$$\lim_{n\to\infty} n\frac{\left(\sum_{k=1}^{n}\sqrt{k}\right)^2}{\left(\sum_{k=1}^{n}\sqrt[3]{k}\right)^3} + \sum_{k=1}^{n-1}\left[\ln\left(1+\frac{1}{n+k}\right)\sin\left(\ln\left(1+\frac{k}{n}\right)\right)\right] =$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\right)^2}{\left(\frac{1}{n}\sum_{k=1}^{n}\sqrt[3]{\frac{k}{n}}\right)^3} + \frac{1}{n}\sum_{k=1}^{n-1}\frac{1}{1+\frac{k}{n}}\sin\left(\ln\left(1+\frac{k}{n}\right)\right) =$$

$$\lim_{n\to\infty}\frac{\left(\int_0^1 x^{\frac{1}{2}}\mathrm{d}x\right)^2}{\left(\int_0^1 x^{\frac{1}{3}}\mathrm{d}x\right)^3} + \int_0^1 \frac{\sin(\ln(1+x))}{1+x}\mathrm{d}x = \frac{\frac{4}{9}}{\frac{27}{64}} + \int_0^1 \sin(\ln(1+x))\mathrm{d}(\ln(x+1)) =$$

$$\frac{256}{243} - \cos u|_0^{\ln 2} = \frac{499}{243} - \cos(\ln 2)$$

$$p(x) = \frac{\mathrm{d}^n}{\mathrm{d}x^n}(1-x^m)^n = \left(\sum_{k=0}^{n} C_n^k(-x^m)^k\right)^{(n)} = \left(1 + C_n^1(-x^m) + C_n^2(-x^m)^2 + ...\right)^{(n)}$$

$$\exists k_0 \in \{1,2,3,...\}, mk_0 > n$$

$$\int_0^{\frac{\pi}{2}} \ln\left[\left(\sin^2 x + 99\cos^2 x\right)\left(999\sin^2 x + \cos^2 x\right)\right]\mathrm{d}x$$

$$\begin{cases} x^2 + y^2 = z \\ y = x\tan z \end{cases} = \begin{cases} x = \sqrt{\theta}\cos\theta \\ y = \sqrt{\theta}\sin\theta \\ z = \theta \end{cases}, \theta \in (0,c)$$

$$s = \int \mathrm{d}s = \int_0^c \sqrt{\left(\frac{\cos\theta}{2\sqrt{\theta}} - \sqrt{\theta}\sin\theta\right)^2 + \left(\frac{\sin\theta}{2\sqrt{\theta}} + \sqrt{\theta}\cos\theta\right)^2 + 1}\mathrm{d}\theta =$$

$$\int_0^c \sqrt{\frac{1}{4\theta} + \theta + 1}\mathrm{d}\theta = \int_0^c \frac{2\theta+1}{2\sqrt{\theta}}\mathrm{d}\theta = \frac{2}{3}\theta^{\frac{3}{2}} + \theta^{\frac{1}{2}}|_0^c = \frac{2}{3}c^{\frac{3}{2}} + c^{\frac{1}{2}} = \sqrt{c}\left(\frac{2c}{3} + 1\right)$$

$$f(x) = \sec x, x \in (-\frac{\pi}{4}, \frac{\pi}{4})$$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos 4nx$$

$$a_0 = \frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\mathrm{d}x = \frac{8}{\pi}\ln|\sec x + \tan x||_0^{\frac{\pi}{4}} = \frac{8}{\pi}\ln(\sqrt{2}+1)$$

$$a_n = \frac{8}{\pi}\int_0^{\frac{\pi}{4}}\sec x\cos 4nx\mathrm{d}x = \frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\frac{\mathrm{d}(\sin 4nx)}{\cos x} = \frac{4\sin 4nx}{n\pi\cos x}|_0^{\frac{\pi}{4}} - \frac{4}{n\pi}\int_0^{\frac{\pi}{4}}\sin 4nx\sec x\tan x\mathrm{d}x =$$

$$2m = n, \frac{x^2}{2} - \frac{\sqrt{2}xy}{m} + \frac{3y^2}{2m} = 1$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{2}}{2m} \\ \frac{-\sqrt{2}}{2m} & \frac{3}{2m} \end{bmatrix} \quad (x, y)^T = Q(u, v)^T$$

$$|\lambda E - A| = (\lambda - \frac{1}{2})(\lambda - \frac{3}{2m}) - \frac{1}{2m^2} = \lambda^2 - \frac{m+3}{2m}\lambda + \frac{3m-2}{4m^2} = 0$$

$$\lambda = \frac{\frac{m+3}{2m} \pm \sqrt{\left(\frac{m+3}{2m}\right)^2 - 4\frac{3m-2}{4m^2}}}{2} \notin Z$$

$$\lim_{x \to 0} \frac{\int_0^x t \cos t \mathrm{d}t - 1 + \cos x}{\sqrt{1 + x \tan x} - \sqrt{1 + x \sin x}} =$$

$$\lim_{x \to 0} \frac{2(\int_0^x t \cos t \mathrm{d}t - 1 + \cos x)}{x(\tan x - \sin x)} = \lim_{x \to 0} \frac{2(x \cos x - \sin x)}{2x^3} = -\frac{1}{3}$$

$$\sum_{n=0}^{\infty} \frac{n^2 + 1}{(\frac{1}{2})^n n!} x^n = \sum_{n=0}^{\infty} \frac{n^2 (2x)^n}{n!} + \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \lim_{n \to \infty} 2\frac{(n+1)^2 + 1}{(n^2+1)(n+1)} = 0, x \in (-\infty, +\infty)$$

$$S(x) = \sum_{n=1}^{\infty} \frac{n(2x)^n}{(n-1)!} + \mathrm{e}^{2x} = \sum_{n=0}^{\infty} \frac{(n+1)(2x)^{n+1}}{n!} + \mathrm{e}^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^{n+2}}{n!} + \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n!} + \mathrm{e}^{2x}$$

$$= (4x^2 + 2x + 1)\mathrm{e}^{2x}, x \in (-\infty, +\infty)$$

$$\begin{cases} x + y - z = \mathrm{e}^z \\ x\mathrm{e}^x = \tan t \\ y = \cos t \end{cases}$$

$$x'\mathrm{e}^x + x\mathrm{e}^x x' = \sec t \to \mathrm{e}^x(x' + x'x) = \sec t, \mathrm{e}^x(x' + x'x + x'' + x''x + x'x') = \sec t \tan t$$

$$y' = -\sin t, y'' = -\cos t$$

$$x' + y' - z' = \mathrm{e}^z z' \to z' = \frac{x' + y'}{\mathrm{e}^z + 1}$$

$$t = 0, x = 0, y = 1, z = 0, x' = 1, y' = 0, z' = \frac{1}{2}$$

$$1 + 0 + x'' + 0 + 1 = 0, x'' = -2, y'' = -1$$

$$z'' = \frac{(x'' + y'')(\mathrm{e}^z + 1) - (x' + y')(\mathrm{e}^z z')}{(\mathrm{e}^z + 1)^2} = \frac{-3*2 - \frac{1}{2}}{4} = -\frac{13}{8}$$

