Amazing Integral Sum Limit

$$\lim_{n\to\infty}\sum_{k=0}^{n}\sum_{i=1}^{4}\frac{1}{(ik)!}$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}\Longrightarrow S_1(x)=\sum_{n=0}^{\infty}\frac{x^n}{n!},\quad S_1(1)=\mathrm{e}$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n)!}\Longrightarrow S_2(x)=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!},\quad S_2(1)=\frac{\mathrm{e}^{-1}+\mathrm{e}}{2}$$

$$\sum_{n=0}^{\infty}\frac{1}{(3n)!}\Longrightarrow S_3(x)=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!},S_3(1)=\frac{\mathrm{e}}{3}+\frac{2}{3}\cos\left(\frac{\sqrt{3}}{2}\right)\mathrm{e}^{-\frac{1}{2}}$$

$$\sum_{n=0}^{\infty}\frac{1}{(4n)!}\Longrightarrow S_4(x)=\sum_{n=0}^{\infty}\frac{x^{4n}}{(4n)!},S_4(1)=\frac{\mathrm{e}+\mathrm{e}^{-1}}{4}+\frac{\cos 1}{2}$$

$$\lim_{n\to\infty}\sum_{k=0}^{n}\sum_{i=0}^{4}\frac{1}{(ik)!}=\frac{25}{12}\mathrm{e}+\frac{3}{4}\mathrm{e}^{-1}+\frac{2}{3}\cos\left(\frac{\sqrt{3}}{2}\right)\mathrm{e}^{-\frac{1}{2}}+\frac{\cos 1}{2}$$

$$\iint_{D_{xy}}(x+y)\mathrm{d}\sigma, D=\{(x,y)|y^2\le x+2, x^2\le y+2\}$$

$$\mathbf{A_0}=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sum_{m=0}^{\infty}\frac{1}{n2^m+1}$$

$$\mathbf{A_1}=\int_0^{x^2}\pi(\sqrt[4]{1+t}-1)\sin t^4\mathrm{d}x$$

$$\mathbf{A_2}=\sum_{n=1}^{\infty}\frac{((n-1)!)^2(2t)^{2n}}{(2n)!}$$

$$\mathbf{A_3}=\int_0^1\frac{(1-2x)\ln(1-x)}{x^2-x+1}\mathrm{d}x$$

$$\mathbf{A_4}=x^2(x-\tan x)\ln(x^2+1)[\left(\frac{2\arctan\frac{y}{x}}{\pi}\right)^y-1]$$

$$\lim_{x\to 0^+}\lim_{y\to+\infty}\frac{\mathbf{A_0}\mathbf{A_1}}{\mathbf{A_2}\mathbf{A_3}\mathbf{A_4}}=\frac{27}{32}$$

$$\int \frac{\sec^3 x}{1-\tan^6 x}\mathrm{d}x$$

$$\int \frac{1}{\csc x+\sec x+\tan x+\cot x}\mathrm{d}x$$

$$\lim_{x\to 0}\frac{-2\sin x+\arctan x-\frac{x^3\cos x}{3}+\frac{\ln\left(\frac{1+x}{1-x}\right)}{2}}{(\mathrm{e}^x-1)^2\sum_{n=0}^{\infty}\left(1-\frac{1}{x^2}\right)^n D(\chi^2(x))}=\frac{11}{40}$$

$$\lim_{N\to\infty}\sum_{n=1}^{N}\sum_{k=1}^{n}\frac{(-1)^{k-1}}{k}-\ln 2=\ln 2-\frac{1}{2}$$

$$\iint_D \mathrm{e}^x\cos y\mathrm{d}\sigma, D=\{(x,y)|x^2+y^2\le 1\}$$

$$\iint_D \mathrm{e}^{-y^2}\mathrm{d}x\mathrm{d}y, D=\{(x,y)|x<y<1,0<x<1\}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1-\cos\frac{\pi}{\sqrt{n}}}{1+\cos\frac{i\pi}{\sqrt{2n}}}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{[\sqrt{n}]}}{n}\qquad\sum_{n=1}^{\infty}\frac{n!}{(2n+1)!!}\frac{1}{(n+1)}$$

$$t\frac{\mathrm{d}^3x}{\mathrm{d}t^3}+3\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-t\frac{\mathrm{d}x}{\mathrm{d}t}-x=0$$

