$$\lim_{n\to\infty} n\frac{\left(\sum_{k=1}^{n}\sqrt{k}\right)^2}{\left(\sum_{k=1}^{n}\sqrt[3]{k}\right)^3}+\sum_{k=1}^{n-1}\left[\ln\left(1+\frac{1}{n+k}\right)\sin\left(\ln\left(1+\frac{k}{n}\right)\right)\right]=$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\right)^2}{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt[3]{\frac{k}{n}}\right)^3}+\frac{1}{n}\sum_{k=1}^{n-1}\frac{1}{1+\frac{k}{n}}\sin\left(\ln\left(1+\frac{k}{n}\right)\right)=$$

$$\lim_{n\to\infty}\frac{\left(\int_0^1 x^{\frac{1}{2}}\mathrm{d}x\right)^2}{\left(\int_0^1 x^{\frac{1}{3}}\mathrm{d}x\right)^3}+\int_0^1\frac{\sin(\ln(1+x))}{1+x}\mathrm{d}x=\frac{\frac{4}{9}}{\frac{27}{64}}+\int_0^1\sin(\ln(1+x))\mathrm{d}(\ln(x+1))=$$

$$\frac{256}{243}-\cos u|_0^{\ln 2}=\frac{499}{243}-\cos(\ln 2)$$

$$p(x)=\frac{\mathrm{d}^n}{\mathrm{d}x^n}(1-x^m)^n=\left(\sum_{k=0}^{n}C_n^k(-x^m)^k\right)^{(n)}=\left(1+C_n^1(-x^m)+C_n^2(-x^m)^2+...\right)^{(n)}$$

$$\exists k_0\in\{1,2,3,...\},mk_0>n$$

$$\int_0^{\frac{\pi}{2}}\ln\left[\left(\sin^2 x+99\cos^2 x\right)\left(999\sin^2 x+\cos^2 x\right)\right]\mathrm{d}x$$

$$\left\{\begin{array}{c} x^2+y^2=z \\ y=x\tan z \end{array}\right.=\left\{\begin{array}{c} x=\sqrt{\theta}\cos\theta \\ y=\sqrt{\theta}\sin\theta \\ z=\theta \end{array}\right.,\theta\in(0,c)$$

$$s=\int\mathrm{d}s=\int_0^c\sqrt{\left(\frac{\cos\theta}{2\sqrt{\theta}}-\sqrt{\theta}\sin\theta\right)^2+\left(\frac{\sin\theta}{2\sqrt{\theta}}+\sqrt{\theta}\cos\theta\right)^2+1}\mathrm{d}\theta=$$

$$\int_0^c\sqrt{\frac{1}{4\theta}+\theta+1}\mathrm{d}\theta=\int_0^c\frac{2\theta+1}{2\sqrt{\theta}}\mathrm{d}\theta=\frac{2}{3}\theta^{\frac{3}{2}}+\theta^{\frac{1}{2}}|_0^c=\frac{2}{3}c^{\frac{3}{2}}+c^{\frac{1}{2}}=\sqrt{c}\left(\frac{2c}{3}+1\right)$$

$$f(x)=\sec x,x\in(-\frac{\pi}{4},\frac{\pi}{4})$$

$$S(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos 4nx$$

$$a_0=\frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\mathrm{d}x=\frac{8}{\pi}\ln|\sec x+\tan x||_0^{\frac{\pi}{4}}=\frac{8}{\pi}\ln(\sqrt{2}+1)$$

$$a_n=\frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\cos 4nx\mathrm{d}x=\frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\frac{\mathrm{d}(\sin 4nx)}{\cos x}=\frac{4\sin 4nx}{n\pi\cos x}|_0^{\frac{\pi}{4}}-\frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\sin 4nx\sec x\tan x\mathrm{d}x=$$

$$2m = n, \frac{x^2}{2} - \frac{\sqrt{2}xy}{m} + \frac{3y^2}{2m} = 1$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{2}}{2m} \\ \frac{-\sqrt{2}}{2m} & \frac{3}{2m} \end{bmatrix} \quad (x, y)^T = Q\,(u, v)^T$$

$$|\lambda E - A| = (\lambda - \frac{1}{2})(\lambda - \frac{3}{2m}) - \frac{1}{2m^2} = \lambda^2 - \frac{m+3}{2m}\lambda + \frac{3m-2}{4m^2} = 0$$

$$\lambda = \frac{\frac{m+3}{2m} \pm \sqrt{\left(\frac{m+3}{2m}\right)^2 - 4\frac{3m-2}{4m^2}}}{2} \notin Z$$

$$\lim_{x\to 0}\frac{\int_0^x t\cos t\mathrm{d}t-1+\cos x}{\sqrt{1+x\tan x}-\sqrt{1+x\sin x}}=$$

$$\lim_{x\to 0}\frac{2(\int_0^x t\cos t\mathrm{d}t-1+\cos x)}{x(\tan x-\sin x)}=\lim_{x\to 0}\frac{2(x\cos x-\sin x)}{2x^3}=-\frac{1}{3}$$

$$\sum_{n=0}^{\infty}\frac{n^2+1}{(\frac{1}{2})^n n!}x^n=\sum_{n=0}^{\infty}\frac{n^2(2x)^n}{n!}+\sum_{n=0}^{\infty}\frac{(2x)^n}{n!}$$

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}2\frac{(n+1)^2+1}{(n^2+1)(n+1)}=0,x\in(-\infty,+\infty)$$

$$S(x)=\sum_{n=1}^{\infty}\frac{n(2x)^n}{(n-1)!}+\mathrm{e}^{2x}=\sum_{n=0}^{\infty}\frac{(n+1)(2x)^{n+1}}{n!}+\mathrm{e}^{2x}=\sum_{n=0}^{\infty}\frac{(2x)^{n+2}}{n!}+\sum_{n=0}^{\infty}\frac{(2x)^{n+1}}{n!}+\mathrm{e}^{2x}$$

$$=(4x^2+2x+1)\mathrm{e}^{2x},x\in(-\infty,+\infty)$$

$$\begin{cases} x+y-z=\mathrm{e}^z \\ x\mathrm{e}^x=\tan t \\ y=\cos t \end{cases}$$

$$x'\mathrm{e}^x+x\mathrm{e}^x x'=\sec t\to \mathrm{e}^x(x'+x'x)=\sec t,\mathrm{e}^x(x'+x'x+x''+x''x+x'x')=\sec t\tan t$$

$$y'=-\sin t,y''=-\cos t$$

$$x'+y'-z'=\mathrm{e}^z z'\to z'=\frac{x'+y'}{\mathrm{e}^z+1}$$

$$t=0,x=0,y=1,z=0,x'=1,y'=0,z'=\frac{1}{2}$$

$$1+0+x''+0+1=0,x''=-2,y''=-1$$

$$z''=\frac{(x''+y'')(\mathrm{e}^z+1)-(x'+y')(\mathrm{e}^z z')}{(\mathrm{e}^z+1)^2}=\frac{-3*2-\frac{1}{2}}{4}=-\frac{13}{8}$$

$$\iint_D r^2\sin\theta\sqrt{1-r^2\cos(2\theta)}\mathrm{d}\sigma,D=\{(r,\theta),0\le r\le\sec\theta,0\le\theta\le\frac{\pi}{4}\}$$

$$D=\{(x,y),0\le y\le x,0\le x\le 1\}$$

$$\iint_D y\sqrt{1-x^2+y^2}\mathrm{d}\sigma=\frac{1}{2}\int_0^1\int_{1-x^2}^{x^2+1-x^2}\sqrt{1-x^2+y^2}\mathrm{d}(y^2+1-x^2)\mathrm{d}x=\frac{1}{3}\int_0^1 u^{\frac{3}{2}}|_{1-x^2}^{1}\mathrm{d}x$$

$$=\frac{1}{3}\int_0^1 1-(1-x^2)^{\frac{3}{2}}\mathrm{d}x=\frac{1}{3}-\int_0^{\frac{\pi}{2}}\cos^4 t\mathrm{d}t=\frac{1}{3}-\frac{1*3*1*\pi}{3*4*2*2}=\frac{1}{3}-\frac{\pi}{16}$$

$$u=ax^2+by^2+cz^2,x+y+z=1,(ax^2+by^2+cz^2)(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})\ge(x+y+z)^2=1$$

$$\min_{x>0,y>0,z>0}u(x,y,z)=\frac{1}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$a_n=\int_0^1\frac{x^{n-1}}{1+x}\mathrm{d}x=\int_0^1x^{n-1}\mathrm{d}(\ln(x+1))=x^{n-1}\ln(x+1)|_0^1-(n-1)\int_0^1\ln(x+1)x^{n-2}\mathrm{d}x$$

$$a_n=\frac{1}{n}\int_0^1\frac{\mathrm{d}(x^n)}{1+x}=\frac{x^n}{n(1+x)}|_0^1+\int_0^1\frac{x^n\mathrm{d}x}{n(1+x)^2}=\frac{1}{2n}+\int_0^1\frac{\mathrm{d}(x^{n+1})}{(1+x)^2n(n+1)}=$$

$$\frac{1}{2n}+\frac{x^{n+1}}{(1+x)^2n(n+1)}|_0^1+\int_0^1\frac{2x^{n+1}\mathrm{d}x}{(n+1)(1+x)^3}=\frac{1}{2n}+\frac{1}{4n^2}+o(\frac{1}{n^2})mn\to\infty$$

$$I_1=\iint_S\frac{xz}{a^2}\mathrm{d}y\mathrm{d}z+\frac{yz}{b^2}\mathrm{d}z\mathrm{d}x+\frac{z^2}{c^2}\mathrm{d}x\mathrm{d}y$$

$$I_2=0,S_1=\{(x,y,z),\frac{x^2}{a^2}+\frac{y^2}{b^2}\le1,z=0\},down0$$

$$D_{xy}=\{(x,y),\frac{x^2}{a^2}+\frac{y^2}{b^2}\le1\}$$

$$I_1+I_2=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{c^2}\right)\iiint_\Omega z\mathrm{d}v=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{c^2}\right)\frac{c^2}{2}\iint_{D_{xy}}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)\mathrm{d}\sigma$$

$$\frac{x}{a}=u,\frac{y}{b}=v,J=ab$$

$$I_1+I_2=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{c^2}\right)\frac{c^2}{2}ab\iint_{D_{uv}}(1-u^2-v^2)\mathrm{d}\sigma$$

$$\iint_{D_{uv}}(1-u^2-v^2)\mathrm{d}\sigma=\int_0^1(1-r^2)r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta=\frac{\pi}{2}$$

$$I_1=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{c^2}\right)\frac{c^2\pi}{4}ab$$

