Elektronik

Basic Components and Circuit Theory 3

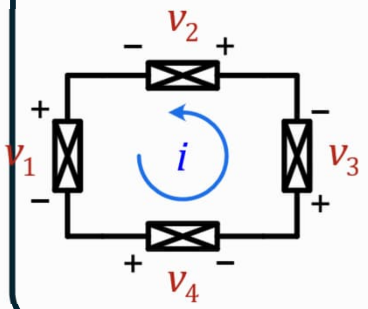
EITA10

Iman Ghotbi

Mars 2025

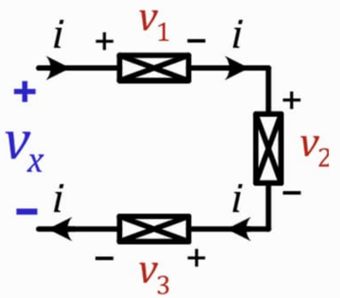
We learned

KVL



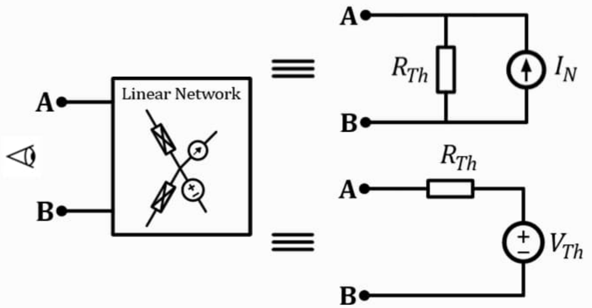
$$\sum_{k=1}^{4} v_k = 0$$

Series Coupling

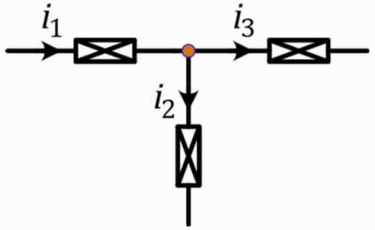


$$R_{eq} = \sum_{k=1}^{N} R_k$$

Thevenin's/Norton's

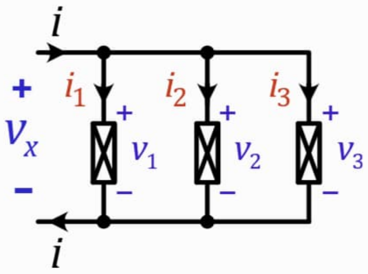


KCL



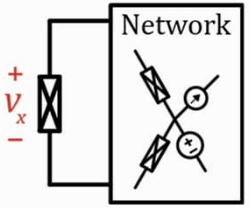
$$\sum I_{in} = \sum I_{out}$$

Parallel Coupling



$$G_{eq} = \sum_{k=1}^{N} G_k$$

Superposition



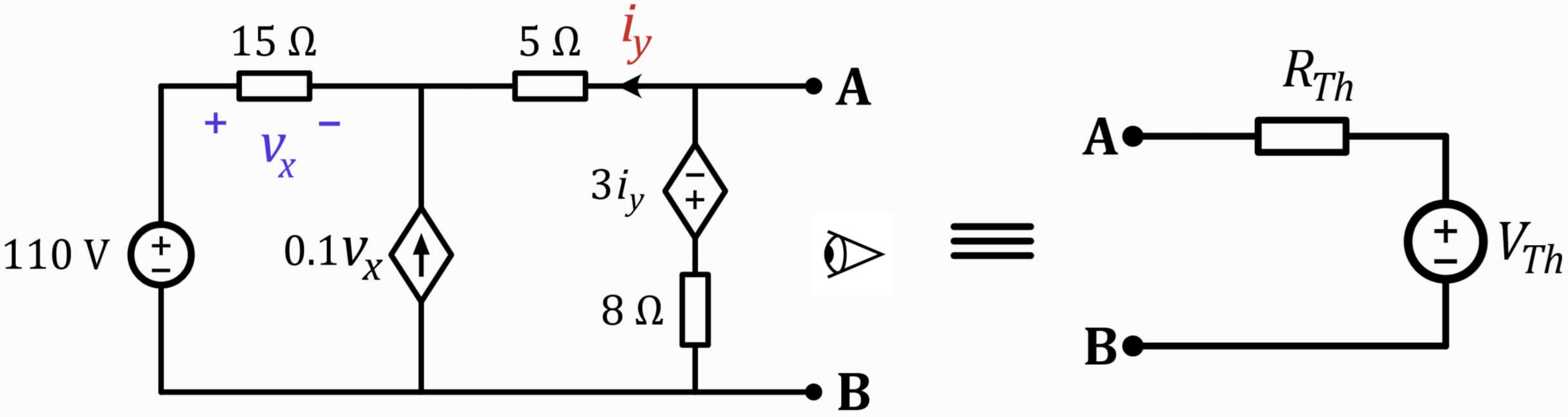
$$v_x = \sum_{n=1}^{N} A_n V_n + \sum_{m=1}^{M} R_m I_m$$

Today we learn

- Circuit analysis – an example
- Measurement techniques
- Capacitance
- Capacitors
- First-order (RC) circuits

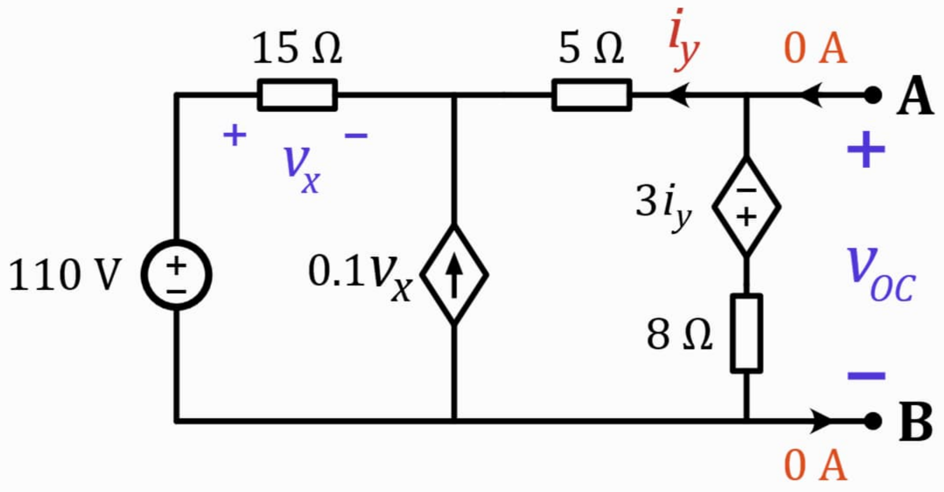
Example

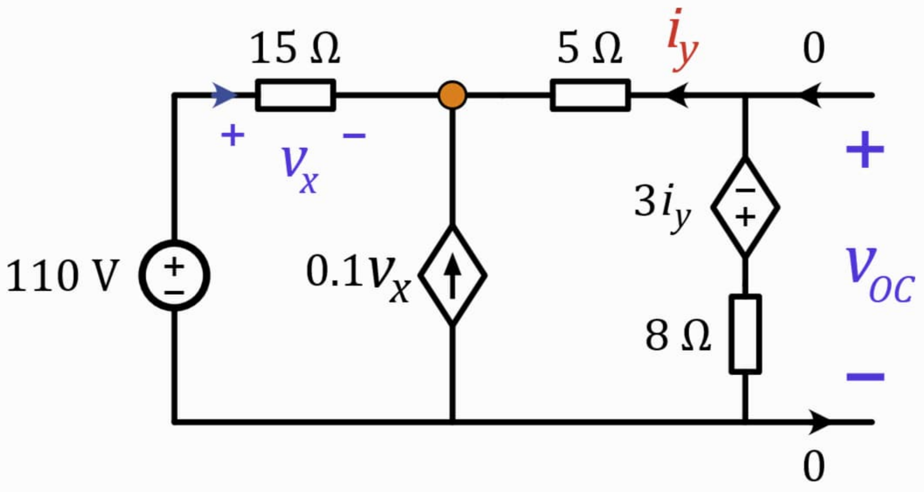
- Determine the Thevenin’s equivalent circuit seen from A-B port.



Nodal Analysis – Example

- Determine the Thevenin’s equivalent circuit seen from A-B port.
  - **Open-circuit voltage ($V_{Th} = V_{OC}$)**

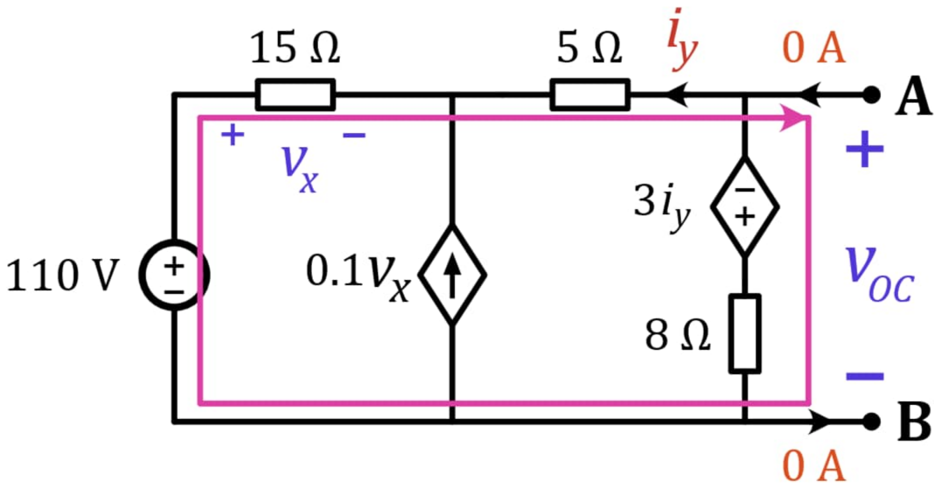




KCL: $i_y + 0.1v_x + \frac{1}{15}v_x = 0$

Nodal Analysis – Example

- Determine the Thevenin's equivalent circuit seen from A-B port.
  - **Open-circuit voltage ($V_{Th} = V_{OC}$)**



$$\begin{cases} \text{KCL: } i_y + \frac{1}{6}v_x = 0 \\ \text{KVL: } v_x - 16i_y = 110 \end{cases} \qquad \begin{cases} i_y = -5\text{ A} \\ v_x = 30\text{ V} \end{cases}$$

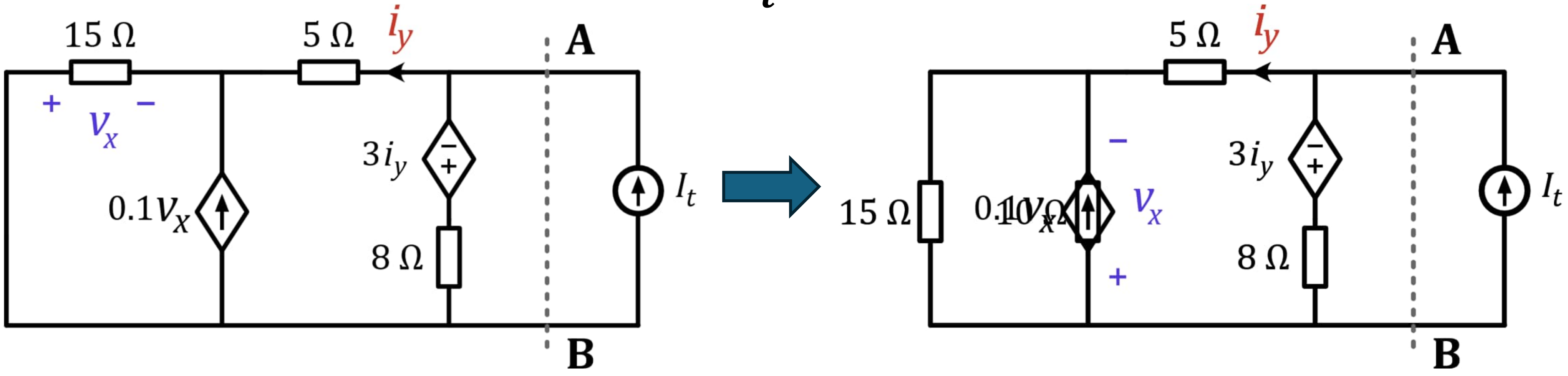
KVL: $-110 + v_x - 5i_y - 3i_y - 8i_y = 0$

$$V_{OC} = -3i_y - 8i_y = -11i_y$$

$$\Rightarrow \boxed{V_{OC} = 55\text{ V}}$$

Nodal Analysis – Example

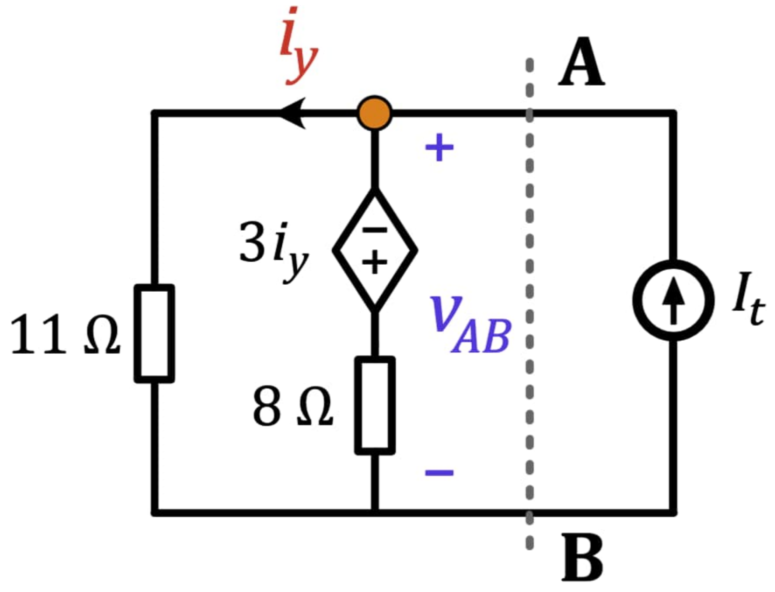
- Determine the Thevenin’s equivalent circuit seen from A-B port.
  - **Thevenin’s resistance (** $\boldsymbol{R_{Th} = \frac{V_{AB}}{I_t}}$ **)**



$$R_x = \frac{v_x}{0.1v_x} = 10\ \Omega$$

Nodal Analysis – Example

- Determine the Thevenin's equivalent circuit seen from A-B port.
  - **Thevenin's resistance ($\boldsymbol{R_{Th}} = \dfrac{\boldsymbol{V_{AB}}}{\boldsymbol{I_t}}$)**