$$\iint_D r^2 \sin\theta \sqrt{1 - r^2 \cos(2\theta)} \mathrm{d}\sigma, D = \{(r, \theta), 0 \le r \le \sec\theta, 0 \le \theta \le \frac{\pi}{4}\}$$

$$D = \{(x, y), 0 \le y \le x, 0 \le x \le 1\}$$

$$\iint_D y\sqrt{1 - x^2 + y^2} \mathrm{d}\sigma = \frac{1}{2}\int_0^1 \int_{1-x^2}^{x^2 + 1 - x^2} \sqrt{1 - x^2 + y^2} \mathrm{d}(y^2 + 1 - x^2) \mathrm{d}x = \frac{1}{3}\int_0^1 u^{\frac{3}{2}}|_{1-x^2}^1 \mathrm{d}x$$

$$= \frac{1}{3}\int_0^1 1 - (1 - x^2)^{\frac{3}{2}} \mathrm{d}x = \frac{1}{3} - \int_0^{\frac{\pi}{2}} \cos^4 t \mathrm{d}t = \frac{1}{3} - \frac{1*3*1*\pi}{3*4*2*2} = \frac{1}{3} - \frac{\pi}{16}$$

$$u = ax^2 + by^2 + cz^2, x + y + z = 1, (ax^2 + by^2 + cz^2)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \ge (x + y + z)^2 = 1$$

$$\min_{x>0, y>0, z>0} u(x, y, z) = \frac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$a_n = \int_0^1 \frac{x^{n-1}}{1+x} \mathrm{d}x = \int_0^1 x^{n-1} \mathrm{d}(\ln(x+1)) = x^{n-1}\ln(x+1)|_0^1 - (n-1)\int_0^1 \ln(x+1)x^{n-2}\mathrm{d}x$$

$$a_n = \frac{1}{n}\int_0^1 \frac{\mathrm{d}(x^n)}{1+x} = \frac{x^n}{n(1+x)}|_0^1 + \int_0^1 \frac{x^n \mathrm{d}x}{n(1+x)^2} = \frac{1}{2n} + \int_0^1 \frac{\mathrm{d}(x^{n+1})}{(1+x)^2 n(n+1)} =$$

$$\frac{1}{2n} + \frac{x^{n+1}}{(1+x)^2 n(n+1)}|_0^1 + \int_0^1 \frac{2x^{n+1}\mathrm{d}x}{(n+1)(1+x)^3} = \frac{1}{2n} + \frac{1}{4n^2} + o(\frac{1}{n^2})mn \to \infty$$

$$I_1 = \iint_S \frac{xz}{a^2}\mathrm{d}y\mathrm{d}z + \frac{yz}{b^2}\mathrm{d}z\mathrm{d}x + \frac{z^2}{c^2}\mathrm{d}x\mathrm{d}y$$

$$I_2 = 0, S_1 = \{(x,y,z), \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1, z = 0\}, down0$$

$$D_{xy} = \{(x,y), \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\}$$

$$I_1 + I_2 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\iiint_\Omega z\mathrm{d}v = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2}{2}\iint_{D_{xy}}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\mathrm{d}\sigma$$

$$\frac{x}{a} = u, \frac{y}{b} = v, J = ab$$

$$I_1 + I_2 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2}{2}ab\iint_{D_{uv}}(1 - u^2 - v^2)\mathrm{d}\sigma$$

$$\iint_{D_{uv}}(1 - u^2 - v^2)\mathrm{d}\sigma = \int_0^1 (1 - r^2)r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta = \frac{\pi}{2}$$

$$I_1 = \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}\right)\frac{c^2\pi}{4}ab$$

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[n]{3}}{2}\right)^n$$

$$\lim_{x\to+\infty}\left((x+1)^a-x^a\right)=a$$

$$\sum_{n=0}^{\infty}x^n=\frac{1}{1-x},\sum_{n=0}^{\infty}(n+1)x^n=\frac{1}{(1-x)^2}$$

$$\lim_{n\to\infty}\frac{(n+1)^2}{n^2}=1,x\in(-1,1),x\neq\pm1$$

$$\sum_{n=1}^{\infty}n^2x^{n-1}=\left(\sum_{n=1}^{\infty}nx^n\right)'=\left(\frac{1}{(1-x)^2}-1-\left(\frac{1}{1-x}-1\right)\right)'=\frac{2}{(1-x)^3}-\frac{1}{(1-x)^2}$$

$$f(x)=\ln(1+x)+axe^{-x},x_1\in(-1,0),x_2\in(0,+\infty)$$

$$f(x),x\in(-1,+\infty)$$

$$f'(x)=\frac{1}{x+1}+a(1-x)e^{-x}=\frac{1+a(1-x^2)e^{-x}}{x+1}$$

$$g(x)=1+a(1-x^2)e^{-x},x\in(-1,+\infty)$$

$$g'(x)=a(x^2-2x-1)e^{-x}=a(x-1+\sqrt{2})(x-1-\sqrt{2})e^{-x}$$

$$\exists a=0,g(x)\equiv1f'(x)>0,f(x)\uparrow,a\neq0$$

$$\exists a<0,g'(x)\leq0,x\in(-1,+\infty),g(x)\downarrow$$

$$\exists a>0,g'(x)\geq0,x\in(-1,+\infty),g(x)\uparrow$$

$$g(-1)=1,g(0)=1+a,g(+\infty)=1$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\frac{i}{n}}=\int_0^1\frac{1}{1+x}dx=\ln(x+1)|_0^1=\ln2$$

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$$\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x}-\cot x\right)=\lim_{x\to 0}\frac{\tan x-x}{x^3}=\frac{1}{3}$$

$$\lim_{n\to\infty}\tan n, n=k\pi \lim=0, n\to\left(k\pi+\frac{\pi}{2}\right)^{-}, \lim=+\infty, n\to\left(k\pi+\frac{\pi}{2}\right)^{+}, \lim=-\infty$$

$$\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x), \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=\ln 2$$

$$\oint_L \frac{1}{x}\arctan\frac{y}{x}dx+\frac{2}{y}\arctan\frac{x}{y}dy=\iint_D\frac{2}{y}\frac{\frac{1}{y}}{\left(\frac{x}{y}\right)^2+1}-\frac{1}{x}\frac{\frac{1}{x}}{\left(\frac{y}{x}\right)^2+1}d\sigma=\iint_D\frac{1}{x^2+y^2}d\sigma$$

$$=\int_1^2\frac{1}{r}dr\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}d\theta=\frac{\pi\ln 2}{12}$$