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[n]{3}}{2}\right)^n$$

$$\lim_{x\to+\infty}\left((x+1)^a - x^a\right) = a$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \sum_{n=0}^{\infty}(n+1)x^n = \frac{1}{(1-x)^2}$$

$$\lim_{n\to\infty}\frac{(n+1)^2}{n^2} = 1, x \in (-1,1), x \neq \pm 1$$

$$\sum_{n=1}^{\infty} n^2 x^{n-1} = \left(\sum_{n=1}^{\infty} n x^n\right)' = \left(\frac{1}{(1-x)^2} - 1 - \left(\frac{1}{1-x} - 1\right)\right)' = \frac{2}{(1-x)^3} - \frac{1}{(1-x)^2}$$

$$f(x) = \ln(1+x) + ax\mathrm{e}^{-x}, x_1 \in (-1,0), x_2 \in (0,+\infty)$$

$$f(x), x \in (-1,+\infty)$$

$$f'(x) = \frac{1}{x+1} + a(1-x)\mathrm{e}^{-x} = \frac{1+a(1-x^2)\mathrm{e}^{-x}}{x+1}$$

$$g(x) = 1 + a(1-x^2)\mathrm{e}^{-x}, x \in (-1,+\infty)$$

$$g'(x) = a(x^2-2x-1)\mathrm{e}^{-x} = a(x-1+\sqrt{2})(x-1-\sqrt{2})\mathrm{e}^{-x}$$

$$\exists a = 0, g(x) \equiv 1 f'(x) > 0, f(x)\uparrow, a \neq 0$$

$$\exists a < 0, g'(x) \leq 0, x \in (-1,+\infty), g(x)\downarrow$$

$$\exists a > 0, g'(x) \geq 0, x \in (-1,+\infty), g(x)\uparrow$$

$$g(-1) = 1, g(0) = 1+a, g(+\infty) = 1$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\frac{i}{n}} = \int_0^1 \frac{1}{1+x}\mathrm{d}x = \ln(x+1)|_0^1 = \ln 2$$

$$\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x}-\cot x\right)=\lim_{x\to 0}\frac{\tan x-x}{x^3}=\frac{1}{3}$$

$$\lim_{n\to\infty}\tan n, n=k\pi \lim=0, n\to\left(k\pi+\frac{\pi}{2}\right)^{-}, \lim=+\infty, n\to\left(k\pi+\frac{\pi}{2}\right)^{+}, \lim=-\infty$$

$$\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x), \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=\ln 2$$

$$\oint_L\frac{1}{x}\arctan\frac{y}{x}\mathrm{d}x+\frac{2}{y}\arctan\frac{x}{y}\mathrm{d}y=\iint_D\frac{2}{y}\frac{\frac{1}{y}}{\left(\frac{x}{y}\right)^2+1}-\frac{1}{x}\frac{\frac{1}{x}}{\left(\frac{y}{x}\right)^2+1}\mathrm{d}\sigma=\iint_D\frac{1}{x^2+y^2}\mathrm{d}\sigma$$

$$=\int_1^2\frac{1}{r}\mathrm{d}r\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\mathrm{d}\theta=\frac{\pi\ln 2}{12}$$

$$\vec{n}=\left(\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right), \mathrm{d}S=\sqrt{(-1)^2+1^2+1}\mathrm{d}x\mathrm{d}y=\sqrt{3}\mathrm{d}x\mathrm{d}y$$

$$\iint_{\Sigma}(x-y+z)\frac{1}{\sqrt{3}}\mathrm{d}S=S_{xy}=\frac{1}{2}$$

$$\iiint_{\Omega}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)\mathrm{d}v=\frac{1}{3}\iiint_{\Omega}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}+\frac{x^2}{c^2}+\frac{z^2}{a^2}+\frac{x^2}{b^2}+\frac{y^2}{c^2}\right)\mathrm{d}v$$

$$\frac{1}{3}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\int_0^1 r^4\mathrm{d}r\int_0^{\pi}\sin\phi\mathrm{d}\phi\int_0^{2\pi}\mathrm{d}\theta=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\frac{1*1*2*2\pi}{3*5}=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\frac{4\pi}{15}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\sin\frac{i\pi}{n}}{n+i}=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{\sin\left(\pi\frac{i}{n}\right)}{1+\frac{i}{n}}=\int_0^1\frac{\sin(\pi x)}{1+x}\mathrm{d}x$$

$$S(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n+2}}{3n+2}, S(0)=0$$

$$S'(x)=\sum_{n=0}^{\infty}(-1)^n x^{3n+1}=x\sum_{n=0}^{\infty}(-x^3)^n=\frac{x}{1+x^3}$$

$$S(x)=\int_0^x S(t)\mathrm{d}t+S(0)=\int_0^x\frac{t}{(t+1)(t^2-t+1)}\mathrm{d}t+0=\frac{1}{3}\int_0^x-\frac{1}{t+1}+\frac{t+1}{t^2-t+1}\mathrm{d}t$$

$$At^2-At+A+Bt^2+Ct+Bt+C\to A+B=0, B+C-A=1, A+C=0, A=-\frac{1}{3}, B=C=\frac{1}{3}$$

$$S(x)=-\frac{\ln(x+1)}{3}+\frac{1}{3}\int_0^x\frac{1}{2}\frac{\mathrm{d}(t^2-t+1)}{t^2-t+1}+\frac{1}{\sqrt{3}}\frac{\mathrm{d}(t-\frac{1}{2})(\frac{2}{\sqrt{3}})}{[(t-\frac{1}{2})(\frac{2}{\sqrt{3}})]^2+1}$$

$$=-\frac{\ln(x+1)}{3}+\frac{\ln(t^2-t+1)}{6}+\frac{1}{\sqrt{3}}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)+C|_0^x$$

$$=-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{1}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{\pi}{6\sqrt{3}}$$

$$S(1)=\sum_{n=0}^{\infty}\frac{(-1)^n}{3n+2}=-\frac{\ln 2}{3}+\frac{\pi\sqrt{3}}{9}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}7\frac{1}{\sqrt{k(n-k+1)}}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{\frac{k}{n}-\frac{k^2}{n^2}+\frac{k}{n^2}}}=\int_0^1\frac{\mathrm{d}x}{\sqrt{x-x^2}}=\pi$$

$$I=\int\sec^3 x\mathrm{d}x=\int\sec x(\tan^2 x+1)\mathrm{d}x=\int\tan x\mathrm{d}(\sec x)+\ln|\sec x+\tan x|$$

$$=\ln|\sec x+\tan x|+\sec x\tan x-\int\sec^3 x\mathrm{d}x$$

$$\int\sec^3 x\mathrm{d}x=\frac{1}{2}\left(\ln|\sec x+\tan x|+\sec x\tan x\right)+C$$

$$y(x)=\sqrt{x+\sqrt[3]{x+\sqrt[4]{x}}},\ln y(x)=\frac{1}{2}\ln\left(x+g(x)\right)$$

$$\frac{y'(x)}{y(x)}=\frac{1}{2}\frac{1+g'(x)}{x+g(x)}$$

$$g(x)=\sqrt[3]{x+\sqrt[4]{x}},\ln\left(g(x)\right)=\frac{1}{3}\ln\left(x+x^{\frac{1}{4}}\right)$$

$$\frac{g'(x)}{g(x)}=\frac{1}{3}\frac{1+\frac{1}{4}x^{-\frac{3}{4}}}{x+x^{\frac{1}{4}}}$$

$$g'(x)=\frac{4x^{\frac{3}{4}}+1}{12(x+x^{\frac{7}{4}})}g(x)=\frac{4x^{\frac{3}{4}}+1}{12(x+x^{\frac{1}{4}})^{\frac{2}{3}}x^{\frac{3}{4}}}$$

$$y'(x)=\frac{1+g'(x)}{2(x+g(x))}y(x)=\frac{1+\frac{4x^{\frac{3}{4}}+1}{12(x+x^{\frac{1}{4}})^{\frac{2}{3}}x^{\frac{3}{4}}}}{2(x+(x+x^{\frac{1}{4}})^{\frac{1}{3}})^{\frac{1}{2}}}$$

$$=\frac{12(x+x^{\frac{1}{4}})^{\frac{2}{3}}x^{\frac{3}{4}}+4x^{\frac{3}{4}}+1}{24(x+x^{\frac{1}{4}})^{\frac{2}{3}}x^{\frac{3}{4}}(x+(x+x^{\frac{1}{4}})^{\frac{1}{3}})^{\frac{1}{2}}}$$

$$I=\lim_{x\to+\infty}y(x)-x^{\frac{1}{2}}=\lim_{x\to+\infty}\frac{\sqrt[3]{x+\sqrt[4]{x}}}{\sqrt{x+\sqrt[3]{x+\sqrt[4]{x}}}+\sqrt{x}}=0$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}=\frac{\pi}{4}$$

$$\lim_{x\to1}\frac{x^x-x}{\ln x-x+1}=\lim_{x\to1}\frac{x^{x-1}-1}{\ln(x-1+1)-x+1}=\lim_{t\to0}\frac{(t+1)^t-1}{\ln(t+1)-t}$$

$$=\lim_{t\to0}\frac{\mathrm{e}^{t\ln(t+1)}-1}{-\frac{t^2}{2}+o(t^2)}=-2$$

$$\int_0^{+\infty}(x+1)\mathrm{e}^{-x^2}\mathrm{d}x=-\frac{1}{2}\int_0^{+\infty}\mathrm{e}^{-x^2}\mathrm{d}(-x^2)+\frac{\sqrt{\pi}}{2}=\frac{1+\sqrt{\pi}}{2}$$

$$
\begin{aligned}
X_1 = X_2 \sim N(0,1), F_{X_1}(x) = F_{X_2}(x) = \Phi(x) \\
F(x,y) = P\{X_1 \leq x, Y \leq y\} = P\{X_1 \leq x, X_3X_1 + (1-X_3)X_2 \leq y\} \\
= P\{X_1 \leq x, \{X_3X_1 + (1-X_3)X_2 \leq y\}, X_3 = 0\} + \\
P\{X_1 \leq x, \{X_3X_1 + (1-X_3)X_2 \leq y\}, X_3 = 1\} \\
= P\{X_1 \leq x, X_2 \leq y\}P\{X_3 = 0\} + P\{X_1 \leq x, X_1 \leq y\}P\{X_3 = 1\} \\
= \frac{1}{2}\left(P\{X_1 \leq x\}P\{X_2 \leq y\} + P\{X_1 \leq x, X_1 \leq y\}\right) \\
\exists x < y, \quad F(x,y) = \frac{1}{2}\left(\Phi(x)\Phi(y) + \Phi(x)\right) \\
\exists x \geq y, \quad F(x,y) = \frac{1}{2}\left(\Phi(x)\Phi(y) + \Phi(y)\right) \\
F_Y(y) = P\{Y \leq y\} = P\{X_3X_1 + (1-X_3)X_2 \leq y\} = \frac{1}{2}\left(P\{X_2 \leq y\} + P\{X_1 \leq y\}\right) \\
= \frac{1}{2}\left(\Phi(y) + \Phi(y)\right) = \Phi(y), Y \sim N(0,1)
\end{aligned}
$$

$$D=\{(x,y)|x^2+y^2\le 1\},\alpha^2+\beta^2=1$$

$$I=\iint_D \frac{\mathrm{d}\sigma}{(1-\alpha x+\beta y)^2+(\beta x+\alpha y)^2}=\iint_D \frac{\mathrm{d}\sigma}{(x-\alpha)^2+(y+\beta)^2}$$

