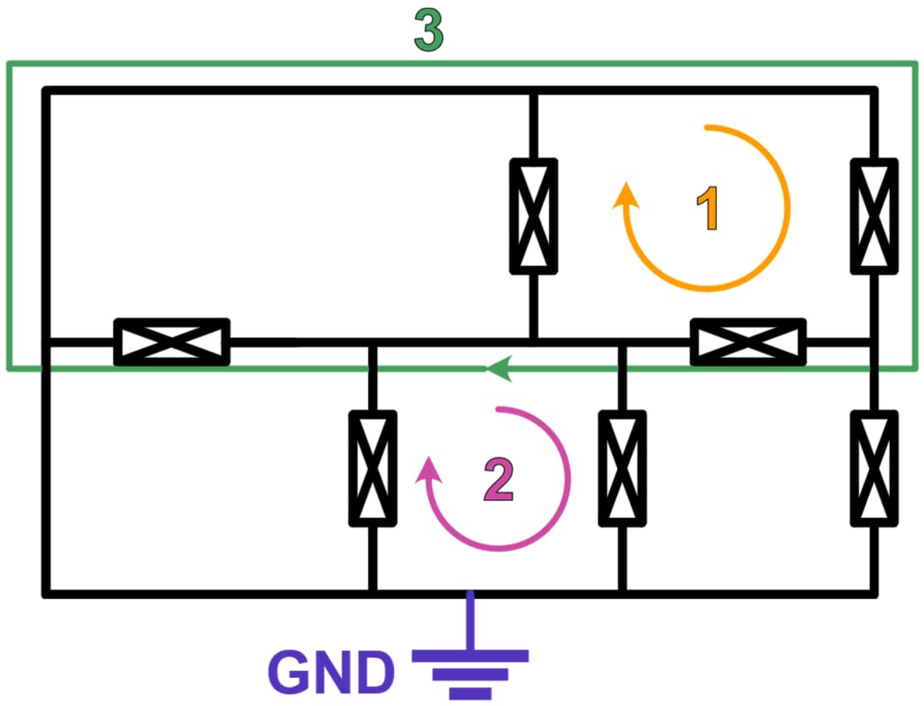
Electric Circuit Essentials

- **Wire**
  - ideal conductor
  - No voltage difference
  - Can handle any current level
- **Circuit element**
  - Sources
  - Passives: Resistors, capacitors, inductors
  - Actives: Transistors, diodes, …
- **Node**
  - A junction where two or more branches intersect
  - Two nodes must have different voltages (no wire connecting them directly)



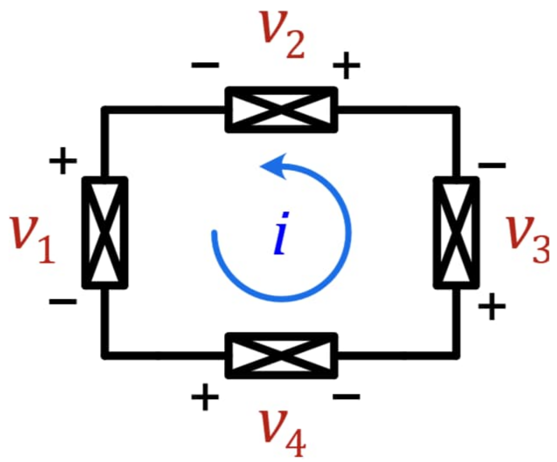
Electric Circuit Essentials

- **Loop**
  - A subset of branches forming a closed path
  - Might pass by several nodes (at least two)
- **Ground Node**
  - The node with the maximum connected branches
  - Voltage reference for our calculations and measurements
  - Denoted by GND
  - V(GND)=0 V



Kirchhoff’s Voltage Law (KVL)

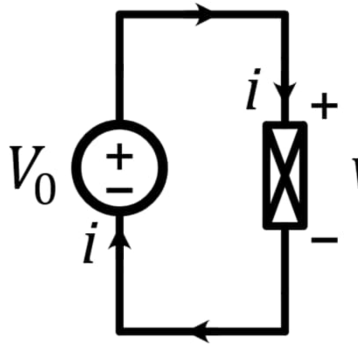
- The algebraic **sum of the voltage** drops or rises around **any loop** in a circuit is always equal to **zero**.



$$\sum_{k=1}^{4} v_k = 0$$

- Current-voltage direction
  - current flows across an element from + to -

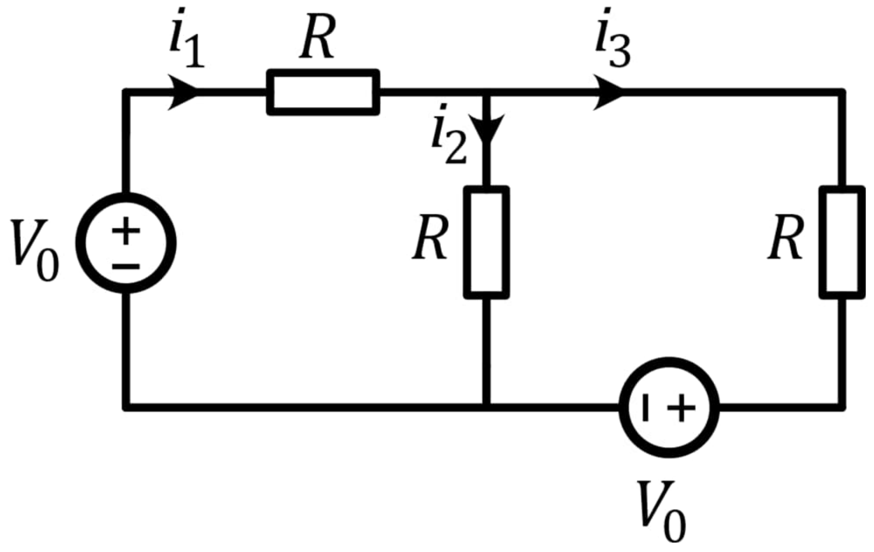
- Voltage sources (pumps of charge)
  - Follow the polarity of the source in the direction of circulating around the loop



$$+v - V_0 = 0$$

KVL– Example

- Write all KVLs in terms of i_1, i_2 and i_3.



KVL– Example

- Write all KVLs in terms of i_1, i_2 and i_3.

KVL 1: $Ri_1 + Ri_2 - V_0 = 0$

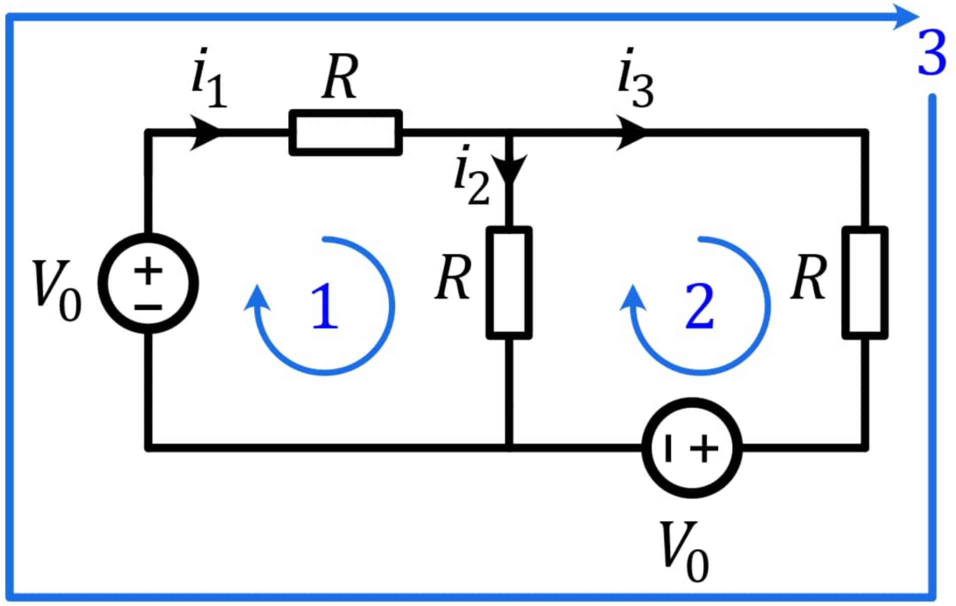
KVL 2: $Ri_3 + V_0 - Ri_2 = 0$

KVL 3: $Ri_1 + Ri_3 + V_0 - V_0 = 0$

Therefore:

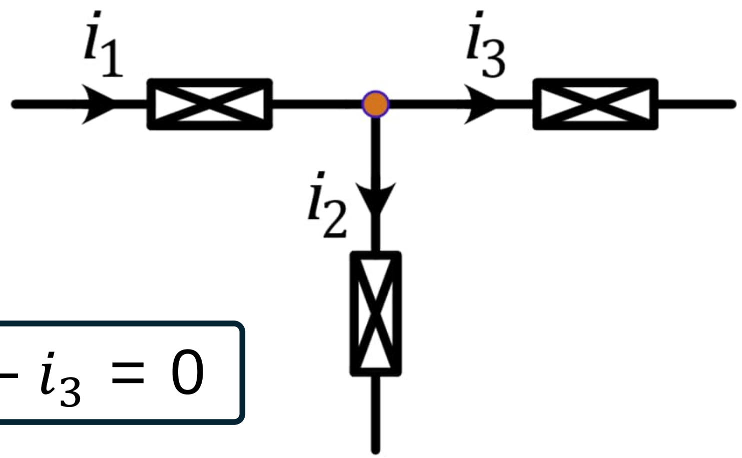
$$i_1 = -i_3$$

$$i_1 + i_2 = \frac{V_0}{R}$$



Kirchhoff's Current Law (KCL)

- The algebraic **sum of the currents** entering/leaving **a node** is always equal to **zero**.
- Current direction
  - Entering → positive
  - Leaving → negative

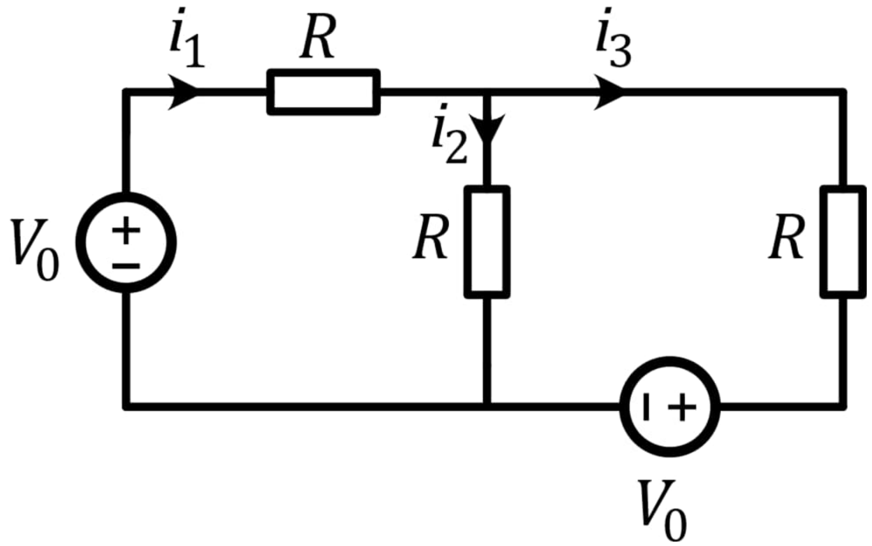


$$\boxed{i_1 - i_2 - i_3 = 0}$$

$$\boxed{\sum I_{in} = \sum I_{out}}$$

KCL– Example

- Write all KCLs in terms of i_1, i_2 and i_3.