$$\vec{n}=\left(\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right), dS=\sqrt{(-1)^2+1^2+1}dxdy=\sqrt{3}dxdy$$

$$\iint_{\Sigma}(x-y+z)\frac{1}{\sqrt{3}}dS=S_{xy}=\frac{1}{2}$$

$$\iiint_{\Omega}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)dv=\frac{1}{3}\iiint_{\Omega}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}+\frac{x^2}{c^2}+\frac{z^2}{a^2}+\frac{x^2}{b^2}+\frac{y^2}{c^2}\right)dv$$

$$\frac{1}{3}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\int_0^1 r^4 dr\int_0^{\pi}\sin\phi d\phi\int_0^{2\pi}d\theta=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\frac{1*1*2*2\pi}{3*5}=\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\frac{4\pi}{15}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\sin\frac{i\pi}{n}}{n+i}=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{\sin\left(\pi\frac{i}{n}\right)}{1+\frac{i}{n}}=\int_0^1\frac{\sin(\pi x)}{1+x}dx$$

$$S(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n+2}}{3n+2}, S(0)=0$$

$$S'(x)=\sum_{n=0}^{\infty}(-1)^n x^{3n+1}=x\sum_{n=0}^{\infty}(-x^3)^n=\frac{x}{1+x^3}$$

$$S(x)=\int_0^x S(t)dt+S(0)=\int_0^x\frac{t}{(t+1)(t^2-t+1)}dt+0=\frac{1}{3}\int_0^x-\frac{1}{t+1}+\frac{t+1}{t^2-t+1}dt$$

$$At^2-At+A+Bt^2+Ct+Bt+C\to A+B=0, B+C-A=1, A+C=0, A=-\frac{1}{3}, B=C=\frac{1}{3}$$

$$S(x)=-\frac{\ln(x+1)}{3}+\frac{1}{3}\int_0^x\frac{1}{2}\frac{d(t^2-t+1)}{t^2-t+1}+\frac{1}{\sqrt{3}}\frac{d(t-\frac{1}{2})(\frac{2}{\sqrt{3}})}{[(t-\frac{1}{2})(\frac{2}{\sqrt{3}})]^2+1}$$

$$=-\frac{\ln(x+1)}{3}+\frac{\ln(t^2-t+1)}{6}+\frac{1}{\sqrt{3}}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)+C|_0^x$$

$$=-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{1}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{\pi}{6\sqrt{3}}$$

$$S(1)=\sum_{n=0}^{\infty}\frac{(-1)^n}{3n+2}=-\frac{\ln 2}{3}+\frac{\pi\sqrt{3}}{9}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt[7]{k(n-k+1)}}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{\frac{k}{n}-\frac{k^2}{n^2}+\frac{k}{n^2}}}=\int_0^1\frac{dx}{\sqrt{x-x^2}}=\pi$$

$$I=\int\sec^3 xdx=\int\sec x(\tan^2 x+1)dx=\int\tan xd(\sec x)+\ln|\sec x+\tan x|$$

$$=\ln|\sec x+\tan x|+\sec x\tan x-\int\sec^3 xdx$$

$$\int\sec^3 xdx=\frac{1}{2}\left(\ln|\sec x+\tan x|+\sec x\tan x\right)+C$$

$$y(x) = \sqrt{x + \sqrt[3]{x + \sqrt[4]{x}}}, \ln y(x) = \frac{1}{2} \ln \left(x + g(x)\right)$$

$$\frac{y'(x)}{y(x)} = \frac{1}{2} \frac{1 + g'(x)}{x + g(x)}$$

$$g(x) = \sqrt[3]{x + \sqrt[4]{x}}, \ln \left(g(x)\right) = \frac{1}{3} \ln \left(x + x^{\frac{1}{4}}\right)$$

$$\frac{g'(x)}{g(x)} = \frac{1}{3} \frac{1 + \frac{1}{4} x^{-\frac{3}{4}}}{x + x^{\frac{1}{4}}}$$

$$g'(x) = \frac{4 x^{\frac{3}{4}} + 1}{12(x + x^{\frac{7}{4}})} g(x) = \frac{4 x^{\frac{3}{4}} + 1}{12(x + x^{\frac{1}{4}})^{\frac{2}{3}} x^{\frac{3}{4}}}$$

$$y'(x) = \frac{1 + g'(x)}{2(x + g(x))} y(x) = \frac{1 + \frac{4 x^{\frac{3}{4}} + 1}{12(x + x^{\frac{1}{4}})^{\frac{2}{3}} x^{\frac{3}{4}}}}{2(x + (x + x^{\frac{1}{4}})^{\frac{1}{3}})^{\frac{1}{2}}}$$

$$= \frac{12(x + x^{\frac{1}{4}})^{\frac{2}{3}} x^{\frac{3}{4}} + 4 x^{\frac{3}{4}} + 1}{24(x + x^{\frac{1}{4}})^{\frac{2}{3}} x^{\frac{3}{4}} (x + (x + x^{\frac{1}{4}})^{\frac{1}{3}})^{\frac{1}{2}}}$$

$$I = \lim_{x \to +\infty} y(x) - x^{\frac{1}{2}} = \lim_{x \to +\infty} \frac{\sqrt[3]{x + \sqrt[4]{x}}}{\sqrt{x + \sqrt[3]{x + \sqrt[4]{x}}} + \sqrt{x}} = 0$$

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1 + \left(\frac{i}{n}\right)^2} = \frac{\pi}{4}$$

$$\lim_{x \to 1} \frac{x^x - x}{\ln x - x + 1} = \lim_{x \to 1} \frac{x^{x-1} - 1}{\ln(x - 1 + 1) - x + 1} = \lim_{t \to 0} \frac{(t + 1)^t - 1}{\ln(t + 1) - t}$$

$$= \lim_{t \to 0} \frac{e^{t \ln(t+1)} - 1}{-\frac{t^2}{2} + o(t^2)} = -2$$

$$\int_0^{+\infty} (x + 1) e^{-x^2} dx = -\frac{1}{2} \int_0^{+\infty} e^{-x^2} d(-x^2) + \frac{\sqrt{\pi}}{2} = \frac{1 + \sqrt{\pi}}{2}$$

$$X_1 = X_2 \sim N(0,1), F_{X_1}(x) = F_{X_2}(x) = \Phi(x)$$

$$F(x,y) = P\{X_1 \leq x, Y \leq y\} = P\{X_1 \leq x, X_3 X_1 + (1 - X_3)X_2 \leq y\}$$

$$= P\{X_1 \leq x, \{X_3 X_1 + (1 - X_3)X_2 \leq y\}, X_3 = 0\} +$$

$$P\{X_1 \leq x, \{X_3 X_1 + (1 - X_3)X_2 \leq y\}, X_3 = 1\}$$

$$= P\{X_1 \leq x, X_2 \leq y\}P\{X_3 = 0\} + P\{X_1 \leq x, X_1 \leq y\}P\{X_3 = 1\}$$