$$\begin{cases} x-\alpha=u \\ y+\beta=v \end{cases}$$

$$D_{uv}=\{(u,v)|(u+\alpha)^2+(v-\beta)^2\le 1\}\to D_{uv}=\{(u,v)|r^2+2\alpha r\cos\theta-2\beta r\sin\theta\le 0\}$$

$$I=\iint_{D_{uv}}\frac{1}{r}\mathrm{d}r\mathrm{d}\theta=\int_{\sqrt{\alpha^2+\beta^2}-1}^{2(\beta\sin\theta-\alpha\cos\theta)}\frac{1}{r}\mathrm{d}r\int_?^?\mathrm{d}\theta$$

$$\int_{-2}^{2}x^3\cos\frac{x}{2}\sqrt{4-x^2}+\frac{1}{2}\sqrt{4-x^2}\mathrm{d}x=\frac{1}{2*2}\pi 4=\pi\approx 3.1415926$$

$$I=\int\frac{\sqrt{\mathrm{e}^{2x}+1}}{\mathrm{e}^x+1}\mathrm{d}x$$

$$\mathrm{e}^x+1=u,\mathrm{e}^{2x}+2\mathrm{e}^x+1=u^2,\mathrm{e}^{2x}+1=u^2-2(u-1),$$

$$x=\ln(u-1),\mathrm{d}x=\frac{\mathrm{d}u}{u-1}$$

$$I=\int\frac{\sqrt{u^2-2u+2}\mathrm{d}u}{u(u-1)}=\int\frac{\sqrt{(u-1)^2+1}}{u-1}-\frac{\sqrt{u^2-2u+2}}{u}\mathrm{d}u$$

$$=\int\frac{\sqrt{(u-1)^2+1}}{u-1}\mathrm{d}(u-1)-\int\frac{\sqrt{(u-1)^2+1}}{u}\mathrm{d}u$$

$$v=u-1,I=I_1-I_2=\int\frac{\sqrt{v^2+1}}{v}\mathrm{d}v-\int\frac{\sqrt{v^2+1}}{v+1}\mathrm{d}v$$

$$v=\tan t,I_1=\int\frac{\sec^3 t\mathrm{d}t}{\tan t}=\int\frac{1}{\cos^2 x\sin x}\mathrm{d}t$$

$$I_2=$$

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$$f(x,y)=A\mathrm{e}^{-2x^2+2xy-y^2}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}x\mathrm{d}y=1$$

$$A\int_{-\infty}^{+\infty}\mathrm{e}^{-2(x^2-xy+\frac{y^2}{4}-\frac{y^2}{4})-y^2}\mathrm{d}x\int_{-\infty}^{+\infty}\mathrm{d}y=A\int_{-\infty}^{+\infty}\mathrm{e}^{-2(x-\frac{y}{2})^2}\mathrm{d}x\int_{-\infty}^{+\infty}\mathrm{e}^{-\frac{y^2}{2}}\mathrm{d}y$$

$$\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2}}\mathrm{d}x=1,\int_{-\infty}^{+\infty}\mathrm{e}^{-\frac{x^2}{2}}\mathrm{d}x=\sqrt{2\pi},\int_{-\infty}^{+\infty}\mathrm{e}^{-2u^2}\mathrm{d}u=\sqrt{\frac{\pi}{2}}$$

$$A=\frac{1}{\pi}$$

$$\lim_{x\to0^+}\tan\left(2x^2\arctan\frac{1}{5x}\right)\sim\tan\left(2x^2\frac{\pi}{2}\right)\sim\pi x^2+o(x^2)$$

$$x\to0^+,\arctan5x\sim5x,x\to0^+,\arctan\left(\frac{1}{5x}\right)=\frac{\pi}{2}$$

$$I=\int\frac{1}{\sqrt{x}+1}\mathrm{d}x,\sqrt{x}=t,x=t^2,\mathrm{d}x=2t\mathrm{d}t$$

$$I=2\int\frac{t+1-1}{t+1}\mathrm{d}t=2t-2\ln(t+1)+C=2\sqrt{x}-2\ln(\sqrt{x}+1)+C$$

$$y=x^{2022},y^{(1)}=2022x^{2021}...y^{(2022)}=2022!x^0,y^{(2023)}=0$$

$$y=\frac{x^{1+x}}{(1+x)^x},x>0$$

$$k=\lim_{x\to+\infty}\frac{y}{x}=\lim_{x\to+\infty}\frac{x^x}{(1+x)^x}=\frac{1}{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x}=\frac{1}{\mathrm{e}}$$

$$\lim_{x\to+\infty}\mathrm{e}-\left(1+\frac{1}{x}\right)^x=\lim_{x\to+\infty}\mathrm{e}-\mathrm{e}^{x\ln\left(1+\frac{1}{x}\right)}=\lim_{x\to+\infty}\mathrm{e}\left(1-\mathrm{e}^{x\ln\left(1+\frac{1}{x}\right)-1}\right)$$

$$=\lim_{x\to+\infty}\mathrm{e}\left(1-x\ln\left(1+\frac{1}{x}\right)\right)=\lim_{x\to+\infty}\mathrm{e}\left(1-x\left(\frac{1}{x}-\frac{1}{2x^2}+o(\frac{1}{x^2})\right)\right)=\frac{\mathrm{e}}{2x}+o(\frac{1}{x})$$

$$b=\lim_{x\to+\infty}(y-kx)=\lim_{x\to+\infty}\left(\frac{x^{1+x}}{(1+x)^x}-\frac{x}{\mathrm{e}}\right)=\lim_{x\to+\infty}\frac{x}{\mathrm{e}}\left(\frac{ex^x-(1+x)^x}{(1+x)^x}\right)$$

$$=\lim_{x\to+\infty}\frac{x}{\mathrm{e}}\frac{\mathrm{e}-\left(1+\frac{1}{x}\right)^x}{\left(1+\frac{1}{x}\right)^x}=\lim_{x\to+\infty}\frac{x\frac{\mathrm{e}}{2x}}{\mathrm{e}^2}=\frac{1}{2\mathrm{e}}$$

$$y=\frac{x}{\mathrm{e}}+\frac{1}{2\mathrm{e}}$$

$$A=\int_0^4f(x)\mathrm{d}x,f(x)=\sin2x+3A$$

$$A=\int_0^4(\sin2x+3A)\mathrm{d}x$$

$$A=-\frac{1}{2}\cos2x|_0^4+12A\to\int_0^4f(x)\mathrm{d}x=A=\frac{\cos8-1}{22}$$

$$tx^2 = u, x = \sqrt{\frac{u}{t}}, \mathrm{d}x = \frac{1}{2\sqrt{ut}}\mathrm{d}u$$

$$\lim_{t\to 0} \frac{\int_0^t \sin(tx^2)\mathrm{d}x}{t^4} = \lim_{t\to 0} \frac{\int_0^{t^3} \frac{\sin u}{\sqrt{u}}\mathrm{d}u}{2t^{\frac{9}{2}}} = \lim_{t\to 0} \frac{3t^2 \sin t^3}{9t^{\frac{7}{2}} t^{\frac{3}{2}}} = \frac{1}{3}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \frac{n}{n+2} = 1, x \in (-1, 1)$$

$$S(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n} - \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n+1}$$

$$= (-x)\sum_{n=1}^{\infty} \frac{(-x)^n}{n} - \sum_{n=2}^{\infty} \frac{(-x)^n}{n} = x\ln(x+1) - (-\ln(x+1) - (-x)) = (x+1)\ln(x+1) - x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x), \sum_{n=1}^{\infty} \frac{(-x)^n}{n} = -\ln(1+x)$$

$$x = -1, S(x) = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1$$

$$x = 1, S(x) = 2\ln 2 - 1$$

$$S(x) = (x+1)\ln(x+1) - x, x \in [-1, 1]$$

$$\lim_{n\to\infty} n^2 \left[1 - \frac{\pi}{2n} \sum_{k=1}^{n} \sin\frac{(2k-1)\pi}{4n}\right]$$

$$\sin\frac{(2k-1)\pi}{4n} = \sin\frac{\pi}{2}\left(\frac{k-1}{2n} + \frac{k}{2n}\right)$$

$$I_{n(k)} = \lim_{n\to\infty} \frac{\pi}{2}\left(\frac{k}{n} - \frac{k-1}{n}\right) \sum_{k=1}^{n} \sin(\frac{\pi}{2}\frac{k-1+k}{2n}) = \frac{\pi}{2}\int_0^1 \sin(\frac{\pi x}{2})\mathrm{d}x = 1$$

$$I_{n(k)} = 1 + \frac{\pi^2}{96n^2} + o(\frac{1}{n^2})$$

$$\int_0^1 \left(\ln\frac{1}{x}\right)^5 \mathrm{d}x = -\int_0^1 (\ln x)^5 \mathrm{d}x = -x(\ln x)^5|_0^1 + 5\int_0^1 (\ln x)^4 \mathrm{d}x = 5x(\ln x)^4|_0^1 - 5\int_0^1 4(\ln x)^3\mathrm{d}x$$

$$= 20x(\ln x)^3|_0^1 - 20\int_0^1 3(\ln x)^2\mathrm{d}x = -60x(\ln x)^2|_0^1 + 60\int_0^1 2\ln x\mathrm{d}x = -120\int_0^1 \mathrm{d}x = -120$$

$$\lim_{x\to -1} \frac{x^3 - ax^2 - x + 4}{x+1} = \lim_{x\to -1} \frac{(x+1)^3 - 3x^2 - 3x - 1 - ax^2 - x + 4}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)^3 - (3+a)(x+1)^2 + 2(a+1)(x+1) + 4 - a}{x+1} = \lim_{t\to 0} \frac{t^3 - (3+a)t^2 + 2(a+1)t + 4 - a}{t}$$

$$2a + 2 = l, 4 - a = 0 \to a = 4, l = 10$$

$$
\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{x\tan x} &= \lim_{x\to 0}\frac{1+x\sin x-\cos x}{(\sqrt{1+x\sin x}+\sqrt{\cos x})x\tan x}\\
&= \lim_{x\to 0}\frac{1+x(x+o(x))-\left(1-\frac{1}{2}x^2+o(x^2)\right)}{2x\tan x}=\frac{3}{4}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n} &= x\sum_{n=1}^{\infty}\frac{x^{2n}}{2n}=x\int_0^x\sum_{n=1}^{\infty}t^{2n-1}\mathrm{d}t=x\int_0^x\frac{t}{1-t^2}\mathrm{d}t\\
&= \frac{x}{2}\int_0^x\frac{1}{1-t}-\frac{1}{1+t}\mathrm{d}t=-\frac{x}{2}\ln(1-x^2), x\in(-1,1)
\end{aligned}
$$

$$
x=\tan u, \mathrm{d}x=\sec^2 u\mathrm{d}u
$$

$$
\int_0^{\sqrt{3}}\frac{1}{\sqrt{x^2+1}}\mathrm{d}x=\int_0^{\frac{\pi}{3}}\frac{\sec^2 u\mathrm{d}u}{\sec u}=\ln|\sec u+\tan u|\,|_0^{\frac{\pi}{3}}=\ln(2+\sqrt{3})
$$

$$
\begin{aligned}
\int xf''(x)\mathrm{d}x = \int x\mathrm{d}(f'(x)) = xf'(x)-\int f'(x)\mathrm{d}x+C=\\
x\left(\frac{1-\ln x}{x^2}\right)-\frac{\ln x}{x}+C=\frac{1-2\ln x}{x}+C
\end{aligned}
$$