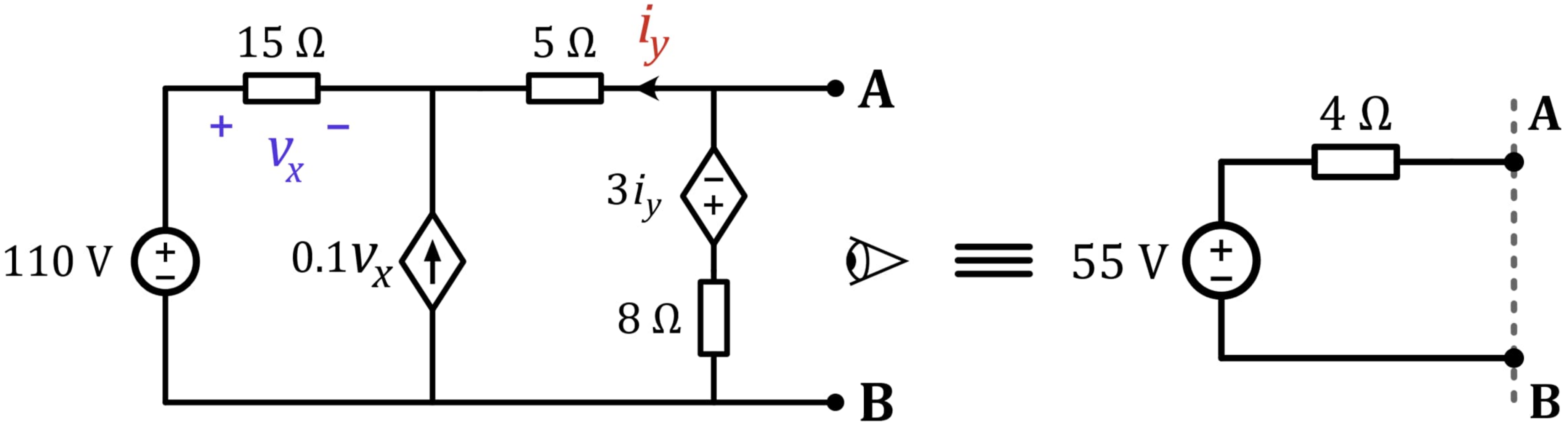


$$\begin{cases} \text{KVL: } -v_{AB} + 11i_y = 0 \\ \text{KVL: } -v_{AB} - 3i_y + 8\overset{\text{KCL}}{(I_t - i_y)} = 0 \end{cases}$$

$$v_{AB} = 4I_t \Rightarrow \boxed{R_{Th} = 4\,\Omega}$$

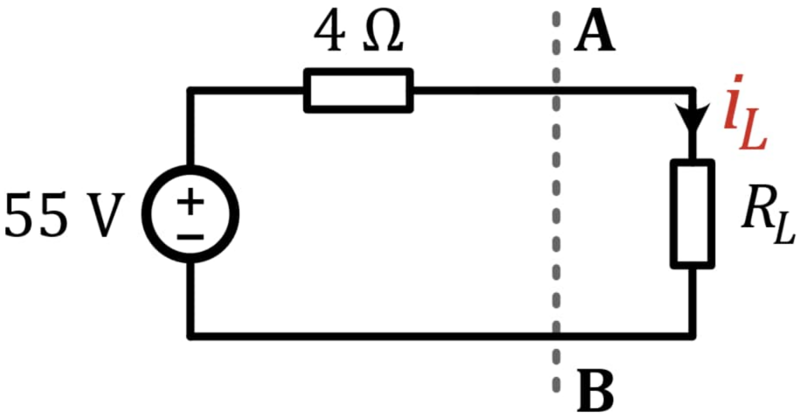
Example

- Determine the Thevenin's equivalent circuit seen from A-B port.



Pause and Ponder 1

- We aim to supply power to a resistive load using the circuit we've just analyzed. What is the **optimal value for the load resistance** to absorb the **maximum power** available?



$$i_L = \frac{55}{4 + R_L} \quad , \quad v_L = \frac{R_L}{4 + R_L} \times 55$$

$$P_L = v_L i_L \quad \Rightarrow \quad P_L = \frac{R_L}{(4 + R_L)^2} \times 55^2$$

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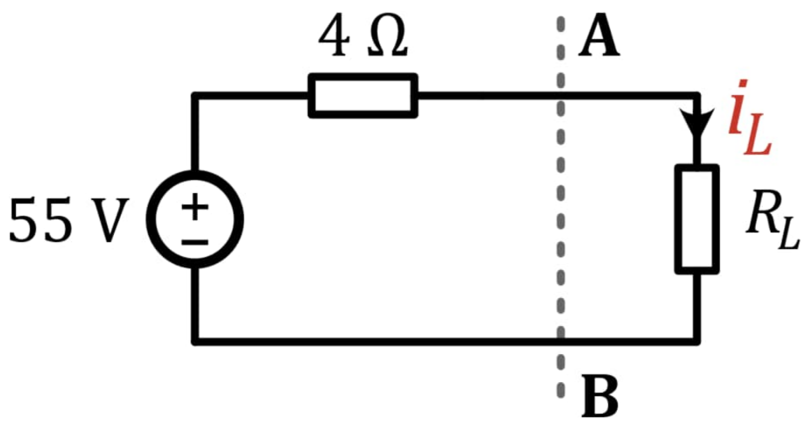

$$\frac{dP_L}{dR_L} = 0 \Rightarrow R_{L,opt} = R_{Th} \Rightarrow \boxed{R_{L,opt} = 4\ \Omega}$$

$$P_{L,max} = \frac{v_{Th}^2}{4R_{Th}} = 189\ \text{W}$$

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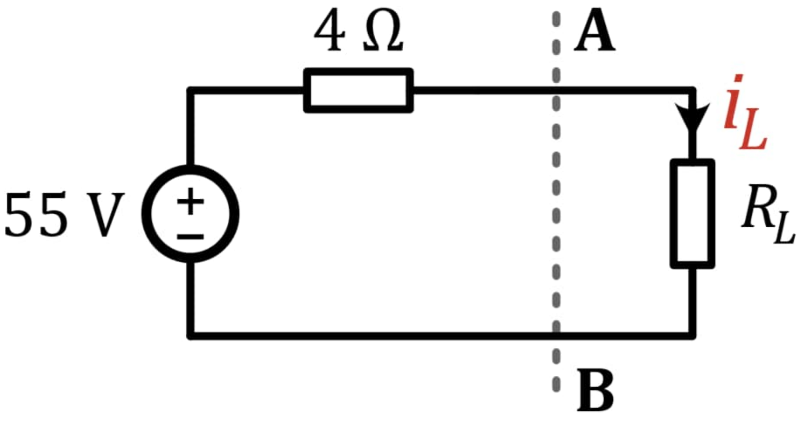

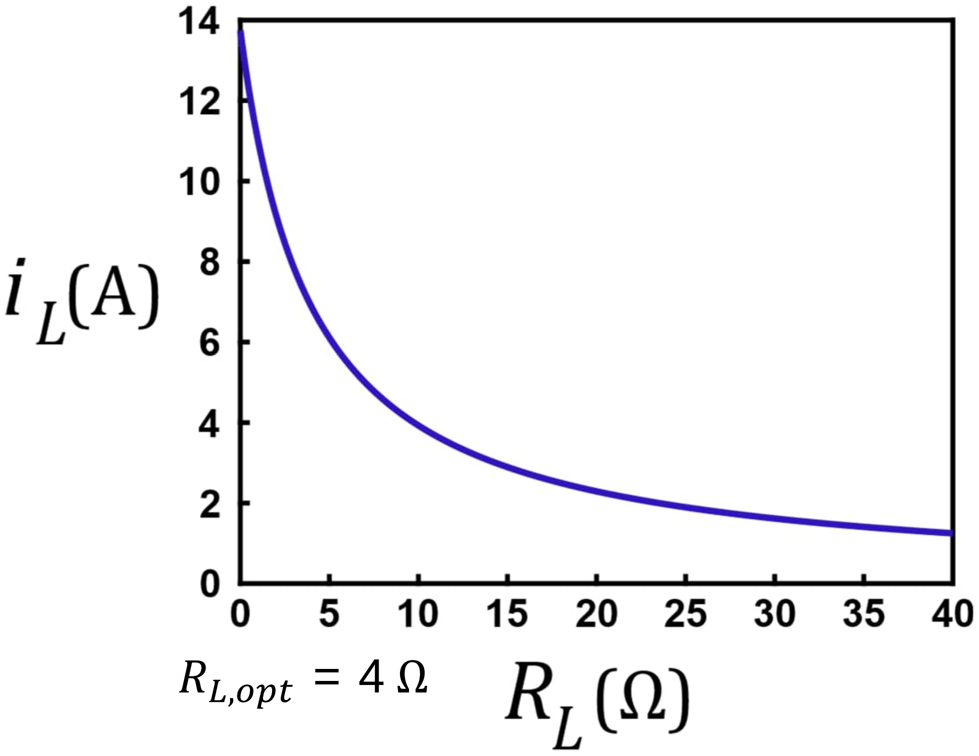

$R_{L,opt} = 4\ \Omega$

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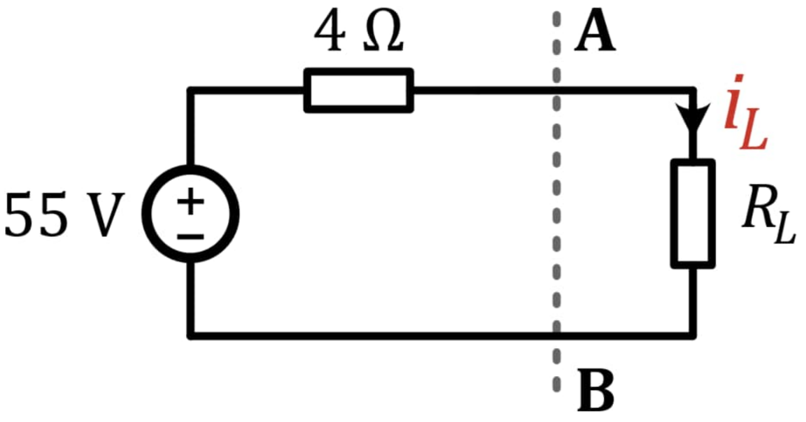