KCL– Example

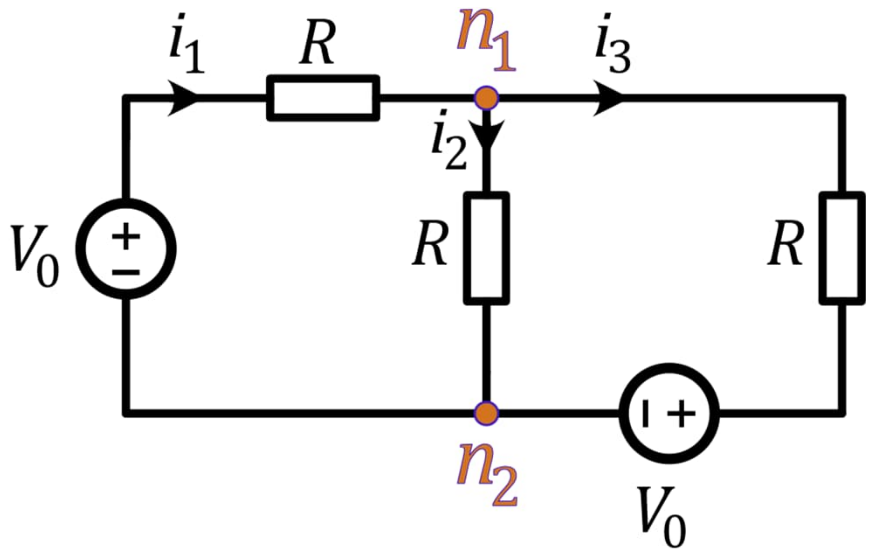
- Write all KVLs in terms of i_1, i_2 and i_3.

KCL @$\mathbf{n_1}$: $i_1 - i_2 - i_3 = 0$

KCL @$\mathbf{n_2}$: $i_3 + i_2 - i_1 = 0$

Therefore:

$$i_1 = i_2 + i_3$$

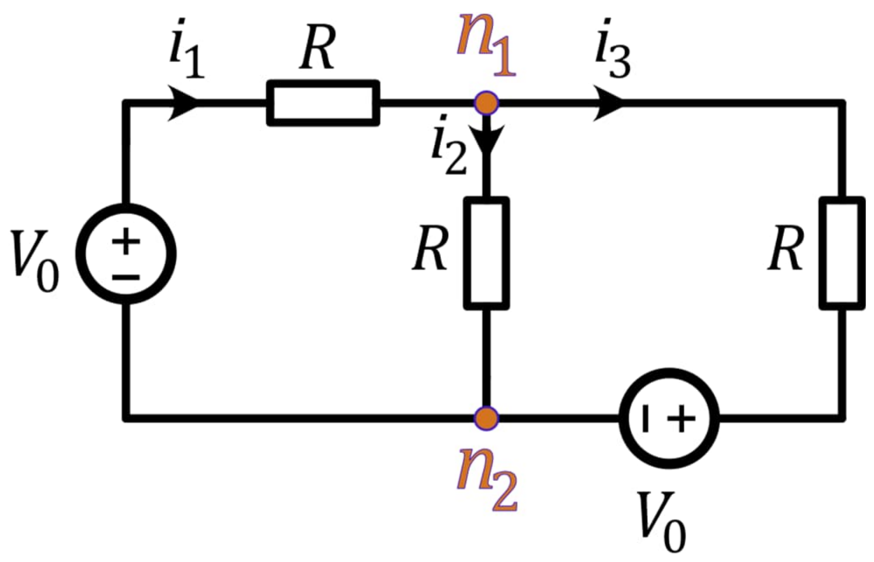


One of KCLs seems redundant!

Pause and Ponder 1

- In a circuit with n_t nodes, how many **independent KCLs** can be written?

$$\boxed{n = n_t - 1}$$

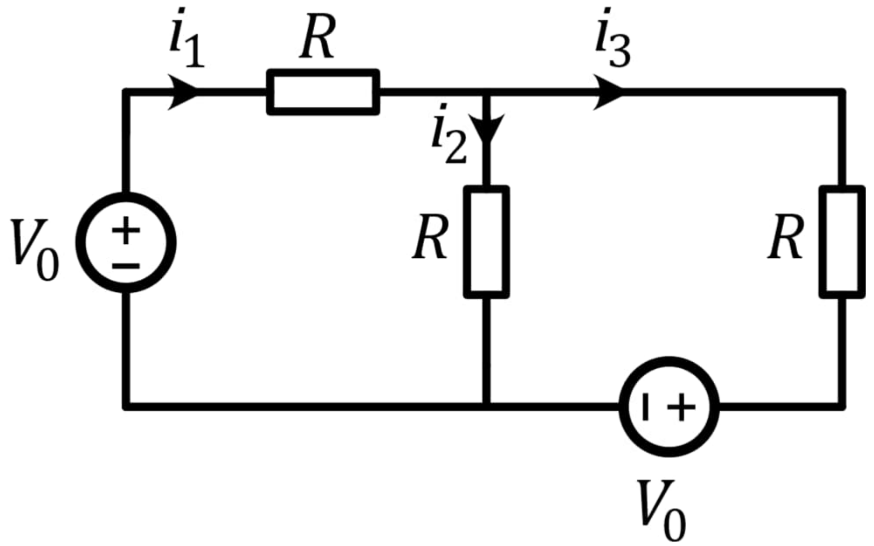


KVL+KCL → System of Equations

- Express i_1, i_2 and i_3 in terms of V_0.

From KVL:

$$i_1 = -i_3 \qquad i_1 + i_2 = \frac{V_0}{R}$$



From KCL:

$$i_1 = i_2 + i_3$$

Therefore:

$$i_2 = 2i_1 \Rightarrow \quad i_1 = \frac{V_0}{3R}, \qquad i_2 = \frac{2V_0}{3R}, \qquad i_3 = -\frac{V_0}{3R}$$

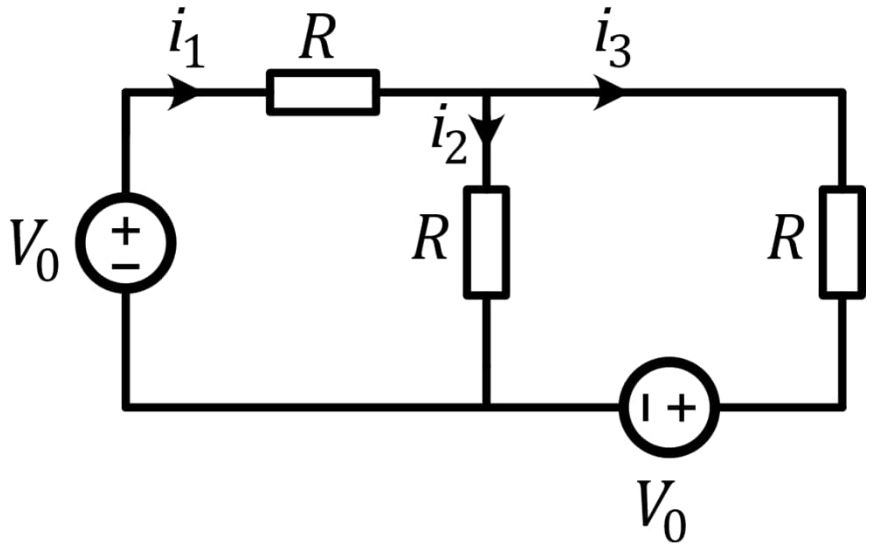
KVL+KCL → System of Equations

- Write a system of equation in matrix format.

KVL: $i_1 + i_3 = 0$

KVL: $i_1 + i_2 = \frac{V_0}{R}$

KCL: $i_1 - i_2 - i_3 = 0$



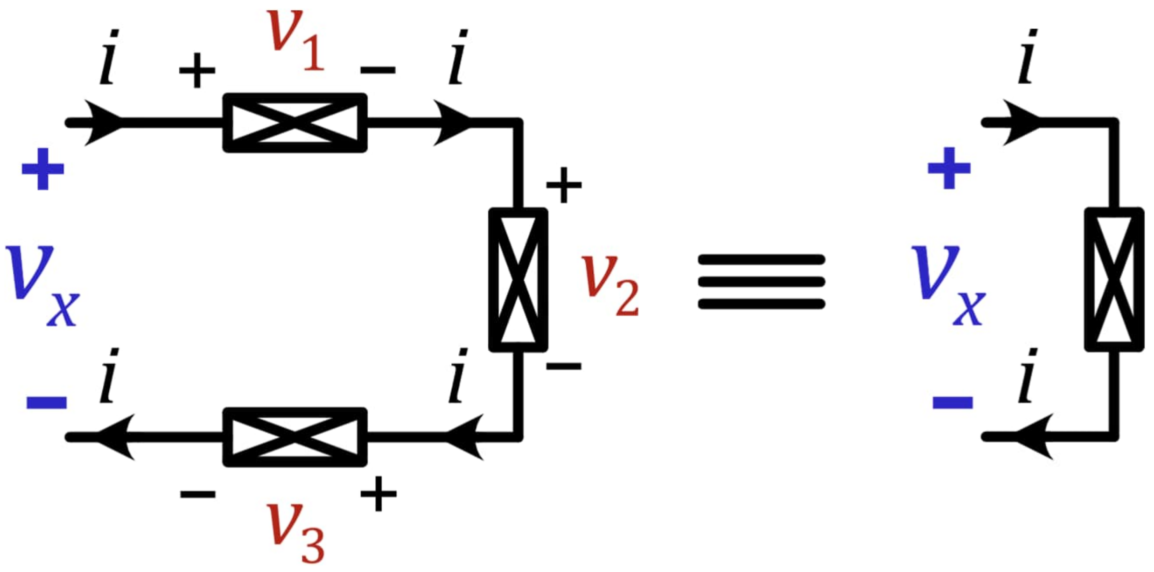
$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{V_0}{R} \\ 0 \end{pmatrix}$$

Series Coupling

- Two or more elements are connected in series if **the same current** flows through them.

$$\boxed{i_1 = i_2 = i_3 = i}$$

$$\boxed{v_1 + v_2 + v_3 = v_x}$$

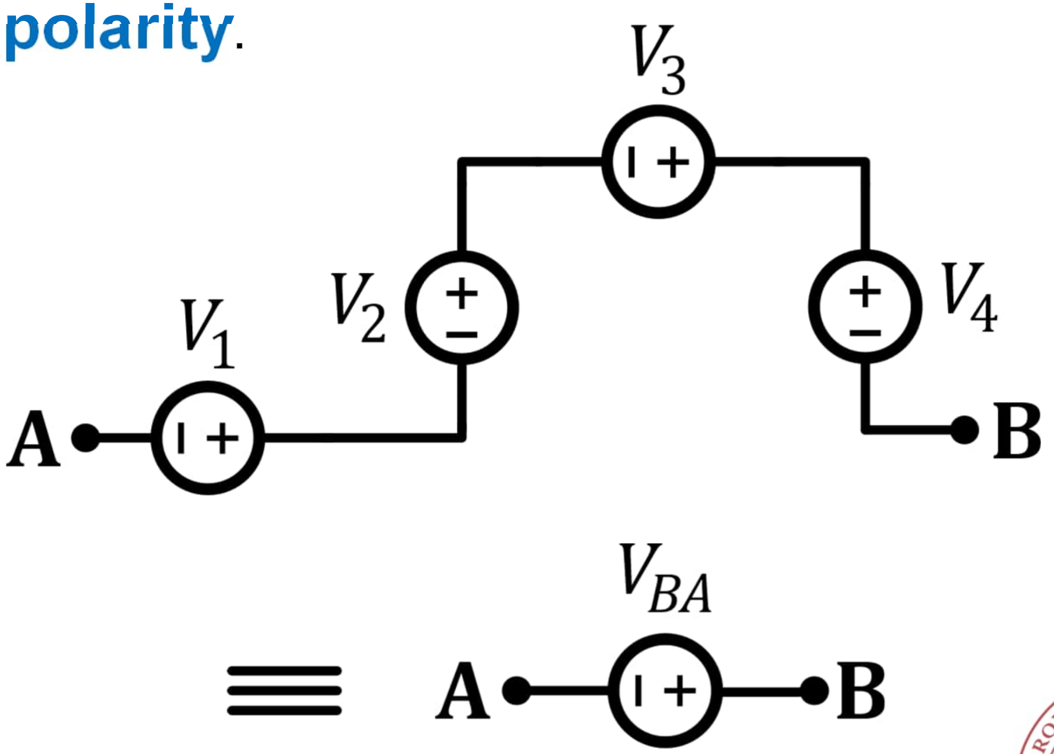


Series Voltage Sources

- The total voltage difference equals **the sum of voltage sources** according to their **polarity**.