Consistent Continuity

$$f(x), x\in I$$

$$\forall\epsilon>0,\exists\delta=\delta(\epsilon)>0,\forall x_1,x_2\in I,|x_1-x_2|<\delta,|f(x_1)-f(x_2)|<\epsilon$$

$$\forall\epsilon>0,\exists\delta>0,|x-x_0|<\delta,|f(x)-f(x_0)|<\epsilon$$

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{a}+\sqrt[n]{b}+\sqrt[n]{c}}{3}\right)^n = \mathrm{e}^{n\ln\left(\frac{a^{\frac{1}{n}}-1+b^{\frac{1}{n}}-1+c^{\frac{1}{n}}-1}{3}+1\right)} = \mathrm{e}^{n\frac{(a^{\frac{1}{n}}-1+b^{\frac{1}{n}}-1+c^{\frac{1}{n}}-1)}{3}} =$$

$$\mathrm{e}^{\ln a^{\frac{1}{3}}+\ln b^{\frac{1}{3}}+\ln c^{\frac{1}{3}}} = \sqrt[3]{abc}$$

$$\lim_{x\to 0}\frac{x\ln(1+x)-ax^2+bx^3}{x-\tan x} = \lim_{x\to 0}\frac{x(x-\frac{1}{2}x^2+\frac{1}{3}x^3+o(x^3))-ax^2+bx^3}{-\frac{1}{3}x^3+o(x^3)} = 0$$

$$1-a=0, b-\frac{1}{2}==0 \to a=1, b=\frac{1}{2}$$

$$\iint_\Sigma x\mathrm{d}y\mathrm{d}z+y\mathrm{d}z\mathrm{d}x+z\mathrm{d}x\mathrm{d}y = 3\iiint_\Omega \mathrm{d}v = 3\iint_D\int_{\sqrt{x^2+y^2}}^{\sqrt{R^2-x^2-y^2}}\mathrm{d}z\mathrm{d}\sigma$$

$$= 3\iint_D(\sqrt{R^2-x^2-y^2}-\sqrt{x^2+y^2})\mathrm{d}\sigma, \quad D=\{(x,y)|x^2+y^2\leq\frac{R^2}{2}\}$$

$$= 3\int_0^{\frac{R}{\sqrt{2}}}(\sqrt{R^2-r^2}-r)r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta = 6\pi\left(-\frac{1}{2}\int_{R^2}^{\frac{R^2}{2}}\sqrt{R^2-r^2}\mathrm{d}(R^2-r^2)-\frac{R^3}{6\sqrt{2}}\right)$$

$$= 6\pi\left(\frac{1}{3}u^{\frac{3}{2}}|_{\frac{R^2}{2}}^{R^2}-\frac{R^3}{6\sqrt{2}}\right) = 2\pi R^3\left(1-\frac{1}{2\sqrt{2}}\right)-\pi R^3\frac{1}{\sqrt{2}} = (2-\sqrt{2})\pi R^3$$

$$\lim_{\Delta x\to 0}\frac{\ln(x+\Delta x)-\ln x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\ln\left(1+\frac{\Delta x}{x}\right)}{\Delta x}=\frac{1}{x}$$

$$\lim_{\Delta x\to 0}\frac{\tan(x+\Delta x)-\tan x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\tan x+\tan\Delta x}{1-\tan x\tan\Delta x}-\tan x}{\Delta x}=\lim_{\Delta x\to 0}\frac{(\tan^2 x+1)\tan\Delta x}{\Delta x}=\sec x$$

$$\lim_{\Delta x\to 0}\frac{\cos(x+\Delta x)-\cos x}{\Delta x}=-\sin x$$

$$\lim_{\Delta x\to 0}\frac{\sec(x+\Delta x)-\sec x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\cos x-\cos(x+\Delta x)}{\Delta x\cos(x+\Delta x)\cos x}=\tan x\sec x$$

$$\lim_{\Delta x\to 0}\frac{\log_a(x+\Delta x)-\log_a x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\log_a(1+\frac{\Delta x}{x})\log_{\mathrm{e}} a}{\Delta x\ln a}=\frac{1}{x\ln a}$$

$$\lim_{\Delta x\to 0}\frac{\arctan(x+\Delta x)-\arctan x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\arctan(\frac{\Delta x}{1+x^2+x\Delta x})}{\Delta x}=\frac{1}{1+x^2}$$

$$\lim_{\Delta x\to 0}\frac{\arcsin(x+\Delta x)-\arcsin x}{\Delta x}=\lim_{\Delta x\to 0}\frac{\arcsin((x+\Delta x)\sqrt{1-x^2}-x\sqrt{1-(x+\Delta x)^2})}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{(x+\Delta x)\sqrt{1-x^2}-x\sqrt{1-(x+\Delta x)^2}}{\Delta x}=\lim_{\Delta x\to 0}\frac{x(\sqrt{1-x^2}-\sqrt{1-(x+\Delta x)^2})}{\Delta x}+\sqrt{1-x^2}$$

$$=\lim_{\Delta x\to 0}\frac{x(2x\Delta x+\Delta^2 x)}{\Delta x(\sqrt{1-x^2}+\sqrt{1-(x+\Delta x)^2})}+\sqrt{1-x^2}=\frac{x^2}{\sqrt{1-x^2}}+\sqrt{1-x^2}=\frac{1}{\sqrt{1-x^2}}$$

$$\lim_{x\to 0^+}\frac{x^2\sin\frac{1}{x}-0}{x-0}=\lim_{x\to 0^+}x\sin\frac{1}{x}=(x+o(x))*M=0,|M|<=1$$

$$f'(x)=2x\sin\frac{1}{x}+x^2(\frac{-1}{x^2})\cos\frac{1}{x}=2x\sin\frac{1}{x}-\cos\frac{1}{x},(x\neq 0)$$

$$\lim_{x\to 0^+}f'(x)=\lim_{x\to 0^+}2x\sin\frac{1}{x}-\cos\frac{1}{x}=0-\nexists=\nexists$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{f(x)}{x^2}*x^2=A*\lim_{x\to 0}x^2=0$$

$$\lim_{x\to 0}f(x)=f(0)=0$$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{f(x)}{x}=f(0)$$

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}}{x^2}$$

$$x\to 0,\ln(1+x)=x-\frac{x^2}{2}+o(x^2),\ln(1+2x)=2x-\frac{4x^2}{2}+o(x^2)$$

$$\lim_{x\to 0}\frac{xf(x)-\ln(1+2x)}{x-\ln(1+x)}=\lim_{x\to 0}\frac{xf(x)-2x+2x^2+o(x^2)}{\frac{x^2}{2}+o(x^2)}=4$$

$$x\to 0,f(x)=2+o(1),f'(0)=0$$

$$f(x)=\frac{1}{x}-\frac{1}{\mathrm{e}^x-1}\qquad \lim_{x\to 0}\frac{f(x)-\lim_{x\to 0}f(x)-\left(\lim_{x\to 0}\frac{f(x)-\lim_{x\to 0}f(x)}{x}\right)x}{x^3}=?$$

$$A=\lim_{x\to 0}f(x)$$

$$B=\lim_{x\to 0}\frac{f(x)-A}{x}$$

$$C=\lim_{x\to 0}\frac{f(x)-A-Bx}{x^3}$$

$$A=\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\mathrm{e}^x-1-x}{x(\mathrm{e}^x-1)}=\lim_{x\to 0}\frac{1+x+\frac{x^2}{2}+o(x^2)-1-x}{x^2+o(x^2)}=\frac{1}{2}$$

$$B=\lim_{x\to 0}\frac{f(x)-\frac{1}{2}}{x}=\lim_{x\to 0}\frac{2\mathrm{e}^x-2-2x-x\mathrm{e}^x+x}{2x^2(\mathrm{e}^x-1)}$$

$$=\lim_{x\to 0}\frac{2(1+x+\frac{x^2}{2}+\frac{x^3}{6}+o(x^3))-2-2x-x(1+x+\frac{x^2}{2}+o(x^2))+x}{2x^3+o(x^3)}=\lim_{x\to 0}\frac{-\frac{1}{6}x^3+o(x^3)}{2x^3+o(x^3)}=-\frac{1}{12}$$

$$C=\lim_{x\to 0}\frac{f(x)-\frac{1}{2}+\frac{1}{12}x}{x^3}=\lim_{x\to 0}\frac{12(\mathrm{e}^x-x-1)+(x^2-6x)(\mathrm{e}^x-1)}{12x^4(\mathrm{e}^x-1)}$$

$$=\lim_{x\to 0}\frac{x^2\mathrm{e}^x-6x\mathrm{e}^x+12\mathrm{e}^x-x^2-6x-12}{12x^5+o(x^5)}=\lim_{x\to 0}\frac{x^2(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+o(x^3))}{12x^5}$$

$$+\frac{-6x(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+o(x^4))}{12x^5}+\frac{12(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+o(x^5))}{12x^5}$$

$$+\frac{-x^2-6x-12}{12x^5}=\lim_{x\to 0}\frac{\frac{1}{60}x^5+o(x^5)}{12x^5+o(x^5)}=\frac{1}{720}$$

$$\lim_{x\to+\infty}\sqrt{x^2+x+1}\frac{x-\ln(\mathrm{e}^x+x)}{x}\sim\frac{\infty*\infty}{\infty}=\lim_{x\to+\infty}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\ln\left(\frac{\mathrm{e}^x}{\mathrm{e}^x+x}\right)=1*0=0$$

$$2\sin(x+2y-3z)=x+2y-3z$$

$$2\cos(x+2y-3z)(1-3z'_x)=1-3z'_x$$

$$2\cos(x+2y-3z)(2-3z'_y)=2-3z'_y$$

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\frac{1}{3}+\frac{2}{3}=1$$

$$F(x)=\int_0^x f(t)\mathrm{d}t$$

$$\lim_{x\to0}\frac{2\int_0^x(x-t)f(t)\mathrm{d}t}{x\int_0^x f(x-t)\mathrm{d}t}=\lim_{x\to0}\frac{2x\int_0^x f(t)\mathrm{d}t-2\int_0^x tf(t)\mathrm{d}t}{x\int_0^x f(u)\mathrm{d}u}=\lim_{x\to0}\frac{2F(x)+2xf(x)-2xf(x)}{F(x)+xf(x)}$$

$$=\lim_{x\to0,\xi\to0}\frac{2xf(\xi)}{xf(\xi)+xf(x)}=1$$

$$ABA^{-1}=BA^{-1}+3I, AB=B+3A, (A-I)B=3A, B=3(A-I)^{-1}A$$

$$A^*A=|A|I, |A^*|=16, |A|=4, A^*=4A^{-1}, A=4\left(A^*\right)^{-1}$$

$$A=diag(4,\frac{1}{4},4), B=3diag(\frac{1}{3},-\frac{4}{3},\frac{1}{3})diag(4,\frac{1}{4},4)=diag(4,-1,4)$$

