$$\lim_{n\to\infty} n\frac{\left(\sum_{k=1}^{n}\sqrt{k}\right)^2}{\left(\sum_{k=1}^{n}\sqrt[3]{k}\right)^3}+\sum_{k=1}^{n-1}\left[\ln\left(1+\frac{1}{n+k}\right)\sin\left(\ln\left(1+\frac{k}{n}\right)\right)\right]=$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\right)^2}{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt[3]{\frac{k}{n}}\right)^3}+\frac{1}{n}\sum_{k=1}^{n-1}\frac{1}{1+\frac{k}{n}}\sin\left(\ln\left(1+\frac{k}{n}\right)\right)=$$

$$\lim_{n\to\infty}\frac{\left(\int_0^1 x^{\frac{1}{2}}\mathrm{d}x\right)^2}{\left(\int_0^1 x^{\frac{1}{3}}\mathrm{d}x\right)^3}+\int_0^1\frac{\sin(\ln(1+x))}{1+x}\mathrm{d}x=\frac{\frac{4}{9}}{\frac{27}{64}}+\int_0^1\sin(\ln(1+x))\mathrm{d}(\ln(x+1))=$$

$$\frac{256}{243}-\cos u|_0^{\ln 2}=\frac{499}{243}-\cos(\ln 2)$$

$$p(x)=\frac{\mathrm{d}^n}{\mathrm{d}x^n}(1-x^m)^n=\left(\sum_{k=0}^{n}C_n^k(-x^m)^k\right)^{(n)}=\left(1+C_n^1(-x^m)+C_n^2(-x^m)^2+...\right)^{(n)}$$

$$\exists k_0\in\{1,2,3,...\},mk_0>n$$

$$\int_0^{\frac{\pi}{2}}\ln\left[\left(\sin^2 x+99\cos^2 x\right)\left(999\sin^2 x+\cos^2 x\right)\right]\mathrm{d}x$$

$$\left\{\begin{array}{c} x^2+y^2=z \\ y=x\tan z\end{array}\right.=\left\{\begin{array}{c} x=\sqrt{\theta}\cos\theta \\ y=\sqrt{\theta}\sin\theta \\ z=\theta\end{array}\right.,\theta\in(0,c)$$

$$s=\int\mathrm{d}s=\int_0^c\sqrt{\left(\frac{\cos\theta}{2\sqrt{\theta}}-\sqrt{\theta}\sin\theta\right)^2+\left(\frac{\sin\theta}{2\sqrt{\theta}}+\sqrt{\theta}\cos\theta\right)^2+1}\mathrm{d}\theta=$$

$$\int_0^c\sqrt{\frac{1}{4\theta}+\theta+1}\mathrm{d}\theta=\int_0^c\frac{2\theta+1}{2\sqrt{\theta}}\mathrm{d}\theta=\frac{2}{3}\theta^{\frac{3}{2}}+\theta^{\frac{1}{2}}|_0^c=\frac{2}{3}c^{\frac{3}{2}}+c^{\frac{1}{2}}=\sqrt{c}\left(\frac{2c}{3}+1\right)$$

$$f(x)=\sec x,x\in(-\frac{\pi}{4},\frac{\pi}{4})$$

$$S(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos 4nx$$

$$a_0=\frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\mathrm{d}x=\frac{8}{\pi}\ln|\sec x+\tan x||_0^{\frac{\pi}{4}}=\frac{8}{\pi}\ln(\sqrt{2}+1)$$

$$a_n=\frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\cos 4nx\mathrm{d}x=\frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\frac{\mathrm{d}(\sin 4nx)}{\cos x}=\frac{4\sin 4nx}{n\pi\cos x}|_0^{\frac{\pi}{4}}-\frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\sin 4nx\sec x\tan x\mathrm{d}x=$$

$$2m=n,\frac{x^2}{2}-\frac{\sqrt{2}xy}{m}+\frac{3y^2}{2m}=1$$

$$A=\left[\begin{array}{cc}\frac{1}{2} & \frac{-\sqrt{2}}{2m}\\ \frac{-\sqrt{2}}{2m} & \frac{3}{2m}\end{array}\right]\quad (x,y)^T=Q\,(u,v)^T$$

$$|\lambda E-A|=(\lambda-\frac{1}{2})(\lambda-\frac{3}{2m})-\frac{1}{2m^2}=\lambda^2-\frac{m+3}{2m}\lambda+\frac{3m-2}{4m^2}=0$$

$$\lambda=\frac{\frac{m+3}{2m}\pm\sqrt{\left(\frac{m+3}{2m}\right)^2-4\frac{3m-2}{4m^2}}}{2}\notin Z$$

$$\lim_{x\to 0}\frac{\int_0^x t\cos t\mathrm{d}t-1+\cos x}{\sqrt{1+x\tan x}-\sqrt{1+x\sin x}}=$$

$$\lim_{x\to 0}\frac{2(\int_0^x t\cos t\mathrm{d}t-1+\cos x)}{x(\tan x-\sin x)}=\lim_{x\to 0}\frac{2(x\cos x-\sin x)}{2x^3}=-\frac{1}{3}$$

$$\sum_{n=0}^{\infty}\frac{n^2+1}{(\frac{1}{2})^n n!}x^n=\sum_{n=0}^{\infty}\frac{n^2(2x)^n}{n!}+\sum_{n=0}^{\infty}\frac{(2x)^n}{n!}$$

