Elementary   Function

$$(a+b)^n = \sum_{k=0}^{n} \mathbf{C_n^k} a^{n-k} b^k$$

$$a^n - b^n = (a-b) \cdot \sum_{k=0}^{n-1} a^{n-1-k} b^k$$

$$\arcsin x + \arccos x = \arctan x + \operatorname{arccot} x = \frac{\pi}{2}$$

$$f(x) = \int_a^x f'(t)\mathrm{d}t + f(a)$$

$$K = \frac{|y''x' - y'x''|}{(x'^2 + y'^2)^{\frac{3}{2}}}, \quad \rho = \frac{1}{K}$$

$$\int_0^1 f(x)\mathrm{d}x = \sum_{i=1}^{n} \left(\frac{i+1}{n} - \frac{i}{n}\right) f\left(\frac{i}{n}\right) = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{2i+1}{2n}\right)$$

$$\int_{t1}^{t2} f(x(t), y(t))\mathrm{d}s = \int_a^b f(x, y(x))\sqrt{x'^2 + y'^2}\mathrm{d}x = \int_\alpha^\beta f(r, \theta)\sqrt{r^2 + r'^2}\mathrm{d}\theta$$

$$S = \int_a^b y(x)\mathrm{d}x = \int_{t1}^{t2} y(t)x'(t)\mathrm{d}t = \frac{1}{2}\int_\alpha^\beta r^2(\theta)\mathrm{d}\theta$$

$$S_{side} = \int_a^b 2\pi y(x)\sqrt{x'^2(x) + y'^2(x)}\mathrm{d}x = \int_\alpha^\beta 2\pi r(\theta)\sin\theta\sqrt{r^2(\theta) + r'^2(\theta)}\mathrm{d}\theta$$

$$[\overrightarrow{abc}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Gamma Function Integral

$$\left(-\frac{1}{2}\right)! = \sqrt{\pi}$$

$$\left(\frac{2n+1}{2}\right)! = \Pi_{i=n}^{0}\frac{2i+1}{2}\sqrt{\pi} = \frac{2n+1}{2}\frac{2n-1}{2}\cdots\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^a \mathrm{e}^{-x} = a!$$

$$\int_0^\infty x^{-\frac{1}{2}}\mathrm{e}^{-x} = \sqrt{\pi}$$

$$\int_0^\infty x^3 \mathrm{e}^{-x} = (3)! = 6$$

$$\int_0^\infty x^{\frac{5}{2}}\mathrm{e}^{-x} = \frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^a \mathrm{e}^{-x^2} = \frac{1}{2}\left(\frac{a-1}{2}\right)!$$

$$\int_0^\infty x^1 \mathrm{e}^{-x^2} = \frac{1}{2}\left(\frac{1-1}{2}\right)! = \frac{1}{2}$$

$$\int_0^\infty x^4 \mathrm{e}^{-x^2} = \frac{1}{2}\left(\frac{4-1}{2}\right)! = \frac{1}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^7 \mathrm{e}^{-x^2} = \frac{1}{2}\left(\frac{7-1}{2}\right)! = \frac{1}{2}(3)! = \frac{1}{2}6$$

$$\iiint_{\Omega_{xyz}} f(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z = \iiint_{\Omega_{uvw}} f(u,v,w) \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} \mathrm{d}u\mathrm{d}v\mathrm{d}w$$

$$\left.\frac{\partial f}{\partial l}\right|_{P_0=(x_0,y_0,z_0)} = f_x(P_0)\cos\alpha + f_y(P_0)\cos\beta + f_z(P_0)\cos\gamma$$

$$\mathbf{grad}f(x_0,y_0,z_0) = f_x(x_0,y_0,z_0)\vec{i} + f_y(x_0,y_0,z_0)\vec{j} + f_z(x_0,y_0,z_0)\vec{k}$$