$$\int\frac{x^2}{1+x^2}\arctan x\mathrm{d}x=\int\left(1-\frac{1}{1+x^2}\right)\arctan x\mathrm{d}x=\int\arctan x\mathrm{d}x-\int\arctan x\mathrm{d}(\arctan x)$$

$$=x\arctan x-\int\frac{x}{1+x^2}\mathrm{d}x-\frac{1}{2}(\arctan x)^2=x\arctan x-\frac{1}{2}\ln(1+x^2)-\frac{1}{2}(\arctan x)^2+C$$

$$\int \frac{\mathrm{d}x}{\mathrm{e}^x-\mathrm{e}^{-x}} = \int \frac{\mathrm{d}\mathrm{e}^x}{\mathrm{e}^{2x}-1} = \frac{1}{2}\left(\int \frac{\mathrm{d}(\mathrm{e}^x-1)}{\mathrm{e}^x-1} - \int \frac{\mathrm{d}(\mathrm{e}^x+1)}{\mathrm{e}^x+1}\right) = \frac{1}{2}\ln\left|\frac{\mathrm{e}^x-1}{\mathrm{e}^x+1}\right| + C$$

$$\int \frac{x}{(1-x)^3}\mathrm{d}x = \int \frac{1-(1-x)}{(1-x)^3}\mathrm{d}x = \int \frac{1}{(1-x)^3} - \frac{1}{(1-x)^2}\mathrm{d}x = \frac{1}{2(1-x)^2} - \frac{1}{1-x} + C$$

$$\int \frac{x^2}{a^6-x^6}\mathrm{d}x = \int \frac{\mathrm{d}x^3}{3(a^3-x^3)(a^3+x^3)} = \int -\frac{\mathrm{d}(a^3-x^3)}{6a^3(a^3-x^3)} + \int \frac{\mathrm{d}(a^3+x^3)}{6a^3(a^3+x^3)} = \frac{1}{6a^3}\ln\left|\frac{a^3+x^3}{a^3-x^3}\right| + C$$

$$\int \frac{1+\cos x}{x+\sin x}\mathrm{d}x = \ln|x+\sin x| + C$$

$$\int \frac{\ln(\ln x)}{x}\mathrm{d}x = \int \ln(\ln x)\mathrm{d}\ln x = \ln x\ln|\ln x| - \int \frac{\ln x}{x\ln x}\mathrm{d}x = \ln x\ln|\ln x| - \ln x + C$$

$$\int \frac{\sin x\cos x}{1+\sin^4 x}\mathrm{d}x = \int \frac{\sin 2x}{2(1+\frac{(1-\cos 2x)^2}{4})}\mathrm{d}x = \int \frac{1}{2}\frac{\mathrm{d}(\frac{1-\cos 2x}{2})}{1+\left(\frac{(1-\cos 2x)}{2}\right)^2} = \frac{1}{2}\arctan\sin^2 x + C$$

$$\int \tan^4 x\mathrm{d}x = \int (\sec^2 x-1)^2\mathrm{d}x = \int (\tan^2 x+1)\mathrm{d}\tan x - 2\tan x + x = \frac{1}{3}\tan^2 x - \tan x + x + C$$

$$\cos(a+b)-\cos(a-b) = -2\sin a\sin b, \sin(a+b)+\sin(a-b) = 2\sin a\cos b$$

$$\int \sin x\sin 2x\sin 3x\mathrm{d}x = \frac{1}{2}\int (\cos x-\cos 3x)\sin 3x\mathrm{d}x = -\frac{1}{4}\int \sin 6x\mathrm{d}x + \frac{1}{4}\int (\sin 4x+\sin 2x)\mathrm{d}x$$

$$= \frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + C$$

$$\int \frac{\mathrm{d}x}{x(x^6+4)} = \frac{1}{4}\int \frac{1}{x} - \frac{x^5}{x^6+4}\mathrm{d}x = \frac{1}{4}\ln|x| - \frac{1}{24}\ln(x^6+4) + C$$

$$a>0, \int \sqrt{\frac{a+x}{a-x}}\mathrm{d}x = \int \frac{a+x}{\sqrt{a^2-x^2}}\mathrm{d}x = \int \frac{a\mathrm{d}\frac{x}{a}}{\sqrt{1-\left(\frac{x}{a}\right)^2}} - \frac{\mathrm{d}(a^2-x^2)}{2\sqrt{a^2-x^2}} = a\arcsin\frac{x}{a} - \sqrt{a^2-x^2} + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{x(1+x)}} = \int \frac{\mathrm{d}(2x+1)}{\sqrt{(2x+1)^2-1}} = \ln(2x+1+\sqrt{(2x+1)^2-1}) + C = \ln|2x+\sqrt{4x^2+4x}+1| + C$$

$$\frac{1}{2}\int x(\cos 2x+1)\mathrm{d}x = \frac{1}{4}\int x\mathrm{d}\sin 2x + \frac{x^2}{4} = \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4} + C$$

$$I = \int \mathrm{e}^{ax}\cos bx\mathrm{d}x = \frac{1}{a}\int \cos bx\mathrm{d}(\mathrm{e}^{ax}) = \frac{\cos bx\mathrm{e}^{ax}}{a} + \frac{b}{a}\int \mathrm{e}^{ax}\sin bx\mathrm{d}x = A(x,a,b) + \frac{b}{a^2}\int \sin bx\mathrm{d}(\mathrm{e}^{ax})$$

$$= A(x,a,b) + \frac{b\sin bx\mathrm{e}^{ax}}{a^2} - \frac{b^2}{a^2}\int \mathrm{e}^{ax}\cos bx\mathrm{d}x = A(x,a,b) + B(x,a,b) - \frac{b^2}{a^2}I + C_1 = I$$

$$\frac{b^2+a^2}{a^2}I = \frac{a\mathrm{e}^{ax}\cos bx + b\mathrm{e}^{ax}\sin bx}{a^2} + C_2, \quad I = \frac{\mathrm{e}^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C$$

$$I = \int \frac{\mathrm{d}x}{\sqrt{1+\mathrm{e}^x}}, \sqrt{1+\mathrm{e}^x} = u, x = \ln(u^2-1), \mathrm{d}x = \frac{2u\mathrm{d}u}{u^2-1}$$

$$I = \int \frac{2\mathrm{d}u}{(u+1)(u-1)} = \int \frac{\mathrm{d}u}{u-1} - \int \frac{\mathrm{d}u}{u+1} = \ln\left|\frac{u-1}{u+1}\right| + C = \ln\left|\frac{\sqrt{1+\mathrm{e}^x}-1}{\sqrt{1+\mathrm{e}^x}+1}\right| + C$$

$$I = \int \frac{\mathrm{d}x}{x^2\sqrt{x^2-1}}, x = \sec u, \mathrm{d}x = \sec u\tan u\mathrm{d}u, \cos u = \frac{1}{x}, \sin u = \frac{\sqrt{x^2-1}}{x}$$

$$I = \int \frac{\sec u\tan u\mathrm{d}u}{\sec^2 u\tan u} = \sin u + C = \frac{\sqrt{x^2-1}}{x} + C$$

$$I = \int \frac{\mathrm{d}x}{(a^2-x^2)^{\frac{5}{2}}}, x = a\sin u, \mathrm{d}x = a\cos u\mathrm{d}u, \tan u = \frac{x}{\sqrt{a^2-x^2}}$$

$$I = \int \frac{a\cos u\mathrm{d}u}{a^5\cos^5 u} = \frac{1}{a^4}\int \sec^4 u\mathrm{d}u = \frac{1}{a^4}\left(\frac{1}{3}\tan^3 u + \tan u\right) + C = \frac{x^3}{3a^4(a^2-x^2)^{\frac{3}{2}}} + \frac{x}{a^4\sqrt{a^2-x^2}} + C$$

$$I = \int \frac{\mathrm{d}x}{x^4\sqrt{1+x^2}}, x = \tan u, \mathrm{d}x = \sec^2 u\mathrm{d}u, (\csc x)' = -\csc x\cot x, 1+\cot^2 x = \csc^2 x, \csc u = \frac{\sqrt{x^2+1}}{x}$$

$$I = \int \frac{\sec^2 u\mathrm{d}u}{\tan^4 u\sec u} = \int \frac{\cos^3 u\mathrm{d}u}{\sin^4 u} = \int \cot^3 u\csc u\mathrm{d}u = \int (1-\csc^2 u)\mathrm{d}(\csc u) = \csc u - \frac{\csc^3 u}{3} + C$$

$$I = \frac{\sqrt{x^2+1}}{x} - \frac{(x^2+1)^{\frac{3}{2}}}{3x^3} + C$$

$$I = \int \sqrt{x}\sin\sqrt{x}\mathrm{d}x, \sqrt{x} = u, x = u^2, \mathrm{d}x = 2u\mathrm{d}u$$

$$I = \int 2u^2\sin u\mathrm{d}u = \int -2u^2\mathrm{d}(\cos u) = -2u^2\cos u + \int 4u\mathrm{d}\sin u = -2u^2\cos u + 4u\sin u + 4\cos u + C$$

$$I = -2x\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + 4\cos\sqrt{x} + C$$

$$\int \ln(1+x^2)\mathrm{d}x = x\ln(1+x^2) - 2\int\left(1 - \frac{1}{1+x^2}\right)\mathrm{d}x = x\ln(1+x^2) - 2x + 2\arctan x + C$$

$$\int \frac{\sin^2 x}{\cos^3 x}\mathrm{d}x = \int \tan^2 x\sec x\mathrm{d}x = \int \tan x\mathrm{d}\sec x = \tan x\sec x - \int \sec^3 x\mathrm{d}x$$

$$= \frac{1}{2}(\tan x\sec x - \ln|\sec x + \tan x|) + C$$

$$I=\int \arctan\sqrt{x}\mathrm{d}x, \sqrt{x}=u, x=u^2, \mathrm{d}x=2u\mathrm{d}u$$

$$I=\int 2u\arctan u\mathrm{d}u=\int \arctan u\mathrm{d}(u^2+1)=(u^2+1)\arctan u-u+C=(x+1)\arctan\sqrt{x}-\sqrt{x}+C$$

$$\sin x=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}=\frac{2\tan\frac{x}{2}}{\tan^2\frac{x}{2}+1}, \tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}, \cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\tan\frac{x}{2}=u, x=2\arctan u, \mathrm{d}x=\frac{2\mathrm{d}u}{u^2+1}$$

$$I=\int\frac{\sqrt{1+\cos x}}{\sin x}\mathrm{d}x=\int\frac{\sqrt{1+\frac{1-u^2}{1+u^2}}\frac{2}{u^2+1}}{\frac{2u}{u^2+1}}\mathrm{d}u=\int\frac{\sqrt{2}}{u\sqrt{u^2+1}}\mathrm{d}u$$