$$\boxed{V_{BA} = V_1 + V_2 + V_3 - V_4}$$

$$\boxed{V_{BA} = \sum_{k=1}^{N} V_k}$$



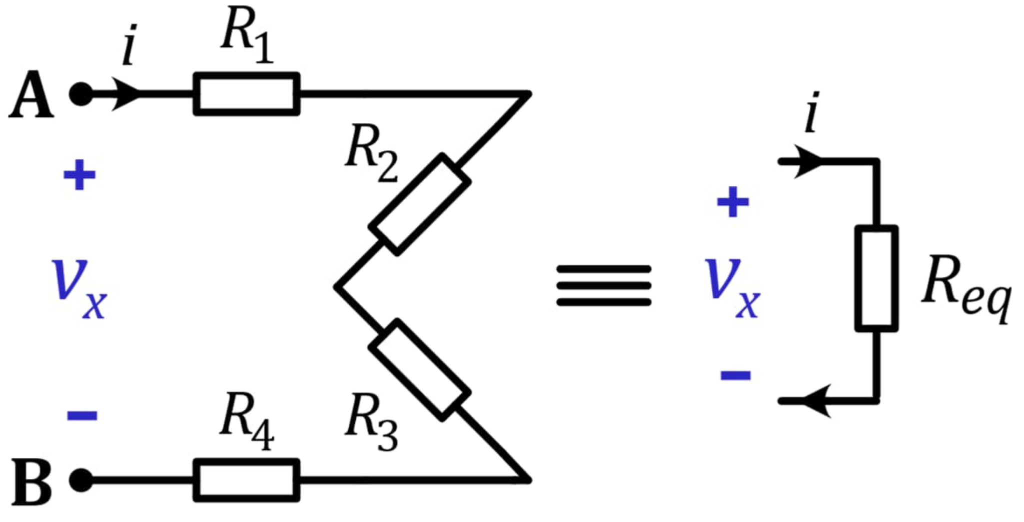
Series Resistors

- Series resistors can be replaced by an equivalent resistor whose value is equal to the **sum of their individual resistances**.

$$\boxed{V_x = R_1 i + R_2 i + R_3 i + R_4 i}$$

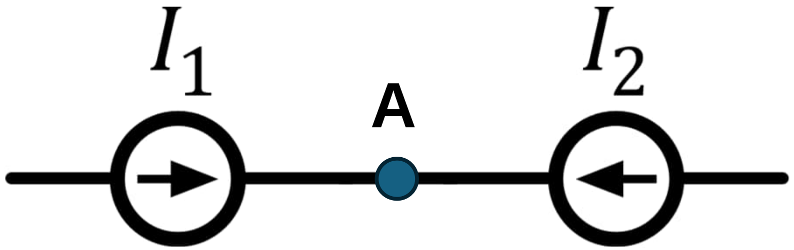
$$\boxed{V_x = \left(\sum_{k=1}^{4} R_k \right) i}$$

$$\boxed{R_{eq} = \sum_{k=1}^{N} R_k}$$



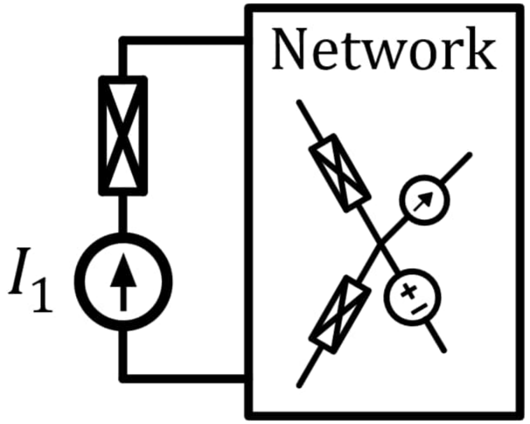
Pause and Ponder 2

- What would happen if a student were to connect two ideal independent current sources in series?



KCL: $I_1 = -I_2$

- Does connecting any circuit element in series with an ideal independent current source make any difference to the rest of the circuit?

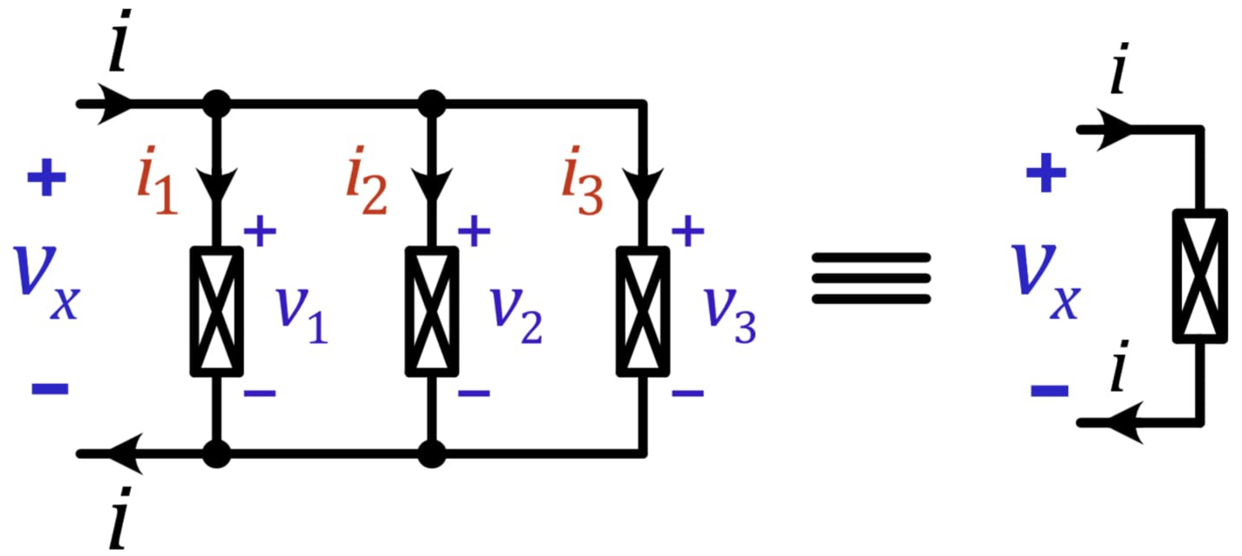


Parallel Coupling

- Two or more components are connected **across the same two nodes**.

$$\boxed{v_1 = v_2 = v_3 = v_x}$$

$$\boxed{i_1 + i_2 + i_3 = i}$$

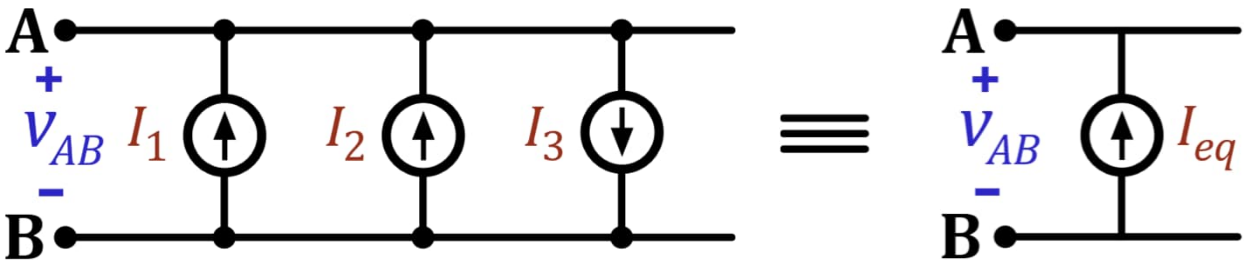


Parallel Current Sources

- Parallel current sources can be replaced by an equivalent current source whose value is equal to **the sum of their individual currents, according their direction.**

$$\boxed{I_{eq} = I_1 + I_2 - I_3}$$

$$\boxed{I_{eq} = \sum_{k=1}^{N} I_k}$$



Parallel Resistors

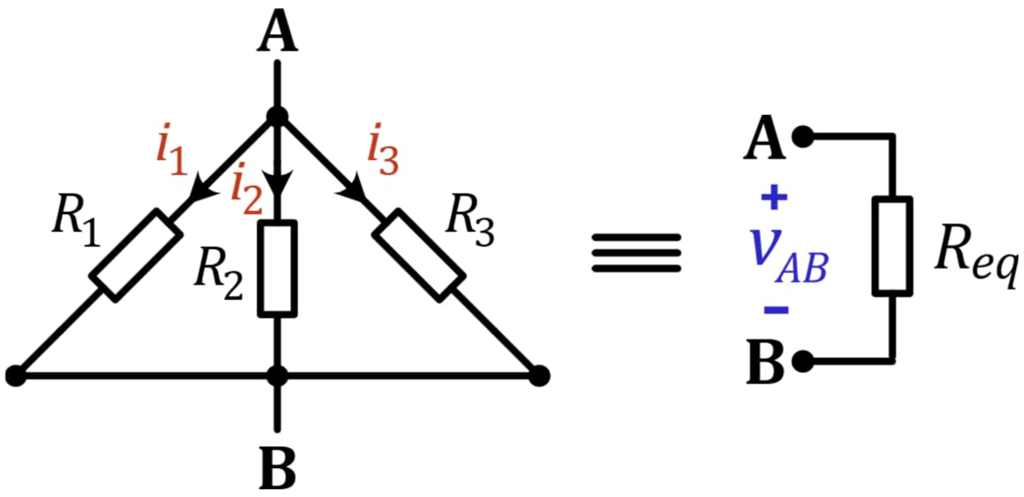
- Parallel resistors can be replaced by an equivalent resistor whose conductance (G_{eq}) is equal to the **sum of their individual conductances**.

$$i_x = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2} + \frac{V_{AB}}{R_3} \quad \& \quad G_k = \frac{1}{R_k}$$

$$\boxed{i_x = (G_1 + G_2 + G_3) \cdot V_{AB}}$$

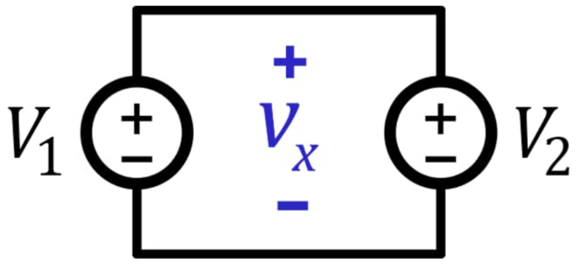
$$\boxed{G_{eq} = \sum_{k=1}^{N} G_k}$$

$$\boxed{R_{eq} = \frac{1}{G_{eq}}}$$



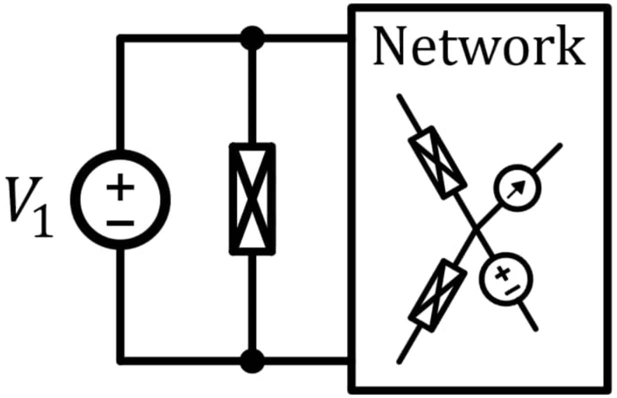
Pause and Ponder 3

- What would happen if a student were to connect two ideal independent voltage sources in parallel?



