$\boldsymbol{Elementary \quad Function}$

$$(a+b)^n=\sum_{k=0}^{n}\boldsymbol{C_n^k}a^{n-k}b^k$$

$$a^n-b^n=(a-b)\cdot\sum_{k=0}^{n-1}a^{n-1-k}b^k$$

$$\arcsin x+\arccos x=\arctan x+\mathrm{arccot}x=\frac{\pi}{2}$$

$$f(x)=\int_a^x f'(t)\mathrm{d}t+f(a)$$

$$K=\frac{|y''x'-y'x''|}{(x'^2+y'^2)^{\frac{3}{2}}},\quad \rho=\frac{1}{K}$$

$$\int_a^b f(x)\mathrm{d}x=\lim_{\lambda\to 0}\sum_{i=1}^{n}f(\xi_i)\Delta x_i$$

$$\int_{t1}^{t2}f(x(t),y(t))\mathrm{d}s=\int_a^b f(x,y(x))\sqrt{x'^2+y'^2}\mathrm{d}x=\int_\alpha^\beta f(r,\theta)\sqrt{r^2+r'^2}\mathrm{d}\theta,\quad \theta=\alpha\to\beta$$

$$S=\int_a^b y(x)\mathrm{d}x=\int_{t1}^{t2}y(t)x'(t)\mathrm{d}t=\frac{1}{2}\int_\alpha^\beta r^2(\theta)\mathrm{d}\theta,\quad \theta=\alpha\to\beta$$

$$S_{side}=\int_a^b 2\pi y(x)\sqrt{x'^2(x)+y'^2(x)}\mathrm{d}x=\int_\alpha^\beta 2\pi r(\theta)\sin\theta\sqrt{r^2(\theta)+r'^2(\theta)}\mathrm{d}\theta$$

$$[\overrightarrow{abc}]=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}$$

Gamma Function Integral

$$(-\frac{1}{2})! = \sqrt{\pi}$$

$$(\frac{2n+1}{2})! = \Pi_{i=n}^{0}\frac{2i+1}{2}\sqrt{\pi} = \frac{2n+1}{2}\frac{2n-1}{2}\dots\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^a \mathrm{e}^{-x} = a!$$

$$\int_0^\infty x^{-\frac{1}{2}} \mathrm{e}^{-x} = \sqrt{\pi}$$

$$\int_0^\infty x^3 \mathrm{e}^{-x} = (3)! = 6$$

$$\int_0^\infty x^{\frac{5}{2}} \mathrm{e}^{-x} = \frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^a \mathrm{e}^{-x^2} = \frac{1}{2}(\frac{a-1}{2})!$$

$$\int_0^\infty x^1 \mathrm{e}^{-x^2} = \frac{1}{2}(\frac{1-1}{2})! = \frac{1}{2}$$

$$\int_0^\infty x^4 \mathrm{e}^{-x^2} = \frac{1}{2}(\frac{4-1}{2})! = \frac{1}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^7 \mathrm{e}^{-x^2} = \frac{1}{2}(\frac{7-1}{2})! = \frac{1}{2}(3)! = \frac{1}{2}6$$

Multivariate Integral

$$\iiint_{\Omega_{xyz}} f(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z = \iiint_{\Omega_{uvw}} f(u,v,w)\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} \mathrm{d}u\mathrm{d}v\mathrm{d}w$$

$$\left.\frac{\partial f}{\partial l}\right|_{P_0=(x_0,y_0,z_0)} = f_x(P_0)\cos\alpha + f_y(P_0)\cos\beta + f_z(P_0)\cos\gamma$$

$$\mathbf{grad} f(x_0,y_0,z_0) = f_x(x_0,y_0,z_0)\vec{\boldsymbol{i}} + f_y(x_0,y_0,z_0)\vec{\boldsymbol{j}} + f_z(x_0,y_0,z_0)\vec{\boldsymbol{k}}$$