$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{\iiint_{\Omega} x\rho\mathrm{d}v}{\iiint_{\Omega} \rho\mathrm{d}v}, \frac{\iiint_{\Omega} y\rho\mathrm{d}v}{\iiint_{\Omega} \rho\mathrm{d}v}, \frac{\iiint_{\Omega} z\rho\mathrm{d}v}{\iiint_{\Omega} \rho\mathrm{d}v}\right), \quad J_{k_j} = \iiint_{\Omega}((\sum_{i=1}^{n} k_i^2) - k_j^2)\rho\mathrm{d}v$$

$$S = \iint_{\Sigma} \mathrm{d}S = \iint_{D_{xy}} \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}\mathrm{d}x\mathrm{d}y$$

$$S = \iint_{D_{zx}} \sqrt{1 + (\frac{\partial y}{\partial z})^2 + (\frac{\partial y}{\partial x})^2}\mathrm{d}z\mathrm{d}x = \iint_{D_{yz}} \sqrt{1 + (\frac{\partial x}{\partial y})^2 + (\frac{\partial x}{\partial z})^2}\mathrm{d}y\mathrm{d}z$$

$$\iint_{-\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = -\iint_{\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y$$

$$\oiint_{\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = \iiint_{\Omega}(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})\mathrm{d}v, \quad \mathrm{div}\vec{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\Sigma : F(x,y,z) = 0 \quad \vec{n} = \left(\frac{F_x}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_y}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_z}{\sqrt{F_x^2 + F_y^2 + F_z^2}}\right) = (\cos\alpha, \cos\beta, \cos\gamma)$$

$$\oiint_{\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = \iint_{\Sigma}(P\cos\alpha + Q\cos\beta + R\cos\gamma)\mathrm{d}S$$

$$\oint_{\Gamma} P\mathrm{d}x + Q\mathrm{d}y + R\mathrm{d}z = \iint_{\Sigma}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathrm{d}y\mathrm{d}z + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathrm{d}z\mathrm{d}x + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathrm{d}x\mathrm{d}y$$

$$\mathbf{rot}\vec{A} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

$$\boldsymbol{Infinite \quad Series}$$

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \rho \quad , R = \frac{1}{\rho}$$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l})$$

$$a_0 = \frac{1}{l} \int_{-l}^{l} f(x)\mathrm{d}x, \quad a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l}\mathrm{d}x, \quad b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l}\mathrm{d}x$$

$$S(x) = \begin{cases} f(x), x \in consecutive \\ \frac{f(x-0)+f(x+0)}{2}, x \in discontinuity \\ \frac{f(l-0)+f(l+0)}{2}, x = \{-l, l\} \end{cases}$$

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$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \alpha^{\mathbf{T}} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} \quad |\mathbf{A}| = \det \mathbf{A} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|\mathbf{A}| = \Pi_{i=1}^{n}\lambda_i \quad |k\mathbf{A}| = k^n|\mathbf{A}| \quad |\mathbf{AB}| = |\mathbf{A}||\mathbf{B}| \quad |\mathbf{A}^n| = |\mathbf{A}|^n$$

$$|\mathbf{A^T}| = |\mathbf{A}| \quad (k\mathbf{A})^{\mathbf{T}} = k\mathbf{A^T} \quad (\mathbf{A^T})^{\mathbf{T}} = \mathbf{A} \quad (\mathbf{A}^n)^{\mathbf{T}} = (\mathbf{A^T})^n$$

$$|\mathbf{A}^{-1}| = \frac{1}{|\mathbf{A}|} \quad (k\mathbf{A})^{-1} = \frac{1}{k}\mathbf{A}^{-1} \quad (\mathbf{A}^{-1})^{-1} = \mathbf{A} \quad (\mathbf{A}^n)^{-1} = (\mathbf{A}^{-1})^n$$

$$|\mathbf{A}^*| = |\mathbf{A}|^{n-1} \quad (k\mathbf{A})^* = k^{n-1}\mathbf{A}^* \quad (\mathbf{A}^*)^* = |\mathbf{A}|^{n-2}\mathbf{A} \quad \mathbf{AA}^* = \mathbf{A}^*\mathbf{A} = |\mathbf{A}|\mathbf{E}$$

$$(\mathbf{ABC})^T = \mathbf{C^T B^T A^T} \quad (\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1} \quad (\mathbf{ABC})^* = \mathbf{C}^*\mathbf{B}^*\mathbf{A}^*$$

$$\mathbf{diag}(a_1,...,a_n)\mathbf{diag}(b_1,...,b_n) = \mathbf{diag}(a_1b_1,...,a_nb_n)$$

