

Elementary Function

$$(a+b)^n=\sum_{k=0}^nC_n^ka^{n-k}b^k$$

$$a^n-b^n=(a-b)\cdot\sum_{k=0}^{n-1}a^{n-1-k}b^k$$

$$\arcsin x+\arccos x=\arctan x+\operatorname{arccot} x=\frac{\pi}{2}$$

$$f(x)=\int_a^xf'(t)\mathrm{d}t+f(a)$$

$$K=\frac{|y''x'-y'x''|}{(x'^2+y'^2)^{\frac{3}{2}}},\quad\rho=\frac{1}{K}$$

$$\int_0^1f(x)\mathrm{d}x=\sum_{i=1}^nf(\frac{i}{n})(\frac{i+1}{n}-\frac{i}{n})=\sum_{i=1}^nf(\frac{2i+1}{2n})\frac{1}{n}$$

$$\int_{t_1}^{t_2}f(x(t),y(t))\mathrm{d}s=\int_a^bf(x,y(x))\sqrt{x'^2+y'^2}\mathrm{d}x=\int_{\alpha}^{\beta}f(r,\theta)\sqrt{r^2+r'^2}\mathrm{d}\theta$$

$$S=\int_a^by(x)\mathrm{d}x=\int_{t_1}^{t_2}y(t)x'(t)\mathrm{d}t=\frac{1}{2}\int_{\alpha}^{\beta}r^2(\theta)\mathrm{d}\theta$$

$$S_{side}=\int_a^b2\pi y(x)\sqrt{x'^2(x)+y'^2(x)}\mathrm{d}x=\int_{\alpha}^{\beta}2\pi r(\theta)\sin\theta\sqrt{r^2(\theta)+r'^2(\theta)}\mathrm{d}\theta$$

$$[\overrightarrow{abc}]=\begin{vmatrix}a_x&a_y&a_z\\b_x&b_y&b_z\\c_x&c_y&c_z\end{vmatrix}$$

Gamma Function Integral

$$\begin{aligned}
 \left(-\frac{1}{2}\right)! &= \sqrt{\pi} \\
 \left(\frac{2n+1}{2}\right)! &= \prod_{i=n}^0 \frac{2i+1}{2} \sqrt{\pi} = \frac{2n+1}{2} \frac{2n-1}{2} \cdots \frac{1}{2} \sqrt{\pi} \\
 \int_0^{\infty} x^a e^{-x} &= a! \\
 \int_0^{\infty} x^{-\frac{1}{2}} e^{-x} &= \sqrt{\pi} \\
 \int_0^{\infty} x^3 e^{-x} &= (3)! = 6 \\
 \int_0^{\infty} x^{\frac{5}{2}} e^{-x} &= \frac{5}{2} \frac{3}{2} \frac{1}{2} \sqrt{\pi} \\
 \int_0^{\infty} x^a e^{-x^2} &= \frac{1}{2} \left(\frac{a-1}{2}\right)! \\
 \int_0^{\infty} x^1 e^{-x^2} &= \frac{1}{2} \left(\frac{1-1}{2}\right)! = \frac{1}{2} \\
 \int_0^{\infty} x^4 e^{-x^2} &= \frac{1}{2} \left(\frac{4-1}{2}\right)! = \frac{1}{2} \frac{3}{2} \frac{1}{2} \sqrt{\pi} \\
 \int_0^{\infty} x^7 e^{-x^2} &= \frac{1}{2} \left(\frac{7-1}{2}\right)! = \frac{1}{2} (3)! = \frac{1}{2} 6
 \end{aligned}$$