$$(\bar{x},\bar{y},\bar{z}) = (\frac{\iiint_\Omega x\rho\mathrm{d}v}{\iiint_\Omega \rho\mathrm{d}v}, \frac{\iiint_\Omega y\rho\mathrm{d}v}{\iiint_\Omega \rho\mathrm{d}v}, \frac{\iiint_\Omega z\rho\mathrm{d}v}{\iiint_\Omega \rho\mathrm{d}v}), \quad J_{k_j} = \iiint_\Omega ((\sum_{i=1}^n k_i^2) - k_j^2)\rho\mathrm{d}v$$

$$S = \iint_\Sigma \mathrm{d}S = \iint_{D_{xy}} \sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}\mathrm{d}x\mathrm{d}y$$

$$S = \iint_{D_{zx}} \sqrt{1 + (\frac{\partial y}{\partial z})^2 + (\frac{\partial y}{\partial x})^2}\mathrm{d}z\mathrm{d}x = \iint_{D_{yz}} \sqrt{1 + (\frac{\partial x}{\partial y})^2 + (\frac{\partial x}{\partial z})^2}\mathrm{d}y\mathrm{d}z$$

$$\iint_{-\Sigma} P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = -\iint_\Sigma P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y$$

$$\oint_\Sigma P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = \iiint_\Omega (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})\mathrm{d}v, \quad \mathrm{div}\vec{\boldsymbol{A}} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\Sigma : F(x,y,z) = 0 \quad \vec{n} = (\frac{F_x}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_y}{\sqrt{F_x^2 + F_y^2 + F_z^2}}, \frac{F_z}{\sqrt{F_x^2 + F_y^2 + F_z^2}}) = (\cos\alpha, \cos\beta, \cos\gamma)$$

$$\oint_\Sigma P\mathrm{d}y\mathrm{d}z + Q\mathrm{d}z\mathrm{d}x + R\mathrm{d}x\mathrm{d}y = \iint_\Sigma (P\cos\alpha + Q\cos\beta + R\cos\gamma)\mathrm{d}S$$

$$\oint_\Gamma P\mathrm{d}x + Q\mathrm{d}y + R\mathrm{d}z = \iint_\Sigma (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\mathrm{d}y\mathrm{d}z + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\mathrm{d}z\mathrm{d}x + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\mathrm{d}x\mathrm{d}y$$

$$\mathbf{rot}\vec{\boldsymbol{A}} = (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\vec{\boldsymbol{i}} + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\vec{\boldsymbol{j}} + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\vec{\boldsymbol{k}}$$

Infinite Series

$$\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}|=\rho \quad ,R=\frac{1}{\rho}$$

$$S(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l})$$

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)\mathrm{d}x, \quad a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}\mathrm{d}x, \quad b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}\mathrm{d}x$$

$$S(x)=\begin{cases} f(x), x\in consecutive \\ \frac{f(x-0)+f(x+0)}{2}, x\in discontinuity \\ \frac{f(l-0)+f(l+0)}{2}, x=\{-l,l\} \end{cases}$$

Traditional Probability Theory

$$A-B=A\bar{B}=A\cap\bar{B}\Rightarrow P(A-B)=P(A)-P(AB)$$

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}},x\in(-\infty,+\infty)$$

$$\Phi(x)=\int_{-\infty}^{x}f(t)\mathrm{d}t,X\sim N(0,1),\quad \Phi(a)+\Phi(-a)=1,\quad F(x)=\Phi(\frac{x-\mu}{\sigma})$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)},f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}$$

$$(X,Y)\sim U(D),f(x,y)=\begin{cases}\frac{1}{S_D},(x,y)\in D\\ 0,others\end{cases}$$

$$(X,Y)\sim N(\mu_1,\mu_2;\sigma_1,\sigma_2,\rho),f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\mathrm{e}^{-\frac{1}{2(1-\rho^2)}[(\frac{x-\mu_1}{\sigma_1})^2-\frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+(\frac{y-\mu_2}{\sigma_2})^2]}$$