$$\Lambda = \mathbf{diag}(\lambda_1,...,\lambda_n), |\Lambda| = \Pi_{i=1}^{n}\lambda_i, \varphi(\Lambda) = \mathbf{diag}(\varphi(\lambda_1),...,\varphi(\lambda_n))$$

$$\Lambda = \begin{pmatrix} 0 & 0 & \lambda_1 \\ 0 & \lambda_i & 0 \\ \lambda_n & 0 & 0 \end{pmatrix} \quad |\Lambda| = (-1)^{\frac{n(n-1)}{2}}\Pi_{i=1}^{n}\lambda_i \quad \Lambda^{-1} = \begin{pmatrix} 0 & 0 & \frac{1}{\lambda_1} \\ 0 & \frac{1}{\lambda_i} & 0 \\ \frac{1}{\lambda_n} & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} \mathbf{A} & * \\ \mathbf{O} & \mathbf{B} \end{vmatrix} = \begin{vmatrix} \mathbf{A} & \mathbf{O} \\ * & \mathbf{B} \end{vmatrix} = |\mathbf{A}||\mathbf{B}|$$

$$\begin{vmatrix} \mathbf{O} & \mathbf{A}_{mm} \\ \mathbf{B}_{nn} & * \end{vmatrix} = \begin{vmatrix} * & \mathbf{A}_{mm} \\ \mathbf{B}_{nn} & \mathbf{O} \end{vmatrix} = (-1)^{mn}|\mathbf{A}||\mathbf{B}|$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}^{\mathbf{T}} = \begin{pmatrix} \mathbf{A^T} & \mathbf{B^T} \\ \mathbf{C^T} & \mathbf{D^T} \end{pmatrix} \quad \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & \mathbf{B} & 0 \\ 0 & 0 & \mathbf{C} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A}^{-1} & 0 & 0 \\ 0 & \mathbf{B}^{-1} & 0 \\ 0 & 0 & \mathbf{C}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix}^{\mathbf{T}} = (\mathbf{A^T}, \mathbf{B^T}) \quad \begin{pmatrix} 0 & 0 & \mathbf{A} \\ 0 & \mathbf{B} & 0 \\ \mathbf{C} & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & \mathbf{A}^{-1} \\ 0 & \mathbf{B}^{-1} & 0 \\ \mathbf{C}^{-1} & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{O} & \mathbf{B} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{C}\mathbf{B}^{-1} \\ \mathbf{O} & \mathbf{B}^{-1} \end{pmatrix} \quad \begin{pmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{C} & \mathbf{B} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A}^{-1} & \mathbf{O} \\ -\mathbf{B}^{-1}\mathbf{C}\mathbf{A}^{-1} & \mathbf{B}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{C} & \mathbf{A} \\ \mathbf{B} & \mathbf{O} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{O} & \mathbf{B}^{-1} \\ \mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{C}\mathbf{B}^{-1} \end{pmatrix} \quad \begin{pmatrix} \mathbf{O} & \mathbf{A} \\ \mathbf{B} & \mathbf{C} \end{pmatrix}^{-1} = \begin{pmatrix} -\mathbf{B}^{-1}\mathbf{C}\mathbf{A}^{-1} & \mathbf{B}^{-1} \\ \mathbf{A}^{-1} & \mathbf{O} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{O} & \mathbf{B} \end{pmatrix}^* = \begin{pmatrix} |\mathbf{B}|\mathbf{A}^* & \mathbf{O} \\ \mathbf{O} & |\mathbf{A}|\mathbf{B}^* \end{pmatrix} \quad \begin{pmatrix} \mathbf{O} & \mathbf{A_{mm}} \\ \mathbf{B_{nn}} & \mathbf{O} \end{pmatrix}^* = (-1)^{mn}\begin{pmatrix} \mathbf{O} & |\mathbf{A}|\mathbf{B}^* \\ |\mathbf{B}|\mathbf{A}^* & \mathbf{O} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{O} & \mathbf{B} \end{pmatrix}^n = \begin{pmatrix} \mathbf{A}^n & \mathbf{O} \\ \mathbf{O} & \mathbf{B}^n \end{pmatrix} \quad \begin{pmatrix} \mathbf{O} & \mathbf{A} \\ \mathbf{B} & \mathbf{O} \end{pmatrix}^n = \begin{pmatrix} \text{not} & \text{important} \\ \text{at} & \text{all} \end{pmatrix}$$