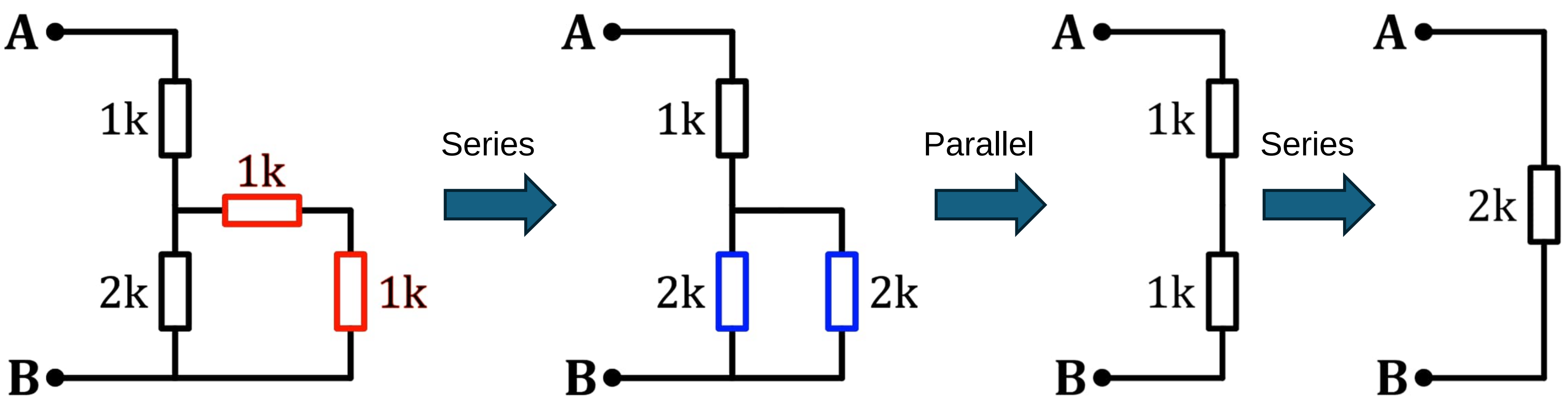
KVL: $V_1 = V_2$

- Does connecting any circuit element in parallel with an ideal independent voltage source make any difference to the rest of the circuit?



Example of Equivalent Resistance

- Find the equivalent resistance.



S: $1k + 1k = 2k$

P: $2k \parallel 2k = 1k$

S: $1k + 1k = 2k$

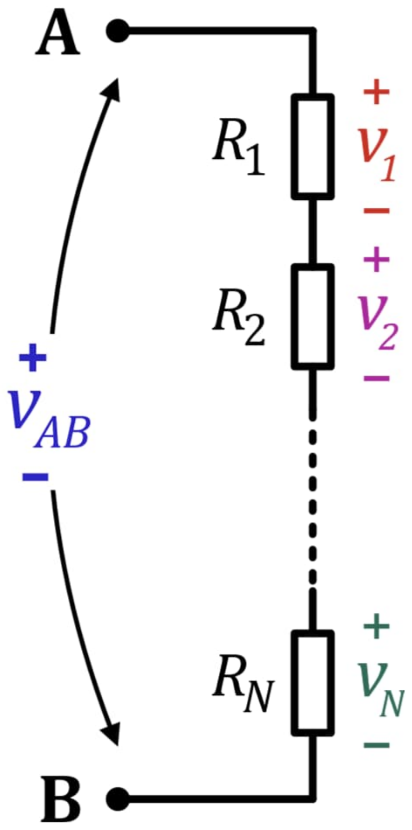
Voltage Division

- Within a **series combination**, the voltage is distributed among the individual elements according to **their respective resistances.**

$$v_{AB} = \sum_{n=1}^{N} v_n = i \cdot \sum_{n=1}^{N} R_n = i \cdot R_{tot}$$

$$v_k = R_k i = \frac{R_k}{R_{tot}} v_{AB}$$

$$\boxed{v_k = \frac{R_k}{\sum_{n=1}^{N} R_n} v_{AB}}$$



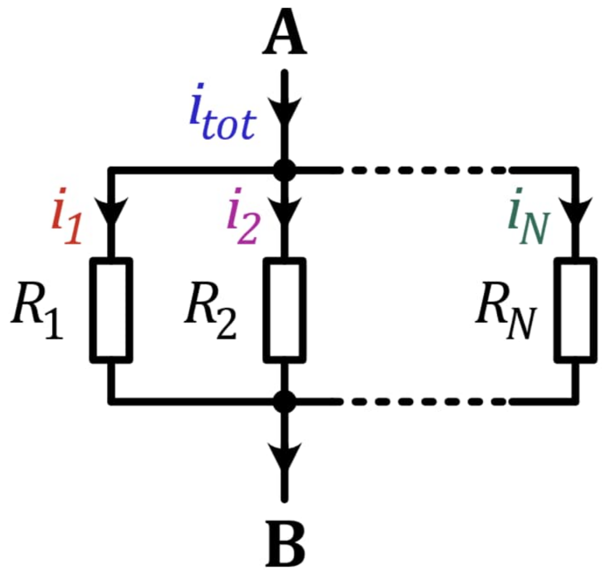
Current Division

- Within a **parallel combination**, the current is distributed among the individual elements according to **their respective conductances.**

$$i_{tot} = \sum_{n=1}^{N} G_n \cdot v_{AB}$$

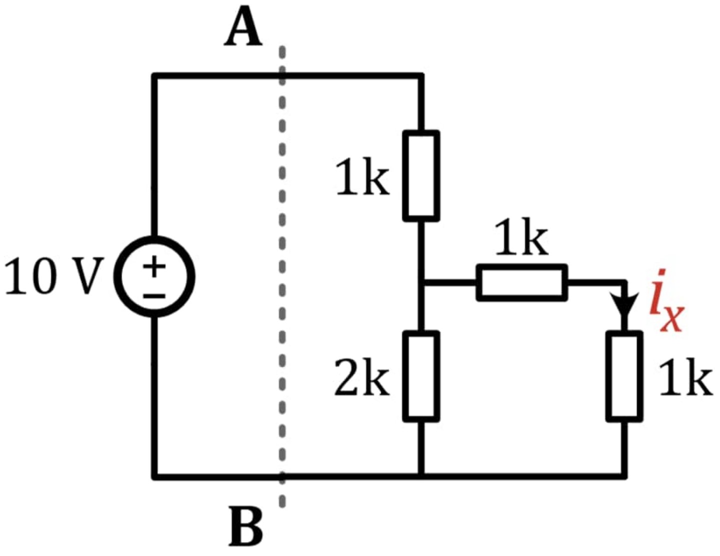
$$i_k = G_k v_{AB}$$

$$\boxed{i_k = \frac{G_k}{\sum_{n=1}^{N} G_n} i_{tot}}$$



Example

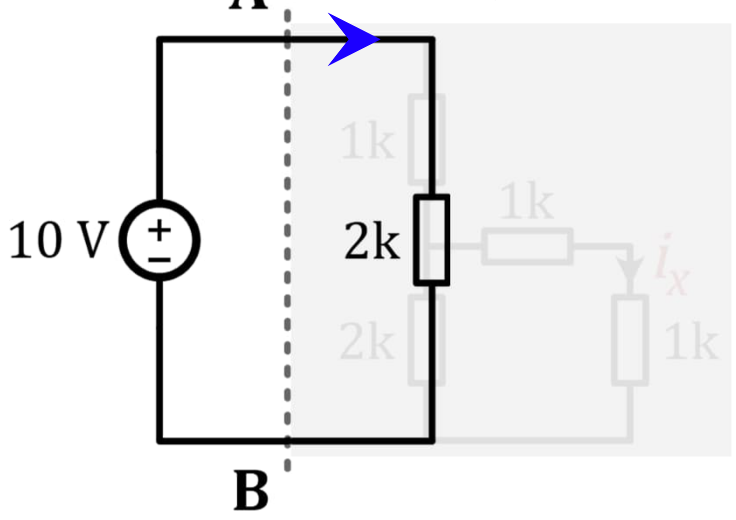
- Find the value of i_x.

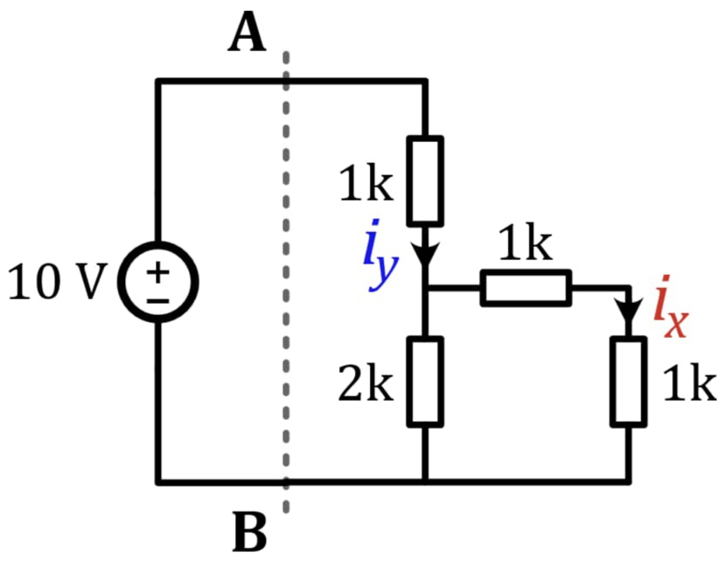


Example

- Find the value of i_x.

$$i_y = \frac{10}{2k} = 5\text{ mA}$$



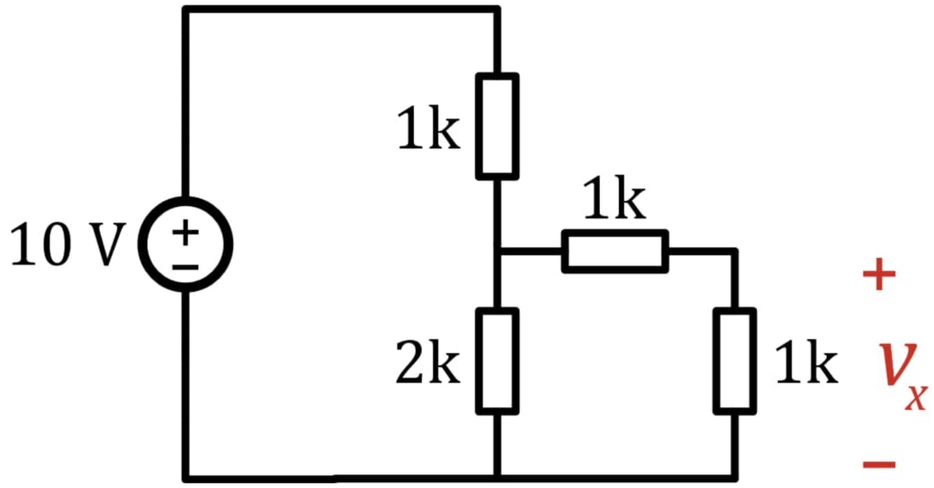


$$i_x = \frac{G_{(1k+1k)}}{G_{(1k+1k)} + G_{2k}} i_y$$

$$i_x = \frac{\frac{1}{1k+1k}}{\frac{1}{1k+1k} + \frac{1}{2k}} i_y = 2.5\text{ mA}$$

Pause and Ponder 4

- In the previous example, find v_x by using voltage division.