$$= \frac{1}{2}\left(P\{X_1 \leq x\}P\{X_2 \leq y\} + P\{X_1 \leq x, X_1 \leq y\}\right)$$

$$\exists x < y, \quad F(x,y) = \frac{1}{2}\left(\Phi(x)\Phi(y) + \Phi(x)\right)$$

$$\exists x \geq y, \quad F(x,y) = \frac{1}{2}\left(\Phi(x)\Phi(y) + \Phi(y)\right)$$

$$F_Y(y) = P\{Y \leq y\} = P\{X_3 X_1 + (1 - X_3)X_2 \leq y\} = \frac{1}{2}\left(P\{X_2 \leq y\} + P\{X_1 \leq y\}\right)$$

$$= \frac{1}{2}\left(\Phi(y) + \Phi(y)\right) = \Phi(y), Y \sim N(0,1)$$

$$D = \{(x,y)|x^2 + y^2 \leq 1\}, \alpha^2 + \beta^2 = 1$$

$$I = \iint_D \frac{\mathrm{d}\sigma}{(1 - \alpha x + \beta y)^2 + (\beta x + \alpha y)^2} = \iint_D \frac{\mathrm{d}\sigma}{(x - \alpha)^2 + (y + \beta)^2}$$

$$\begin{cases} x - \alpha = u \\ y + \beta = v \end{cases}$$

$$D_{uv} = \{(u,v)|(u + \alpha)^2 + (v - \beta)^2 \leq 1\} \rightarrow D_{uv} = \{(u,v)|r^2 + 2\alpha r \cos\theta - 2\beta r \sin\theta \leq 0\}$$

$$I = \iint_{D_{uv}} \frac{1}{r} \mathrm{d}r\mathrm{d}\theta = \int_{\sqrt{\alpha^2 + \beta^2} - 1}^{2(\beta \sin\theta - \alpha \cos\theta)} \frac{1}{r} \mathrm{d}r \int_?^? \mathrm{d}\theta$$

$$\int_{-2}^2 x^3 \cos\frac{x}{2}\sqrt{4 - x^2} + \frac{1}{2}\sqrt{4 - x^2}\mathrm{d}x = \frac{1}{2*2}\pi 4 = \pi \approx 3.1415926$$

$$I = \int \frac{\sqrt{\mathrm{e}^{2x} + 1}}{\mathrm{e}^x + 1}\mathrm{d}x$$

$$\mathrm{e}^x + 1 = u, \mathrm{e}^{2x} + 2\mathrm{e}^x + 1 = u^2, \mathrm{e}^{2x} + 1 = u^2 - 2(u - 1),$$

$$x = \ln(u - 1), \mathrm{d}x = \frac{\mathrm{d}u}{u - 1}$$

$$I = \int \frac{\sqrt{u^2 - 2u + 2}\mathrm{d}u}{u(u - 1)} = \int \frac{\sqrt{(u - 1)^2 + 1}}{u - 1} - \frac{\sqrt{u^2 - 2u + 2}}{u}\mathrm{d}u$$

$$= \int \frac{\sqrt{(u - 1)^2 + 1}}{u - 1}\mathrm{d}(u - 1) - \int \frac{\sqrt{(u - 1)^2 + 1}}{u}\mathrm{d}u$$

$$v = u - 1, I = I_1 - I_2 = \int \frac{\sqrt{v^2 + 1}}{v}\mathrm{d}v - \int \frac{\sqrt{v^2 + 1}}{v + 1}\mathrm{d}v$$

$$v = \tan t, I_1 = \int \frac{\sec^3 t\mathrm{d}t}{\tan t} = \int \frac{1}{\cos^2 x \sin x}\mathrm{d}t$$

$$I_2 =$$

$$f(x,y) = Ae^{-2x^2+2xy-y^2}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\mathrm{d}x\mathrm{d}y = 1$$

$$A\int_{-\infty}^{+\infty} e^{-2(x^2-xy+\frac{y^2}{4}-\frac{y^2}{4})-y^2}\mathrm{d}x \int_{-\infty}^{+\infty}\mathrm{d}y = A\int_{-\infty}^{+\infty} e^{-2(x-\frac{y}{2})^2}\mathrm{d}x \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2}}\mathrm{d}y$$

$$\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x = 1, \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}}\mathrm{d}x = \sqrt{2\pi}, \int_{-\infty}^{+\infty} e^{-2u^2}\mathrm{d}u = \sqrt{\frac{\pi}{2}}$$

$$A = \frac{1}{\pi}$$

$$\lim_{x\to 0^+} \tan\left(2x^2\arctan\frac{1}{5x}\right) \sim \tan\left(2x^2\frac{\pi}{2}\right) \sim \pi x^2 + o(x^2)$$

$$x \to 0^+, \arctan 5x \sim 5x, x \to 0^+, \arctan\left(\frac{1}{5x}\right) = \frac{\pi}{2}$$

$$I = \int \frac{1}{\sqrt{x}+1}\mathrm{d}x, \sqrt{x} = t, x = t^2, \mathrm{d}x = 2t\mathrm{d}t$$

$$I = 2\int \frac{t+1-1}{t+1}\mathrm{d}t = 2t - 2\ln(t+1) + C = 2\sqrt{x} - 2\ln(\sqrt{x}+1) + C$$

$$y = x^{2022}, y^{(1)} = 2022x^{2021}...y^{(2022)} = 2022!x^0, y^{(2023)} = 0$$

$$y = \frac{x^{1+x}}{(1+x)^x}, x > 0$$

$$k = \lim_{x\to+\infty}\frac{y}{x} = \lim_{x\to+\infty}\frac{x^x}{(1+x)^x} = \frac{1}{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x} = \frac{1}{e}$$

$$\lim_{x\to+\infty} e - \left(1+\frac{1}{x}\right)^x = \lim_{x\to+\infty} e - e^{x\ln\left(1+\frac{1}{x}\right)} = \lim_{x\to+\infty} e\left(1 - e^{x\ln\left(1+\frac{1}{x}\right)-1}\right)$$

$$= \lim_{x\to+\infty} e\left(1 - x\ln\left(1+\frac{1}{x}\right)\right) = \lim_{x\to+\infty} e\left(1 - x\left(\frac{1}{x} - \frac{1}{2x^2} + o(\frac{1}{x^2})\right)\right) = \frac{e}{2x} + o(\frac{1}{x})$$

$$b = \lim_{x\to+\infty}(y - kx) = \lim_{x\to+\infty}\left(\frac{x^{1+x}}{(1+x)^x} - \frac{x}{e}\right) = \lim_{x\to+\infty}\frac{x}{e}\left(\frac{ex^x - (1+x)^x}{(1+x)^x}\right)$$

$$= \lim_{x\to+\infty}\frac{x}{e}\frac{e - \left(1+\frac{1}{x}\right)^x}{\left(1+\frac{1}{x}\right)^x} = \lim_{x\to+\infty}\frac{x\frac{e}{2x}}{e^2} = \frac{1}{2e}$$