Matrix   Rank

$$r(\mathbf{A}^*) = \begin{cases} n & \text{if } r(\mathbf{A}) = n, \\ 1 & \text{if } r(\mathbf{A}) = n - 1, \\ 0 & \text{if } r(\mathbf{A}) < n - 1. \end{cases}$$

$$0 \leq r(\mathbf{A_{mn}}) \leq \max\{m, n\}$$

$$\max\{r(\mathbf{A}), r(\mathbf{B})\} \leq \mathbf{r(A, B)} \leq r(\mathbf{A}) + r(\mathbf{B})$$

$$\max\{r(\mathbf{A}), r(\mathbf{B})\} \leq r \begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix} \leq r(\mathbf{A}) + r(\mathbf{B})$$

$$r(\mathbf{A} \pm \mathbf{B}) \leq r(\mathbf{A} \pm \mathbf{B}, \mathbf{B}) = r(\mathbf{A}, \mathbf{B}) \leq r(\mathbf{A}) + r(\mathbf{B})$$

$$r(\mathbf{A}, \mathbf{B}) \leq r \begin{pmatrix} \mathbf{A^T} \\ \mathbf{B^T} \end{pmatrix} \neq r \begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix} = r(\mathbf{A}, \mathbf{B})$$

$$r(\mathbf{A}) + r(\mathbf{B}) - n \leq r(\mathbf{AB}) \leq \min\{r(\mathbf{A}), r(\mathbf{B})\}$$

$$r(\mathbf{A^T A}) = r(\mathbf{A A^T}) = r(\mathbf{A^T}) = r(\mathbf{A}) = r(k\mathbf{A}), (\forall k \neq 0)$$

$$\exists \mathbf{A_{mn} B_{ns}} = \mathbf{O}, r(\mathbf{A}) + r(\mathbf{B}) \leq n$$

$$\exists \mathbf{A_{mn} B_{ns}} = \mathbf{C}, \exists r(\mathbf{A} = n), r(\mathbf{B}) = r(\mathbf{C})$$

$$\exists \mathbf{A_{mn} B_{ns}} = \mathbf{C}, \exists r(\mathbf{B} = n), r(\mathbf{A}) = r(\mathbf{C})$$

$$\exists \mathbf{A_{nn}}, \forall k \in \mathbf{N}^*, r(\mathbf{A}^n) = r(\mathbf{A}^{n+k}) \Longrightarrow r(\mathbf{A}) = ... = r(\mathbf{A}^n)$$

Similarity   Theory   and   Feature   Vector

$$|\lambda \mathbf{E} - \mathbf{A}| = 0 \Rightarrow \lambda_i, i \in [1, n]$$

$$|\lambda \mathbf{E} - \mathbf{A}| = \begin{pmatrix} \lambda - a_{11} & ... & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & ... & \lambda - a_{nn} \end{pmatrix} = \lambda^n + \sum_{i=1}^{n} a_{ii}\lambda^{n-1} + ...$$

$$\forall i \in [0, n], \exists \lambda_i, f(\lambda_i) = 0 \Longrightarrow \Pi_{i=1}^{n}(\lambda - \lambda_i) = 0$$

$$\lambda^n + \sum_{i=1}^{n} \lambda_i \lambda^{n-1} + ... + (-1)^n \Pi_{i=1}^{n}\lambda_i = 0 \Rightarrow \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i$$

$$\exists \lambda = 0, (-1)^n \Pi_{i=1}^{n}\lambda_i = |-\mathbf{A}| = (-1)^n |\mathbf{A}| \Rightarrow \Pi_{i=1}^{n}\lambda_i = |\mathrm{A}|$$