Dependent Sources

	Voltage Source	Current Source
Voltage-Controlled	$v = A_v(v_x)$ **VCVS**	$i = G_m(v_x)$ **VCCS**
Current-Controlled	$v = R_m(i_x)$ **CCVS**	$i = A_i(i_x)$ **CCCS**

Linear Dependent Sources

	Voltage Source	Current Source
Voltage-Controlled	$v = A_v v_x$ **VCVS**	$i = G_m v_x$ **VCCS**
Current-Controlled	$v = R_m i_x$ **CCVS**	$i = A_i i_x$ **CCCS**

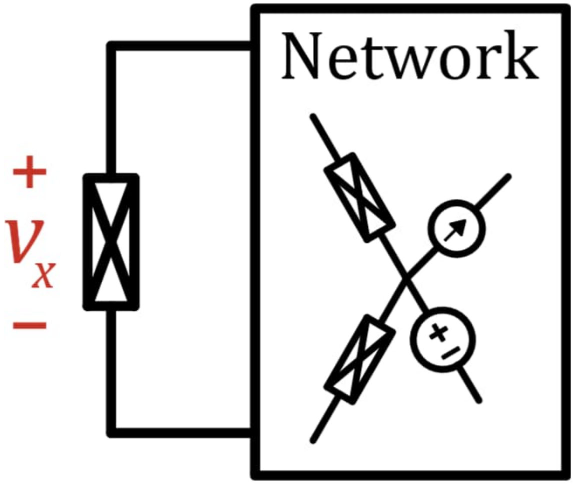
Superposition

- In a linear circuit, the response (such as voltage or current) due to **multiple sources** can be determined by considering **the effects of each independent source individually** and then **summing up** these individual responses.

$$\boxed{v_x = \sum_{n=1}^{N} A_n V_n + \sum_{m=1}^{M} R_m I_m}$$

N: number of independent voltage sources

M: number of independent current sources

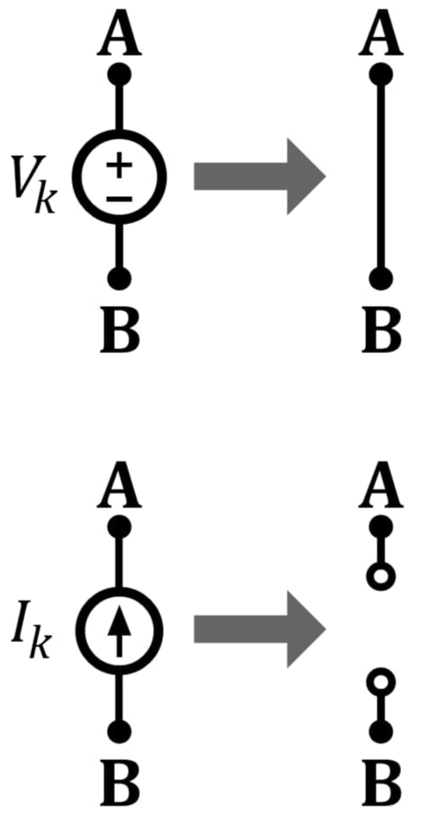


Using Superposition in Circuit Analysis

Step 1: Calculate the contribution of each independent source, one by one.

- Turn off all other independent sources.
  - **Voltage sources → Short-circuit** (a wire)
  - **Current source → Open-circuit** (remove it)
- Use nodal analysis, current/voltage division, etc.

Step 2: Sum up individual responses.



Example of Superposition

- Find the value of i_x.
  - Effect of 10 V source (already calculated):

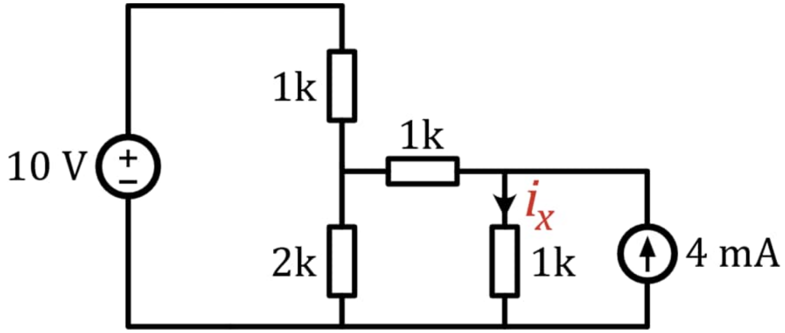
$$\boxed{i_{x\_10V} = 2.5\text{ mA}}$$

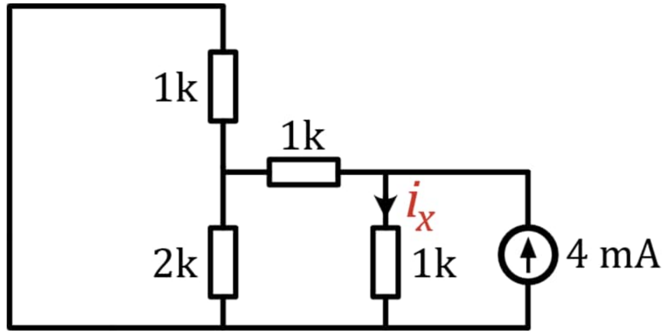
  - Effect of 2 mA source:

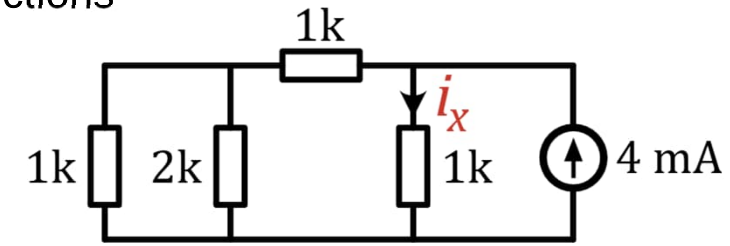
    - Short-circuit 10 V source

    - Use of current division and parallel/series connections

$$\boxed{i_{x\_4mA} = 2.5\text{ mA}}$$



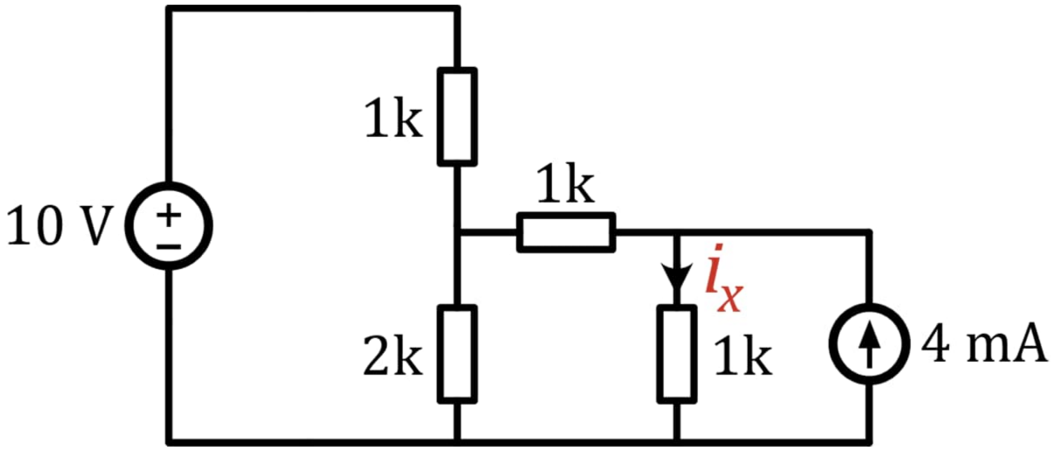




Example of Superposition

- Find the value of i_x.
  - Summing them up:

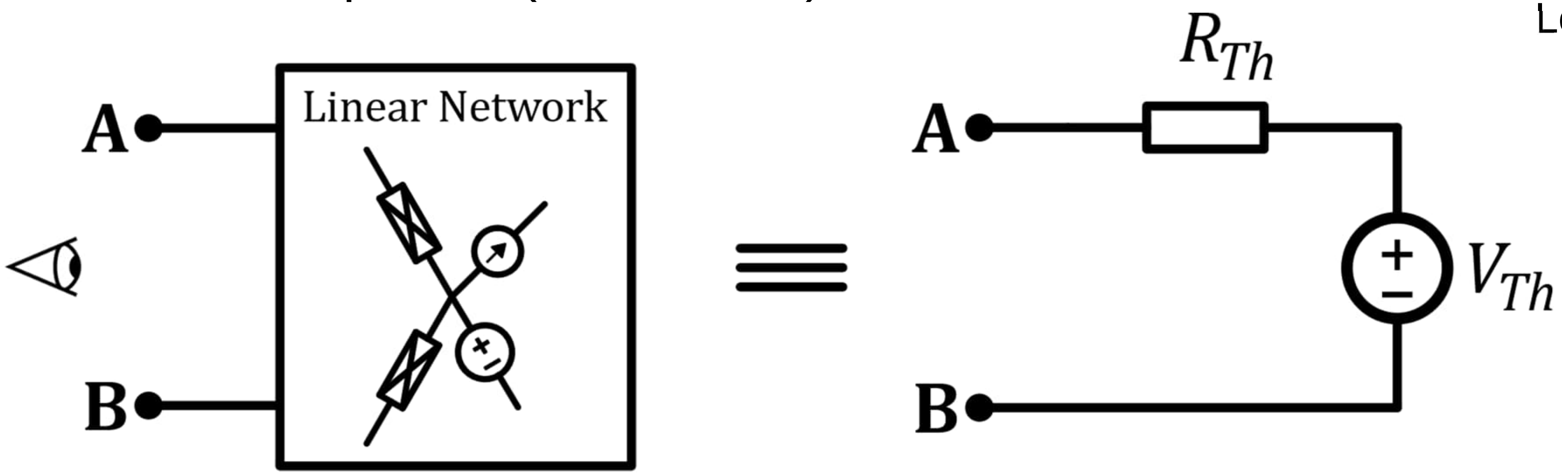
$$i_x = i_{x\_10V} + i_{x\_4m} = 5\text{ mA}$$



Thevenin's Theorem

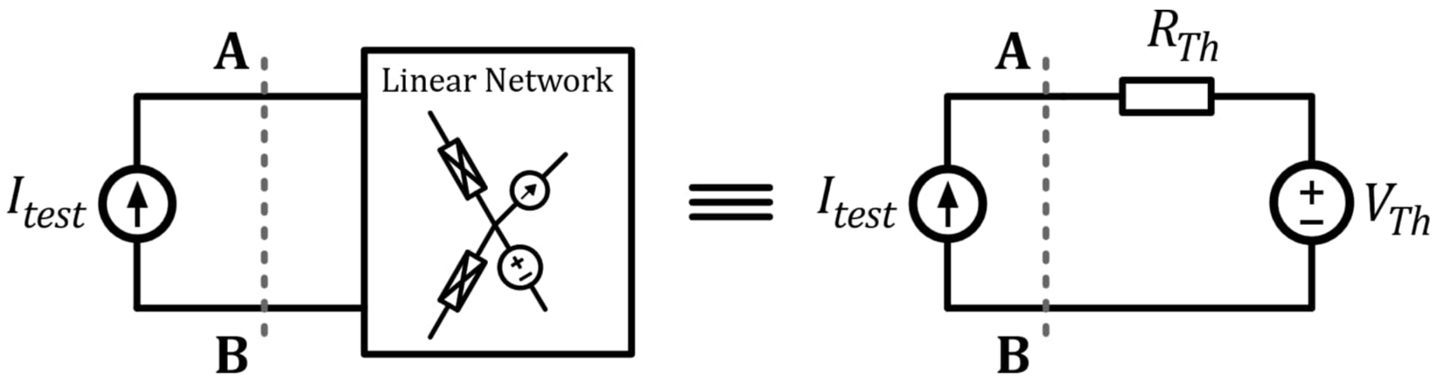
- Any linear circuit can be modeled as **a voltage source** (V_{Th}) in series with **a resistor** (R_{Th}).
  - Simplifies circuit analysis.
  - Useful in time-constant calculations and circuit with only one nonlinear component (like a diode).