$$y = \frac{x}{e} + \frac{1}{2e}$$

$$A = \int_0^4 f(x)\mathrm{d}x, f(x) = \sin 2x + 3A$$

$$A = \int_0^4 (\sin 2x + 3A)\mathrm{d}x$$

$$A = -\frac{1}{2}\cos 2x|_0^4 + 12A \to \int_0^4 f(x)\mathrm{d}x = A = \frac{\cos 8 - 1}{22}$$

$$tx^2 = u, x = \sqrt{\frac{u}{t}}, \mathrm{d}x = \frac{1}{2\sqrt{ut}}\mathrm{d}u$$

$$\lim_{t \to 0} \frac{\int_0^t \sin(tx^2)\mathrm{d}x}{t^4} = \lim_{t \to 0} \frac{\int_0^{t^3} \frac{\sin u}{\sqrt{u}}\mathrm{d}u}{2t^{\frac{9}{2}}} = \lim_{t \to 0} \frac{3t^2 \sin t^3}{9t^{\frac{7}{2}}t^{\frac{3}{2}}} = \frac{1}{3}$$

$$\lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \to \infty} \frac{n}{n+2} = 1, x \in (-1,1)$$

$$S(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n} - \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n+1}$$

$$= (-x)\sum_{n=1}^{\infty} \frac{(-x)^n}{n} - \sum_{n=2}^{\infty} \frac{(-x)^n}{n} = x\ln(x+1) - (-\ln(x+1) - (-x)) = (x+1)\ln(x+1) - x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x), \sum_{n=1}^{\infty} \frac{(-x)^n}{n} = -\ln(1+x)$$

$$x = -1, S(x) = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1$$

$$x = 1, S(x) = 2\ln 2 - 1$$

$$S(x) = (x+1)\ln(x+1) - x, x \in [-1,1]$$

$$\lim_{n \to \infty} n^2 \left[1 - \frac{\pi}{2n} \sum_{k=1}^{n} \sin \frac{(2k-1)\pi}{4n} \right]$$

$$\sin \frac{(2k-1)\pi}{4n} = \sin \frac{\pi}{2} \left(\frac{k-1}{2n} + \frac{k}{2n} \right)$$

$$I_{n(k)} = \lim_{n \to \infty} \frac{\pi}{2} \left(\frac{k}{n} - \frac{k-1}{n} \right) \sum_{k=1}^{n} \sin(\frac{\pi}{2}\frac{k-1+k}{2n}) = \frac{\pi}{2} \int_0^1 \sin(\frac{\pi x}{2})\mathrm{d}x = 1$$

$$I_{n(k)} = 1 + \frac{\pi^2}{96n^2} + o(\frac{1}{n^2})$$

$$\int_0^1 \left(\ln \frac{1}{x} \right)^5 \mathrm{d}x = -\int_0^1 (\ln x)^5 \mathrm{d}x = -x(\ln x)^5|_0^1 + 5\int_0^1 (\ln x)^4 \mathrm{d}x = 5x(\ln x)^4|_0^1 - 5\int_0^1 4(\ln x)^3 \mathrm{d}x$$

$$= 20x(\ln x)^3|_0^1 - 20\int_0^1 3(\ln x)^2 \mathrm{d}x = -60x(\ln x)^2|_0^1 + 60\int_0^1 2\ln x\mathrm{d}x = -120\int_0^1 \mathrm{d}x = -120$$

$$\lim_{x \to -1} \frac{x^3 - ax^2 - x + 4}{x+1} = \lim_{x \to -1} \frac{(x+1)^3 - 3x^2 - 3x - 1 - ax^2 - x + 4}{x+1}$$

$$= \lim_{x \to -1} \frac{(x+1)^3 - (3+a)(x+1)^2 + 2(a+1)(x+1) + 4 - a}{x+1} = \lim_{t \to 0} \frac{t^3 - (3+a)t^2 + 2(a+1)t + 4 - a}{t}$$

$$2a + 2 = l, 4 - a = 0 \to a = 4, l = 10$$

$$\lim_{x \to 0} \frac{\sqrt{1 + x \sin x} - \sqrt{\cos x}}{x \tan x} = \lim_{x \to 0} \frac{1 + x \sin x - \cos x}{(\sqrt{1 + x \sin x} + \sqrt{\cos x}) x \tan x}$$

$$= \lim_{x \to 0} \frac{1 + x(x + o(x)) - \left(1 - \frac{1}{2}x^2 + o(x^2)\right)}{2x \tan x} = \frac{3}{4}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n+1}}{2n} = x \sum_{n=1}^{\infty} \frac{x^{2n}}{2n} = x \int_0^x \sum_{n=1}^{\infty} t^{2n-1} dt = x \int_0^x \frac{t}{1 - t^2} dt$$

$$= \frac{x}{2} \int_0^x \frac{1}{1 - t} - \frac{1}{1 + t} dt = -\frac{x}{2} \ln(1 - x^2), x \in (-1, 1)$$

$$x = \tan u, dx = \sec^2 u \, du$$

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{x^2 + 1}} dx = \int_0^{\frac{\pi}{3}} \frac{\sec^2 u \, du}{\sec u} = \ln|\sec u + \tan u| \Big|_0^{\frac{\pi}{3}} = \ln(2 + \sqrt{3})$$

$$\int x f''(x) dx = \int x d(f'(x)) = x f'(x) - \int f'(x) dx + C =$$

$$x \left(\frac{1 - \ln x}{x^2}\right) - \frac{\ln x}{x} + C = \frac{1 - 2 \ln x}{x} + C$$

$$\textbf{\textit{Consistent \quad Continuity}}$$

$$f(x), x \in I$$

$$\forall \epsilon > 0, \exists \delta = \delta(\epsilon) > 0, \forall x_1, x_2 \in I, |x_1 - x_2| < \delta, |f(x_1) - f(x_2)| < \epsilon$$

$$\forall \epsilon > 0, \exists \delta > 0, |x - x_0| < \delta, |f(x) - f(x_0)| < \epsilon$$