Specially $n = 3$

$$|\lambda \mathbf{E} - \mathbf{A}| = \begin{pmatrix} \lambda - a_{11} & a_{12} & a_{13} \\ a_{21} & \lambda - a_{22} & a_{23} \\ a_{31} & a_{32} & \lambda - a_{33} \end{pmatrix} = \lambda^3 - \left(\sum_{i=1}^{3} a_{ii}\right)\lambda^2 + \left(\sum_{i=1}^{3} \mathbf{A_{ii}}\right)\lambda - |\mathbf{A}|$$

$$\lambda^3 - (a_{11} + a_{22} + a_{33})\lambda^2 + (\mathbf{A_{11}} + \mathbf{A_{22}} + \mathbf{A_{33}})\lambda - |\mathbf{A}| = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = \lambda^3 - (\lambda_1 + \lambda_2 + \lambda_3)\lambda^2 + (\lambda_2\lambda_3 + \lambda_1\lambda_3 + \lambda_1\lambda_2)\lambda - (\lambda_1\lambda_2\lambda_3)$$

$$\sum_{i=1}^{3} \mathbf{A_{ii}} = \mathbf{tr}(\mathbf{A}^*) = (\lambda_2\lambda_3 + \lambda_1\lambda_3 + \lambda_1\lambda_2)$$

$$\mathbf{A} \sim \mathbf{B} \Longrightarrow \mathbf{A^T} \sim \mathbf{B^T}, \mathbf{A}^{-1} \sim \mathbf{B}^{-1}, \mathbf{A}^* \sim \mathbf{B}^*, f(\mathbf{A}) \sim f(\mathbf{B})$$

$$f(\mathbf{A}) = 0 \Rightarrow f(\lambda) = 0(E \sim 1)$$

$$\lambda_{\mathbf{A}_i^*}\lambda_{\mathbf{A}_i} = |\mathbf{A}|, i \in [1, n]$$

$\mathbf{A}$	$\mathbf{A^T}$	$\mathbf{A}^{-1}$	$\mathbf{A}^*$	$f(\mathbf{A})$	$\mathbf{P}^{-1}\mathbf{AP}$	$\mathbf{PAP}^{-1}$
λ	λ	$\frac{1}{\lambda}$	$\frac{|\mathbf{A}|}{\lambda}$	$f(\lambda)$	λ	λ
α		α	α	α	$\mathbf{P}^{-1}\alpha$	$\mathbf{P}\alpha$

$$Base \quad \sigma$$

$$(\eta_1, \eta_2, ... \eta_n) = (\xi_1, \xi_2, ... \xi_n)\mathbf{M}$$

$$\begin{cases} \eta_1 = a_{11}\xi_1 + a_{21}\xi_2 + ... + a_{n1}\xi n \\ \vdots \quad \vdots \quad \ddots \qquad\qquad\qquad \vdots \quad \vdots \\ \eta_n = a_{1n}\xi_1 + a_{2n}\xi_2 + ... + a_{nn}\xi n \end{cases} \Longrightarrow \mathbf{AM} = \mathbf{B} \Longleftrightarrow \mathbf{M} = \mathbf{A^{-1}B}$$

$$\mathbf{A}\xi_{\mathbf{A}} = \mathbf{B}\xi_{\mathbf{B}} \Longrightarrow \xi_{\mathbf{A}} = \mathbf{A^{-1}B}\xi_{\mathbf{B}}$$

$$\boldsymbol{Traditional \quad Probability \quad Theory}$$

$$\boldsymbol{Opposition} : P(A) + P(\bar{A}) = 1 \quad \boldsymbol{Exclusive} : A \cap B = \varnothing \Rightarrow P(AB) = 0$$

$$\boldsymbol{Independent} : P(AB) = P(A)P(B) \quad \boldsymbol{Equal} : A = B \Rightarrow P(A) = P(B)$$

$$A - B = A\bar{B} = A \cap \bar{B} \Rightarrow P(A - B) = P(A) - P(AB)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(BC) - P(AC) - P(AB) + P(ABC)$$

$$P(B|A) = \frac{P(AB)}{P(A)} \quad P(A_1 A_2 ... A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 A_2)...P(A_n|A_1...A_{n-1})$$

$$\boldsymbol{Bayes} \quad P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