$$u=\tan k, \cot k=\frac{1}{u}, \csc k=\frac{\sqrt{u^2+1}}{u}, \mathrm{d}u=\sec^2 k\mathrm{d}k$$

$$I=\sqrt{2}\int\csc k\mathrm{d}k=-\sqrt{2}\ln|\cot k+\csc k|+C=\sqrt{2}\ln\left|\frac{u}{\sqrt{u^2+1}+1}\right|+C$$

$$=\sqrt{2}\ln\left|\frac{\tan\frac{x}{2}}{\sqrt{\tan^2\frac{x}{2}+1}+1}\right|+C \quad \boldsymbol{or}, k=\frac{x}{2}, I=-\sqrt{2}\ln\left|\cot\frac{x}{2}+\csc\frac{x}{2}\right|+C$$

$$\int\frac{x^3}{(1+x^8)^2}\mathrm{d}x=\int\frac{\mathrm{d}x^4}{4(1+(x^4)^2)^2}\xrightarrow{x^4=u}I=\int\frac{(1+u^2)+(1-u^2)\mathrm{d}u}{8(1+u^2)^2}=\int\frac{\mathrm{d}u}{8(1+u^2)}-\int\frac{\mathrm{d}(\frac{1}{u}+u)}{8(\frac{1}{u}+u)^2}$$

$$=\frac{1}{8}\arctan u+\frac{u}{8(1+u^2)}+C=\frac{1}{8}\arctan x^4+\frac{x^4}{8(1+x^8)}+C$$

$$\int\frac{x^{11}}{x^8+3x^4+2}\mathrm{d}x=\int\frac{x^{11}}{(x^4+1)(x^4+2)}\mathrm{d}x=\int\frac{\mathrm{d}(x^{12})}{12(x^4+1)}-\int\frac{\mathrm{d}(x^{12})}{12(x^4+2)}\xrightarrow{x^4=u}$$

$$\frac{1}{4}\left(\int\frac{(u+1)(u-1)+1}{u+1}\mathrm{d}u-\int\frac{(u+2)(u-2)+4}{u+2}\mathrm{d}u\right)=\frac{1}{4}\left(u+\ln\left|\frac{u+1}{(u+2)^4}\right|\right)+C$$

$$=\frac{x^4}{4}+\frac{1}{4}\ln\left|\frac{x^4+1}{(x^4+2)^4}\right|+C$$

$$\int\frac{\mathrm{d}x}{16-x^4}=\int\frac{\mathrm{d}x}{(4-x^2)(4+x^2)}=\int\frac{\mathrm{d}x}{32(2-x)}+\int\frac{\mathrm{d}x}{32(2+x)}+\int\frac{\mathrm{d}(\frac{x}{2})}{16(1+(\frac{x}{2})^2)}$$

$$=\frac{1}{32}\left(\ln\left|\frac{2+x}{2-x}\right|+2\arctan\frac{x}{2}\right)+C$$

$$\int\frac{\sin x}{1+\sin x}\mathrm{d}x\xrightarrow{\tan\frac{x}{2}=u}\int\frac{\frac{4u}{(u^2+1)^2}}{1+\frac{2u}{u^2+1}}\mathrm{d}u=\int\frac{4u\mathrm{d}u}{(u^2+1)(u+1)^2}=\int\frac{2\mathrm{d}u}{u^2+1}-\int\frac{2\mathrm{d}u}{(u+1)^2}$$

$$=2\arctan u+\frac{2}{u+1}+C=x+\frac{2}{\tan\frac{x}{2}+1}+C$$

$$\int \frac{x+\sin x}{1+\cos x}\mathrm{d}x = \int x\mathrm{d}(\tan\frac{x}{2}) - \int \frac{\mathrm{d}(1+\cos x)}{1+\cos x} = x\tan\frac{x}{2} - \int \tan\frac{x}{2}\mathrm{d}x - \ln|1+\cos x|$$

$$= x\tan\frac{x}{2} + 2\ln|\cos\frac{x}{2}| - \ln|1+\cos x| + C = x\tan\frac{x}{2} + C$$

$$\int \mathrm{e}^{\sin x}\frac{x\cos^3 x - \sin x}{\cos^2 x}\mathrm{d}x = \int x\mathrm{d}(\mathrm{e}^{\sin x}) - \int \mathrm{e}^{\sin x}\mathrm{d}(\sec x) = x\mathrm{e}^{\sin x} - \int \mathrm{e}^{\sin x}\mathrm{d}x - \sec x\mathrm{e}^{\sin x} + \int \mathrm{e}^{\sin x}\mathrm{d}x$$

$$= (x-\sec x)\mathrm{e}^{\sin x} + C$$

$$\int \frac{x^{\frac{1}{3}}}{x(x^{\frac{1}{2}}+x^{\frac{1}{3}})}\mathrm{d}x, x^{\frac{1}{6}} = u, x = u^6, \mathrm{d}x = 6u^5\mathrm{d}u$$

$$\int \frac{6u^7}{u^6(u^3+u^2)}\mathrm{d}u = \int \frac{6\mathrm{d}u}{u(u+1)} = \int \frac{6\mathrm{d}u}{u} - \int \frac{6\mathrm{d}u}{u+1} = 6\ln\left|\frac{u}{u+1}\right| + C = 6\ln\left|\frac{x^{\frac{1}{6}}}{x^{\frac{1}{6}}+1}\right| + C$$

$$I = \int \frac{\mathrm{d}x}{(1+\mathrm{e}^x)^2}, \mathrm{e}^x = u, x = \ln u, \mathrm{d}x = \frac{\mathrm{d}u}{u}$$

$$I = \int \frac{\mathrm{d}u}{u(1+u)^2} = \int \frac{\mathrm{d}u}{u} - \int \frac{(u+2)\mathrm{d}u}{(1+u)^2} = \ln|u| + \frac{2}{1+u} + \int u\mathrm{d}\left(\frac{1}{1+u}\right) = \ln|u| + \frac{2+u}{1+u} - \ln|1+u|$$

$$= \ln\left|\frac{\mathrm{e}^x}{1+\mathrm{e}^x}\right| + \frac{1}{1+\mathrm{e}^x} + C$$

$$\int \frac{\mathrm{e}^{3x}+\mathrm{e}^x}{\mathrm{e}^{4x}-\mathrm{e}^{2x}+1}\mathrm{d}x = \int \frac{\mathrm{e}^x+\mathrm{e}^{-x}}{\mathrm{e}^{2x}+\mathrm{e}^{-2x}-1}\mathrm{d}x = \int \frac{\mathrm{d}(\mathrm{e}^x-\mathrm{e}^{-x})}{(\mathrm{e}^x-\mathrm{e}^{-x})^2+1} = \arctan(\mathrm{e}^x-\mathrm{e}^{-x}) + C$$

$$\int \frac{x\mathrm{e}^x}{(1+\mathrm{e}^x)^2}\mathrm{d}x = -\int \ln u\mathrm{d}\left(\frac{1}{1+u}\right) = -\frac{\ln u}{1+u} + \int \frac{1}{u} - \frac{1}{1+u}\mathrm{d}u = \ln\left|\frac{u}{1+u}\right| - \frac{\ln u}{1+u} + C$$

$$= \ln\frac{\mathrm{e}^x}{1+\mathrm{e}^x} - \frac{x}{1+\mathrm{e}^x} + C$$

$$\int \ln^2(x+\sqrt{1+x^2})\mathrm{d}x = x\ln^2(x+\sqrt{1+x^2}) - 2\int \ln(x+\sqrt{1+x^2})\mathrm{d}(\sqrt{1+x^2})$$

$$= x\ln^2(x+\sqrt{1+x^2}) - 2\sqrt{1+x^2}\ln^2(x+\sqrt{1+x^2}) + 2x + C$$

$$x=\tan u, u=\arctan x, \mathrm{d}x=\sec^2 u\mathrm{d}u, \sin u=\frac{x}{\sqrt{1+x^2}}, \cos u=\frac{1}{\sqrt{1+x^2}}$$

$$\int\frac{\ln x}{(1+x^2)^{\frac{3}{2}}}\mathrm{d}x=\int(\ln(\sin u)-\ln(\cos u))\mathrm{d}\sin u=\sin u(\ln(\sin u)-1)-\sin u\ln(\cos u)$$

$$-\int\frac{\sin^2 u}{\cos u}\mathrm{d}u=A(u)+\int\cos u-\sec u\mathrm{d}u=A(u)+\sin u-\ln|\sec u+\tan u|+C$$

$$=\frac{x}{\sqrt{1+x^2}}\ln x-\ln(x+\sqrt{1+x^2})+C$$

$$x=\sin u, u=\arcsin x, \mathrm{d}x=\cos u\mathrm{d}u, \sin 2u=2x\sqrt{1-x^2}, \cos 2u=1-2x^2$$

$$\int\sqrt{1-x^2}\arcsin x\mathrm{d}x=\int u\cos^2 u\mathrm{d}u=\frac{u\sin 2u}{4}+\frac{\cos 2u}{8}+\frac{u^2}{4}+C$$

$$=\frac{x\sqrt{1-x^2}\arcsin x}{2}-\frac{x^2}{4}+\frac{\arcsin^2 x}{4}+C$$

$$x=\cos u, u=\arccos x, \mathrm{d}x=-\sin u\mathrm{d}u$$

$$I_{36}=\int\frac{x^3\arccos x}{\sqrt{1-x^2}}\mathrm{d}x=-\int\frac{u\cos^3 u\sin u}{\sin u}\mathrm{d}u=-\int u\cos^3 u\mathrm{d}u=-\frac{1}{2}\int u\cos u(\cos 2u+1)\mathrm{d}u$$

$$=-\frac{1}{4}\int u(\cos 3u+\cos u)\mathrm{d}u-\frac{1}{2}\int u\cos u\mathrm{d}u=-\frac{1}{36}\int 3u\cos 3u\mathrm{d}(3u)-\frac{3}{4}\int u\cos u\mathrm{d}u$$

$$=-\frac{1}{36}\left(3u\sin 3u+\cos 3u\right)-\frac{3}{4}\left(u\sin u+\cos u\right)+C$$

$$=-\frac{1}{12}((4x^2-1)\sqrt{1-x^2}\arccos x)-\frac{x(4x^2-3)}{36}-\frac{3}{4}(\sqrt{1-x^2}\arccos x+x)+C$$

$$\sin 3u=\sin 2u\cos u+\cos 2u\sin u=(4x^2-1)\sqrt{1-x^2}$$

$$\cos 3u=\cos 2u\cos u-\sin 2u\sin u=(4x^2-3)x$$

$$I_{36}=-\frac{1}{3}\sqrt{1-x^2}(x^2+2)\arccos x-\frac{x(x^2+6)}{9}+C$$

$$\int\frac{\cot x}{1+\sin x}\mathrm{d}x=\int\frac{\mathrm{d}(\sin x)}{\sin x(1+\sin x)}\xrightarrow{\sin x=u}\int\frac{\mathrm{d}u}{u}-\int\frac{\mathrm{d}u}{1+u}=\ln\left|\frac{\sin x}{1+\sin x}\right|+C$$