Multivariate Integral

$$\iiint_{\Omega_{xyz}} f(x, y, z) dx dy dz = \iiint_{\Omega_{uvw}} f(u, v, w) \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} du dv dw$$

$$\left. \frac{\partial f}{\partial l} \right|_{P_0=(x_0, y_0, z_0)} = f_x(P_0) \cos \alpha + f_y(P_0) \cos \beta + f_z(P_0) \cos \gamma$$

$$\mathbf{grad} f(x_0, y_0, z_0) = f_x(x_0, y_0, z_0) \vec{i} + f_y(x_0, y_0, z_0) \vec{j} + f_z(x_0, y_0, z_0) \vec{k}$$

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{\iiint_{\Omega} x \rho dv}{\iiint_{\Omega} \rho dv}, \frac{\iiint_{\Omega} y \rho dv}{\iiint_{\Omega} \rho dv}, \frac{\iiint_{\Omega} z \rho dv}{\iiint_{\Omega} \rho dv} \right), \quad J_{k_j} = \iiint_{\Omega} \left(\left(\sum_{i=1}^n k_i^2 \right) - k_j^2 \right) \rho dv$$

$$S = \iint_{\Sigma} dS = \iint_{D_{xy}} \sqrt{1 + \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2} dx dy$$

$$S = \iint_{D_{zx}} \sqrt{1 + \left(\frac{\partial y}{\partial z} \right)^2 + \left(\frac{\partial y}{\partial x} \right)^2} dz dx = \iint_{D_{yz}} \sqrt{1 + \left(\frac{\partial x}{\partial y} \right)^2 + \left(\frac{\partial x}{\partial z} \right)^2} dy dz$$

$$\iint_{-\Sigma} P dy dz + Q dz dx + R dx dy = - \iint_{\Sigma} P dy dz + Q dz dx + R dx dy$$

$$\oint_{\Sigma} P dy dz + Q dz dx + R dx dy = \iiint_{\Omega} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dv, \quad \operatorname{div} \vec{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\Sigma : F(x, y, z) = 0 \quad \vec{n} = \left(\frac{F_x}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_y}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_z}{\sqrt{F_x^2 + F_y^2 + F_z^2}} \right) = (\cos \alpha, \cos \beta, \cos \gamma)$$

$$\oint_{\Sigma} P dy dz + Q dz dx + R dx dy = \iint_{\Sigma} (P \cos \alpha + Q \cos \beta + R \cos \gamma) dS$$

$$\oint_{\Gamma} P dx + Q dy + R dz = \iint_{\Sigma} \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) dy dz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) dz dx + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

$$\mathbf{rot} \vec{A} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \vec{k}$$

Infinite Series

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \rho, R = \frac{1}{\rho}$$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_0 = \frac{1}{l} \int_{-l}^l f(x) dx, \quad a_n = \frac{1}{l} \int_{-l}^l f(x) \cos \frac{n\pi x}{l} dx, \quad b_n = \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi x}{l} dx$$

$$S(x) = \begin{cases} f(x), x \in \text{consecutive} \\ \frac{f(x-0)+f(x+0)}{2}, x \in \text{discontinuity} \\ \frac{f(l-0)+f(l+0)}{2}, x = \{-l, l\} \end{cases}$$

Traditional Probability Theory

$$A - B = A\bar{B} = A \cap \bar{B} \Rightarrow P(A - B) = P(A) - P(AB)$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in (-\infty, +\infty)$$

$$\Phi(x) = \int_{-\infty}^x f(t)dt, X \sim N(0, 1), \quad \Phi(a) + \Phi(-a) = 1, \quad F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}, f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)}$$

$$(X, Y) \sim U(D), f(x, y) = \begin{cases} \frac{1}{S_D}, & (x, y) \in D \\ 0, & \text{others} \end{cases}$$

$$(X, Y) \sim N(\mu_1, \mu_2; \sigma_1, \sigma_2, \rho), f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}[(\frac{x-\mu_1}{\sigma_1})^2 - \frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + (\frac{y-\mu_2}{\sigma_2})^2]}$$