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}2\frac{(n+1)^2+1}{(n^2+1)(n+1)}=0,x\in(-\infty,+\infty)$$

$$S(x)=\sum_{n=1}^{\infty}\frac{n(2x)^n}{(n-1)!}+\mathrm{e}^{2x}=\sum_{n=0}^{\infty}\frac{(n+1)(2x)^{n+1}}{n!}+\mathrm{e}^{2x}=\sum_{n=0}^{\infty}\frac{(2x)^{n+2}}{n!}+\sum_{n=0}^{\infty}\frac{(2x)^{n+1}}{n!}+\mathrm{e}^{2x}$$

$$=(4x^2+2x+1)\mathrm{e}^{2x},x\in(-\infty,+\infty)$$

$$\begin{cases} x+y-z=\mathrm{e}^z \\ x\mathrm{e}^x=\tan t \\ y=\cos t\end{cases}$$

$$x'\mathrm{e}^x+x\mathrm{e}^x x'=\sec t\to\mathrm{e}^x(x'+x'x)=\sec t,\mathrm{e}^x(x'+x'x+x''+x''x+x'x')=\sec t\tan t$$

$$y'=-\sin t,y''=-\cos t$$

$$x'+y'-z'=\mathrm{e}^z z'\to z'=\frac{x'+y'}{\mathrm{e}^z+1}$$

$$t=0,x=0,y=1,z=0,x'=1,y'=0,z'=\frac{1}{2}$$

$$1+0+x''+0+1=0,x''=-2,y''=-1$$

$$z''=\frac{(x''+y'')(\mathrm{e}^z+1)-(x'+y')(\mathrm{e}^z z')}{(\mathrm{e}^z+1)^2}=\frac{-3*2-\frac{1}{2}}{4}=-\frac{13}{8}$$

$$\iint_D r^2\sin\theta\sqrt{1-r^2\cos(2\theta)}\mathrm{d}\sigma,D=\{(r,\theta),0\le r\le\sec\theta,0\le\theta\le\frac{\pi}{4}\}$$

$$D=\{(x,y),0\le y\le x,0\le x\le 1\}$$

$$\iint_D y\sqrt{1-x^2+y^2}\mathrm{d}\sigma=\frac{1}{2}\int_0^1\int_{1-x^2}^{x^2+1-x^2}\sqrt{1-x^2+y^2}\mathrm{d}(y^2+1-x^2)\mathrm{d}x=\frac{1}{3}\int_0^1 u^{\frac{3}{2}}|_{1-x^2}^1\mathrm{d}x$$

$$=\frac{1}{3}\int_0^1 1-(1-x^2)^{\frac{3}{2}}\mathrm{d}x=\frac{1}{3}-\int_0^{\frac{\pi}{2}}\cos^4 t\mathrm{d}t=\frac{1}{3}-\frac{1*3*1*\pi}{3*4*2*2}=\frac{1}{3}-\frac{\pi}{16}$$

$$u=ax^2+by^2+cz^2,x+y+z=1,(ax^2+by^2+cz^2)(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})\ge(x+y+z)^2=1$$

$$\min_{x>0,y>0,z>0}u(x,y,z)=\frac{1}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$a_n = \int_0^1 \frac{x^{n-1}}{1+x}\mathrm{d}x = \int_0^1 x^{n-1}\mathrm{d}(\ln(x+1)) = x^{n-1}\ln(x+1)|_0^1 - (n-1)\int_0^1 \ln(x+1)x^{n-2}\mathrm{d}x$$

$$a_n = \frac{1}{n}\int_0^1 \frac{\mathrm{d}(x^n)}{1+x} = \frac{x^n}{n(1+x)}|_0^1 + \int_0^1 \frac{x^n\mathrm{d}x}{n(1+x)^2} = \frac{1}{2n} + \int_0^1 \frac{\mathrm{d}(x^{n+1})}{(1+x)^2 n(n+1)} =$$

$$\frac{1}{2n} + \frac{x^{n+1}}{(1+x)^2 n(n+1)}|_0^1 + \int_0^1 \frac{2x^{n+1}\mathrm{d}x}{(n+1)(1+x)^3} = \frac{1}{2n} + \frac{1}{4n^2} + o(\frac{1}{n^2})mn \to \infty$$

$$I_1 = \iint_S \frac{xz}{a^2}\mathrm{d}y\mathrm{d}z + \frac{yz}{b^2}\mathrm{d}z\mathrm{d}x + \frac{z^2}{c^2}\mathrm{d}x\mathrm{d}y$$

$$I_2 = 0, S_1 = \{(x,y,z), \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, z = 0\}, down0$$

$$D_{xy} = \{(x,y), \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$$

$$I_1 + I_2 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\iiint_\Omega z\mathrm{d}v = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2}{2}\iint_{D_{xy}}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\mathrm{d}\sigma$$

$$\frac{x}{a} = u, \frac{y}{b} = v, J = ab$$

$$I_1 + I_2 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2}{2}ab\iint_{D_{uv}}(1 - u^2 - v^2)\mathrm{d}\sigma$$

$$\iint_{D_{uv}}(1 - u^2 - v^2)\mathrm{d}\sigma = \int_0^1 (1 - r^2)r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta = \frac{\pi}{2}$$

$$I_1 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2\pi}{4}ab$$