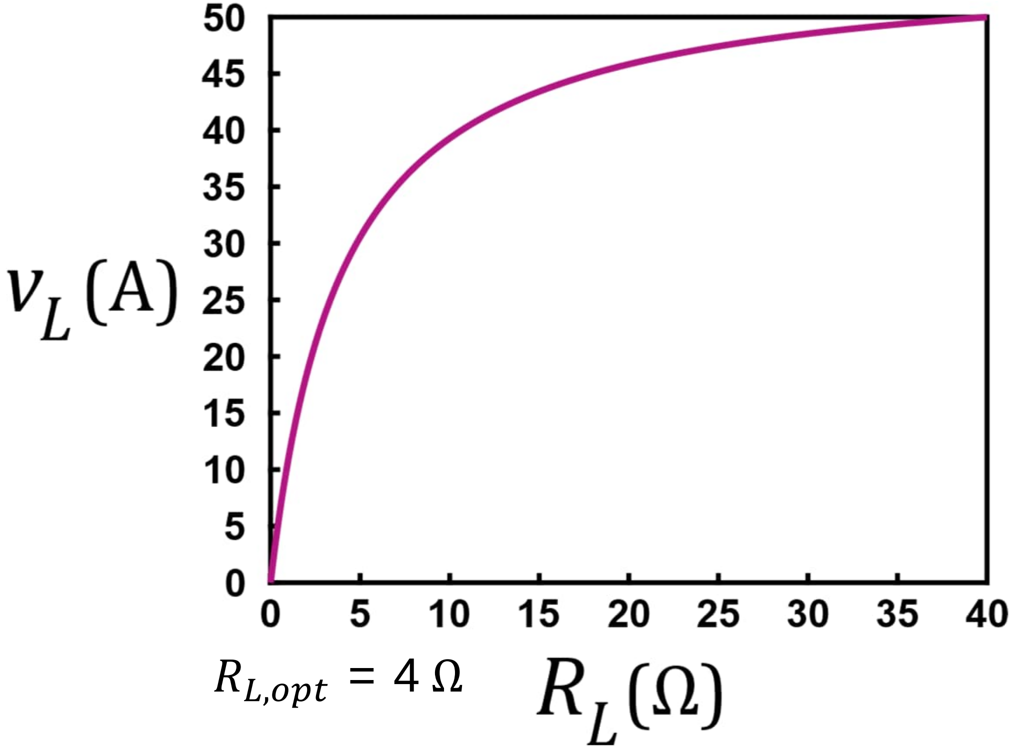

$R_{L,opt} = 4\ \Omega$

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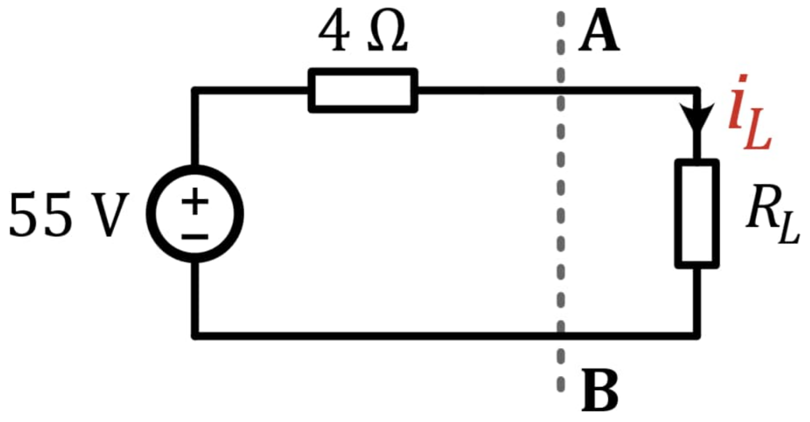
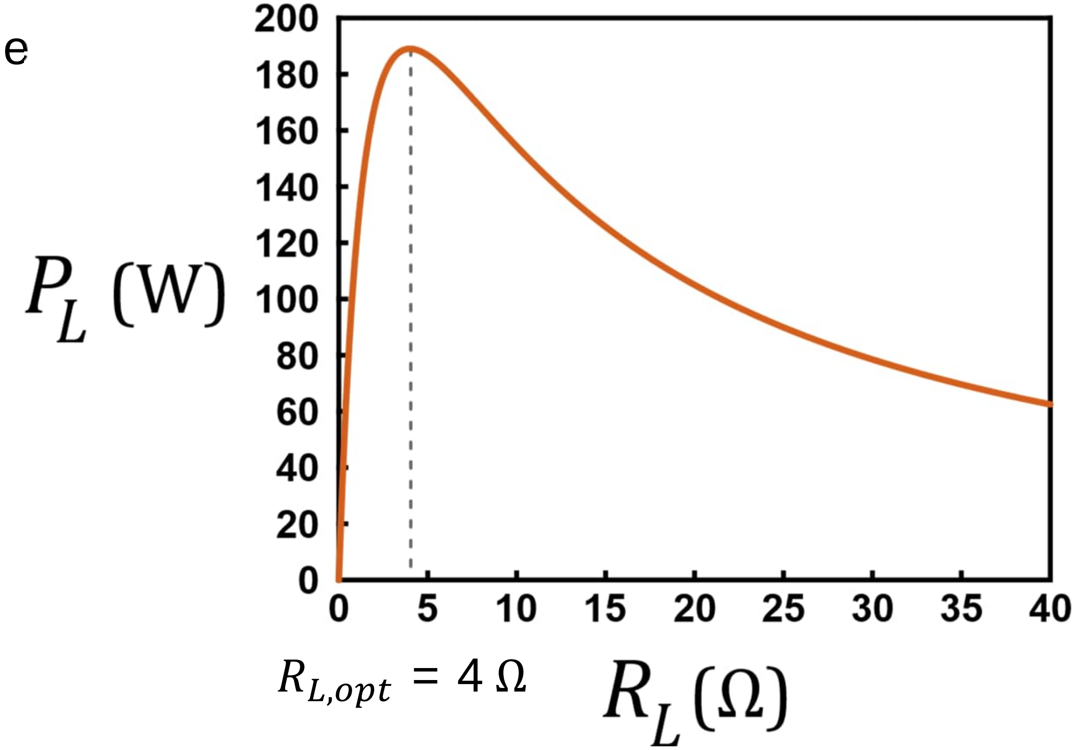
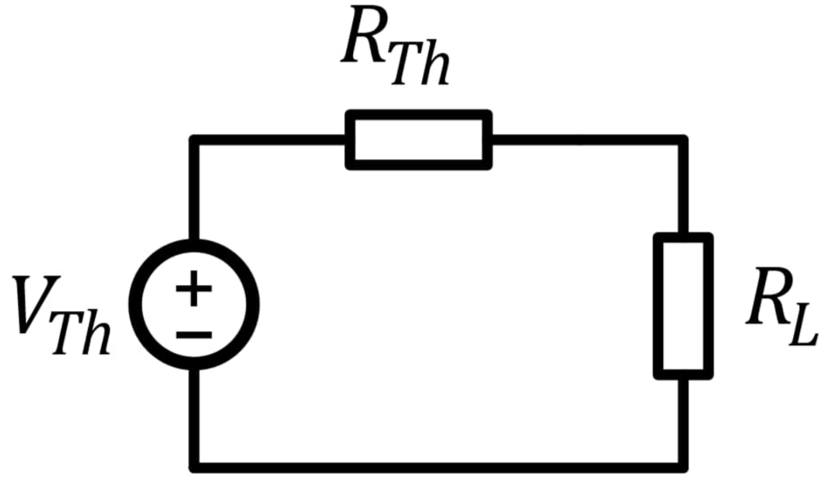




Pause and Ponder 2

- Calculate the power efficiency when the optimal load is connected.



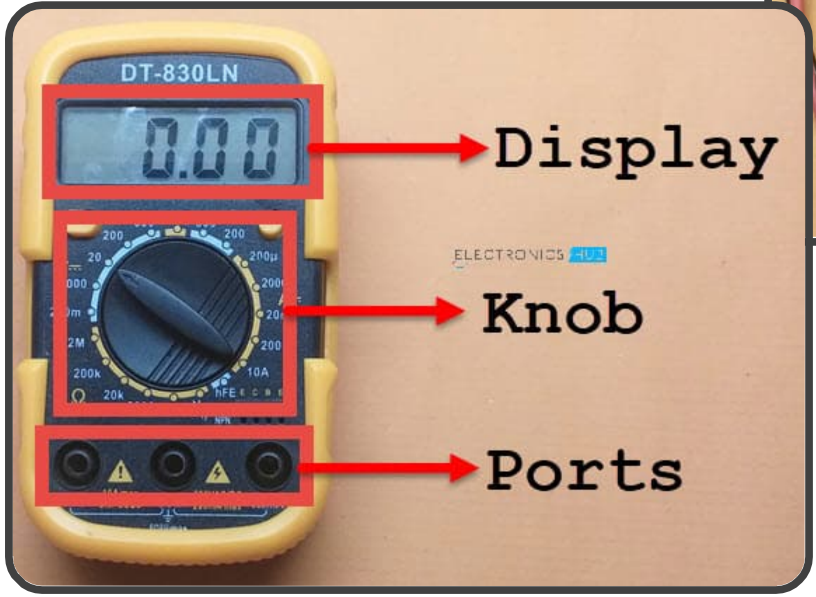
$$P_{in} = v_{th} \times i_L = \frac{v_{Th}^2}{2R_{Th}}$$

$$P_{L,max} = \frac{v_{Th}^2}{4R_{Th}}$$

$$\boxed{\eta\% = \frac{P_L}{P_{in}} \times 100 = 50\%}$$

Measurement Techniques

- A typical multimeter

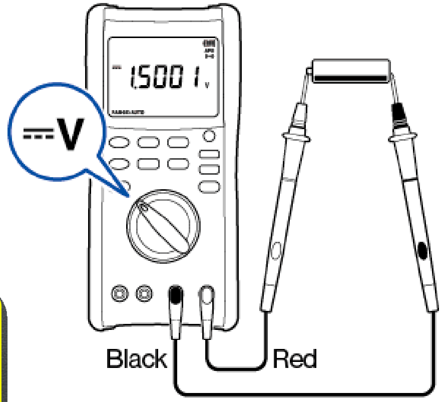


Measurement Techniques

- A typical multimeter

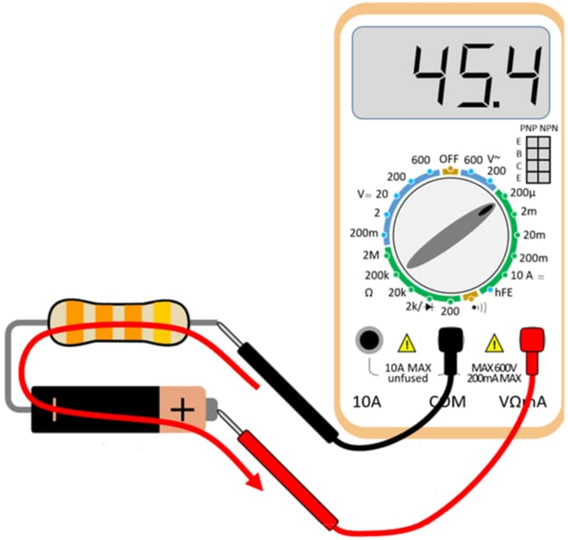
- **DC voltage measurement**
  - The **black probe** is connected between the **COM** jack and **GND** of the circuit.
  - The **red probe** is connected to the **node-of-interest.**
  - Differential measurement: **parallel connection**

    (positive → red, negative → black)



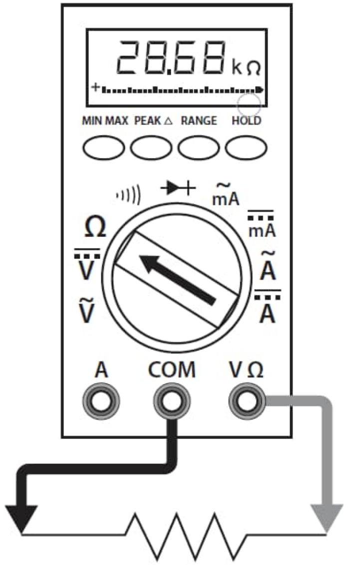
Measurement Techniques

- A typical multimeter
- DC voltage measurement
- **DC current measurement**
  - **Series** connection
  - The probes are placed on the way of the current of interest.



Measurement Techniques

- A typical multimeter
- DC voltage measurement
- DC current measurement
- **Resistance measurement**
  - Connect the probes to the two sides of the resistor.



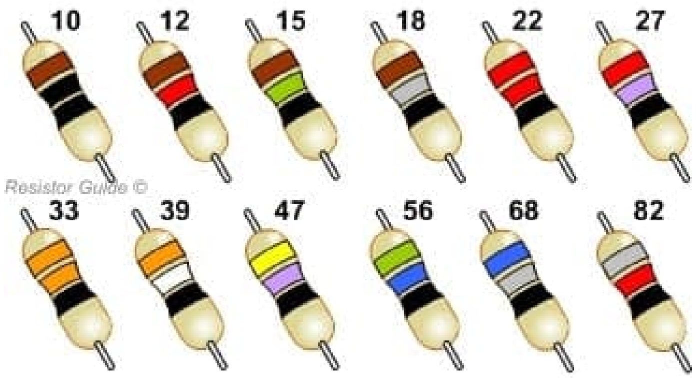
Measurement Techniques

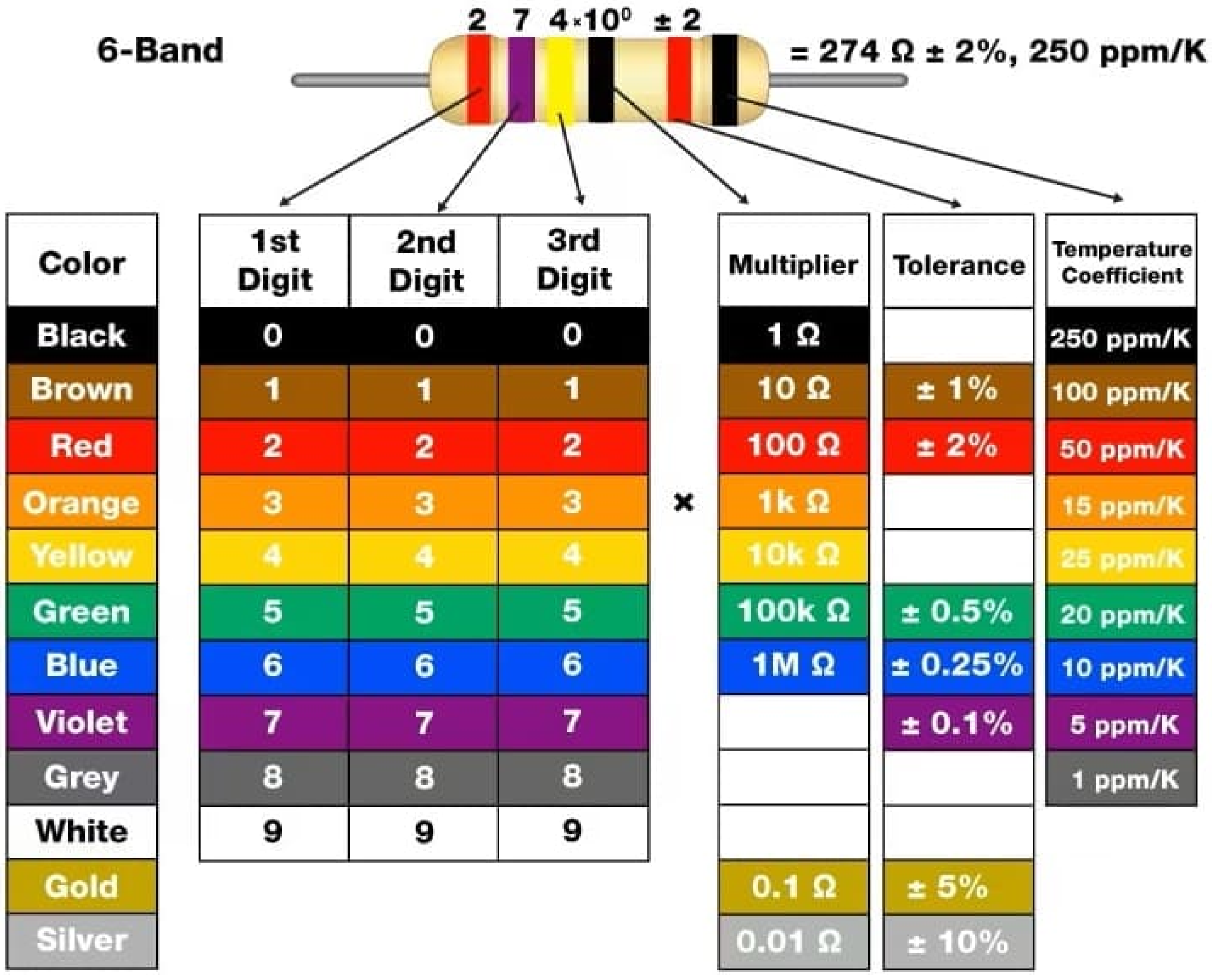
- A typical multimeter
- DC voltage measurement
- DC current measurement
- Resistance measurement
- **Short-circuit test**
  - Kind of resistance measurement
  - Continuity Beeper

Don't forget to **turn it off** before leaving the lab!

Standard Resistors and Color Code

- Resistors are manufactured in a variety of shapes and sizes.
- Commercially available resistors are typically **limited to standard values.**
- Non-standard resistor values are implemented by **creating series and parallel combinations**.