Variable Digital Properties

Distr	Mark	EX	DX
Bin	B(n,p)	np	np(1-p)
Poi	P(λ)	λ	λ
Geo	G(p)	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hyp	H(n,M,N)	$\frac{nM}{N}$	$\frac{nM}{N}(1-\frac{M}{N})(\frac{N-n}{N-1})$
Uni	U(a,b)	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp	$E(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Nor	$N(\mu,\sigma^2)$	μ	σ^2

Large Number Law Central Limit Theorem

$$P\{|X-E(X)|\geq\epsilon\}\leq\frac{D(X)}{\epsilon^2}$$

$$\lim_{n\to\infty}P\{|\frac{X_n}{n}-p|<\epsilon\}=1$$

$$\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{k=1}^{n}x_k-\mu|<\epsilon\}=1$$

$$\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{k=1}^{n}X_k-\frac{1}{n}\sum_{k=1}^{n}E(X_k)|<\epsilon\}=1$$

$$\sum_{i=1}^{n}X_i\sim N(n\mu,n\sigma^2),\lim_{n\to\infty}P\{\frac{\sum_{i=1}^{n}X_i-n\mu}{\sqrt{n}\sigma}\leq x\}=F_n(x)=\Phi(x)$$

$$X\sim N(np,np(1-p)),\lim_{n\to\infty}P\{\frac{X_n-np}{\sqrt{np(1-p)}}\leq x\}=\Phi(x)$$

Mathematical Statistics

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}), \frac{\bar{X}-u}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}X_i^2 - n\bar{X}^2]$$

$$E(S^2)=\sigma^2, D(S^2)=\frac{2\sigma^4}{n-1}$$

$$\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2 \sim \chi^2(n), E(\chi^2(n))=n, D(\chi^2(n))=2n$$

$$T \sim t(n), T=\frac{X}{\sqrt{Y/n}}\frac{\sim}{\sim}\frac{N(0,1)}{\sqrt{\chi^2(n)/n}}$$

$$F \sim F(n1,n2), F=\frac{X/n1}{Y/n2}\frac{\sim}{\sim}\frac{\chi^2(n1)/n1}{\chi^2(n2)/n2}$$

$$(X,Y) \sim N(\mu_1,\mu_2;\sigma_1^2,\sigma_2^2,0)$$

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2}/(n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2}/(n_2-1)} \sim F(n_1-1,n_2-1)$$

$$\frac{\frac{\sum_{i=1}^{n_1}(X_i-\mu_1)^2}{\sigma_1^2}/n_1}{\frac{\sum_{i=1}^{n_2}(X_i-\mu_2)^2}{\sigma_2^2}/n2} \sim F(n1,n2)$$

$$\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n2}}} \sim N(0,1)$$

$$\exists\sigma_1=\sigma_2, \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{S_\omega\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

$$S_\omega=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$$

Constant Series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} = -\frac{\pi}{4}$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \quad \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90} \quad \sum_{k=1}^{\infty} \frac{1}{k^6} = \frac{\pi^6}{945}$$

$$\sum_{i=0}^{n} a_i \cdot \sum_{j=0}^{n} b_j = \sum_{i=0}^{n} \sum_{j=0}^{n} (a_i \cdot b_j) \quad \sum_{n=s}^{t} \ln f(n) = \ln \prod_{n=s}^{t} f(n)$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n} i)^2 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i} = \sum_{i=0}^{\infty} \frac{1}{(2i+1)(2i+2)} = \sum_{i=1}^{\infty} \frac{1}{2^i i} = \sum_{n=1}^{\infty} (\frac{1}{3^i} + \frac{1}{4^i}) \frac{1}{i} = \ln 2$$

Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \ln(1+x)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x)$$