Variable Digital Properties

Distr	Mark	EX	DX
Bin	B(n,p)	np	np(1-p)
Poi	P(λ)	λ	λ
Geo	G(p)	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hyp	H(n,M,N)	$\frac{nM}{N}$	$\frac{nM}{N}(1 - \frac{M}{N})(\frac{N-n}{N-1})$
Uni	U(a,b)	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp	E(λ)	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Nor	N(μ, σ^2)	μ	σ^2

Large Number Law Central Limit Theorem

$$P\{|X - E(X)| \geq \epsilon\} \leq \frac{D(X)}{\epsilon^2}$$

$$\lim_{n \rightarrow \infty} P\{|\frac{X_n}{n} - p| < \epsilon\} = 1$$

$$\lim_{n \rightarrow \infty} P\{|\frac{1}{n} \sum_{k=1}^n x_k - \mu| < \epsilon\} = 1$$

$$\lim_{n \rightarrow \infty} P\{|\frac{1}{n} \sum_{k=1}^n X_k - \frac{1}{n} \sum_{k=1}^n E(X_k)| < \epsilon\} = 1$$

$$\sum_{i=1}^n X_i \sim N(n\mu, n\sigma^2), \lim_{n \rightarrow \infty} P\{\frac{\sum_{i=1}^n X_i - n\mu}{\sqrt{n}\sigma} \leq x\} = F_n(x) = \Phi(x)$$

$$X \sim N(np, np(1-p)), \lim_{n \rightarrow \infty} P\{\frac{X_n - np}{\sqrt{np(1-p)}} \leq x\} = \Phi(x)$$

Mathematical Statistics

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}), \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 = \frac{1}{n-1} [\sum_{i=1}^n X_i^2 - n\bar{X}^2]$$

$$E(S^2) = \sigma^2, D(S^2) = \frac{2\sigma^4}{n-1}$$

$$\frac{\sum_{i=1}^n (X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2 \sim \chi^2(n), E(\chi^2(n)) = n, D(\chi^2(n)) = 2n$$

$$T \sim t(n), T = \frac{X}{\sqrt{Y/n}} \sim \frac{N(0, 1)}{\sqrt{\chi^2(n)/n}}$$

$$F \sim F(n_1, n_2), F = \frac{X/n_1}{Y/n_2} \sim \frac{\chi^2(n_1)/n_1}{\chi^2(n_2)/n_2}$$

$$(X, Y) \sim N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2, 0)$$

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2}/(n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2}/(n_2-1)} \sim F(n_1-1, n_2-1)$$

$$\frac{\frac{\sum_{i=1}^{n_1} (X_i - \mu_1)^2}{\sigma_1^2}/n_1}{\frac{\sum_{i=1}^{n_2} (X_i - \mu_2)^2}{\sigma_2^2}/n_2} \sim F(n_1, n_2)$$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$$

$$\exists \sigma_1 = \sigma_2, \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{S_\omega \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

$$S_\omega = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

Constant Series

$$\begin{aligned}
& \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} = -\frac{\pi}{4} \\
& \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \quad \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90} \quad \sum_{k=1}^{\infty} \frac{1}{k^6} = \frac{\pi^6}{945} \\
& \sum_{i=0}^n a_i \cdot \sum_{j=0}^n b_j = \sum_{i=0}^n \sum_{j=0}^n (a_i \cdot b_j) \quad \sum_{n=s}^t \ln f(n) = \ln \prod_{n=s}^t f(n) \\
& \sum_{i=0}^n i = \frac{n(n+1)}{2} \quad \sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=0}^n i^3 = \left(\sum_{i=0}^n i\right)^2 = \frac{n^2(n+1)^2}{4} \\
& \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i} = \sum_{i=0}^{\infty} \frac{1}{(2i+1)(2i+2)} = \sum_{i=1}^{\infty} \frac{1}{2^i i} = \sum_{n=1}^{\infty} \left(\frac{1}{3^i} + \frac{1}{4^i}\right) \frac{1}{i} = \ln 2
\end{aligned}$$

Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \ln(1+x)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x)$$

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan x$$

$$\sum_{n=0}^{\infty} (n+1)x^n = \frac{1}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} (n+1)(n+2)x^n = \frac{2}{(1-x)^3}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n} = -\ln(1-x^2)$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sin x$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos x$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

Transcendental Equation

$$\sum_{i=0}^n a_i x^i = 0 \implies \prod_{i=0}^n (x - x_i) = 0$$

$$\prod_{i=0}^n x_i = (-1)^n \frac{a_0}{a_n}, \quad \sum_{i=0}^n \frac{\prod_{j=0}^n x_j}{x_i} = (-1)^{n-1} \frac{a_1}{a_n}$$

$$\sum_{i=0}^n \frac{1}{x_i} = -\frac{a_1}{a_0}$$

eg. $\tan x = x \implies \sin x = \cos x \cdot x$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \dots = x \cdot \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \dots\right)$$

$$\frac{1}{3}x^3 - \frac{1}{30}x^5 + \dots = 0 \quad \xrightarrow{x \neq 0} \quad \frac{1}{3}x^2 - \frac{1}{30}x^4 + \dots = 0$$

$$\sum_{i=0}^n \frac{1}{t_i} = \frac{1}{10} \quad t = x^2 \quad \lim_{n \rightarrow \infty} \sum_{i=0}^n \frac{1}{x_i^2} = \frac{1}{10}$$

Beyond Integral

$$\sum_{n=0}^{\infty} \frac{1}{(3n)!} \Rightarrow S(x) = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}, S(1) = \frac{e}{3} + \frac{2}{3} \cos \frac{\sqrt{3}}{2} e^{-\frac{1}{2}}$$

$$\iint_{D_{xy}} (e^x \cos y) d\sigma, D = \{(x, y) | x^2 + y^2 \leq 1\}$$

$$\iint_{D_{xy}} (x + y) d\sigma, D = \{(x, y) | y^2 \leq x + 2, x^2 \leq y + 2\}$$

$$\mathbf{A_0} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sum_{m=0}^{\infty} \frac{1}{n2^m + 1}$$

$$\mathbf{A_1} = \int_0^{x^2} \pi(\sqrt[4]{1+t} - 1) \sin t^4 dx$$

$$\mathbf{A_2} = \sum_{n=1}^{\infty} \frac{((n-1)!)^2 (2t)^{2n}}{(2n)!}$$

$$\mathbf{A_3} = \int_0^1 \frac{(1-2x) \ln(1-x)}{x^2 - x + 1} dx$$

$$\mathbf{A_4} = x^2(x - \tan x) \ln(x^2 + 1) \left[\left(\frac{2 \arctan \frac{y}{x}}{\pi} \right)^y - 1 \right]$$

$$\lim_{x \rightarrow 0^+} \lim_{y \rightarrow +\infty} \frac{\mathbf{A_0 A_1}}{\mathbf{A_2 A_3 A_4}} = \frac{27}{32}$$

$$\int \frac{\sec^3 x}{1 - \tan^6 x} dx$$

$$\int \frac{1}{\csc x + \sec x + \tan x + \cot x} dx$$

$$\lim_{N\rightarrow\infty}\sum_{n=1}^N\sum_{k=1}^n\frac{(-1)^{k-1}}{k}-\ln 2=\ln 2-\frac{1}{2}$$

$$\lim_{n\rightarrow\infty}\sum_{i=1}^n\frac{1-\cos\frac{\pi}{\sqrt{n}}}{1+\cos\frac{i\pi}{\sqrt{2n}}}$$

$$\sum_{n=1}^\infty \frac{(-1)^{[\sqrt{n}]}}{n}$$

$$t\frac{\mathrm{d}^3x}{\mathrm{d}t^3}+3\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-t\frac{\mathrm{d}x}{\mathrm{d}t}-x=0$$