Leon Charles Thévenin
(1857-1926)



How to determine V_{Th} and R_{Th}?

- When applying a **test current**, the response of the model must match the network's response.



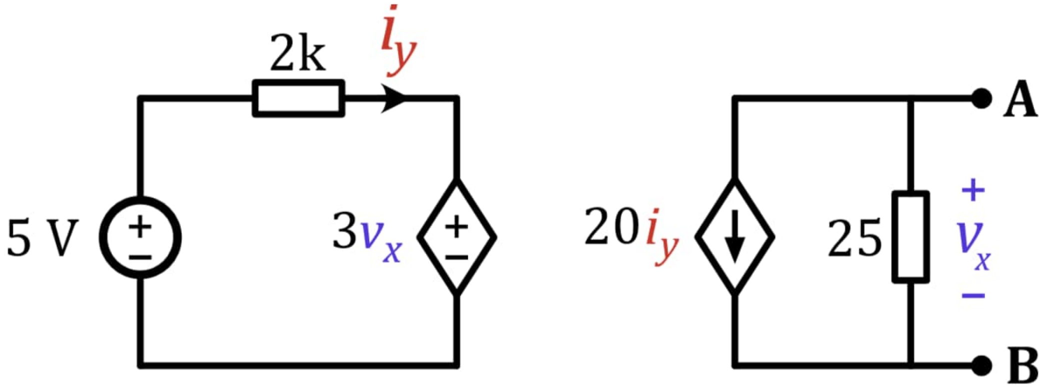
$$V_{AB} = R_{Th} I_{test} + V_{Th}$$

- When I_{test} = 0, meaning open-circuit: $V_{AB} = V_{Th} \Rightarrow \boxed{V_{Th} = V_{oc}}$
- When all independent sources are off: $V_{AB} = R_{Th} I_{test} \Rightarrow \boxed{R_{Th} = \frac{V_{AB}}{I_{test}}}$

Example of Thevenin's Theorem

- Draw an equivalent circuit seen between A and B.
  - Open circuit voltage: (from KVL in the loops)

$$\begin{cases} v_x = -20i_y \times 25 = -500i_y \\ 2000i_y + 3v_x - 5 = 0 \end{cases}$$



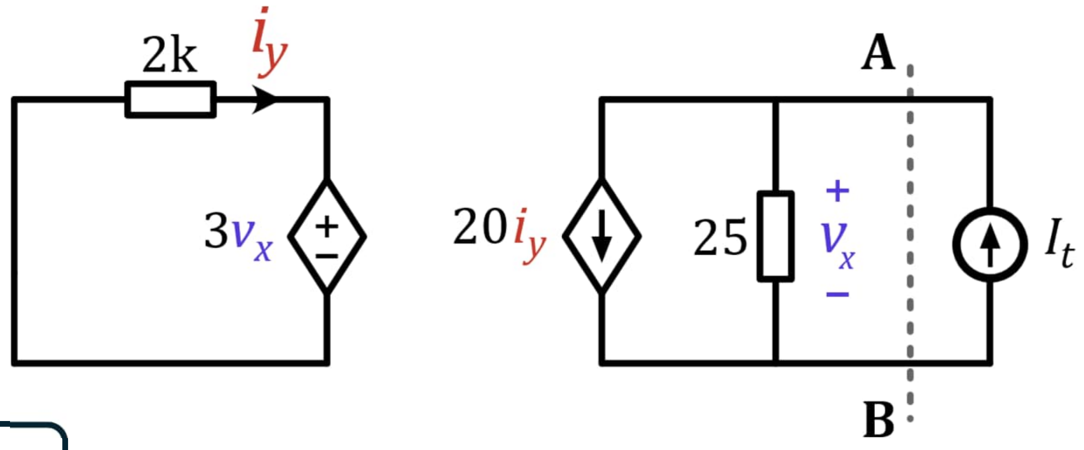
$i_y = 10\text{ mA}$

$$\boxed{V_{OC} = v_x = -5\text{ V}}$$

Example of Thevenin’s Theorem

- Draw an equivalent circuit seen between A and B.
  - Equivalent Resistance: (5 V source off)

$$\begin{cases} 20i_y + \dfrac{v_x}{25} = I_t \\ 2000i_y + 3v_x = 0 \end{cases}$$

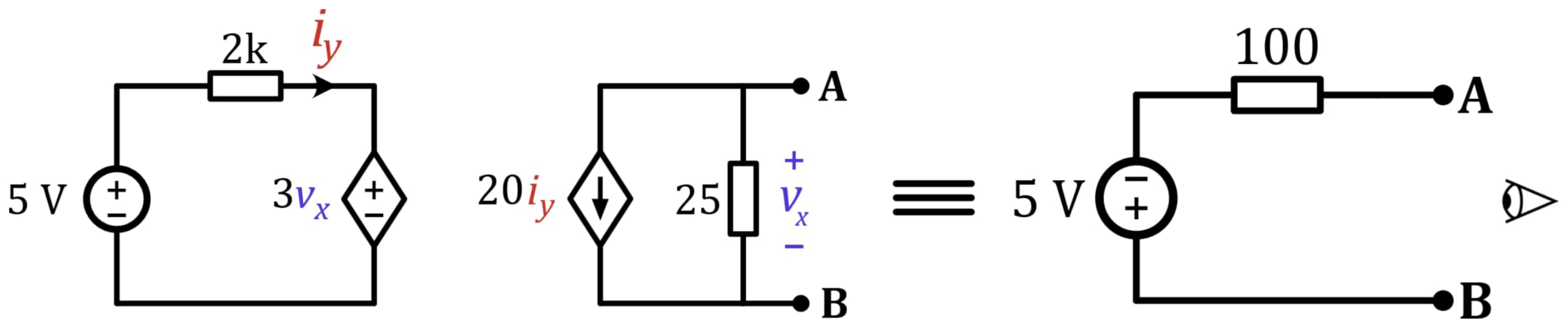


$$v_{AB} = v_x = 100I_t$$

$$\boxed{R_{Th} = \frac{V_{AB}}{I_{test}} = 100\ \Omega}$$

Example of Thevenin's Theorem

- Draw an equivalent circuit seen between A and B.

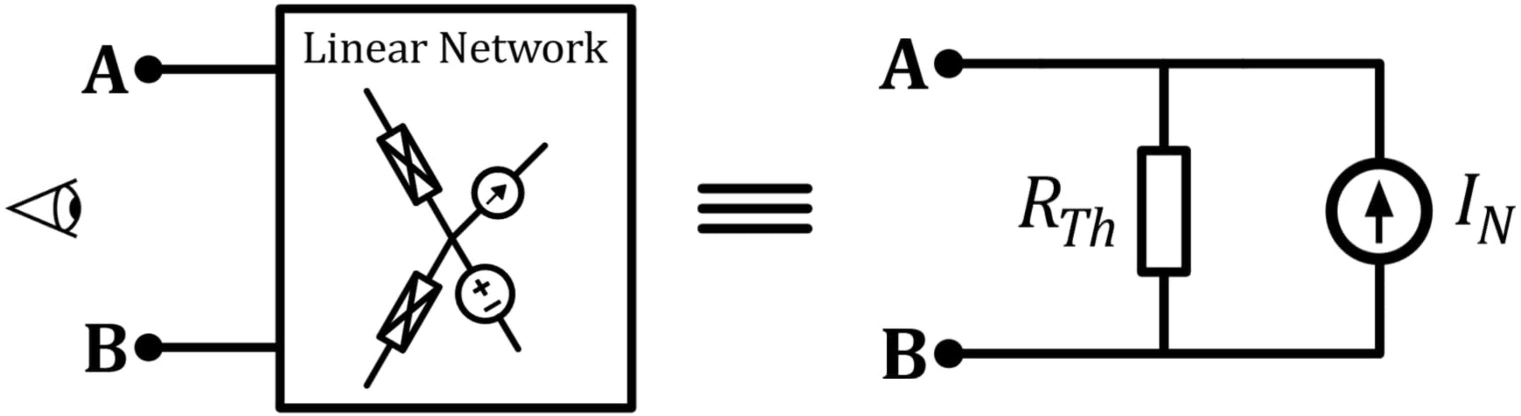


Norton's Equivalent Circuit

- The same concept, but this time we model a linear network with **a current source** (I_N) **in parallel with a resistor** (R_{Th}).

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

Edward Lawry Norton
(1898-1983)

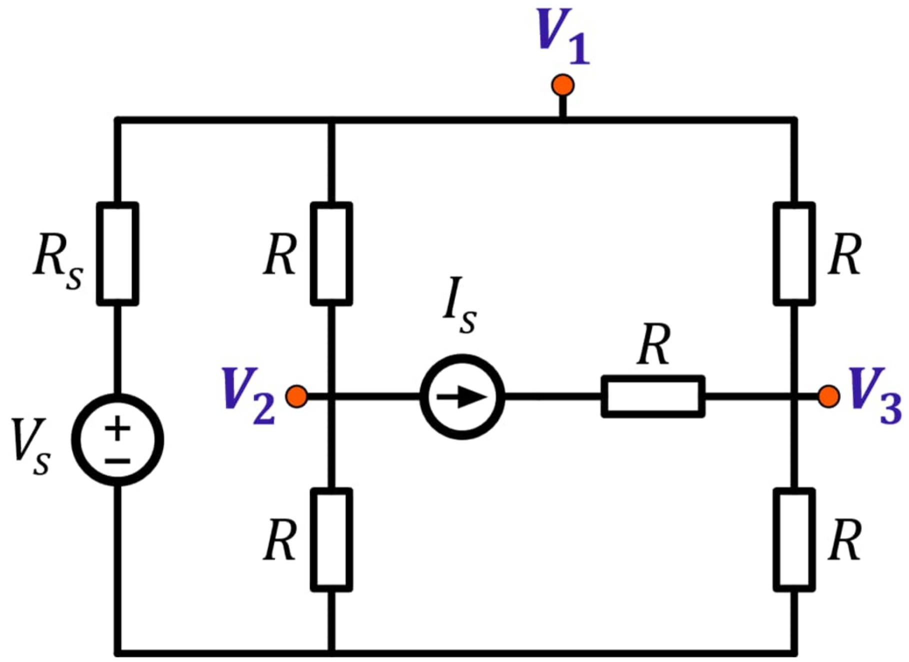


Nodal Analysis

- Systematic approach to derive a system of equations for any circuit.
- Powerful tool for algorithmic simulators.
- Only relies on KCL!