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{a}+\sqrt[n]{b}+\sqrt[n]{c}}{3}\right)^n = \mathrm{e}^{n\ln\left(\frac{a^{\frac{1}{n}}-1+b^{\frac{1}{n}}-1+c^{\frac{1}{n}}-1}{3}+1\right)} = \mathrm{e}^{n\frac{(a^{\frac{1}{n}}-1+b^{\frac{1}{n}}-1+c^{\frac{1}{n}}-1)}{3}} =$$

$$\mathrm{e}^{\ln a^{\frac{1}{3}}+\ln b^{\frac{1}{3}}+\ln c^{\frac{1}{3}}} = \sqrt[3]{abc}$$

$$\lim_{x\to0}\frac{x\ln(1+x)-ax^2+bx^3}{x-\tan x} = \lim_{x\to0}\frac{x(x-\frac{1}{2}x^2+\frac{1}{3}x^3+o(x^3))-ax^2+bx^3}{-\frac{1}{3}x^3+o(x^3)} = 0$$

$$1-a=0, b-\frac{1}{2}==0 \to a=1, b=\frac{1}{2}$$

$$\iint_{\Sigma} x\mathrm{d}y\mathrm{d}z+y\mathrm{d}z\mathrm{d}x+z\mathrm{d}x\mathrm{d}y = 3\iiint_{\Omega}\mathrm{d}v = 3\iint_D\int_{\sqrt{x^2+y^2}}^{\sqrt{R^2-x^2-y^2}}\mathrm{d}z\mathrm{d}\sigma$$

$$= 3\iint_D(\sqrt{R^2-x^2-y^2}-\sqrt{x^2+y^2})\mathrm{d}\sigma, \quad D=\{(x,y)|x^2+y^2\le\frac{R^2}{2}\}$$

$$= 3\int_0^{\frac{R}{\sqrt{2}}}(\sqrt{R^2-r^2}-r)r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta = 6\pi\left(-\frac{1}{2}\int_{R^2}^{\frac{R^2}{2}}\sqrt{R^2-r^2}\mathrm{d}(R^2-r^2)-\frac{R^3}{6\sqrt{2}}\right)$$

$$= 6\pi\left(\frac{1}{3}u^{\frac{3}{2}}|_{\frac{R^2}{2}}^{R^2}-\frac{R^3}{6\sqrt{2}}\right) = 2\pi R^3\left(1-\frac{1}{2\sqrt{2}}\right)-\pi R^3\frac{1}{\sqrt{2}} = (2-\sqrt{2})\pi R^3$$

$$\lim_{\Delta x \to 0} \frac{\ln(x + \Delta x) - \ln x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\ln\left(1 + \frac{\Delta x}{x}\right)}{\Delta x} = \frac{1}{x}$$

$$\lim_{\Delta x \to 0} \frac{\tan(x + \Delta x) - \tan x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{\tan x + \tan \Delta x}{1 - \tan x \tan \Delta x} - \tan x}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\tan^2 x + 1)\tan \Delta x}{\Delta x} = \sec x$$

$$\lim_{\Delta x \to 0} \frac{\cos(x + \Delta x) - \cos x}{\Delta x} = -\sin x$$

$$\lim_{\Delta x \to 0} \frac{\sec(x + \Delta x) - \sec x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos x - \cos(x + \Delta x)}{\Delta x \cos(x + \Delta x)\cos x} = \tan x \sec x$$

$$\lim_{\Delta x \to 0} \frac{\log_a(x + \Delta x) - \log_a x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\log_a(1 + \frac{\Delta x}{x})\log_e a}{\Delta x \ln a} = \frac{1}{x \ln a}$$

$$\lim_{\Delta x \to 0} \frac{\arctan(x + \Delta x) - \arctan x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\arctan(\frac{\Delta x}{1 + x^2 + x\Delta x})}{\Delta x} = \frac{1}{1 + x^2}$$

$$\lim_{\Delta x \to 0} \frac{\arcsin(x + \Delta x) - \arcsin x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\arcsin((x + \Delta x)\sqrt{1 - x^2} - x\sqrt{1 - (x + \Delta x)^2})}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)\sqrt{1 - x^2} - x\sqrt{1 - (x + \Delta x)^2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{x(\sqrt{1 - x^2} - \sqrt{1 - (x + \Delta x)^2})}{\Delta x} + \sqrt{1 - x^2}$$

$$= \lim_{\Delta x \to 0} \frac{x(2x\Delta x + \Delta^2 x)}{\Delta x(\sqrt{1 - x^2} + \sqrt{1 - (x + \Delta x)^2})} + \sqrt{1 - x^2} = \frac{x^2}{\sqrt{1 - x^2}} + \sqrt{1 - x^2} = \frac{1}{\sqrt{1 - x^2}}$$

$$\lim_{x \to 0^+} \frac{x^2 \sin\frac{1}{x} - 0}{x - 0} = \lim_{x \to 0^+} x \sin\frac{1}{x} = (x + o(x)) * M = 0, |M| <= 1$$

$$f'(x) = 2x \sin\frac{1}{x} + x^2(\frac{-1}{x^2})\cos\frac{1}{x} = 2x \sin\frac{1}{x} - \cos\frac{1}{x}, (x \neq 0)$$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} 2x \sin\frac{1}{x} - \cos\frac{1}{x} = 0 - \nexists = \nexists$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{f(x)}{x^2} * x^2 = A * \lim_{x \to 0} x^2 = 0$$

$$\lim_{x \to 0} f(x) = f(0) = 0$$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{f(x)}{x} = f(0)$$

$$\lim_{x \to 0} \frac{1 - \cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}}{x^2}$$

$$x \to 0, \ln(1 + x) = x - \frac{x^2}{2} + o(x^2), \ln(1 + 2x) = 2x - \frac{4x^2}{2} + o(x^2)$$

$$\lim_{x \to 0} \frac{xf(x) - \ln(1 + 2x)}{x - \ln(1 + x)} = \lim_{x \to 0} \frac{xf(x) - 2x + 2x^2 + o(x^2)}{\frac{x^2}{2} + o(x^2)} = 4$$

$$x \to 0, f(x) = 2 + o(1), f'(0) = 0$$

$$f(x) = \frac{1}{x} - \frac{1}{e^x - 1}, \lim_{x \to 0} \frac{f(x) - \lim_{x \to 0} f(x)}{x} = ?$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{e^x - 1 - x}{x(e^x - 1)} = \lim_{x \to 0} \frac{1 + x + \frac{x^2}{2} + o(x^2) - 1 - x}{x^2 + o(x^2)} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{f(x) - \frac{1}{2}}{x}$$

$$= \lim_{x \to 0} \frac{2e^x - 2 - 2x - xe^x + x}{2x^2(e^x - 1)} = \lim_{x \to 0} \frac{2(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)) - 2 - 2x - x(1 + x + \frac{x^2}{2} + o(x^2)) + x}{2x^3 + o(x^3)}$$

$$= \lim_{x \to 0} \frac{-\frac{1}{6}x^3 + o(x^3)}{2x^3 + o(x^3)} = -\frac{1}{12}$$

$$\lim_{x \to 0} \frac{\tan \tan x - \sin \sin x}{x^3} = \lim_{x \to 0} \frac{\tan \tan x - \tan \sin x}{x^3} + \frac{\tan \sin x - \sin \sin x}{x^3} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\lim_{x\to+\infty} \sqrt{x^2+x+1}\frac{x-\ln(e^x+x)}{x} \sim \frac{\infty*\infty}{\infty} = \lim_{x\to+\infty} \sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\ln\left(\frac{e^x}{e^x+x}\right) = 1*0 = 0$$