$$X\sim F(x)\quad is\quad Consecutive\quad Variable:\int_{-\infty}^{+\infty}[F(x+a)-F(x)]\mathrm{d}x=a$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{1+4n^2}=\frac{\pi}{2}\frac{1}{\mathrm{e}^{\frac{\pi}{2}}-\mathrm{e}^{-\frac{\pi}{2}}}-\frac{1}{2}$$

$$\mathbf{L} = \mathbf{L_1} + \mathbf{L_2} + \mathbf{L_3} + \mathbf{L_4}$$

$$\mathbf{L_1} : x \in (0,1), y = 0 \quad \mathbf{L_2} : y \in (0,1), x = 1 \quad \mathbf{L_3} : x \in (1,0), y = 1 \quad \mathbf{L_4} : y \in (1,0), x = 0$$

$$I_1 = \oint_L -xyf'_x(x,y)\mathrm{d}x + xyf'_y(x,y)\mathrm{d}y = \iint_D \left(\frac{\partial \left(xyf'_y(x,y)\right)}{\partial x} - \frac{\partial \left(-xyf'_x(x,y)\right)}{\partial y} \right) \mathrm{d}x\mathrm{d}y$$

$$= 2\iint_D xyf''_{xy}(x,y)\mathrm{d}x\mathrm{d}y + \iint_D \left(xf'_x(x,y) + yf'_y(x,y)\right)\mathrm{d}x\mathrm{d}y$$

$$I_1 = \oint_L -xyf'_x(x,y)\mathrm{d}x + xyf'_y(x,y)\mathrm{d}y = \int_{L_1} + \int_{L_2} + \int_{L_3} + \int_{L_4}$$

$$= 0 + \int_0^1 yf'_y(1,y)\mathrm{d}y + \int_1^0 -xf'_x(x,1)\mathrm{d}x + 0$$

$$= yf(1,y)|_0^1 - \int_0^1 f(1,y)\mathrm{d}y + xf(x,1)|_0^1 - \int_0^1 f(x,1)\mathrm{d}x$$

$$= 0 + 0 + 0 + 0 = 0$$

$$I_2 = \iint_D xf'_x(x,y) + yf'_y(x,y)\mathrm{d}x\mathrm{d}y = I_3 - I_4$$

$$I_3 = \oint_L -yf(x,y)\mathrm{d}x + xf(x,y)\mathrm{d}y = \iint_D \left(xf'_x(x,y) + yf'_y(x,y)\right)\mathrm{d}x\mathrm{d}y + 2\iint_D f(x,y)\mathrm{d}x\mathrm{d}y$$

$$I_3 = \oint_L -yf(x,y)\mathrm{d}x + xf(x,y)\mathrm{d}y = \int_{L_1} + \int_{L_2} + \int_{L_3} + \int_{L_4}$$

$$= 0 + \int_0^1 f(1,y)\mathrm{d}y + \int_1^0 -f(x,1)\mathrm{d}x + 0 = 0 + 0 + 0 + 0 = 0$$

$$I_4 = 2\iint_D f(x,y)\mathrm{d}x\mathrm{d}y = 2a$$

$$I = \frac{1}{2}I_1 - \frac{1}{2}I_2 = \frac{1}{2}I_1 - \frac{1}{2}(I_3 - I_4) = 0 - \frac{1}{2}(0 - 2a) = a$$

$$\frac{1}{\log_a^3 x}+\frac{1}{\log_b^3 x}+\frac{1}{\log_c^3 x}=\frac{3}{\log_a x\log_b x\log_c x}$$
$$(\log_x a)^3+(\log_x b)^3+(\log_x c)^3=3(\log_x a)(\log_x b)(\log_x c)$$
$$\aleph \quad \log_x a=m,\log_x b=n,\log_x c=p\to m^3+n^3+p^3=3mnp$$
$$\log_a x\log_b x\log_c x\neq 0\longrightarrow mnp\neq 0$$
$$\exists p\neq 0\to (\frac{m}{p})^3+(\frac{n}{p})^3+1=3\frac{mn}{p^2}$$
$$\aleph \quad \frac{m}{p}=A,\frac{n}{p}=B\to A^3+B^3-3AB+1=0$$
$$\aleph \quad f(A,B)=A^3+B^3-3AB+1=0$$
$$f'_A(A,B)=3(A^2-B),f'_BA,B=3(B^2-A)$$
$$f''_{AA}(A,B)=6A,f''_{BB}(A,B)=6B,f''_{AB}(A,B)=-3$$
$$\exists f'_A(A,B)=f'_B(A,B)=0\longrightarrow A=B=1,f''_{AA}(A,B)f''_{BB}(1,1)>(f''_{AB}(1,1))^2$$
$$\exists only \quad A=B=1\in\mathbf{R^2},f(A,B)=0\longrightarrow f(A,B)=A^3+B^3-3AB+1=0$$
$$so \quad \exists m=n=p=1\longrightarrow \log_4\left(\frac{a+b}{c}\right)=\frac{1}{2}$$