Variable Digital Properties

Distr	Mark	EX	DX	Addition
Bin	$B(n,p)$	np	$np(1-p)$	$P\{X=k\}=\mathbf{C_n^k}(1-p)^{n-k}p^k$
Poi	$P(\lambda)$	λ	λ	$P\{X=k\}=\frac{\lambda^k}{k!}\mathrm{e}^{-\lambda}, k=0,1,2,...$
Geo	$G(p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$P\{X=k\}=(1-p)^{k-1}p, k=1,2,...$
Hyp	$H(n,M,N)$	$\frac{nM}{N}$	$\frac{nM}{N}(1-\frac{M}{N})(\frac{N-n}{N-1})$	$P\{X=i\}=\frac{\mathbf{C}_M^i\mathbf{C}_{N-M}^{n-i}}{\mathbf{C}_N^n}$
Uni	$U(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Exp	$E(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	
Nor	$N(\mu,\sigma^2)$	μ	σ^2	

$$\textbf{Uni}: f(x)=\begin{cases} \frac{1}{b-a}, a\le x\le b \\ 0, others \end{cases} \quad F(x)=\begin{cases} 0, x<a \\ \frac{x-a}{b-a}, a\le x<b \\ 1, x\ge b \end{cases}$$

$$\textbf{Exp}: f(x)=\begin{cases} \lambda\mathrm{e}^{-\lambda x}, x>0 \\ 0, x\le 0 \end{cases} \quad F(x)=\begin{cases} 1-\lambda\mathrm{e}^{-\lambda x}, x>0 \\ 0, x\le 0 \end{cases}$$

$$\textbf{Nor}: f(x)=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}, F(x)=\int_{-\infty}^x f(t)\mathrm{d}t, x\in(-\infty,+\infty)$$

$$f(\mu+x)=f(\mu-x), F(\mu+x)+F(\mu-x)=1, F(\mu)=\frac{1}{2}$$

$$X\sim N(0,1), \phi(x)=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2}}, \Phi(x)=\int_{-\infty}^x \phi(t)\mathrm{d}t$$

$$\phi(-x)=\phi(x), \quad \Phi(a)+\Phi(-a)=1, \Phi(0)=\frac{1}{2}, \quad F(x)=\Phi\left(\frac{x-\mu}{\sigma}\right)$$

$$f(x,y)\ge 0 \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\mathrm{d}x\mathrm{d}y=1$$

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\mathrm{d}y>0 \quad f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\mathrm{d}x>0$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}, f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}$$

$$\textbf{\textit{Discrete}}: P\{Z=g(x_i,y_j)\}=P\{X=x_i,Y=y_j\}=P_{ij}$$

$$F_Z(z)=P\{Z\le z\}=P\{g(X,Y)\le z\}=\sum_{g(x_i,y_j)\le z} P\{X=x_i,Y=y_j\}$$

$$\textbf{\textit{Continuous}}: Z=g(X,Y) \quad F_Z(z)=P\{Z\le z\}=P\{g(X,Y)\le z\}=\iint_{g(x,y)\le z} f(x,y)\mathrm{d}x\mathrm{d}y$$

$$Z=\max(X,Y), F_{\max}(z)=F_X(z)F_Y(z) \quad Z=\min(X,Y), F_{\min}(z)=1-[1-F_X(z)][1-F_Y(z)]$$

$$(X,Y)\sim U(D), f(x,y)=\begin{cases} \frac{1}{S_D}, (x,y)\in D \\ 0, others \end{cases}$$

$$(X,Y)\sim N(\mu_1,\mu_2;\sigma_1,\sigma_2,\rho), f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\mathrm{e}^{-\frac{1}{2(1-\rho^2)}[(\frac{x-\mu_1}{\sigma_1})^2-\frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+(\frac{y-\mu_2}{\sigma_2})^2]}$$

$$E(X) = \sum_{i=1}^{\infty} x_i p_i \quad E(X) = \int_{-\infty}^{\infty} x f(x) \mathrm{d}x$$

$$E[g(X)] = \sum_{i=1}^{\infty} g(x_i) p_i \quad E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) \mathrm{d}x$$

$$E[g(X,Y)] = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} g(x_i, y_j) P_{ij} \quad E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) \mathrm{d}x \mathrm{d}y$$

$$E(C) = C \quad E(CX) = CE(X) \quad E(X+C) = E(X) + C \quad E(X+Y) = E(X) + E(Y)$$

$$D(X) = \sum_{i} [x_i - E(X)]^2 p_i \quad D(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) \mathrm{d}x$$

$$D(X) = E(X^2) - [E(X)]^2 \quad D(C) = 0 \quad D(CX) = C^2 D(X) \quad D(X+C) = D(X)$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) \quad D(X \pm Y) = D(X) + D(Y) \pm 2Cov(X,Y)$$

$$Cov(X,Y) = Cov(Y,X) \quad Cov(X,X) = D(X) \quad Cov(X,c) = 0$$

$$Cov(aX, bY) = abCov(X,Y) \quad Cov(X_1 + X_2, Y) = Cov(X_1, Y) + Cov(X_2, Y)$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$$