$$\tan x=u, x=\arctan u, \mathrm{d}x=\frac{\mathrm{d}u}{u^2+1}$$

$$\int\frac{\mathrm{d}x}{\sin^3 x\cos x}=\int\frac{4}{(1-\cos 2x)\sin 2x}\mathrm{d}x=\int\frac{4\mathrm{d}u}{(u^2+1)(1-\frac{1-u^2}{1+u^2})\frac{2u}{1+u^2}}=\int\frac{(1+u^2)\mathrm{d}u}{u^3}$$

$$=-\frac{1}{2u^2}+\ln u+C=-\frac{\cot^2 x}{2}+\ln|\tan x|+C$$

$$\int\frac{\mathrm{d}x}{(2+\cos x)\sin x}=\int\frac{\frac{2}{u^2+1}\mathrm{d}u}{(2+\frac{1-u^2}{1+u^2})\frac{2u}{u^2+1}}=\int\frac{(u^2+1)\mathrm{d}u}{(u^2+3)u}=\int\frac{\mathrm{d}u}{3u}+\int\frac{\mathrm{d}(u^2+3)}{3(u^2+3)}$$

$$=\frac{1}{3}\ln|u|+\frac{1}{3}\ln(u^2+3)+C=\frac{1}{3}\ln\left(\tan\frac{x}{2}(\tan^2\frac{x}{2}+3)\right)+C$$

$$\int \frac{\sin x \cos x}{\sin x + \cos x}\mathrm{d}x = ?$$

$$\sin x = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}} = \frac{2u}{u^2+1}, \tan x = \frac{2u}{1-u^2}, \cos x = \frac{1-u^2}{1+u^2}$$

$$\tan\frac{x}{2} = u, x = 2\arctan u, \mathrm{d}x = \frac{2\mathrm{d}u}{u^2+1}$$

$$\int \frac{\sin x \cos x}{\sin x + \cos x}\mathrm{d}x = \int \frac{\frac{2u}{u^2+1}\frac{1-u^2}{1+u^2}\frac{2}{u^2+1}}{\frac{2u}{u^2+1}+\frac{1-u^2}{1+u^2}}\mathrm{d}u = \int \frac{4u(1-u^2)}{(u^2+1)^2(1-u^2+2u)}\mathrm{d}u$$

$$= \int \frac{au^2+bu+c}{(u^2+1)^2} + \frac{d}{1-u^2+2u}\mathrm{d}u \xrightarrow{a=-1,b=2,c=1,d=-1} = \int \frac{-u^2+2u+1}{(u^2+1)^2} - \frac{1}{1-u^2+2u}\mathrm{d}u$$

$$= \int \frac{\mathrm{d}(u^2+1)}{(u^2+1)^2} - \int \frac{\mathrm{d}\left(u+\frac{1}{u}\right)}{(u+\frac{1}{u})^2} + \int \frac{\mathrm{d}u}{(u-1+\sqrt{2})(u-1-\sqrt{2})}$$

$$= \frac{u-1}{u^2+1} + \frac{1}{2\sqrt{2}}\left(\int \frac{\mathrm{d}(u-1-\sqrt{2})}{u-1-\sqrt{2}} - \int \frac{\mathrm{d}(u-1+\sqrt{2})}{u-1+\sqrt{2}}\right)$$

$$= \frac{u-1}{u^2+1} + \frac{\sqrt{2}}{4}\ln\left|\frac{u-1-\sqrt{2}}{u-1+\sqrt{2}}\right| + C = \frac{\tan\frac{x}{2}-1}{\tan^2\frac{x}{2}+1} + \frac{\sqrt{2}}{4}\ln\left|\frac{\tan\frac{x}{2}-1-\sqrt{2}}{\tan\frac{x}{2}-1+\sqrt{2}}\right| + C$$

$$\int\frac{ax^2+bx+c}{\sqrt{mx^2+nx+p}}\mathrm{d}x=\int\frac{\frac{a}{m}(mx^2+nx+p)+\left(b-\frac{an}{m}\right)x+c-\frac{ap}{m}}{A(x)}\mathrm{d}x=\frac{a}{m}\int A(x)\mathrm{d}x$$

$$+\left(b-\frac{an}{m}\right)\int\frac{x}{\sqrt{m(x+\frac{n}{2m})^2+p-\frac{n^2}{4m}}}\mathrm{d}x+\left(c-\frac{ap}{m}\right)\int\frac{\mathrm{d}x}{A(x)}$$

$$\int\frac{\mathrm{d}x}{x-\sqrt{x^2-1}}=\int x+\sqrt{x^2-1}\mathrm{d}x\xrightarrow{x=\sec u}=\frac{x^2}{2}+\int(\sec^3 u-\sec u)\mathrm{d}u+C$$

$$=\frac{x^2}{2}+\frac{1}{2}\left(\sec u\tan u-\ln|\sec u+\tan u|\right)+C=\frac{x^2}{2}+\frac{1}{2}\left(x\sqrt{x^2-1}-\ln|x+\sqrt{x^2-1}|\right)+C$$

$$\int\frac{x\mathrm{d}x}{\sqrt{-x^2+2x+3}}=-\frac{1}{2}\int\frac{\mathrm{d}(-x^2+2x+3)}{\sqrt{-x^2+2x+3}}+\int\frac{\mathrm{d}(\frac{x-1}{2})}{\sqrt{1-(\frac{x-1}{2})^2}}=-\sqrt{-x^2+2x+3}+\arcsin\frac{x-1}{2}+C$$

$$\begin{cases} I_1=\int\frac{\sin x}{\sin x+\cos x}\mathrm{d}x\\ I_2=\int\frac{\cos x}{\sin x+\cos x}\mathrm{d}x\end{cases}$$

$$\begin{cases} I_1+I_2=x+C_1\\ I_2-I_1=\ln|\sin x+\cos x|+C_2\end{cases}$$

$$I_1=\frac{x}{2}-\frac{1}{2}\ln|\sin x+\cos x|+C$$

$$f(\sin^2 x)=\frac{x}{\sin x},\int\frac{\sqrt{x}}{\sqrt{1-x}}f(x)\mathrm{d}x$$

$$\sin^2 x=u,\sin x=\sqrt{u},x=\arcsin\sqrt{u},f(u)=\frac{\arcsin\sqrt{u}}{\sqrt{u}}$$

$$\int\frac{\arcsin\sqrt{x}}{\sqrt{1-x}}\mathrm{d}x,\sqrt{x}=t,x=t^2,\mathrm{d}x=2t\mathrm{d}t$$

$$\int\frac{2t\arcsin t}{\sqrt{1-t^2}}\mathrm{d}t=-2\int\arcsin t\mathrm{d}(\sqrt{1-t^2})=-2\arcsin t\sqrt{1-t^2}+2t+C$$

$$=-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C$$

$$\int xf(x)\mathrm{d}x=\arcsin x+C,xf(x)=\frac{1}{\sqrt{1-x^2}},\frac{1}{f(x)}=x\sqrt{1-x^2}$$

$$-\frac{1}{2}\int\sqrt{1-x^2}\mathrm{d}(1-x^2)=-\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

$$\lim_{n\to\infty}\sum_{i=1}^{2n}\frac{1}{4n+i}=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{2n}\frac{1}{4+\frac{i}{n}}=\int_0^2\frac{1}{4+x}\mathrm{d}x=\ln(4+x)|_0^2=\ln\frac{3}{2}$$

$$\lim_{x\to 0}\frac{\mathrm{e}^{(1+x)^{\frac{1}{x}}}-(1+x)^{\frac{\mathrm{e}}{x}}}{x^2}=\lim_{x\to 0}\frac{\mathrm{e}^{(1+x)^{\frac{1}{x}}}-\mathrm{e}^{\mathrm{e}^{\frac{\ln(1+x)}{x}}}}{x^2}=\lim_{x\to 0}\mathrm{e}^{\mathrm{e}^{\frac{\ln(1+x)}{x}}}*\lim_{x\to 0}\frac{\mathrm{e}^{(1+x)^{\frac{1}{x}}-\mathrm{e}^{\frac{\ln(1+x)}{x}}}-1}{x^2}$$

$$=\mathrm{e}^{\mathrm{e}}\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-\mathrm{e}^{\frac{\ln(1+x)}{x}}}{x^2}=\mathrm{e}^{\mathrm{e}}\lim_{x\to 0}\frac{\mathrm{e}^{\frac{\ln(1+x)}{x}}-\mathrm{e}^{\frac{\ln(1+x)}{x}}}{x^2}=\mathrm{e}^{\mathrm{e}+1}\lim_{x\to 0}\frac{\mathrm{e}^{\frac{\ln(1+x)}{x}-1}-\frac{\ln(1+x)}{x}}{x^2}$$

$$=\mathrm{e}^{\mathrm{e}+1}\lim_{x\to 0}\frac{1-\frac{x}{2}+\frac{x^2}{8}-1+\frac{x}{2}-\frac{x^2}{3}+o(x^2)}{x^2+o(x^2)}=-\frac{5}{24}\mathrm{e}^{\mathrm{e}+1}$$

$$27\ x\to 0,\frac{\ln(1+x)}{x}=\frac{x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)}{x}=1-\frac{x}{2}+\frac{x^2}{3}+o(x^2)$$

$$x\to 0,\mathrm{e}^{\frac{\ln(1+x)}{x}-1}=1-\frac{x}{2}+\frac{x^2}{8}+o(x^2)$$

$$f(x)=(1+x)^{\frac{1}{x}} \qquad \lim_{x\to 0}\frac{f(x)-\lim_{x\to 0}f(x)-\left(\lim_{x\to 0}\frac{f(x)-\lim_{x\to 0}f(x)}{x}\right)x}{x^2}=?$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}(1+x)^{\frac{1}{x}}=\mathrm{e}$$

$$\lim_{x\to 0}\frac{f(x)-\mathrm{e}}{x}=\lim_{x\to 0}\frac{\mathrm{e}^{\frac{\ln(1+x)}{x}}-\mathrm{e}}{x}=\mathrm{e}\lim_{x\to 0}\frac{\ln(1+x)-x}{x^2}=-\frac{\mathrm{e}}{2}$$

$$\mathrm{e}^{\frac{\ln(1+x)}{x}-1}=1+\frac{\ln(1+x)}{x}-1+\frac{\left(\frac{\ln(1+x)}{x}-1\right)^2}{2}+o(x^2)=\frac{x-\frac{x^2}{2}+o(x^2)}{x}+\frac{\left(-\frac{x}{2}+o(x)\right)^2}{2}$$

$$=1-\frac{x}{2}+\frac{x^2}{8}+o(x^2)$$

$$\lim_{x\to 0}\frac{f(x)-\mathrm{e}+\frac{\mathrm{e}}{2}x}{x^2}=\mathrm{e}\lim_{x\to 0}\frac{\mathrm{e}^{\frac{\ln(1+x)}{x}-1}-1+\frac{x}{2}}{x^2+o(x^2)}=\mathrm{e}\lim_{x\to 0}\frac{1-\frac{x}{2}+\frac{x^2}{8}-1+\frac{x}{2}+o(x^2)}{x^2+o(x^2)}=\frac{\mathrm{e}}{8}$$