$$2\sin(x+2y-3z) = x+2y-3z$$

$$2\cos(x+2y-3z)(1-3z_x') = 1-3z_x'$$

$$2\cos(x+2y-3z)(2-3z_y') = 2-3z_y'$$

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y} = \frac{1}{3}+\frac{2}{3} = 1$$

$$F(x) = \int_0^x f(t)\mathrm{d}t$$

$$\lim_{x\to0}\frac{2\int_0^x(x-t)f(t)\mathrm{d}t}{x\int_0^x f(x-t)\mathrm{d}t} = \lim_{x\to0}\frac{2x\int_0^x f(t)\mathrm{d}t-2\int_0^x tf(t)\mathrm{d}t}{x\int_0^x f(u)\mathrm{d}u} = \lim_{x\to0}\frac{2F(x)+2xf(x)-2xf(x)}{F(x)+xf(x)}$$

$$= \lim_{x\to0,\xi\to0}\frac{2xf(\xi)}{xf(\xi)+xf(x)} = 1$$

$$ABA^{-1} = BA^{-1}+3I, AB = B+3A, (A-I)B = 3A, B = 3(A-I)^{-1}A$$

$$A^*A = |A|I, |A^*| = 16, |A| = 4, A^* = 4A^{-1}, A = 4\left(A^*\right)^{-1}$$

$$A = diag(4,\frac{1}{4},4), B = 3diag(\frac{1}{3},-\frac{4}{3},\frac{1}{3})diag(4,\frac{1}{4},4) = diag(4,-1,4)$$

$$\int \frac{x^2}{1+x^2}\arctan x\mathrm{d}x = \int\left(1-\frac{1}{1+x^2}\right)\arctan x\mathrm{d}x = \int\arctan x\mathrm{d}x - \int\arctan x\mathrm{d}(\arctan x)$$

$$= x\arctan x - \int\frac{x}{1+x^2}\mathrm{d}x - \frac{1}{2}(\arctan x)^2 = x\arctan x - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctan x)^2 + C$$

$$\int \frac{\mathrm{d}x}{\mathrm{e}^x - \mathrm{e}^{-x}} = \int \frac{\mathrm{d}\mathrm{e}^x}{\mathrm{e}^{2x} - 1} = \frac{1}{2}\left(\int \frac{\mathrm{d}(\mathrm{e}^x - 1)}{\mathrm{e}^x - 1} - \int \frac{\mathrm{d}(\mathrm{e}^x + 1)}{\mathrm{e}^x + 1}\right) = \frac{1}{2}\ln\left|\frac{\mathrm{e}^x - 1}{\mathrm{e}^x + 1}\right| + C$$

$$\int \frac{x}{(1-x)^3}\mathrm{d}x = \int \frac{1-(1-x)}{(1-x)^3}\mathrm{d}x = \int \frac{1}{(1-x)^3} - \frac{1}{(1-x)^2}\mathrm{d}x = \frac{1}{2(1-x)^2} - \frac{1}{1-x} + C$$

$$\int \frac{x^2}{a^6 - x^6}\mathrm{d}x = \int \frac{\mathrm{d}x^3}{3(a^3 - x^3)(a^3 + x^3)} = \int -\frac{\mathrm{d}(a^3 - x^3)}{6a^3(a^3 - x^3)} + \int \frac{\mathrm{d}(a^3 + x^3)}{6a^3(a^3 + x^3)} = \frac{1}{6a^3}\ln\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

$$\int \frac{1 + \cos x}{x + \sin x}\mathrm{d}x = \ln|x + \sin x| + C$$

$$\int \frac{\ln(\ln x)}{x}\mathrm{d}x = \int \ln(\ln x)\mathrm{d}\ln x = \ln(\ln x)\ln x - \int \frac{\ln x}{x\ln x}\mathrm{d}x = \ln|\ln x|\ln x - \ln x + C$$

$$1 - 2\sin^2 x = \cos 2x, \sin^2 x = \frac{1 - \cos 2x}{2}, \sin^4 x = \frac{(1 - \cos 2x)^2}{4}$$

$$\int \frac{\sin x \cos x}{1 + \sin^4 x}\mathrm{d}x = \int \frac{\sin 2x}{2(1 + \frac{(1-\cos 2x)^2}{4})}\mathrm{d}x = \int \frac{1}{2}\frac{\mathrm{d}(\frac{1-\cos 2x}{2})}{1 + \left(\frac{(1-\cos 2x)}{2}\right)^2} = \frac{1}{2}\int \arctan\sin^2 x + C$$

$$\int \tan^4 x\mathrm{d}x = \int (\sec^2 x - 1)^2\mathrm{d}x = \int (\tan^2 x + 1)\mathrm{d}\tan x - 2\tan x + x = \frac{1}{3}\tan^2 x - \tan x + x + C$$

$$\cos(a+b) - \cos(a-b) = -2\sin a\sin b, \sin(a+b) + \sin(a-b) = 2\sin a\cos b$$

$$\int \sin x\sin 2x\sin 3x\mathrm{d}x = \frac{1}{2}\int (\cos x - \cos 3x)\sin 3x\mathrm{d}x = -\frac{1}{4}\int \sin 6x\mathrm{d}x + \frac{1}{4}\int (\sin 4x + \sin 2x)\mathrm{d}x$$

$$= \frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + C$$

$$\int \frac{\mathrm{d}x}{x(x^6 + 4)} = \frac{1}{4}\int \frac{1}{x} - \frac{x^5}{x^6 + 4}\mathrm{d}x = \frac{1}{4}\ln x - \frac{1}{24}\ln(x^6 + 4) + C$$

$$a > 0, \int \sqrt{\frac{a+x}{a-x}}\mathrm{d}x = \int \frac{a+x}{\sqrt{a^2 - x^2}}\mathrm{d}x = \int \frac{a\mathrm{d}\frac{x}{a}}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} - \frac{\mathrm{d}(a^2 - x^2)}{2\sqrt{a^2 - x^2}} = a\arcsin\frac{x}{a} - \sqrt{a^2 - x^2} + C$$

$$(\ln(x + \sqrt{x^2 - 1}))' = \frac{1 + \frac{2x}{2\sqrt{x^2-1}}}{x + \sqrt{x^2 - 1}} = \frac{1}{\sqrt{x^2 - 1}}$$

$$\int \frac{\mathrm{d}x}{\sqrt{x(1+x)}} = \int \frac{\mathrm{d}(2x+1)}{\sqrt{(2x+1)^2 - 1}} = \ln(2x + 1 + \sqrt{(2x+1)^2 - 1}) + C = \ln|2x + \sqrt{4x^2 + 4x} + 1| + C$$

$$2\cos^2 x - 1 = \cos 2x, \cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\frac{1}{2} \int x(\cos 2x + 1)\mathrm{d}x = \frac{1}{4} \int x \mathrm{d} \sin 2x + \frac{x^2}{4} = \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4} + C$$