Nodal Analysis, step by step

- Find the values of V_1, V_2 and V_3.



Nodal Analysis, step by step

1- Simplify your circuit.

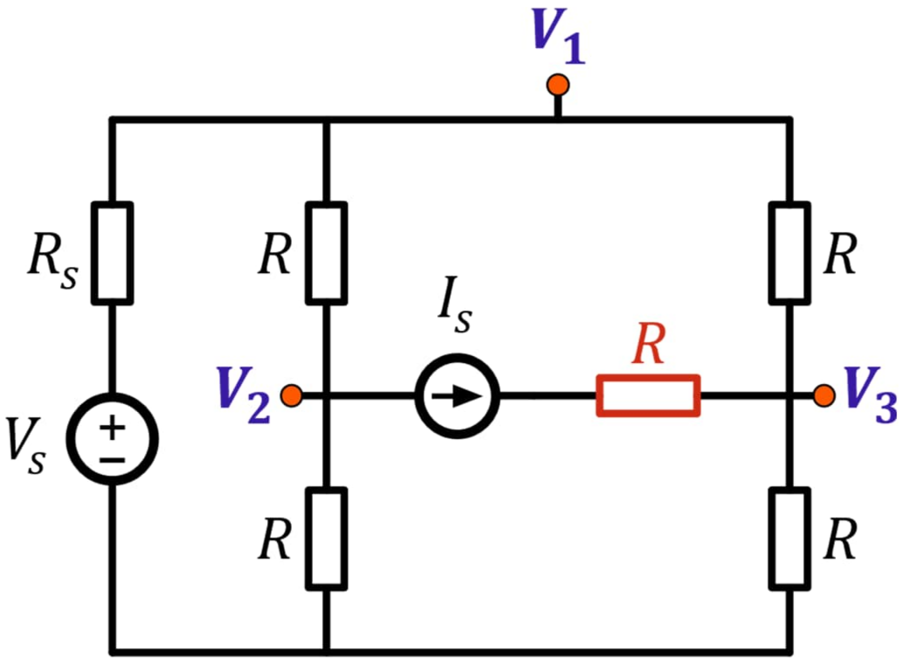
- Remove anything in **series with current sources** or in **parallel with voltage sources**.
- Turn all voltage source into their **Norton equivalent**.
- Series/parallel combinations
- Short-circuits → remove excess wires.

2- Determine the Ground (GND).

- Node with maximum connected branches

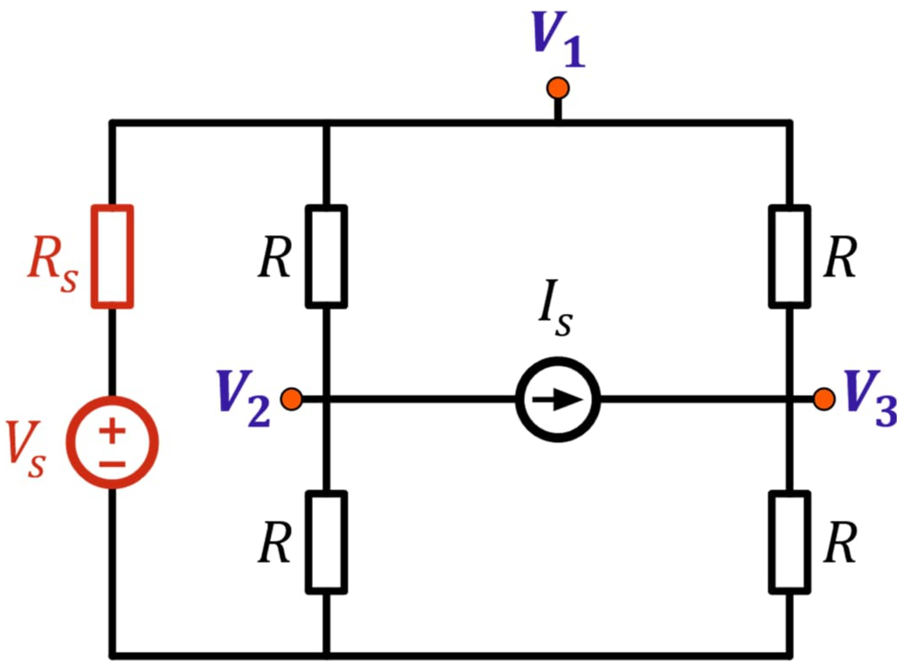
Nodal Analysis, step by step

- Remove R in series with I_s.



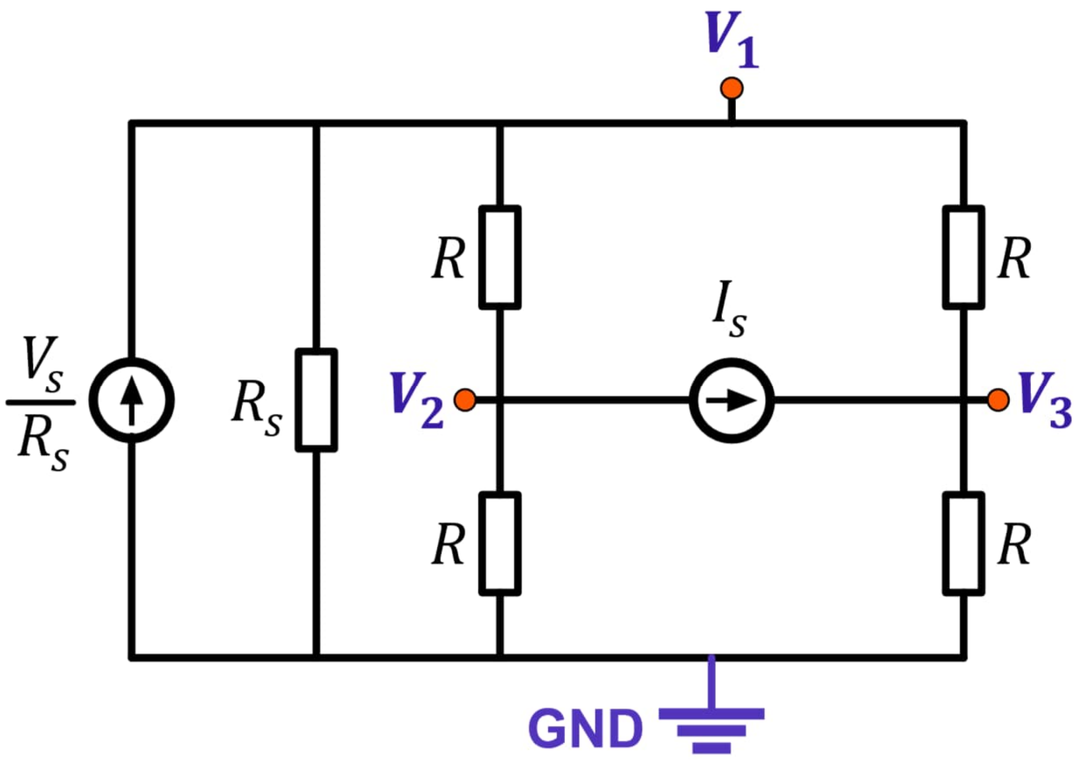
Nodal Analysis, step by step

- Remove R in series with I_s.
- Norton equivalent of V_s



Nodal Analysis, step by step

- Remove R in series with I_s.
- Norton equivalent of V_s.
- Ground node



Nodal Analysis, step by step

3- Write a system of equation in matrix form $\boldsymbol{GV = I}$

- In a circuit with $\boldsymbol{N}$ **nodes**, the number of equations and **independent voltages equals $\boldsymbol{N_t = N - 1}$**. (In our example $N = 4$, meaning 3 independent voltages)

- Vector of unknown variables (node voltages):

  - Its size: $N_t \times 1$

$$\boldsymbol{V} = \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N_t} \end{pmatrix}$$

Nodal Analysis, step by step

3- Write a system of equation in matrix format $\boldsymbol{GV} = \boldsymbol{I}$

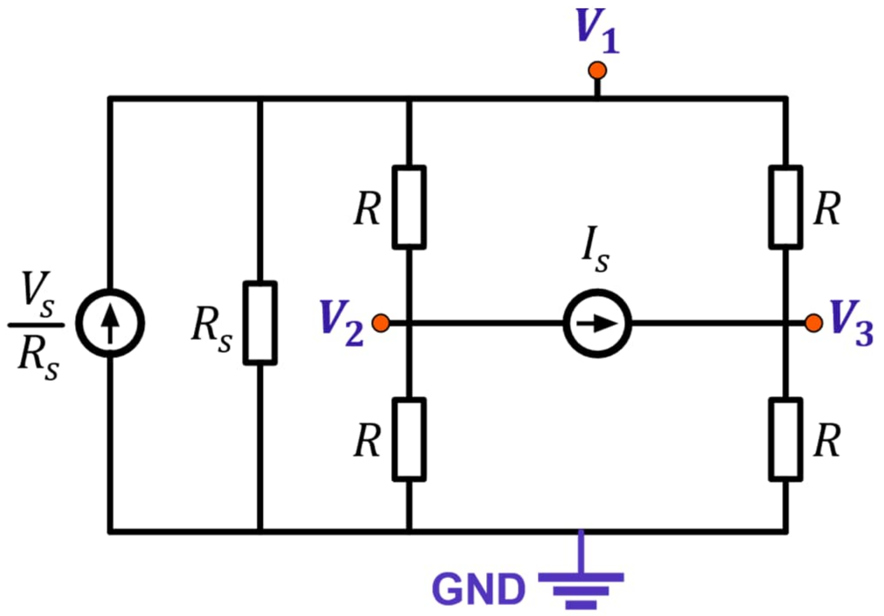
- Coefficient matrix (conductance matrix, $\boldsymbol{G}$)

KCL@n$_1$:

$$\frac{V_1}{R_s} + \frac{V_1 - V_2}{R} + \frac{V_1 - V_3}{R} = 0$$

$$G = \frac{1}{R}, G_s = \frac{1}{R_s}$$

$$\boxed{(G_s + 2G)V_1 - GV_2 - GV_3 = 0}$$



Nodal Analysis, step by step

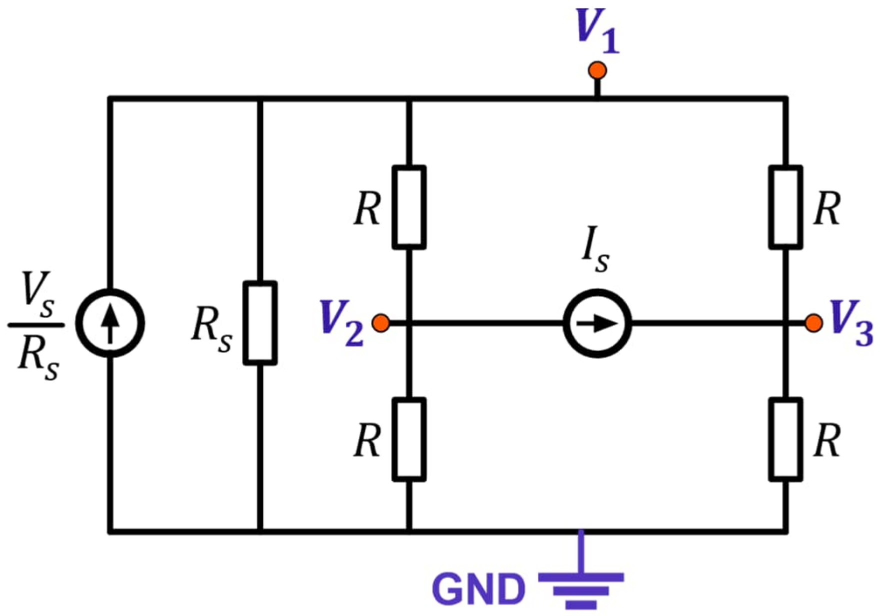
3- Write a system of equation in matrix format $\boldsymbol{G}\boldsymbol{V} = \boldsymbol{I}$