$$\lim_{x\to+\infty}\frac{(1+\frac{1}{x})^{x^2}}{\mathrm{e}^x}=\lim_{x\to+\infty}\frac{\mathrm{e}^{x^2\ln\left(1+\frac{1}{x}\right)}}{\mathrm{e}^x}=\lim_{x\to+\infty}\mathrm{e}^{x^2\left(\ln(1+\frac{1}{x})-\frac{1}{x}\right)}=\mathrm{e}^{\lim_{t\to 0}\frac{\ln(1+t)-t}{t^2}}=\mathrm{e}^{-\frac{1}{2}}$$

$$\int_0^{\ln 2}\frac{\mathrm{e}^x}{1+(\mathrm{e}^x-1)^2}\mathrm{d}x=\int_0^{\ln 2}\frac{\mathrm{d}(\mathrm{e}^x-1)}{1+(\mathrm{e}^x-1)^2}=\int_0^1\frac{\mathrm{d}u}{1+u^2}=\arctan u|_0^1=\frac{\pi}{4}$$

$$I=\int_0^1 r^5\sqrt{1+4r^2}\mathrm{d}r=\frac{1}{2}\int_0^1 r^4\sqrt{1+4r^2}\mathrm{d}(r^2)\xrightarrow{r^2=t}\frac{1}{2}\int_0^1 t^2\sqrt{1+4t}\mathrm{d}t$$

$$1+4t=u, t=\frac{u-1}{4}, \mathrm{d}t=\frac{1}{4}\mathrm{d}u$$

$$I=\frac{1}{128}\int_1^5(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}})\mathrm{d}u=\frac{1}{128}\left(\frac{2}{7}u^2-\frac{4}{5}u+\frac{2}{3}\right)u^{\frac{3}{2}}|_1^5$$

$$I=\frac{1}{128}\left(\frac{50}{7}-4+\frac{2}{3}\right)5\sqrt{5}-\frac{1}{128}\left(\frac{2}{7}-\frac{4}{5}+\frac{2}{3}\right)=\frac{1}{8}\left(\frac{125\sqrt{5}-1}{105}\right)=\frac{125\sqrt{5}-1}{840}$$

$$\int\mathrm{e}^{2x}(\tan x+1)^2\mathrm{d}x=\int\mathrm{e}^{2x}(\sec^2 x+2\tan x)\mathrm{d}x=\int\mathrm{e}^{2x}\mathrm{d}\tan x+\int\tan x\mathrm{d}\mathrm{e}^{2x}=\mathrm{e}^{2x}\tan x+C$$

$$\int \frac{x+\ln(1-x)}{x^2}\mathrm{d}x = \ln x - \int \ln(1-x)\mathrm{d}\left(\frac{1}{x}\right) = \ln x - \frac{\ln(1-x)}{x} - \int \frac{1}{x(1-x)}\mathrm{d}x$$

$$= \frac{(x-1)\ln(1-x)}{x} + C$$

$$\int \frac{x+1}{x(1+x\mathrm{e}^x)}\mathrm{d}x = \int \frac{(x+1)\mathrm{e}^x}{x\mathrm{e}^x(1+x\mathrm{e}^x)}\mathrm{d}x = \int \frac{\mathrm{d}(x\mathrm{e}^x)}{x\mathrm{e}^x(1+x\mathrm{e}^x)} = \int \frac{1}{u} - \frac{1}{u+1}\mathrm{d}u$$

$$= \ln\left|\frac{u}{u+1}\right| + C = \ln\left|\frac{x\mathrm{e}^x}{1+x\mathrm{e}^x}\right| + C$$

$$I = \int \sqrt{\tan x}\mathrm{d}x, \sqrt{\tan x} = u, x = \arctan u^2, \mathrm{d}x = \frac{2u}{u^4+1}\mathrm{d}u$$

$$I = \int \frac{2u^2}{u^4+1}\mathrm{d}u = \int \frac{1+\frac{1}{u^2}+1-\frac{1}{u^2}}{u^2+\frac{1}{u^2}}\mathrm{d}u = \int \frac{\mathrm{d}(u-\frac{1}{u})}{\left(u-\frac{1}{u}\right)^2+2} + \int \frac{\mathrm{d}(u+\frac{1}{u})}{(u+\frac{1}{u})^2-2}$$

$$= \int \frac{\frac{1}{\sqrt{2}}\mathrm{d}(\frac{u-\frac{1}{u}}{\sqrt{2}})}{\left(\frac{u-\frac{1}{u}}{\sqrt{2}}\right)^2+1} + \frac{1}{2\sqrt{2}}\int \frac{\mathrm{d}(u+\frac{1}{u}-\sqrt{2})}{(u+\frac{1}{u}-\sqrt{2})} - \frac{1}{2\sqrt{2}}\int \frac{\mathrm{d}(u+\frac{1}{u}+\sqrt{2})}{(u+\frac{1}{u}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{2}}\arctan\left(\frac{u-\frac{1}{u}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\ln\left|\frac{u+\frac{1}{u}-\sqrt{2}}{u+\frac{1}{u}+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\arctan\left(\frac{u^2-1}{\sqrt{2}u}\right) + \frac{1}{2\sqrt{2}}\ln\left|\frac{u^2+1-\sqrt{2}u}{u^2+1+\sqrt{2}u}\right| + C$$

$$= \frac{1}{\sqrt{2}}\arctan\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\ln\left|\frac{\sec^2 x-\sqrt{2\tan x}}{\sec^2 x+\sqrt{2\tan x}}\right| + C$$

$$I = \int \frac{1}{x^6+1}\mathrm{d}x = \int \frac{1}{(x^2)^3+1}\mathrm{d}x = \int \frac{1}{(x^2+1)(x^4-x^2+1)}\mathrm{d}x$$

$$I=\int x\sqrt{\frac{x}{2a-x}}\mathrm{d}x=\int\frac{x^2\mathrm{d}x}{\sqrt{a^2-(x-a)^2}}\xrightarrow{x-a=t,x=a+t,\mathrm{d}x=\mathrm{d}t}$$

$$I=\int\frac{(a+t)^2\mathrm{d}t}{\sqrt{a^2-t^2}}=\int\frac{a^2\mathrm{d}(\frac{t}{a})}{\sqrt{1-(\frac{t}{a})^2}}-\int\frac{a\mathrm{d}(a^2-t^2)}{\sqrt{a^2-t^2}}+\int\frac{a^2-(\sqrt{a^2-t^2})^2\mathrm{d}t}{\sqrt{a^2-t^2}}$$

$$=2a^2\arcsin\frac{t}{a}-2a\sqrt{a^2-t^2}-\int\sqrt{a^2-t^2}\mathrm{d}t$$

$$t=a\sin u,\mathrm{d}t=a\cos u\mathrm{d}u,\sin u=\frac{t}{a},\cos u=\frac{\sqrt{a^2-t^2}}{a}$$

$$\int\sqrt{a^2-t^2}\mathrm{d}t=\frac{a^2}{2}\int\cos 2u+1\mathrm{d}u=\frac{a^2}{4}\sin 2u+\frac{a^2}{2}u+C=\frac{t\sqrt{a^2-t^2}}{2}+\frac{a^2\arcsin\frac{t}{a}}{2}+C$$

$$I=\frac{3a^2}{2}\arcsin\frac{t}{a}-(2a+\frac{t}{2})\sqrt{a^2-t^2}+C=\frac{3a^2}{2}\arcsin\frac{x-a}{a}-(\frac{x+3a}{2})\sqrt{2ax-x^2}+C$$

$$I_1=\frac{3a^2}{2}\arcsin\frac{x-a}{a}-(\frac{x+3a}{2})\sqrt{2ax-x^2}+C$$

$$I_2=3a^2\arctan\sqrt{\frac{x}{2a-x}}-(\frac{x+3a}{2})\sqrt{2ax-x^2}+C$$

$$\alpha_1=\frac{1}{2}\arcsin\frac{x-a}{a},\sin 2\alpha_1=\frac{x-a}{a},\tan 2\alpha_1=\frac{x-a}{\sqrt{2ax-x^2}}$$

$$\alpha_2=\arctan\sqrt{\frac{x}{2a-x}},\tan\alpha_2=\sqrt{\frac{x}{2a-x}},\tan 2\alpha_2=\frac{2\sqrt{\frac{x}{2a-x}}}{1-\frac{x}{2a-x}}=\frac{\sqrt{2ax-x^2}}{a-x}$$

$$\tan 2\alpha_1*\tan 2\alpha_2=-1,\tan 2\alpha_1=y,\tan 2\alpha_2=-\frac{1}{y}$$

$$\alpha_1=\frac{\arctan y}{2},\alpha_2=-\frac{\arctan\frac{1}{y}}{2},\alpha_1-\alpha_2=\frac{1}{2}\left(\arctan y+\arctan\frac{1}{y}\right)=\frac{\pi}{4}$$

$$\alpha_1=\alpha_2+\frac{\pi}{4},I_1=I_2+C_0$$

$$\int \frac{\mathrm{d}(x^2)}{2(1-x^4)^{\frac{3}{2}}} \xrightarrow{x^2=u} \int \frac{\mathrm{d}u}{2(1-u^2)^{\frac{3}{2}}} \xrightarrow{u=\sin t, \mathrm{d}u=\cos t \mathrm{d}t} \frac{1}{2}\int \sec^2 t \mathrm{d}t = \frac{1}{2}\tan t + C$$

$$= \frac{u}{2\sqrt{1-u^2}} + C = \frac{x^2}{2\sqrt{1-x^4}} + C$$

$$\int \frac{1}{x(1+x^5)^{\frac{1}{3}}} \mathrm{d}x$$

$$V_{y_1} = 2 * \pi \int_0^2 \left(-\frac{1}{4}y^2 + 1\right)^2 \mathrm{d}y = 2\pi \int_0^2 \left(\frac{1}{16}y^4 - \frac{1}{2}y^2 + 1\right) \mathrm{d}y = 2\pi(\frac{32}{16*5} - \frac{8}{2*3} + 2)$$

$$V_{y_2} = 2 * \pi \int_0^2 \left(-\frac{1}{2}y^2 + 2\right)^2 \mathrm{d}y = 2\pi \int_0^2 \left(\frac{1}{4}y^4 - 2y^2 + 4\right) \mathrm{d}y = 2\pi(\frac{32}{4*5} - \frac{2*8}{3} + 8)$$

$$V_{y_2} - V_{y_1} = \frac{32}{5}\pi$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x t f(t^2-x^2)\mathrm{d}t = \frac{\mathrm{d}}{\mathrm{d}x}\int_{0^2-x^2}^{x^2-x^2} \frac{1}{2} f(t^2-x^2)\mathrm{d}(t^2-x^2) = \frac{\mathrm{d}}{\mathrm{d}x}\int_0^{-x^2} -\frac{1}{2}f(u)\mathrm{d}u$$

$$= -2x * \left(-\frac{1}{2}f(-x^2)\right) = xf(-x^2)$$

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$