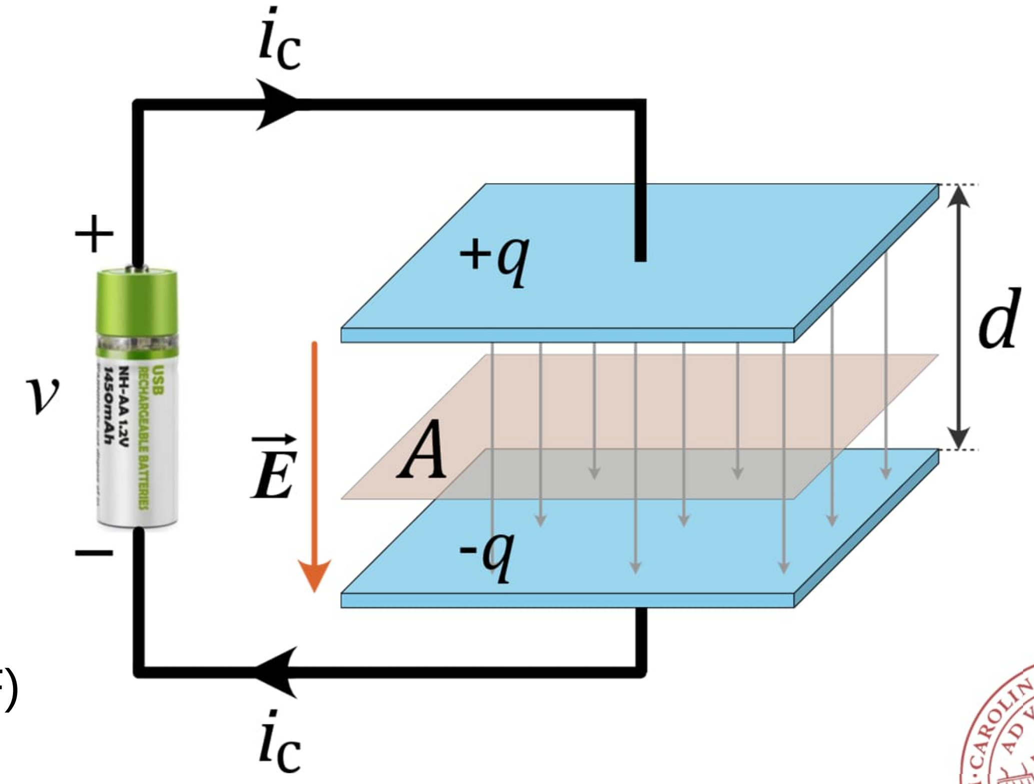


Color	1st Digit	2nd Digit	3rd Digit	Multiplier	Tolerance	Temperature Coefficient
Black	0	0	0	1 Ω		250 ppm/K
Brown	1	1	1	10 Ω	± 1%	100 ppm/K
Red	2	2	2	100 Ω	± 2%	50 ppm/K
Orange	3	3	3	1k Ω		15 ppm/K
Yellow	4	4	4	10k Ω		25 ppm/K
Green	5	5	5	100k Ω	± 0.5%	20 ppm/K
Blue	6	6	6	1M Ω	± 0.25%	10 ppm/K
Violet	7	7	7		± 0.1%	5 ppm/K
Grey	8	8	8			1 ppm/K
White	9	9	9			
Gold				0.1 Ω	± 5%	
Silver				0.01 Ω	± 10%	

Capacitance

- The capability of a device (material) to **store electric charge** in response to an **electric potential**.

$$\boxed{C \triangleq \frac{q}{v}}$$



- Measured in Farad (F).
  - 1 F is a huge capacitance.
  - Normal range of capacitance in circuits:

    μF, nF, and pF.
  - Unwanted (parasitic) capacitances: fF (10^{-15} F)

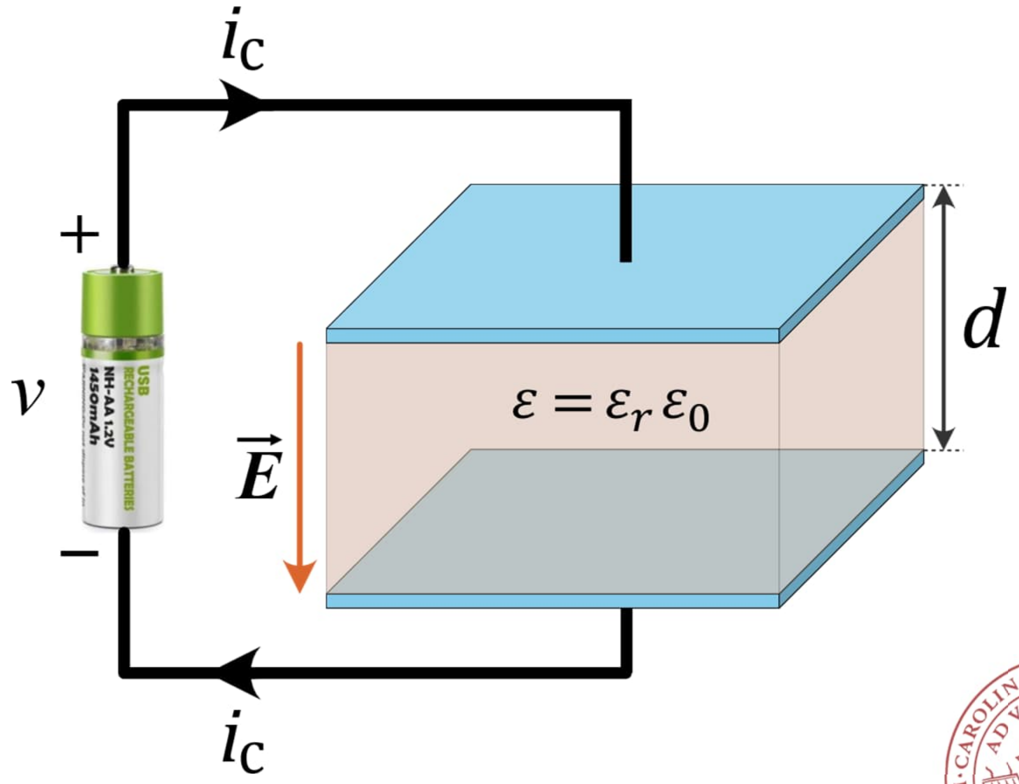
Relative Permittivity (Dielectric constant)

- Permittivity of a dielectric (insulator) measures its **capability to store electric energy** in an electric field.

$$\boxed{\varepsilon = \varepsilon_r \, \varepsilon_0}$$

$$\varepsilon_0 \approx 8.85 \times 10^{-12} \left[\frac{C^2}{N.m^2}\right]$$

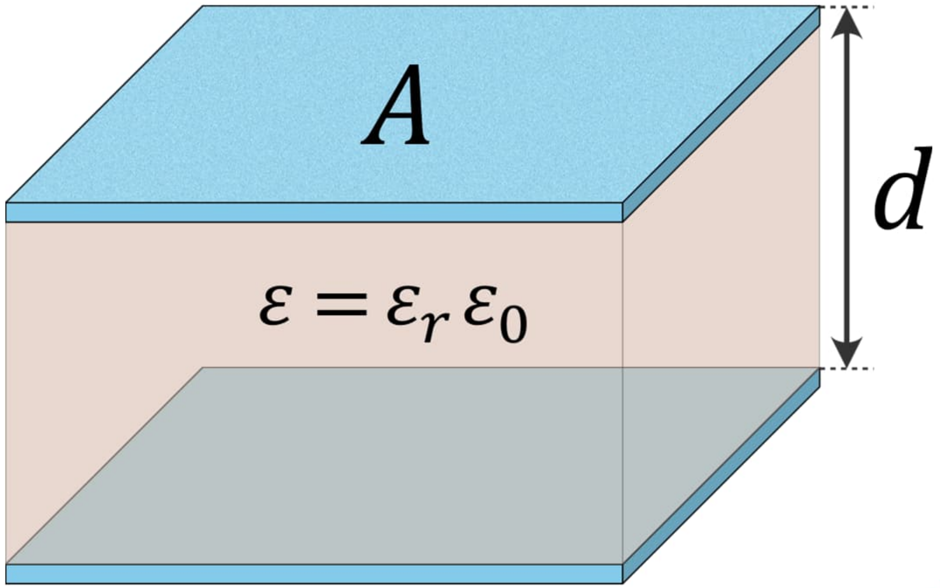
Material	ε_r
Paper	1.4
Mica	3 to 6
Silicon dioxide (SiO_2)	3.9
Graphite	10 to 15
Silicon (Si)	11.68
Water (room temperature)	80
Titanium dioxide (TiO_2)	86 to 173



Parallel-Plate Capacitor

- An insulator is sandwiched between two conductor plates.

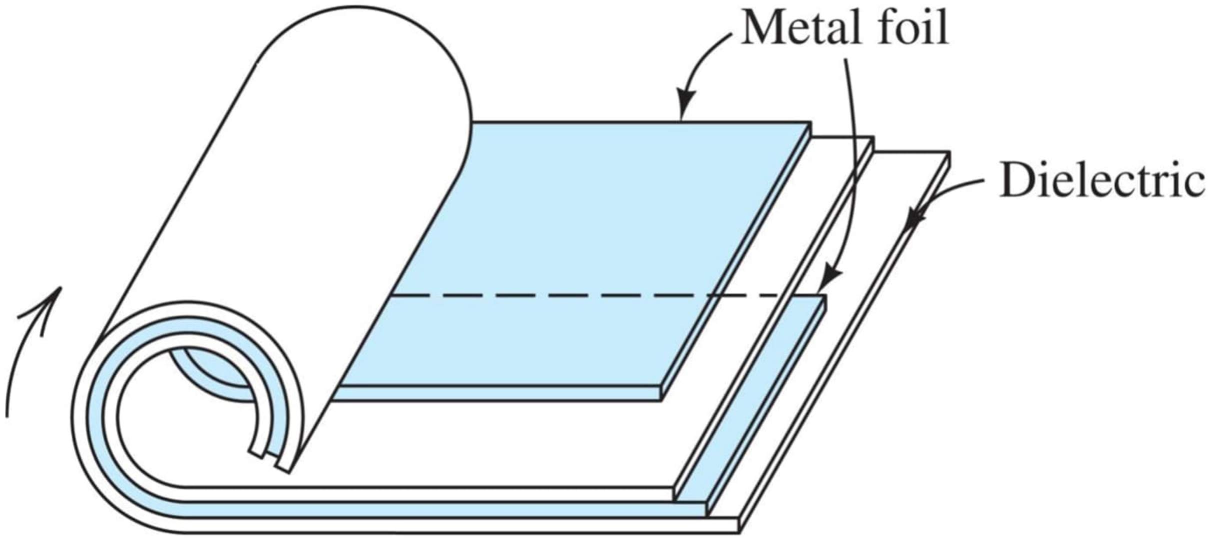
$$\boxed{C = \varepsilon_r \, \varepsilon_0 \frac{A}{d}}$$



Practical Capacitors

- To create sufficiently large capacitance:
  - **Thin insulator**
  - **High ε_r**
  - **Large area**

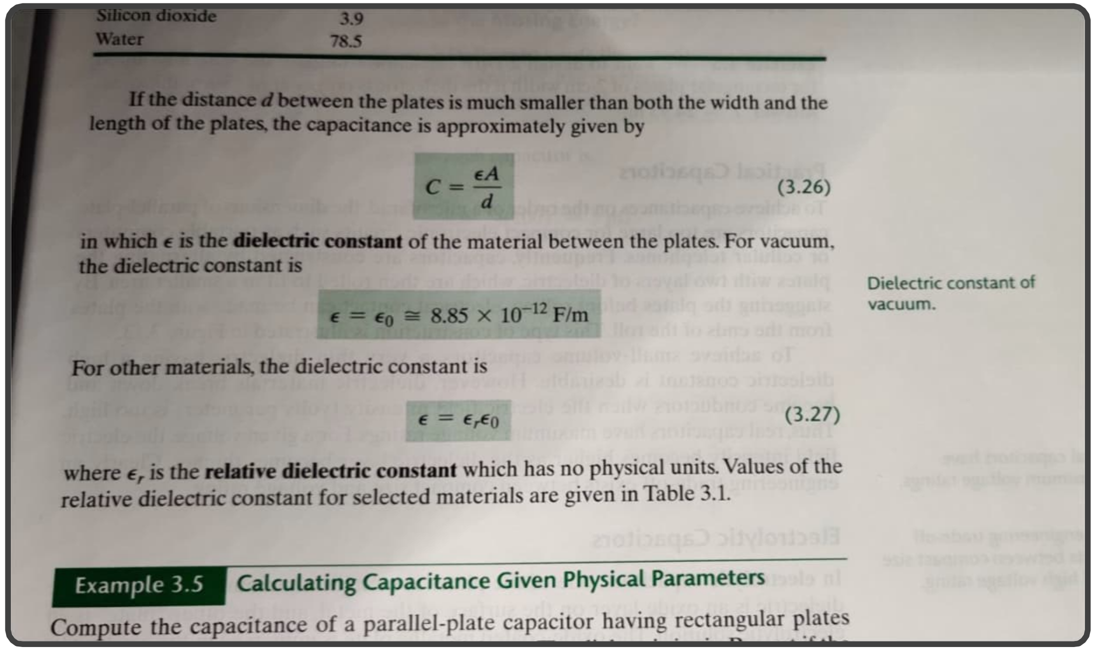
  → **Roll up!**



- We need them for:
  - **Filtering** out interference
  - **Isolating** DC voltages (AC-Coupling)
  - **Bypassing** some components at high frequencies
  - **Compensating** inductances

Coursebook’s Capacitance!

- Coursebook roll!
- Where PDF doesn’t work!



Silicon dioxide	3.9
Water	78.5

If the distance d between the plates is much smaller than both the width and the length of the plates, the capacitance is approximately given by

$$C = \frac{\epsilon A}{d} \qquad (3.26)$$

in which ϵ is the **dielectric constant** of the material between the plates. For vacuum, the dielectric constant is

Dielectric constant of vacuum.

$$\epsilon = \epsilon_0 \cong 8.85 \times 10^{-12}\ \mathrm{F/m}$$

For other materials, the dielectric constant is

$$\epsilon = \epsilon_r \epsilon_0 \qquad (3.27)$$

where ϵ_r is the **relative dielectric constant** which has no physical units. Values of the relative dielectric constant for selected materials are given in Table 3.1.

Example 3.5 Calculating Capacitance Given Physical Parameters

Compute the capacitance of a parallel-plate capacitor having rectangular plates

Coursebook’s Capacitance!