$$I = \int \mathrm{e}^{ax} \cos bx \mathrm{d}x = \frac{1}{a} \int \cos bx \mathrm{d}(\mathrm{e}^{ax}) = \frac{\cos bx \mathrm{e}^{ax}}{a} + \frac{b}{a} \int \mathrm{e}^{ax} \sin bx \mathrm{d}x = A(x, a, b) + \frac{b}{a^2} \int \sin bx \mathrm{d}(\mathrm{e}^{ax})$$

$$= A(x, a, b) + \frac{b \sin bx \mathrm{e}^{ax}}{a^2} - \frac{b^2}{a^2} \int \mathrm{e}^{ax} \cos bx \mathrm{d}x = A(x, a, b) + B(x, a, b) - \frac{b^2}{a^2} I + C_1 = I$$

$$\frac{b^2 + a^2}{a^2} I = \frac{a\mathrm{e}^{ax} \cos bx + b\mathrm{e}^{ax} \sin bx}{a^2} + C_2, \quad I = \frac{\mathrm{e}^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C$$

$$I = \int \frac{\mathrm{d}x}{\sqrt{1 + \mathrm{e}^x}}, \sqrt{1 + \mathrm{e}^x} = u, x = \ln(u^2 - 1), \mathrm{d}x = \frac{2u\mathrm{d}u}{u^2 - 1}$$

$$I = \int \frac{2\mathrm{d}u}{(u+1)(u-1)} = \int \frac{\mathrm{d}u}{u - 1} - \int \frac{\mathrm{d}u}{u + 1} = \ln\left|\frac{u - 1}{u + 1}\right| + C = \ln\left|\frac{\sqrt{1 + \mathrm{e}^x} - 1}{\sqrt{1 + \mathrm{e}^x} + 1}\right| + C$$

$$I = \int \frac{\mathrm{d}x}{x^2\sqrt{x^2 - 1}}, x = \sec u, \mathrm{d}x = \sec u \tan u \mathrm{d}u, \cos u = \frac{1}{x}, \sin u = \frac{\sqrt{x^2 - 1}}{x}$$

$$I = \int \frac{\sec u \tan u \mathrm{d}u}{\sec^2 u \tan u} = \sin u + C = \frac{\sqrt{x^2 - 1}}{x} + C$$

$$I = \int \frac{\mathrm{d}x}{(a^2 - x^2)^{\frac{5}{2}}}, x = a \sin u, \mathrm{d}x = a \cos u \mathrm{d}u, \tan u = \frac{x}{\sqrt{a^2 - x^2}}$$

$$I = \int \frac{a \cos u \mathrm{d}u}{a^5 \cos^5 u} = \frac{1}{a^4} \int \sec^4 u \mathrm{d}u = \frac{1}{a^4} \left(\frac{1}{3} \tan^3 u + \tan u\right) + C = \frac{x^3}{3a^4(a^2 - x^2)^{\frac{3}{2}}} + \frac{x}{a^4\sqrt{a^2 - x^2}} + C$$

$$I = \int \frac{\mathrm{d}x}{x^4\sqrt{1 + x^2}}, x = \tan u, \mathrm{d}x = \sec^2 u \mathrm{d}u, (\csc x)' = -\csc x \cot x, 1 + \cot^2 x = \csc^2 x, \csc u = \frac{\sqrt{x^2 + 1}}{x}$$

$$I = \int \frac{\sec^2 u \mathrm{d}u}{\tan^4 u \sec u} = \int \frac{\cos^3 u \mathrm{d}u}{\sin^4 u} = \int \cot^3 u \csc u \mathrm{d}u = \int (1 - \csc^2 u)\mathrm{d}(\csc u) = \csc u - \frac{\csc^3 u}{3} + C$$

$$I = \frac{\sqrt{x^2 + 1}}{x} - \frac{(x^2 + 1)^{\frac{3}{2}}}{3x^3} + C$$

$$I = \int \sqrt{x} \sin \sqrt{x} \mathrm{d}x, \sqrt{x} = u, x = u^2, \mathrm{d}x = 2u\mathrm{d}u$$

$$I = \int 2u^2 \sin u \mathrm{d}u = \int -2u^2 \mathrm{d}(\cos u) = -2u^2 \cos u + \int 4u \mathrm{d} \sin u = -2u^2 \cos u + 4u \sin u + 4 \cos u + C$$

$$I = -2x \cos \sqrt{x} + 4\sqrt{x} \sin \sqrt{x} + 4 \cos \sqrt{x} + C$$

$$\int \ln(1 + x^2)\mathrm{d}x = x \ln(1 + x^2) - 2 \int \left(1 - \frac{1}{1 + x^2}\right) \mathrm{d}x = x \ln(1 + x^2) - 2x + 2 \arctan x + C$$

$$\int \frac{\sin^2 x}{\cos^3 x} \mathrm{d}x = \int \tan^2 x \sec x \mathrm{d}x = \int \tan x \mathrm{d} \sec x = \tan x \sec x - \int \sec^3 x \mathrm{d}x$$

$$= \frac{1}{2}(\tan x \sec x - \ln|\sec x + \tan x|) + C$$

$$I = \int \arctan \sqrt{x} \mathrm{d}x, \sqrt{x} = u, x = u^2, \mathrm{d}x = 2u\mathrm{d}u$$

$$I = \int 2u \arctan u \mathrm{d}u = \int \arctan u \mathrm{d}(u^2 + 1) = (u^2 + 1) \arctan u - u + C = (x + 1) \arctan \sqrt{x} - \sqrt{x} + C$$

$$\sin x = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}} = \frac{2 \tan \frac{x}{2}}{\tan^2 \frac{x}{2} + 1}, \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\tan \frac{x}{2} = u, x = 2 \arctan u, \mathrm{d}x = \frac{2\mathrm{d}u}{u^2 + 1}$$

$$I = \int \frac{\sqrt{1 + \cos x}}{\sin x} \mathrm{d}x = \int \frac{\sqrt{1 + \frac{1 - u^2}{1 + u^2}} \frac{2}{u^2 + 1}}{\frac{2u}{u^2 + 1}} \mathrm{d}u = \int \frac{\sqrt{2}}{u\sqrt{u^2 + 1}} \mathrm{d}u$$

$$u = \tan k, \cot k = \frac{1}{u}, \csc k = \frac{\sqrt{u^2 + 1}}{u}, \mathrm{d}u = \sec^2 k \mathrm{d}k$$

$$I = \sqrt{2} \int \csc k \mathrm{d}k = -\sqrt{2} \ln |\cot k + \csc k| + C = \sqrt{2} \ln \left| \frac{u}{\sqrt{u^2 + 1} + 1} \right| + C = \sqrt{2} \ln \left| \frac{\tan \frac{x}{2}}{\sqrt{\tan^2 \frac{x}{2} + 1} + 1} \right| + C$$