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan x$$

$$\sum_{n=0}^{\infty} (n+1) x^n = \frac{1}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) x^n = \frac{2}{(1-x)^3}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n} = -\ln(1-x^2)$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = \frac{1}{2}\ln(\frac{1+x}{1-x})$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sin x$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos x$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \mathrm{e}^x$$

Transcendental Equation

$$\sum_{i=0}^{n} a_i x^i = 0 \Longrightarrow \prod_{i=0}^{n}(x - x_i) = 0$$

$$\prod_{i=0}^{n} x_i = (-1)^n \frac{a_0}{a_n}, \quad \sum_{i=0}^{n} \frac{\prod_{i=0}^{n} x_i}{x_i} = (-1)^{n-1} \frac{a_1}{a_n}$$

$$\sum_{i=0}^{n} \frac{1}{x_i} = -\frac{a_1}{a_0}$$

eg. $\tan x = x \Longrightarrow \sin x = \cos x \cdot x$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + ... = x \cdot (1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + ...)$$

$$\frac{1}{3}x^3 - \frac{1}{30}x^5 + ... = 0 \quad \xrightarrow{x \neq 0} \quad \frac{1}{3}x^2 - \frac{1}{30}x^4 + ... = 0$$

$$\sum_{i=0}^{n} \frac{1}{t_i} = \frac{1}{10} \quad \xrightarrow{t = x^2} \quad \lim_{n \to \infty} \sum_{i=0}^{n} \frac{1}{x_i^2} = \frac{1}{10}$$

Beyond Integral

$$\sum_{n=0}^{\infty}\frac{1}{(3n)!}\Longrightarrow S(x)=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!},S(1)=\frac{\mathrm{e}}{3}+\frac{2}{3}\cos\frac{\sqrt{3}}{2}\mathrm{e}^{-\frac{1}{2}}$$

$$\iint_{D_{xy}}(\mathrm{e}^{x}\cos y)\mathrm{d}\sigma, D=\{(x,y)|x^2+y^2\leq 1\}$$

$$\iint_{D_{xy}}(x+y)\mathrm{d}\sigma, D=\{(x,y)|y^2\leq x+2, x^2\leq y+2\}$$

$$\boldsymbol{A_0}=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sum_{m=0}^{\infty}\frac{1}{n2^m+1}$$

$$\boldsymbol{A_1}=\int_0^{x^2}\pi(\sqrt[4]{1+t}-1)\sin t^4\mathrm{d}x$$

$$\boldsymbol{A_2}=\sum_{n=1}^{\infty}\frac{((n-1)!)^2(2t)^{2n}}{(2n)!}$$

$$\boldsymbol{A_3}=\int_0^1\frac{(1-2x)\ln(1-x)}{x^2-x+1}\mathrm{d}x$$

$$\boldsymbol{A_4}=x^2(x-\tan x)\ln(x^2+1)[(\frac{2\arctan\frac{y}{x}}{\pi})^y-1]$$

$$\lim_{x\to 0^+}\lim_{y\to+\infty}\frac{\boldsymbol{A_0A_1}}{\boldsymbol{A_2A_3A_4}}=\frac{27}{32}$$

$$\int\frac{\sec^3 x}{1-\tan^6 x}\mathrm{d}x$$

$$\int\frac{1}{\csc x+\sec x+\tan x+\cot x}\mathrm{d}x$$

$$\lim_{N\to\infty}\sum_{n=1}^{N}\sum_{k=1}^{n}\frac{(-1)^{k-1}}{k}-\ln 2=\ln 2-\frac{1}{2}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1-\cos\frac{\pi}{\sqrt{n}}}{1+\cos\frac{i\pi}{\sqrt{2n}}}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{[\sqrt{n}]}}{n}$$

$$t\frac{\mathrm{d}^3x}{\mathrm{d}t^3}+3\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-t\frac{\mathrm{d}x}{\mathrm{d}t}-x=0$$