- Metal plates:
  - Area:

    $$A = 16 \times 5\ \mathrm{cm}^2 = 80\ \mathrm{cm}^2$$

  - Page thickness:

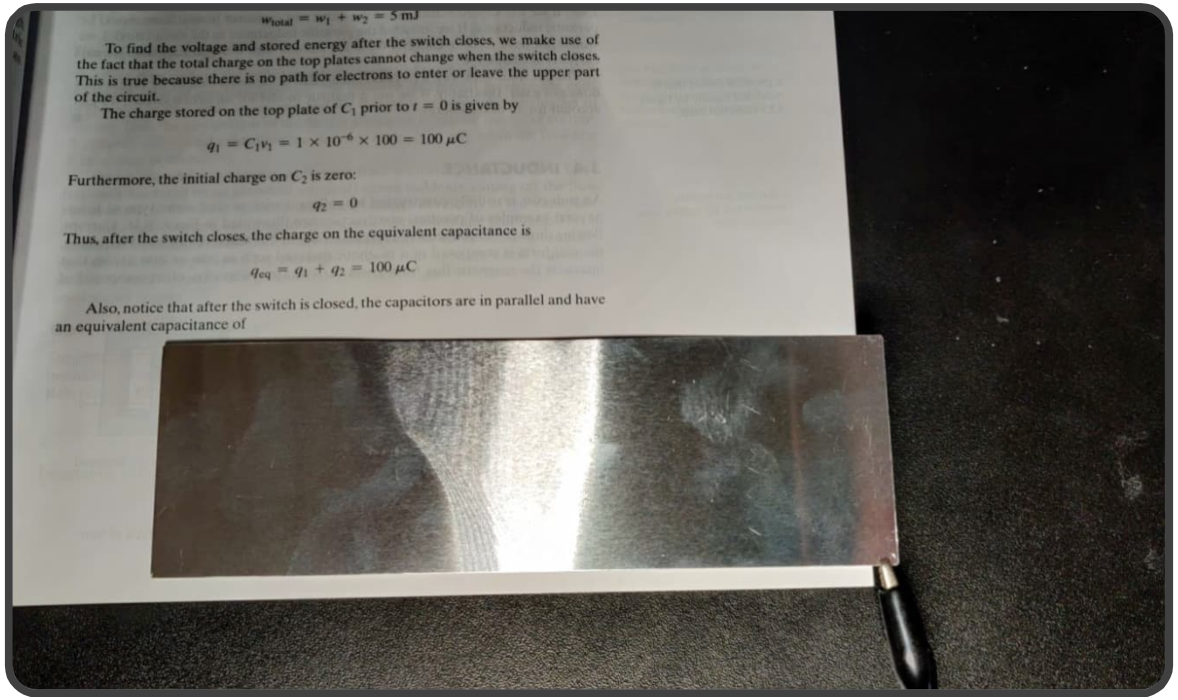
    $$d = 50\ \mu m$$

  - Permittivity:

    $$\varepsilon_r \approx 1.4$$

Theory:

$$\boxed{C = \varepsilon_r\, \varepsilon_0 \frac{A}{d} \approx 2\ nF}$$



Coursebook’s Capacitance!

- First measurement:

$$C_{meas1} = 1.4\, nF$$

- Slightly uneven plates, air gap → (inconsistent d)

- Press it down!

$$C_{meas2} = 2.2\, nF$$

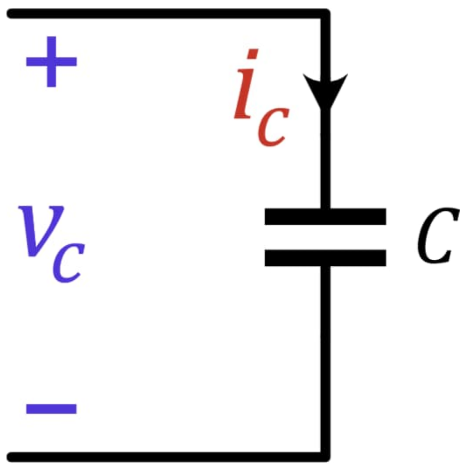
- The theory is correct!

$i - v$ relationship of a Capacitor

- **Time domain:**

$$q_c = C v_c \quad \Rightarrow \frac{dq_c}{dt} = \frac{dC}{dt} v_c + C \frac{dv_c}{dt}$$

$$\boxed{i_c = \frac{dC}{dt} v_c + C \frac{dv_c}{dt}}$$



For a **time-invariant** (constant) capacitor:

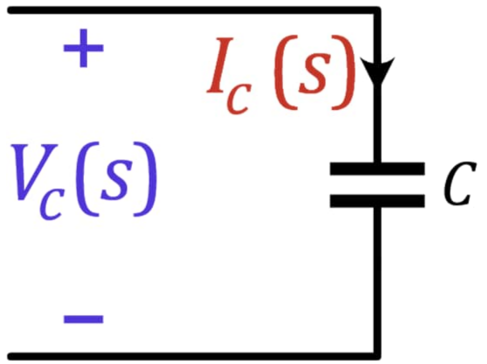
$$\boxed{i_c = C \frac{dv_c}{dt}}$$

$I(s) - V(s)$ relationship of a Capacitor

- **Frequency domain:**

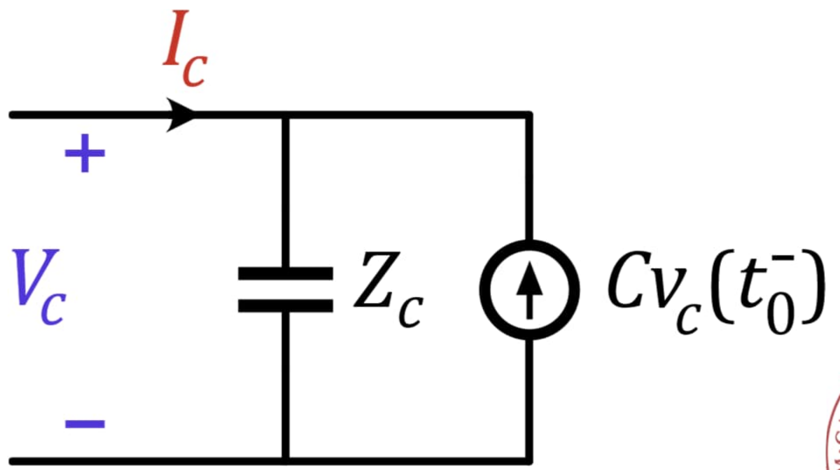
$$i_c = C\frac{dv_c}{dt} \xrightarrow{\mathcal{L}} I_C(s) = C\Big(sV_C(s) - \overbrace{v_c(t_0^-)}^{\text{Initial condition}}\Big)$$

t_0: The time of change in the circuit's state (switching)



$$\Rightarrow \quad \boxed{I_C = (Cs)V_C - Cv_c(t_0^-)}$$

Impedance $\boxed{Z_C = \frac{V_C(s)}{I_C(s)}\Big|_{v_c(t_0^-)=0} = \frac{1}{CS}}$

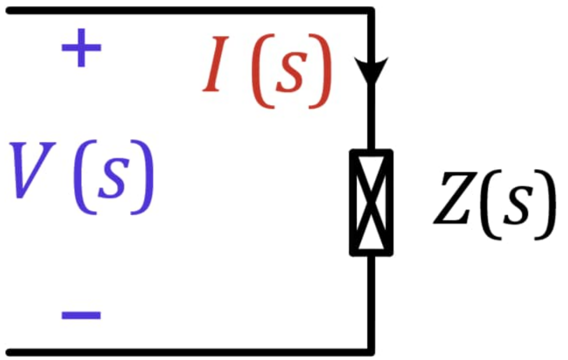


Impedance (Z) and Admittance (Y)

- Ohm's law in frequency domain
- Useful for dynamic circuits

Static circuit	$V = RI$	$I = GV$
Dynamic circuit	$V(s) = Z(s)I(s)$	$I(s) = Y(s)V(s)$

- The same techniques can be applied:
  - Series/parallel combination
  - Voltage/current division, superposition
  - Thevenin's equivalent circuit



$$\boxed{V(s) = Z(s)I(s)}$$

$$\boxed{I(s) = Y(s)V(s)}$$

Laplace Transform and S-Plane

Laplace transform

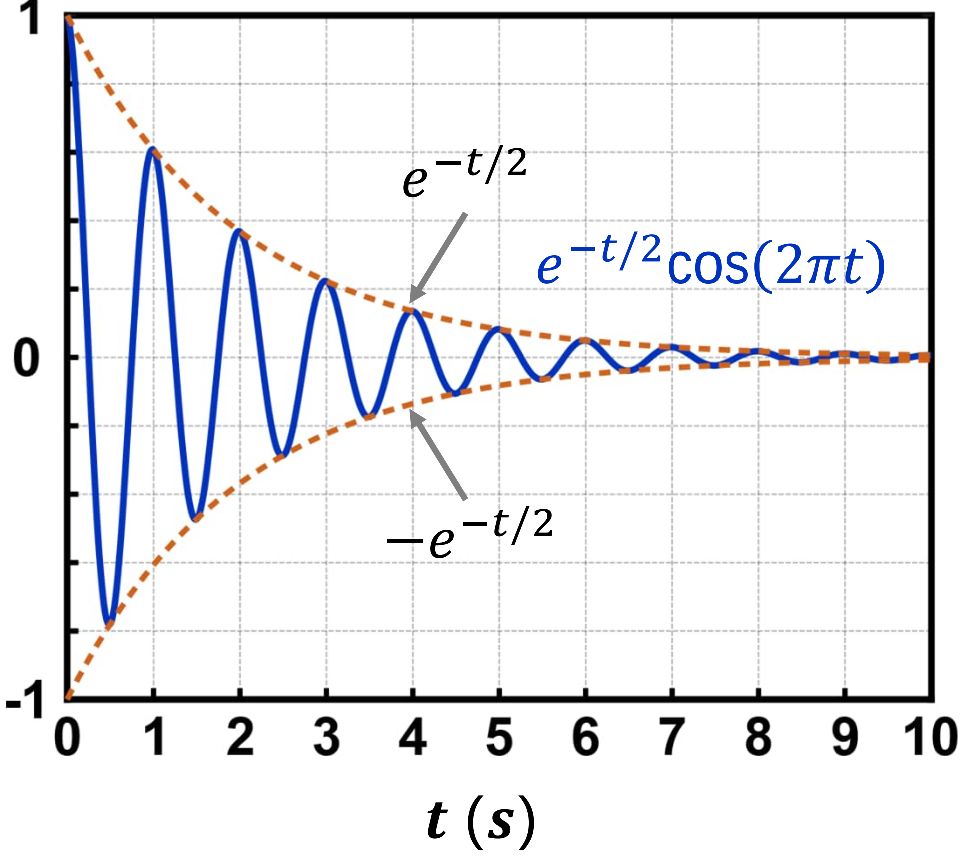
$$\boxed{F(s) = \int_0^\infty f(t)\, e^{-st} dt}$$

$$s = \frac{1}{\tau} + j\omega$$

$$\Rightarrow e^{-st} = \exp\left(-\frac{t}{\tau}\right)\left(\cos(\omega t) - j\sin(\omega t)\right)$$

$$\Rightarrow \boxed{\text{Re}\{e^{-st}\} = \exp\left(-\frac{t}{\tau}\right)\cos(\omega t)}$$

$$s = \frac{1}{2} + j$$



Laplace Transform and S-Plane

Laplace transform

$$F(s) = \int_0^{\infty} f(t)\, e^{-st} dt$$

Derivative

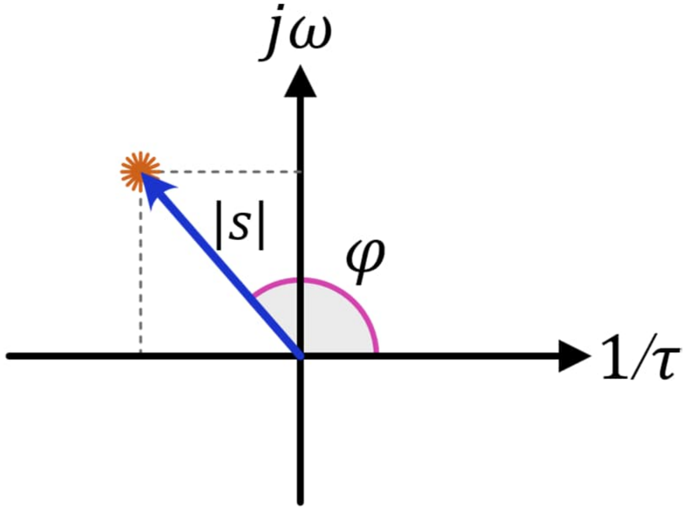
$$\mathcal{L}\{f'(t)\} = sF(s) - f(0^-)$$

Integration

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{1}{s} F(s)$$

Convolution

$$\mathcal{L}\{f(t) * g(t)\} = F(s) \cdot G(s)$$



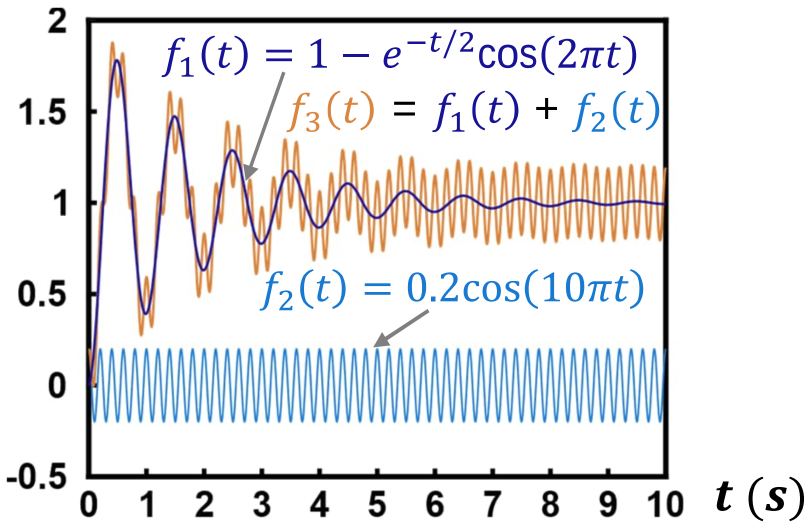
$$|s| = \sqrt{\left(\frac{1}{\tau}\right)^2 + \omega^2}$$

$$\varphi = \arctan(\omega\tau)$$

Steady-state vs. Transient Response

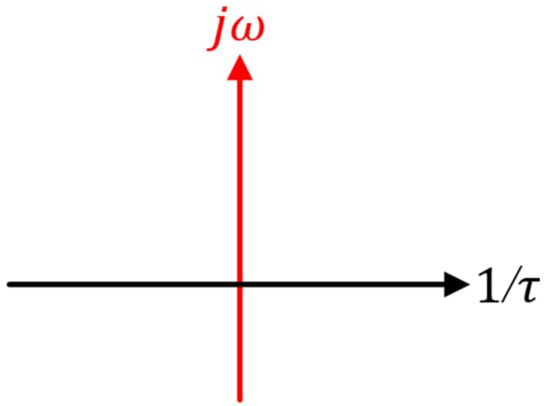
- If we give enough time to a circuit, the effect of **the exponential response** becomes negligible:

$$t \gg \tau \;\Rightarrow \exp\left(-\frac{t}{\tau}\right) \approx 0$$



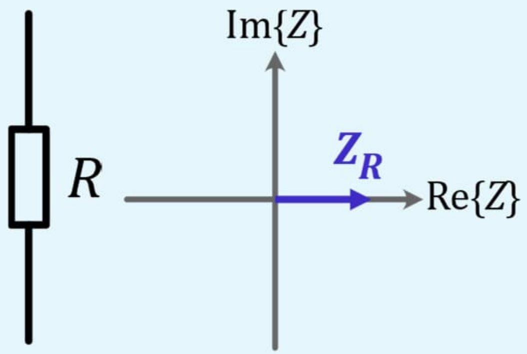
- In the frequency domain, we limit s to the **imaginary axis**:

$$s = j\omega \quad \Rightarrow \quad F(s)|_{s=j\omega} = F(j\omega)$$



Impedance (Z) and Admittance (Y)