Large   Number   Law   Central   Limit   Theorem

$$P\{|X - E(X)| \geq \epsilon\} \leq \frac{D(X)}{\epsilon^2}$$

$$\lim_{n \to \infty} P\{|\frac{X_n}{n} - p| < \epsilon\} = 1$$

$$\lim_{n \to \infty} P\{|\frac{1}{n} \sum_{k=1}^{n} x_k - \mu| < \epsilon\} = 1$$

$$\lim_{n \to \infty} P\{|\frac{1}{n} \sum_{k=1}^{n} X_k - \frac{1}{n} \sum_{k=1}^{n} E(X_k)| < \epsilon\} = 1$$

$$\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2), \lim_{n \to \infty} P\{\frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma} \leq x\} = F_n(x) = \Phi(x)$$

$$X \sim N(np, np(1-p)), \lim_{n \to \infty} P\{\frac{X_n - np}{\sqrt{np(1-p)}} \leq x\} = \Phi(x)$$

Mathematical   Statistics

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}), \frac{\bar{X}-u}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}X_i^2 - n\bar{X}^2]$$

$$E(S^2) = \sigma^2, D(S^2) = \frac{2\sigma^4}{n-1}$$

$$\frac{\sum_{i=1}^{n}(X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2 \sim \chi^2(n), E(\chi^2(n)) = n, D(\chi^2(n)) = 2n$$

$$T \sim t(n), T = \frac{X}{\sqrt{Y/n}} \underset{\sim}{\sim} \frac{N(0,1)}{\sqrt{\chi^2(n)/n}}$$

$$F \sim F(n1, n2), F = \frac{X/n1}{Y/n2} \underset{\sim}{\sim} \frac{\chi^2(n1)/n1}{\chi^2(n2)/n2}$$

$$(X, Y) \sim N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2, 0)$$

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2}/(n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2}/(n_2-1)} \sim F(n_1-1, n_2-1)$$

$$\frac{\frac{\sum_{i=1}^{n_1}(X_i-\mu_1)^2}{\sigma_1^2}/n_1}{\frac{\sum_{i=1}^{n_2}(X_i-\mu_2)^2}{\sigma_2^2}/n2} \sim F(n1, n2)$$

$$\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n2}}} \sim N(0,1)$$

$$\exists \sigma_1 = \sigma_2, \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{S_\omega\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

$$S_\omega = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \quad \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90} \quad \sum_{k=1}^{\infty} \frac{1}{k^6} = \frac{\pi^6}{945}$$

$$\sum_{i=0}^{n} a_i \cdot \sum_{j=0}^{n} b_j = \sum_{i=0}^{n}\sum_{j=0}^{n} (a_i \cdot b_j) \quad \sum_{n=s}^{t} \ln f(n) = \ln \prod_{n=s}^{t} f(n)$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n} i)^2 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i} = \sum_{i=0}^{\infty} \frac{1}{(2i+1)(2i+2)} = \sum_{i=1}^{\infty} \frac{1}{2^i i} = \sum_{i=1}^{\infty} (\frac{1}{3^i} + \frac{1}{4^i})\frac{1}{i} = \ln 2$$

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Power   Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \ln(1+x)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x)$$