$$X \sim N(0,1) \quad E(X^{2k}) = (2k-1)!! \quad E(X^{2k-1}) = 0$$

$$X \sim N(0,\sigma^2), Y \sim N(0,2\sigma^2)$$

$$\frac{X}{\sigma} \sim N(0,1) \frac{Y}{\sqrt{2}\sigma} \sim N(0,1)$$

My Method

$$E(\left(\frac{X}{\sigma}\right)^4) = 3, E(\left(\frac{X}{\sigma}\right)^2) = 1$$

$$E(\left(\frac{Y}{\sqrt{2}\sigma}\right)^4) = 3, E(\left(\frac{Y}{\sqrt{2}\sigma}\right)^2) = 1$$

$$E(X^4) = 3\sigma^4, E(X^2) = \sigma^2, D(X^2) = 2\sigma^4$$

$$E(Y^4) = 12\sigma^4, E(Y^2) = 2\sigma^2, D(Y^2) = 8\sigma^4$$

$$D(\hat{\sigma}^2) = D(\frac{1}{2n}\sum_{i=1}^{n} X_i^2 + \frac{1}{4n}\sum_{i=1}^{n} Y_i^2) = \frac{1}{4n^2}\sum_{i=1}^{n} D(X_i^2) + \frac{1}{16n^2}\sum_{i=1}^{n} D(Y_i^2) = \frac{\sigma^4}{n}$$

AnswerMethod

$$(\frac{X}{\sigma})^2 \sim \chi^2(1), (\frac{Y}{\sqrt{2}\sigma})^2 \sim \chi^2(1)$$

$$D(X^2) = 2\sigma^4, D(Y^2) = 8\sigma^4$$

$$D(\hat{\sigma}^2) = D(\frac{1}{2n}\sum_{i=1}^{n} X_i^2 + \frac{1}{4n}\sum_{i=1}^{n} Y_i^2) = \frac{1}{4n^2}\sum_{i=1}^{n} D(X_i^2) + \frac{1}{16n^2}\sum_{i=1}^{n} D(Y_i^2) = \frac{\sigma^4}{n}$$

$$\lim_{x\to+\infty}\int_x^{2x} \sin\frac{1}{x+t}\mathrm{d}t = \lim_{x\to+\infty}\int_{2x}^{3x} \sin\frac{1}{u}\mathrm{d}u \quad \xrightarrow{\sin\frac{1}{u} \sim \frac{1}{u} + o(\frac{1}{u})} = \ln(\frac{3}{2})$$

$$\mathrm{e}^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} = \sum_{k=0}^{\infty}\left(\frac{x^{2k-1}}{(2k-1)!} + \frac{x^{2k}}{(2k)!}\right) = \sum_{k=0}^{\infty}\frac{x^{2k-1}}{(2k-1)!} + \sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}$$

$$\begin{aligned}
\lim_{n\to\infty}\sum_{i=1}^{4}\sum_{k=0}^{n}\frac{1}{(ik)!} &= \sum_{k=0}^{n}\frac{1}{(k)!} + \sum_{k=0}^{n}\frac{1}{(2k)!} + \sum_{k=0}^{n}\frac{1}{(3k)!} + \sum_{k=0}^{n}\frac{1}{(4k)!} \\
&= \mathrm{e} + \frac{\mathrm{e}+\mathrm{e}^{-1}}{2} + \frac{\mathrm{e}}{3} + \frac{2}{3}\cos\frac{\sqrt{3}}{2}\mathrm{e}^{-\frac{1}{2}} + \frac{\mathrm{e}+\mathrm{e}^{-1}}{4} + \frac{\cos 1}{2} \\
&= \frac{2}{3}\cos\frac{\sqrt{3}}{2}\mathrm{e}^{-\frac{1}{2}} + \frac{\cos 1}{2} + \frac{3}{4}\mathrm{e}^{-1} + \frac{25}{12}\mathrm{e}
\end{aligned}$$