Resistor

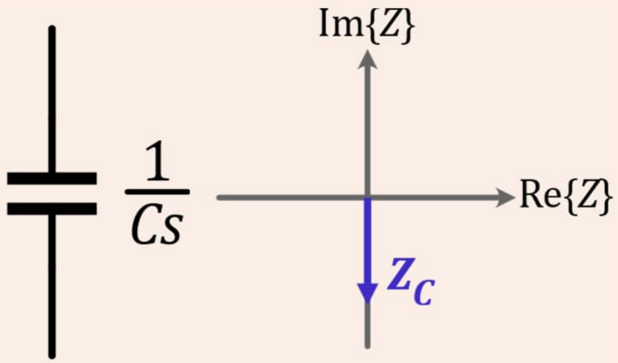


$$Z_R = R \qquad Y_R = \frac{1}{R} = G$$

$$|Z_R(j\omega)| = R$$

$$\angle Z_R(j\omega) = 0$$

Capacitor

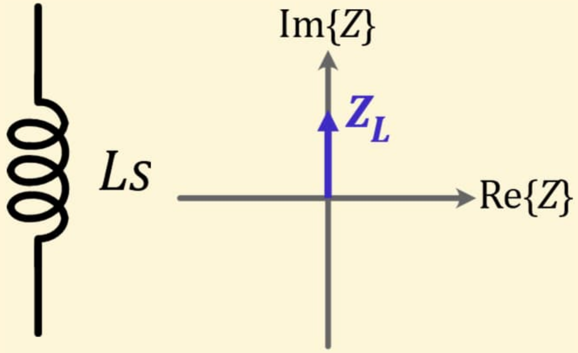


$$Z_C = \frac{1}{Cs} \qquad Y_C = Cs$$

$$|Z_C(j\omega)| = \frac{1}{\omega C}$$

$$\angle Z_C(j\omega) = -\frac{\pi}{2}$$

Inductor

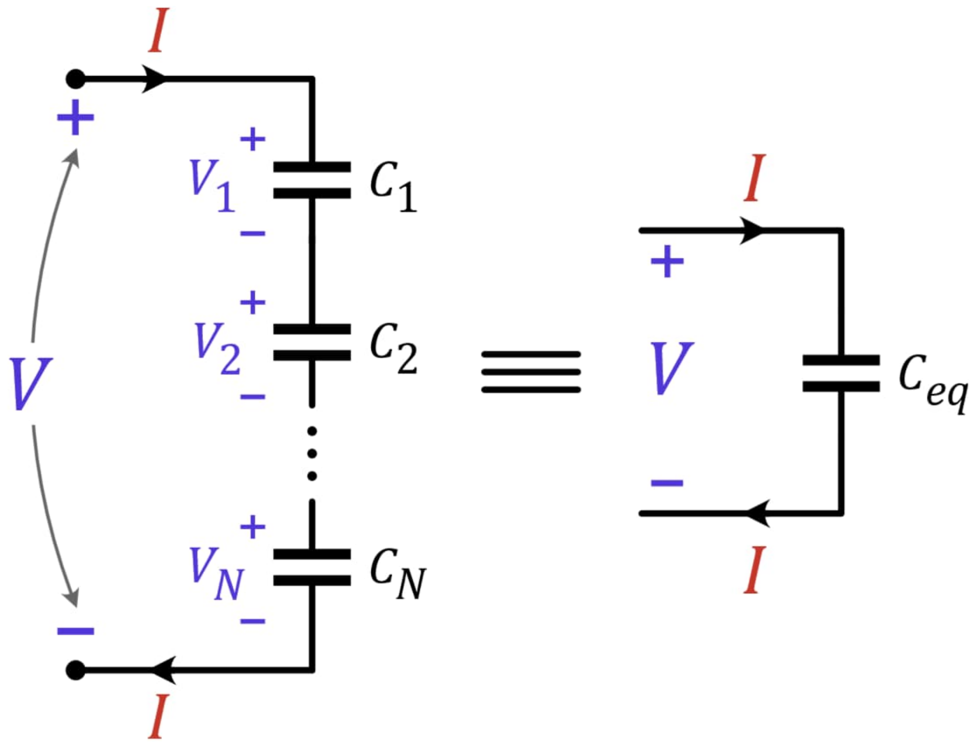


$$Z_L = Ls \qquad Y_L = \frac{1}{Ls}$$

$$|Z_L(j\omega)| = \omega L$$

$$\angle Z_L(j\omega) = +\frac{\pi}{2}$$

Series Capacitors

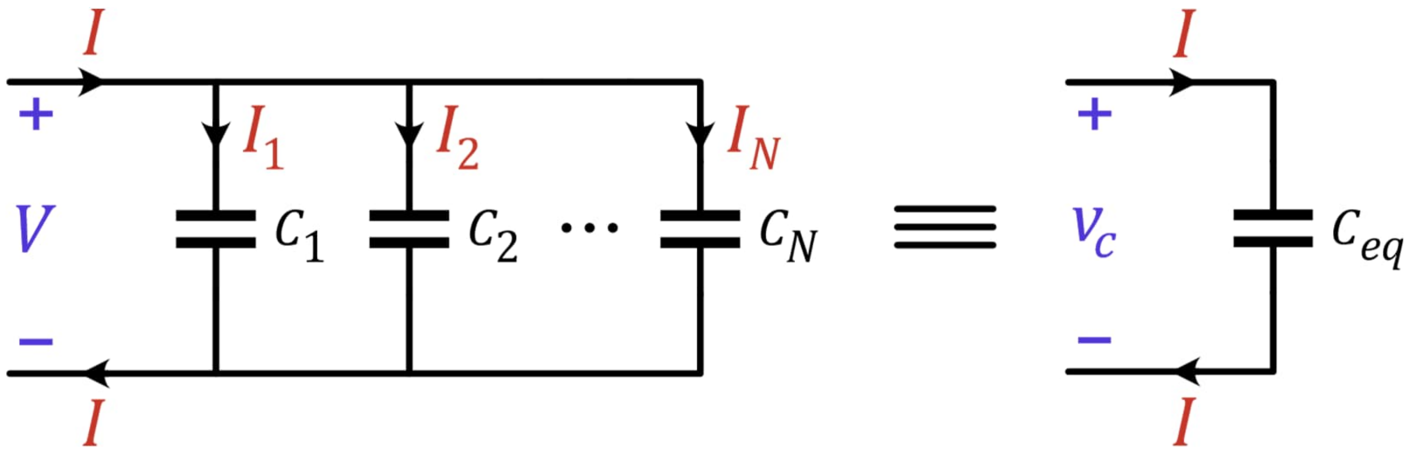


$$\begin{cases} V_k = Z_{Ck} I = \dfrac{1}{C_k s} I \\ V = \displaystyle\sum_{k=1}^{N} V_k = \sum_{k=1}^{N} Z_{Ck} I \end{cases}$$

$$\Rightarrow V = \frac{I}{s} \sum_{k=1}^{N} \frac{1}{C_k} = \frac{I}{C_{eq} s}$$

$$\Rightarrow \boxed{C_{eq} = \frac{1}{\sum_{k=1}^{N} \frac{1}{C_k}}}$$

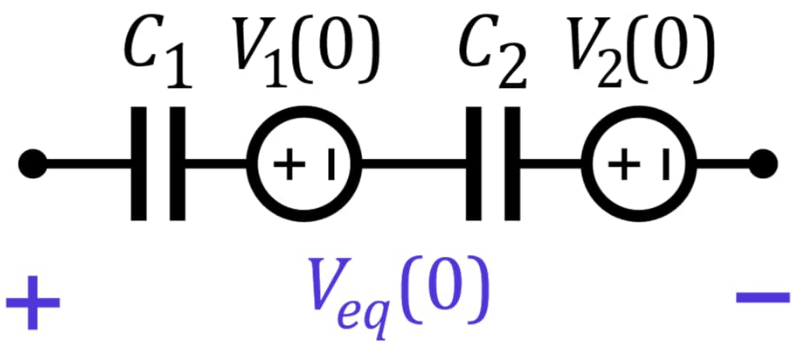
Parallel Capacitors



$$\begin{cases} I_k = \dfrac{V}{Z_{Ck}} = C_k sV \\ I = \displaystyle\sum_{k=1}^{N} I_k = \sum_{k=1}^{N} C_k sV \end{cases} \Rightarrow I = sV \sum_{k=1}^{N} C_k = C_{eq} sV \quad \Rightarrow \boxed{C_{eq} = \sum_{k=1}^{N} C_k}$$

Pause and Ponder 3

- In a **series connection** of N capacitors, assume they have initial voltages as $V_1(0), V_2(0)$, …, $V_N(0)$. What initial condition should we consider for the equivalent capacitor?

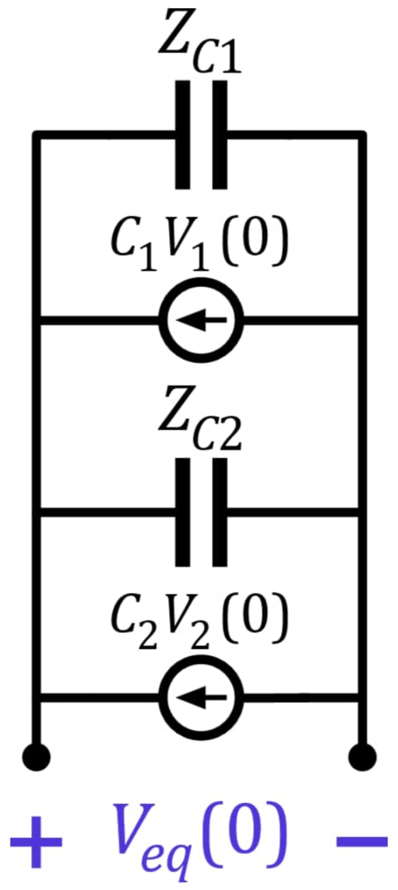


$$V_{eq}(0) = \sum_{k=1}^{N} V_k(0)$$

Pause and Ponder 4

- In a **parallel connection** of N capacitors, assume they have initial voltages as $V_1(0), V_2(0)$, …, $V_N(0)$. What initial condition should we consider for the equivalent capacitor?

$$V_{eq}(0) = \frac{\sum_{k=1}^{N} C_k V_k(0)}{\sum_{k=1}^{N} C_k} = \frac{\sum_{k=1}^{N} q_k(0)}{C_{eq}}$$



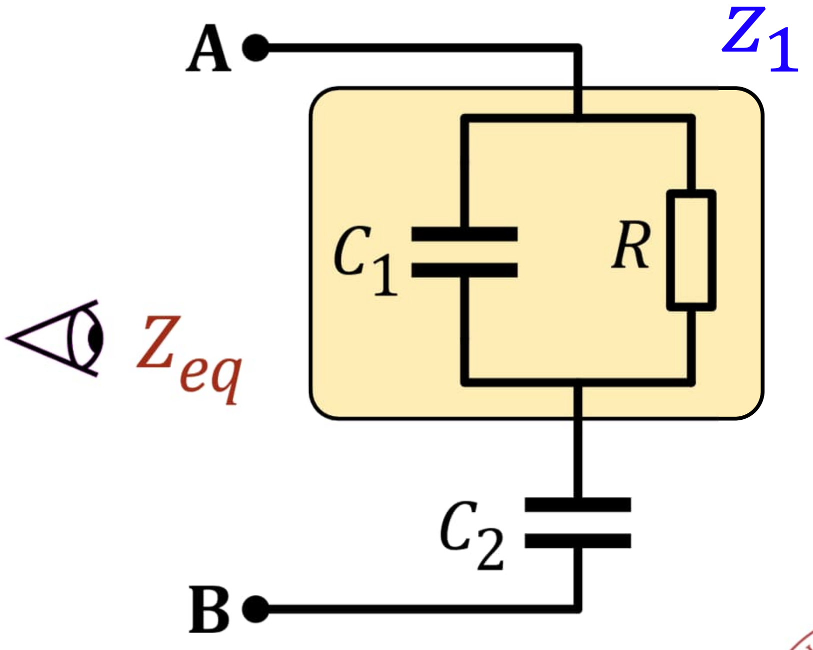
Example: Equivalent Impedance

- Find the equivalent impedance of the following circuit.
  - **C_1 parallel with R:**

$$Z_1 = \frac{1}{\frac{1}{R} + C_1 s} = \frac{R}{1 + RC_1 s}$$

  - **Z_1 series with C_2:**

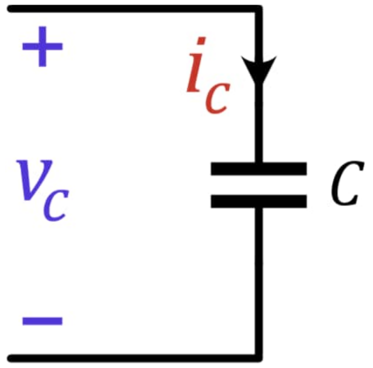
$$Z_{eq} = Z_1 + \frac{1}{C_2 s} = \frac{1 + R(C_1 + C_2)s}{C_2 s(1 + RC_1 s)}$$