- Coefficient matrix (conductance matrix, $\boldsymbol{G}$)

KCL@n$_1$:

$$\boxed{(G_s + 2G)V_1 - GV_2 - GV_3 = 0}$$

$$\begin{pmatrix} G_s + 2G & -G & -G \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ ? \\ ? \end{pmatrix}$$



Nodal Analysis, step by step

3- Write a system of equation in matrix format $\color{red}{\boldsymbol{G}}\boldsymbol{V} = \boldsymbol{I}$

- Coefficient matrix (conductance matrix, $\boldsymbol{G}$)

$$\begin{pmatrix} G_s + 2G & -G & -G \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ ? \\ ? \end{pmatrix}$$

$$\boxed{\boldsymbol{G} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}}$$

$$G_{kk} = \sum (\text{all connected conductances to node } \boldsymbol{n_k})$$

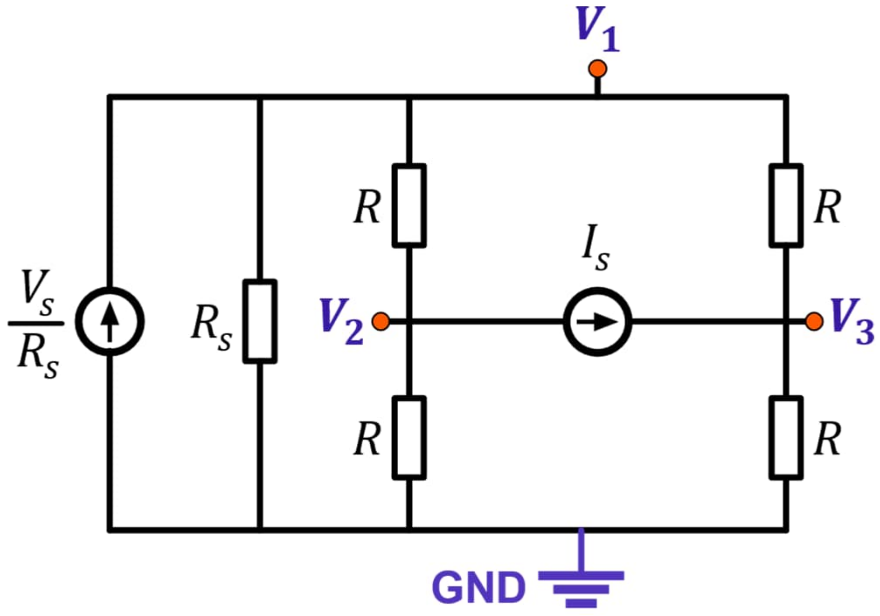
$$G_{jk}\ (j \neq k) = -\sum (\text{all connected conductances between nodes } \boldsymbol{n_j} \text{ and } \boldsymbol{n_k})$$

Nodal Analysis, step by step

3- Write a system of equation in matrix format $\boldsymbol{GV} = \boldsymbol{I}$

$$\begin{pmatrix} G_s + 2G & ? & ? \\ ? & 2G & ? \\ ? & ? & 2G \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix}$$

$$G_{kk} = \sum(\text{all connected conductances to node } \boldsymbol{n_k})$$

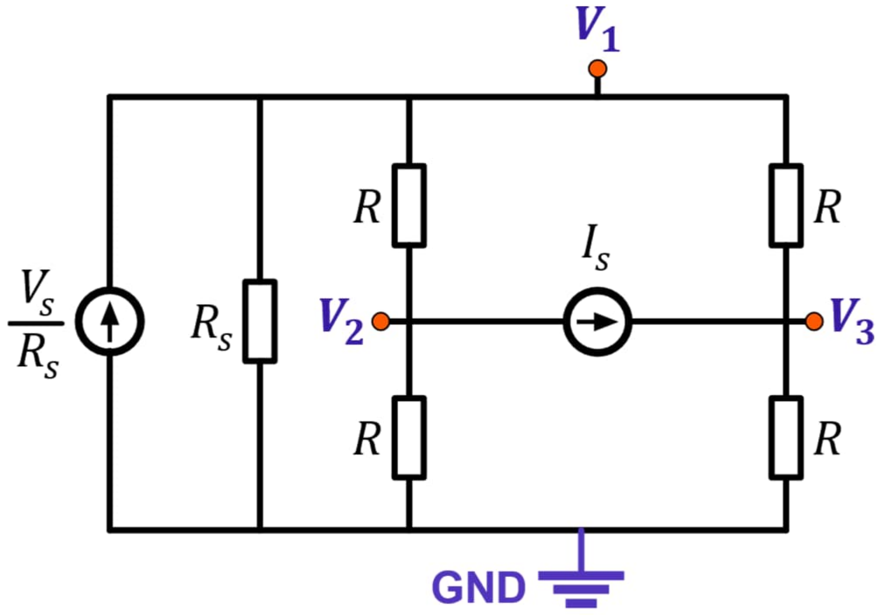


Nodal Analysis, step by step

3- Write a system of equation in matrix format $\boldsymbol{GV} = \boldsymbol{I}$

$$\begin{pmatrix} G_s + 2G & -G & -G \\ -G & 2G & 0 \\ -G & 0 & 2G \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix}$$

$$G_{jk}\ (j \neq k) = -\sum \begin{pmatrix}\text{all connected conductances} \\ \text{between nodes } \boldsymbol{n_j} \text{ and } \boldsymbol{n_k}\end{pmatrix}$$



Nodal Analysis, step by step

3- Write a system of equation in matrix format $\boldsymbol{GV} = \boldsymbol{I}$

- Constants matrix (current matrix, $\boldsymbol{I}$)

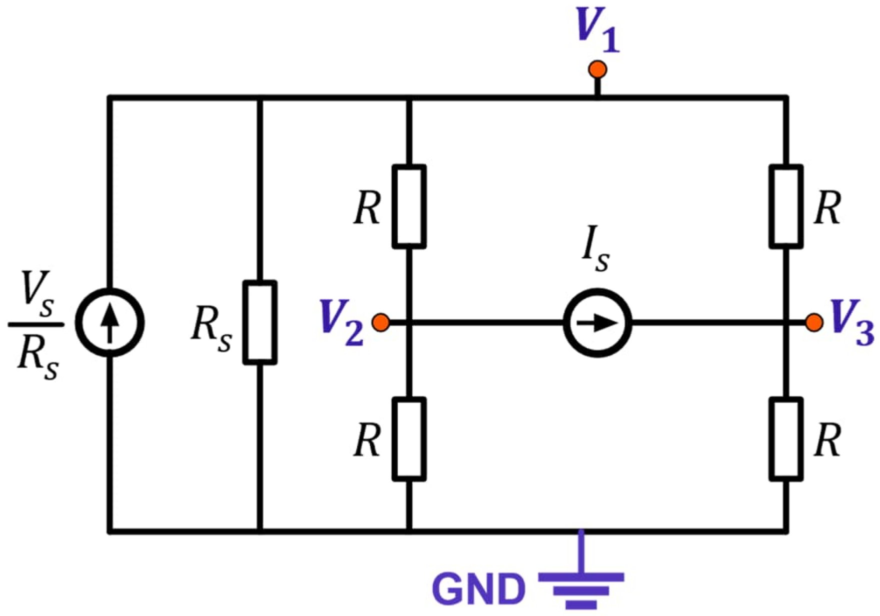
$$I_k = \sum(\text{all connected current sources entering node } \boldsymbol{n_k})$$

$$\boldsymbol{I} = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_{N_t} \end{pmatrix}$$

Nodal Analysis, step by step

3- Write a system of equation in matrix format $\boldsymbol{GV} = \boldsymbol{I}$

- Constants matrix (current matrix, $\boldsymbol{I}$)



$$\begin{pmatrix} G_s + 2G & -G & -G \\ -G & 2G & 0 \\ -G & 0 & 2G \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{V_s}{R_s} \\ -I_s \\ I_s \end{pmatrix}$$

Nodal Analysis, step by step

4- Solve the linear system as you wish!

- Multiplication by the inverse of $\boldsymbol{G}$

$$\boxed{\boldsymbol{V} = \boldsymbol{G}^{-1}\boldsymbol{I}}$$

- Cramer's rule

$$\boxed{V_k = \frac{\det(\boldsymbol{G_k})}{\det(\boldsymbol{G})}}$$

$\boldsymbol{G_k}$: The matrix formed by replacing the k-th column of $\boldsymbol{G}$ by $\boldsymbol{I}$

Nodal Analysis, step by step

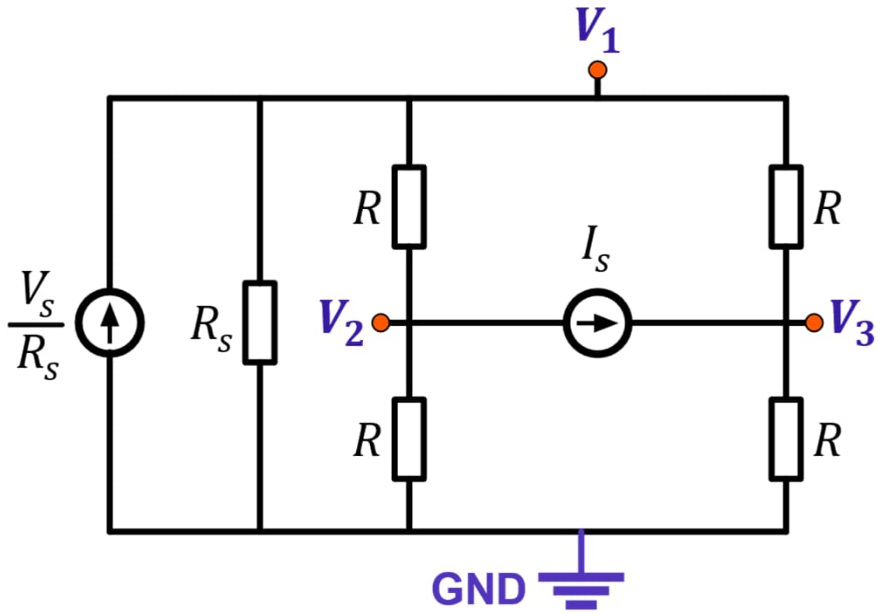
4- Solve the linear system as you wish!

$$\begin{pmatrix} G_s + 2G & -G & -G \\ -G & 2G & 0 \\ -G & 0 & 2G \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{V_s}{R_s} \\ -I_s \\ I_s \end{pmatrix}$$

$$\boxed{V_1 = \frac{R}{R + R_S} V_s}$$

$$\boxed{V_2 = \frac{1}{2}\left(\frac{R}{R + R_S} V_s - R I_s\right)}$$

$$\boxed{V_3 = \frac{1}{2}\left(\frac{R}{R + R_S} V_s + R I_s\right)}$$



Pause and Ponder 5

- Solve the previous question using the principle of superposition.