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan x$$

$$\sum_{n=0}^{\infty} (n+1)x^n = \frac{1}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} (n+1)(n+2)x^n = \frac{2}{(1-x)^3}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n} = -\ln(1-x^2)$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = \frac{1}{2}\ln(\frac{1+x}{1-x})$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sin x$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos x$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$$\textbf{\textit{Basel\quad Problem}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$f(x) = x^2 \quad \underrightarrow{Fourier\,Expansion} \quad S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \cos nx)$$

$$a_0 = \frac{2}{3}\pi^2 \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos nx \mathrm{d}x = (-1)^n \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin nx \mathrm{d}x = 0$$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}(-1)^n \frac{4}{n^2} \cos nx = \frac{1}{3}\pi^2 + \sum_{n=1}^{n} \frac{4}{n^2}$$

$$\exists x = 0, S(0) = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}(-1)^n \frac{4}{n^2} = f(0) = 0 \Longrightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$$

$$\exists x = \pi, S(\pi) = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2} = f(\pi) = \pi^2 \Longrightarrow \sum_{n=1}^{\infty} \frac{4}{n^2} = \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \quad \sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{\pi^2}{24}$$

Transcendental Equation

$$\sum_{i=0}^{n} a_i x^i = 0 \implies \prod_{i=0}^{n} (x - x_i) = 0$$

$$\prod_{i=0}^{n} x_i = (-1)^n \frac{a_0}{a_n}, \quad \sum_{i=0}^{n} \frac{\prod_{i=0}^{n} x_i}{x_i} = (-1)^{n-1} \frac{a_1}{a_n}$$

$$\sum_{i=0}^{n} \frac{1}{x_i} = -\frac{a_1}{a_0}$$

eg. $\quad \tan x = x \implies \sin x = \cos x \cdot x$

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 + ... = x \cdot (1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 + ...)$$

$$\frac{1}{3} x^3 - \frac{1}{30} x^5 + ... = 0 \quad \underset{\longrightarrow}{x \neq 0} \quad \frac{1}{3} x^2 - \frac{1}{30} x^4 + ... = 0$$

$$\sum_{i=0}^{n} \frac{1}{t_i} = \frac{1}{10} \quad \underset{\longrightarrow}{t = x^2} \quad \lim_{n \to \infty} \sum_{i=0}^{n} \frac{1}{x_i^2} = \frac{1}{10}$$

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$$\sum_{n=0}^{\infty} \frac{1}{(3n)!} \implies S(x) = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}, S(1) = \frac{e}{3} + \frac{2}{3}\cos\frac{\sqrt{3}}{2}e^{-\frac{1}{2}}$$

$$\iint_{D_{xy}} (e^x \cos y)\mathrm{d}\sigma, D = \{(x,y)|x^2 + y^2 \leq 1\}$$

$$\iint_{D_{xy}} (x+y)\mathrm{d}\sigma, D = \{(x,y)|y^2 \leq x+2, x^2 \leq y+2\}$$

$$\boldsymbol{A_0} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sum_{m=0}^{\infty} \frac{1}{n2^m + 1}$$

$$\boldsymbol{A_1} = \int_0^{x^2} \pi(\sqrt[4]{1+t} - 1)\sin t^4 \mathrm{d}x$$

$$\boldsymbol{A_2} = \sum_{n=1}^{\infty} \frac{((n-1)!)^2 (2t)^{2n}}{(2n)!}$$

$$\boldsymbol{A_3} = \int_0^1 \frac{(1-2x)\ln(1-x)}{x^2 - x + 1}\mathrm{d}x$$

$$\boldsymbol{A_4} = x^2(x - \tan x)\ln(x^2+1)[\left(\frac{2\arctan\frac{y}{x}}{\pi}\right)^y - 1]$$

$$\lim_{x\to 0^+} \lim_{y\to+\infty} \frac{\boldsymbol{A_0 A_1}}{\boldsymbol{A_2 A_3 A_4}} = \frac{27}{32}$$

$$\int \frac{\sec^3 x}{1 - \tan^6 x}\mathrm{d}x$$

$$\int \frac{1}{\csc x + \sec x + \tan x + \cot x}\mathrm{d}x$$

$$\lim_{N \to \infty} \sum_{n=1}^{N} \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k} - \ln 2 = \ln 2 - \frac{1}{2}$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1 - \cos \frac{\pi}{\sqrt{n}}}{1 + \cos \frac{i\pi}{\sqrt{2n}}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{[\sqrt{n}]}}{n} \qquad \sum_{n=1}^{\infty} \frac{(n)!}{(2n+1)!} \frac{1}{(n+1)}$$

$$t \frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + 3 \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - t \frac{\mathrm{d}x}{\mathrm{d}t} - x = 0$$