Capacitors: Energy and Power

- We have learned so far:

$$i_c(t) = C\frac{dv_c(t)}{dt} \qquad p(t) = v(t)i(t)$$



- Power:

$$p_c(t) = v_c(t)i_c(t) = Cv_c(t)\frac{dv_c(t)}{dt}$$

- Energy consumed to charge C from 0 V at $t = 0$ to V_1 at $t = t_1$:

$$W(t_1) = \int_0^{t_1} p_c(t)\,dt = \int_0^{t_1} Cv_c(t)\frac{dv_c(t)}{dt}dt = C\int_0^{V_1} v_c(t)dv_c(t) = \frac{1}{2}CV_1^2$$

Capacitors: Average Power

- **Energy:**

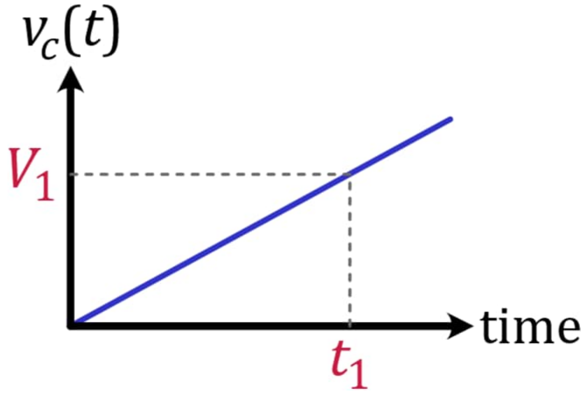
$$\boxed{W(t_1) = \frac{1}{2} C V_1^2}$$

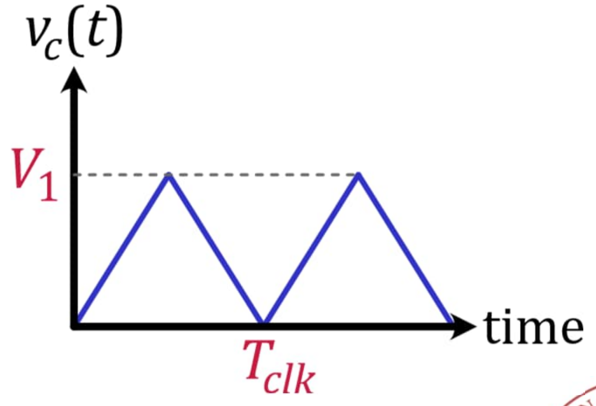
- **Average Power:**

$$\boxed{P_{avg} = \frac{W(t_1)}{t1} = \frac{1}{2t_1} C V_1^2}$$

- **Periodic charge/discharge with a clock frequency (f_{clk}):**

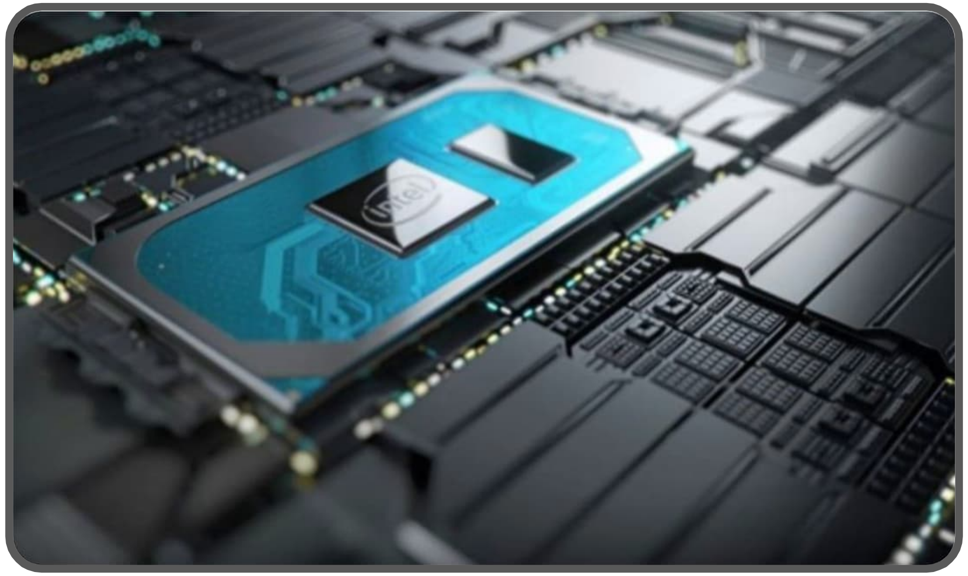
$$\boxed{P_{avg} = \frac{1}{2\frac{T_{clk}}{2}} C V_1^2 = f_{clk} C V_1^2}$$





Pause and Ponder 5

- **A real-world example:**
  - 10 Billion transistors
  - 10 fF capacitance per transistor
  - Total capacitance: $C = 100\ \mu\mathrm{F}$
  - Power supply: $V_{DD} = 1$ V
  - Clock frequency: $f_{clk} = 2$ GHz
  - Dynamic power: $P_{avg} = f_{clk} C V_C^2 = 200$ kW
  - How could it be possible? What’s wrong?!



$$\boxed{P_{avg} = \alpha f_{clk} C V_C^2}$$

Pause and Ponder 6

- How long does it take to charge a Tesla's battery from 20% to 90% of its full capacity?

$$I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta t = \frac{\Delta Q}{I} = \frac{(0.9 - 0.2)Q_{max}}{I_{ch}}$$

$$\Rightarrow T_{ch} = \frac{(0.9 - 0.2)Q_{max}}{I_{ch}} = \frac{0.7 \times 230}{20} = 8\text{ hr} + 3\text{ min}$$

- How much capacitance does this battery have?

$$C = \frac{Q_{max}}{V_{max}} = \frac{230 \times 3600}{22.5} = 36.8\text{ kF}$$

Tremendous!

Specification	Value
Capacity	230 Ah
Max. voltage	22.5 V
Charger current	20 A

Pause and Ponder 6

- How much energy is consumed in this process?

$$W_1 = \frac{1}{2}CV_1^2 = \frac{1}{2}C \times (0.2V_{max})^2$$

$$W_2 = \frac{1}{2}CV_2^2 = \frac{1}{2}C \times (0.9V_{max})^2$$

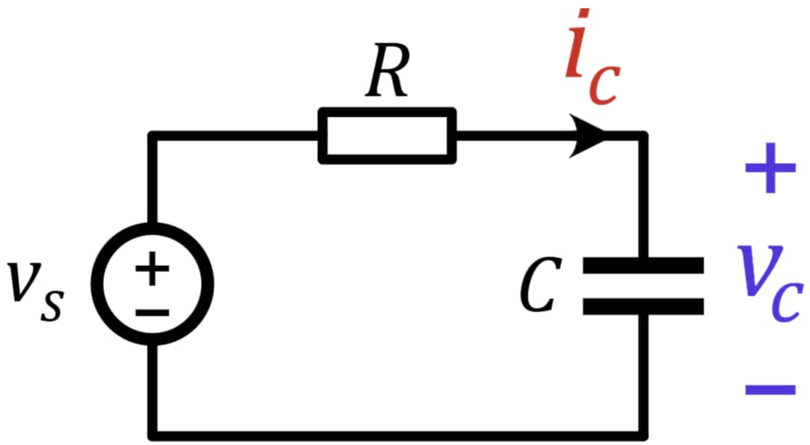
$$\Rightarrow \Delta W = \frac{1}{2}C(V_2^2 - V_1^2) = \frac{1}{2}CV_{max}^2 \times (0.9^2 - 0.2^2)$$

$$\Rightarrow \Delta W \approx 7.2 \text{ MJ} = 2 \text{ kWh}$$

Specification	Value
Capacity	230 Ah
Max. voltage	22.5 V
Charger current	20 A

First-order RC Circuits

- **Time-domain analysis**
  - Initial condition (IC): $v_c(0^-) = V_0$



KVL: $Ri_c + v_c = v_s \;\Rightarrow RC\dfrac{dv_c}{dt} + v_c = v_s$

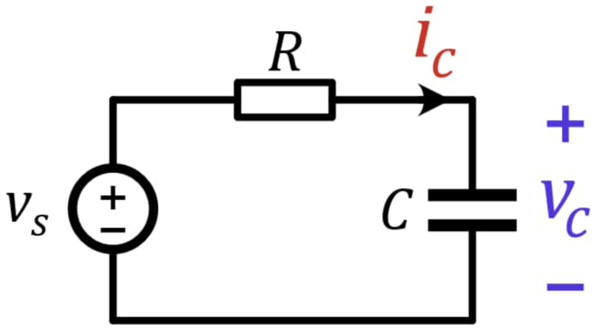
$$\mathcal{L}\left\{RC\frac{dv_c}{dt} + v_c\right\} = \mathcal{L}\{v_s\}$$

$$RCsV_c + V_c = V_s + RCv_c(0^-) \;\Rightarrow V_c = \underbrace{\frac{1}{RCs+1}V_s}_{\text{Source-dependent response}} + \underbrace{\frac{RC}{RCs+1}V_0}_{\text{IC-dependent response}}$$

First-order RC Circuits

- **impulse response ($h(t)$)**

$$v_s(t) = \delta(t) \Rightarrow V_s = 1$$



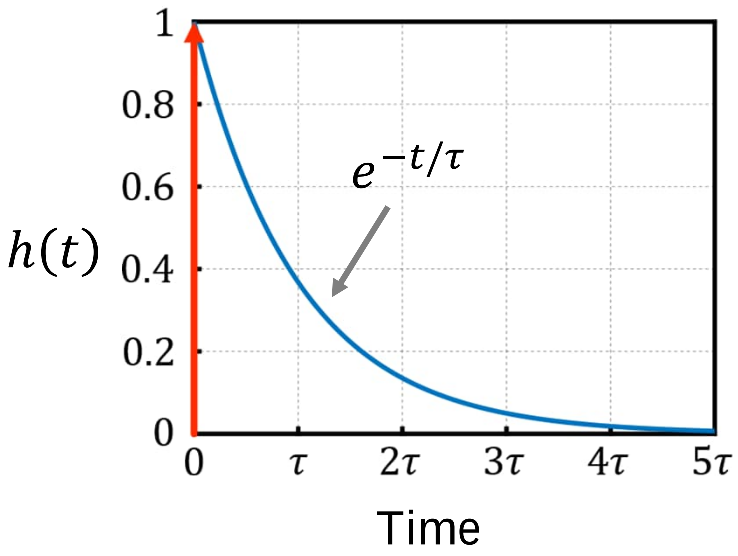
$x(t)$	$X(s)$
$e^{-at}u(t)$	$\dfrac{1}{s+a}$

Transfer function:

$$V_c = H(s) = \frac{1}{RCs+1} = \frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}}$$

$$\boxed{h(t) = \frac{1}{RC}\exp\left(-\frac{t}{RC}\right)u(t)}$$

Time constant: $\tau = RC$



First-order RC Circuits

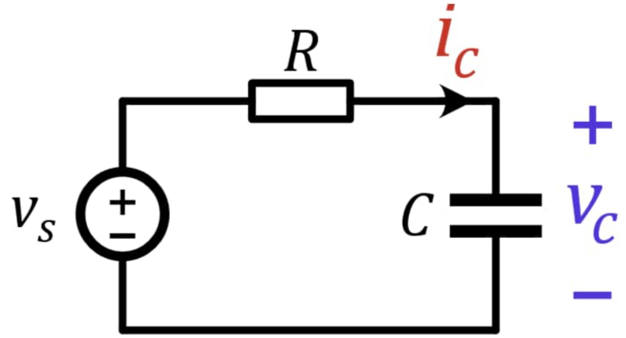
- **IC-dependent response**

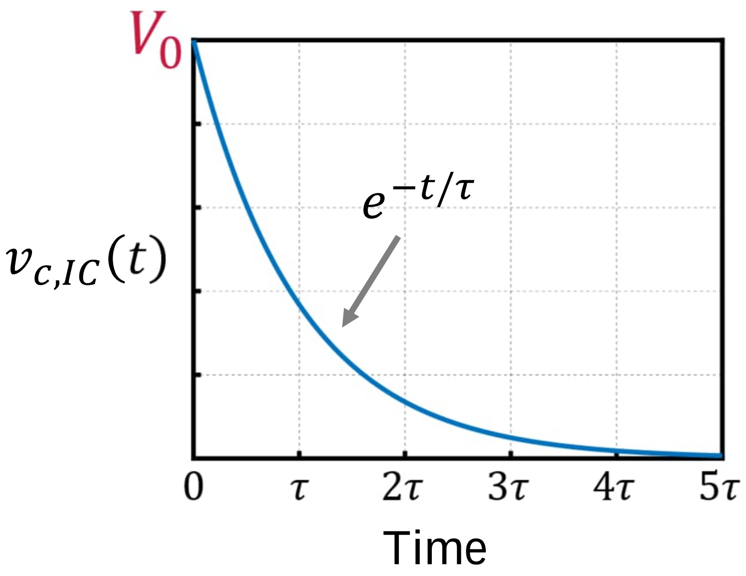
$$V_{c,IC} = \frac{RC}{RCs+1} V_0$$

$$V_{c,IC} = \frac{1}{s + \frac{1}{RC}} V_0$$

$$\boxed{v_{c,IC}(t) = V_0 \exp\left(-\frac{t}{RC}\right) u(t)}$$

Time constant: $\tau = RC$





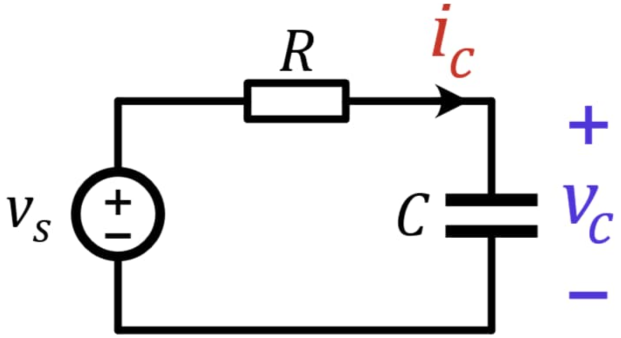
First-order RC Circuits

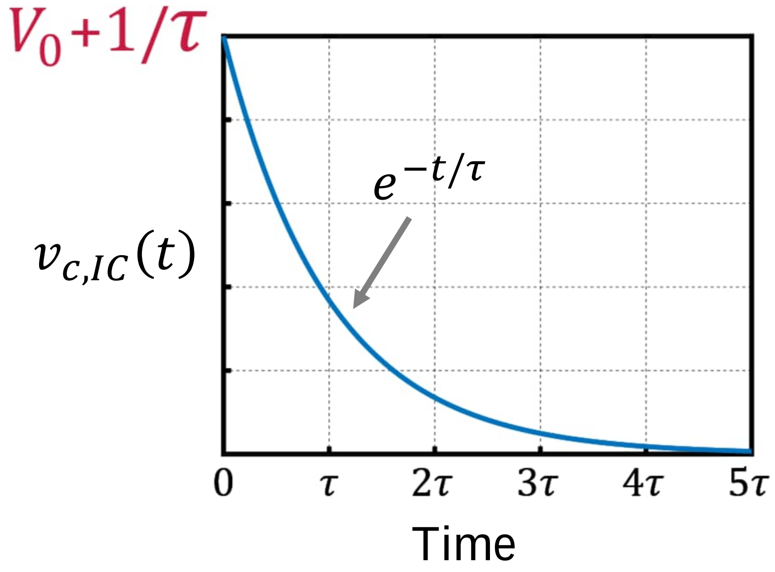
- **Total response**

$$v_c(t) = h(t) + v_{c,IC}(t)$$

$$v_c(t) = \frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)\,u(t) + V_0\exp\left(-\frac{t}{\tau}\right)\,u(t)$$

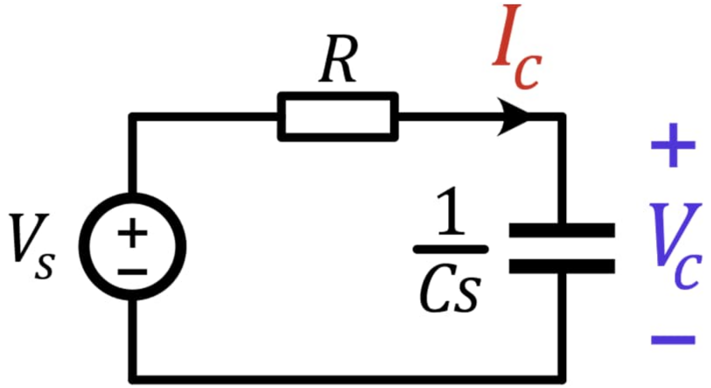
$$\boxed{v_c(t) = \left(\frac{1}{\tau} + V_0\right)\exp\left(-\frac{t}{\tau}\right)\,u(t)}$$





First-order RC Circuits

- **Frequency-domain analysis**
  - Simple voltage division if we consider impedances:



$$V_c(s) = \frac{Z_c}{Z_c + Z_R} V_s(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R} V_s(s) = \frac{1}{1 + RCs} V_s(s)$$

  - Impulse response ($V_s(s) = 1$):

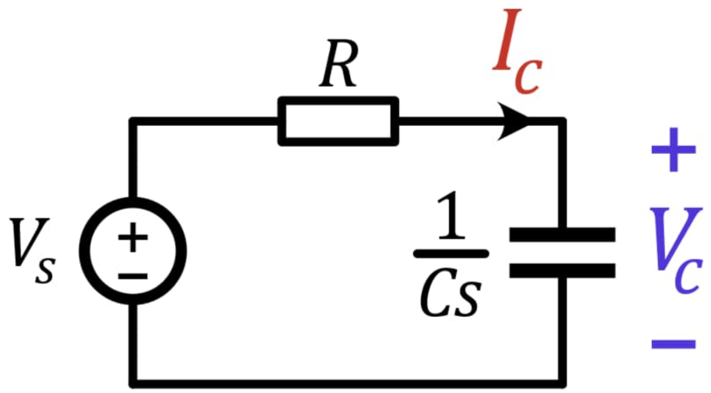
$$\boxed{H(s) = \frac{1}{1 + \tau s}}$$

The same result

Pause and Ponder 7

- Find a transfer function for $I_c(s)$.

$$I_c(s) = \frac{1}{Z_c} V_c(s) = \frac{1}{\frac{1}{Cs}} V_c(s) = \frac{Cs}{1 + RCs}$$



- Find the impulse response of $v_R(t)$.

$$V_R(s) = RI_c(s) = \frac{RCs}{1 + RCs}$$

$$v_R(t) = \mathcal{L}^{-1}\{V_R(s)\} = (RC)\frac{d}{dt}\left(\mathcal{L}^{-1}\left\{\frac{1}{1 + RCs}\right\}\right) = \delta(t) - \frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)u(t)$$

First-order RC Circuits

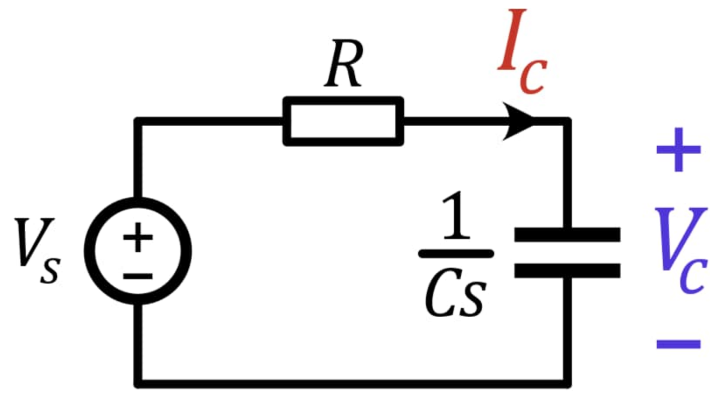
- **Step Response**

$$v_s(t) = u(t) \Rightarrow V_s(s) = \frac{1}{s}$$

$$V_c(s) = H(s) \cdot V_s(s) = \frac{1}{1 + \tau s} \cdot \frac{1}{s}$$

$$\Rightarrow V_c(s) = \frac{1}{\tau} \cdot \frac{1}{s\left(s + \frac{1}{\tau}\right)} = \frac{1}{s} - \frac{1}{s + \frac{1}{\tau}}$$

$$\Rightarrow v_c(t) = u(t) - \exp\left(-\frac{t}{\tau}\right) u(t) = \left(1 - \exp\left(-\frac{t}{\tau}\right)\right) u(t)$$



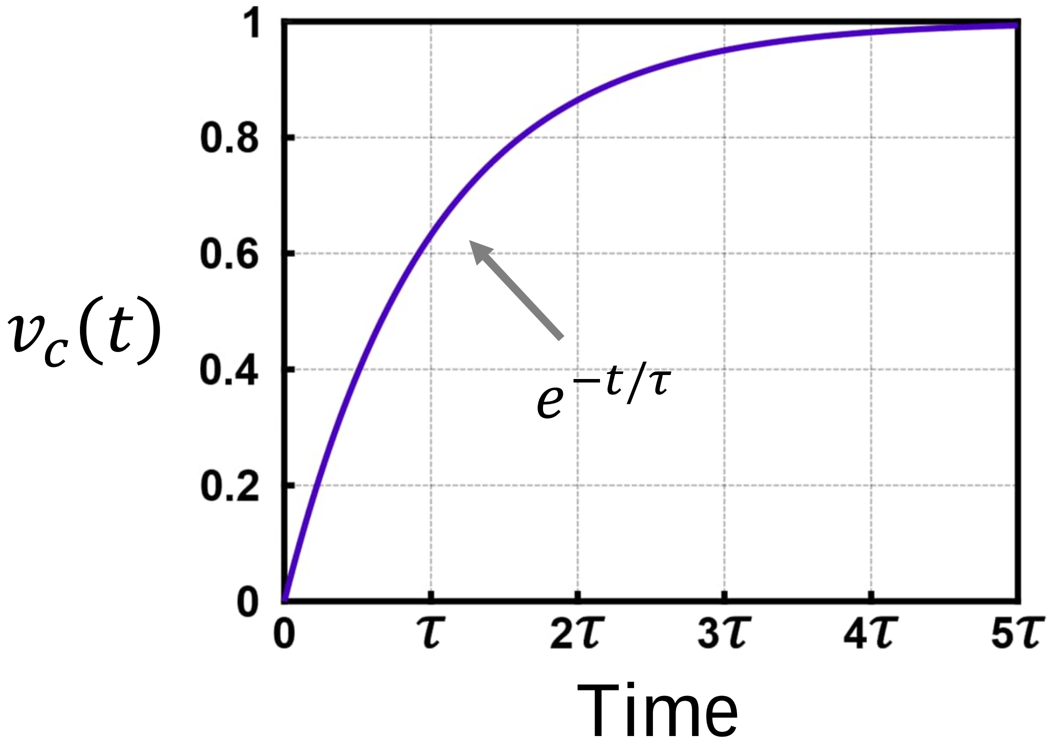
First-order RC Circuits

- **Step Response**

$$\boxed{v_c(t) = \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)u(t)}$$

Initial value: $v_c(0) = 0$

Final value: $v_c(\infty) = 1$



First-order RC Circuits

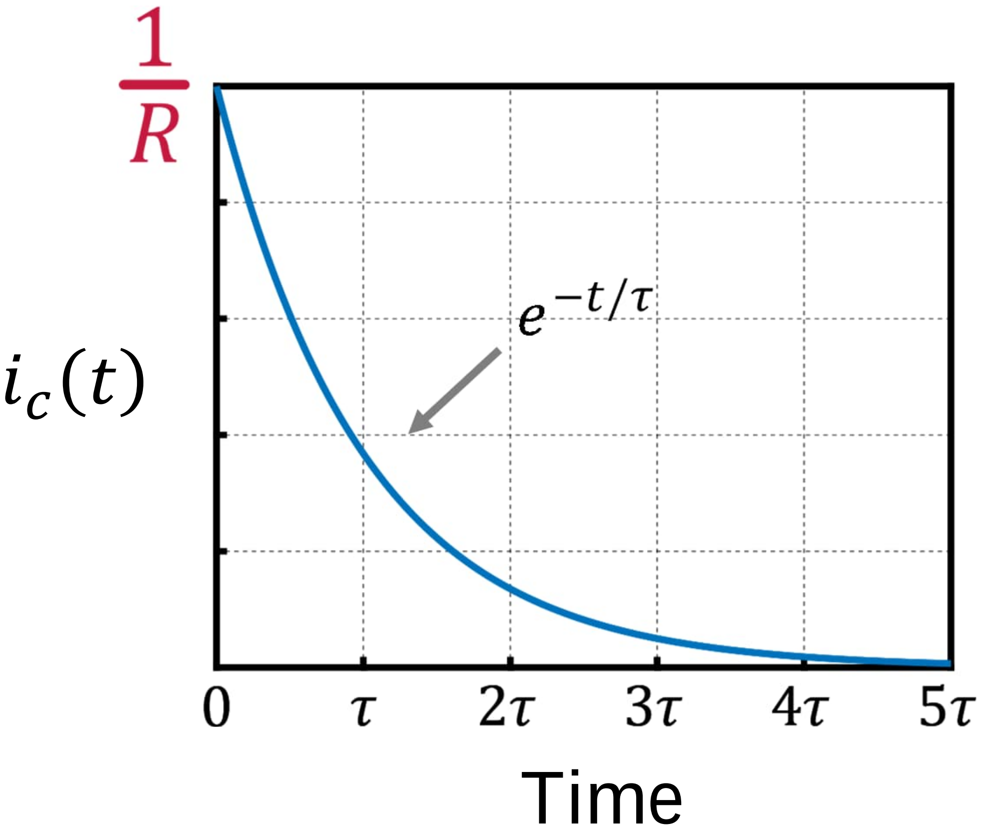
- **Current's Step Response**

$$i_c(t) = c\frac{dv_c}{dt}$$

$$\boxed{i_c(t) = \frac{1}{R}\exp\left(-\frac{t}{\tau}\right)u(t)}$$

Initial value: $i_c(0) = \frac{1}{R}$

Final value: $i_c(\infty) = 0$



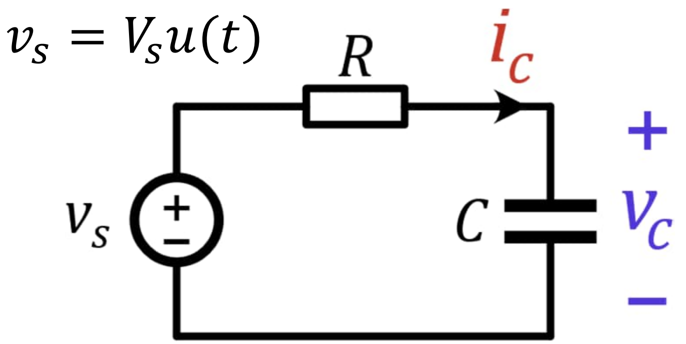
Capacitor's Model when $t \to 0^+$

- Capacitor's **voltage** must be **continuous**.

$$i_c(t) = c\frac{dv_c}{dt} \Rightarrow if\ v_c = \Delta V u(t):\quad i_c(t) \to \infty$$

Physically Impossible

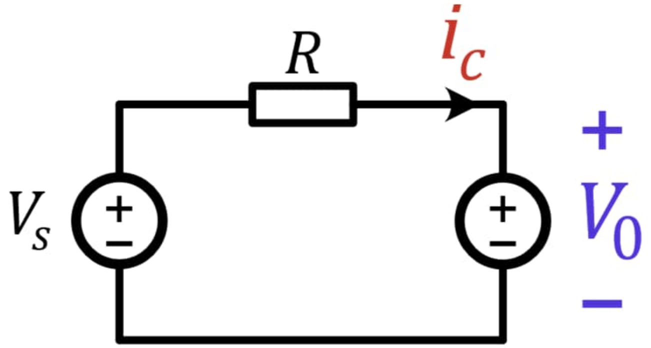
$$\Rightarrow v_c(0^+) = v_c(0^-) = V_0$$



- What about current?

$$i_c(0^+) = \frac{V_s - V_0}{R}$$

Capacitor acts like a voltage source, showing zero impedance



Capacitor's Model when $t \to \infty$

- After a sufficiently long time ($t > 5\tau$), the voltage across the capacitor reaches a stable states.
- This duration is called **settling time**.
- **No current** flows into the circuit anymore.

$$\frac{dv_c}{dt}\Big|_{t\to\infty} = 0 \;\Rightarrow i_c(\infty) = 0, v_c(\infty) = V_s$$

Capacitor acts like an open-circuit, showing infinite impedance