**Traditional Probability Theory**

**Opposition** : $P(A)+P(\bar{A})=1$ **Exclusive** : $A\cap B=\varnothing\Rightarrow P(AB)=0$

**Independent** : $P(AB)=P(A)P(B)$ **Equal** : $A=B\Rightarrow P(A)=P(B)$

$$A-B=A\bar{B}=A\cap\bar{B}\Rightarrow P(A-B)=P(A)-P(AB)$$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(BC)-P(AC)-P(AB)+P(ABC)$$

$$P(B|A)=\frac{P(AB)}{P(A)}\quad P(A_1A_2...A_n)=P(A_1)P(A_2|A_1)P(A_3|A_1A_2)...P(A_n|A_1...A_{n-1})$$

**Bayes** $P(B_j|A)=\dfrac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n}P(A|B_i)P(B_i)}$

**Random Variable Digital Properties**

| Distr | Mark | EX | DX | Addition |
|---|---|---|---|---|
| Bin | $B(n,p)$ | $np$ | $np(1-p)$ | $P\{X=k\}=\mathbf{C_n^k}(1-p)^{n-k}p^k$ |
| Poi | $\pi(\lambda)$ | $\lambda$ | $\lambda$ | $P\{X=k\}=\frac{\lambda^k}{k!}\mathrm{e}^{-\lambda},k=0,1,2,...$ |
| Geo | $G(p)$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ | $P\{X=k\}=(1-p)^{k-1}p,k=1,2,...$ |
| Hyp | $H(n,M,N)$ | $\frac{nM}{N}$ | $\frac{nM}{N}(1-\frac{M}{N})(\frac{N-n}{N-1})$ | $P\{X=i\}=\frac{\mathbf{C}_M^i\mathbf{C}_{N-M}^{n-i}}{\mathbf{C}_N^n}$ |
| Uni | $U(a,b)$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ | |
| Exp | $E(\lambda)$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | |
| Nor | $N(\mu,\sigma^2)$ | $\mu$ | $\sigma^2$ | |

$$\textbf{Uni}: f(x)=\begin{cases}\frac{1}{b-a}, a\le x\le b\\ 0, others\end{cases}\qquad F(x)=\begin{cases}0, x<a\\ \frac{x-a}{b-a}, a\le x<b\\ 1, x\ge b\end{cases}$$

$$\textbf{Exp}: f(x)=\begin{cases}\lambda \mathrm{e}^{-\lambda x}, x>0\\ 0, x\le 0\end{cases}\qquad F(x)=\begin{cases}1-\mathrm{e}^{-\lambda x}, x>0\\ 0, x\le 0\end{cases}$$

$$\textbf{Nor}: f(x)=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}, F(x)=\int_{-\infty}^{x} f(t)\mathrm{d}t, x\in(-\infty,+\infty)$$

$$f(\mu+x)=f(\mu-x), F(\mu+x)+F(\mu-x)=1, F(\mu)=\frac{1}{2}$$

$$X\sim N(0,1), \phi(x)=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2}}, \Phi(x)=\int_{-\infty}^{x}\phi(t)\mathrm{d}t$$

$$\phi(-x)=\phi(x),\quad \Phi(a)+\Phi(-a)=1, \Phi(0)=\frac{1}{2},\quad F(x)=\Phi\left(\frac{x-\mu}{\sigma}\right)$$

$$f(x,y)\ge 0\qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\mathrm{d}x\mathrm{d}y=1$$

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\mathrm{d}y>0\quad f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\mathrm{d}x>0$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}, f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}$$

$$\textbf{Discrete}: P\{Z=g(x_i,y_j)\}=P\{X=x_i, Y=y_j\}=P_{ij}$$

$$F_Z(z)=P\{Z\le z\}=P\{g(X,Y)\le z\}=\sum_{g(x_i,y_j)\le z} P\{X=x_i, Y=y_j\}$$

$$\textbf{Continuous}: Z=g(X,Y)\quad F_Z(z)=P\{Z\le z\}=P\{g(X,Y)\le z\}=\iint_{g(x,y)\le z} f(x,y)\mathrm{d}x\mathrm{d}y$$

$$Z=\max(X,Y), F_{\max}(z)=F_X(z)F_Y(z)\quad Z=\min(X,Y), F_{\min}(z)=1-[1-F_X(z)][1-F_Y(z)]$$

$$Z_1=\max(X,Y)=\frac{X+Y+|X-Y|}{2}\quad Z_2=\min(X,Y)=\frac{X+Y-|X-Y|}{2}\quad Z_1Z_2=XY$$

$$(X,Y)\sim U(D), f(x,y)=\begin{cases}\frac{1}{S_D}, (x,y)\in D\\ 0, others\end{cases}$$

$$(X,Y)\sim N(\mu_1,\mu_2;\sigma_1,\sigma_2,\rho), f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\mathrm{e}^{-\frac{1}{2(1-\rho^2)}[(\frac{x-\mu_1}{\sigma_1})^2-\frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+(\frac{y-\mu_2}{\sigma_2})^2]}$$

$$F(x) = P\{X \le x\} = \int_{-\infty}^{x} f(x)\mathrm{d}x$$

$$F(x,y) = P\{X \le x, Y \le y\} = \int_{-\infty}^{x}\int_{-\infty}^{y} f(u,v)\mathrm{d}v\mathrm{d}u$$

$$1 = \sum_{i=1}^{\infty} x_i p_i \quad 1 = \int_{-\infty}^{+\infty} f(x)\mathrm{d}x$$

$$E(X) = \sum_{i=1}^{\infty} x_i p_i \quad E(X) = \int_{-\infty}^{+\infty} x f(x)\mathrm{d}x$$

$$E[g(X)] = \sum_{i=1}^{\infty} g(x_i) p_i \quad E[g(X)] = \int_{-\infty}^{+\infty} g(x) f(x)\mathrm{d}x$$

$$E[g(X,Y)] = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} g(x_i,y_j) P_{ij} \quad E[g(X,Y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y) f(x,y)\mathrm{d}x\mathrm{d}y$$

$$\exists X \sim F(x), \forall X > 0, E(X) = \int_{0}^{+\infty} (1 - F(x))\mathrm{d}x = \int_{0}^{+\infty} P\{X > x\}\mathrm{d}x$$

$$E(C) = C \quad E(CX) = CE(X) \quad E(X+C) = E(X) + C \quad E(X+Y) = E(X) + E(Y)$$

$$D(X) = \sum_{i=1}^{n} [x_i - E(X)]^2 p_i \quad D(X) = \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x)\mathrm{d}x$$

$$D(X) = E(X^2) - [E(X)]^2 \quad D(C) = 0 \quad D(CX) = C^2 D(X) \quad D(X+C) = D(X)$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) \quad D(X \pm Y) = D(X) + D(Y) \pm 2Cov(X,Y)$$

$$Cov(X,Y) = Cov(Y,X) \quad Cov(X,X) = D(X) \quad Cov(X,c) = 0$$

$$Cov(aX,bY) = abCov(X,Y) \quad Cov(X_1 + X_2, Y) = Cov(X_1,Y) + Cov(X_2,Y)$$

$$\rho_{XY} = \frac{E(XY) - E(X)E(Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}$$

$$X \sim N(0,1) \quad E(X^{2k}) = (2k-1)!! \quad E(X^{2k-1}) = 0, k \in \{1,2,3,...\}$$

**Large Number Law Central Limit Theorem**

$$P\{|X - E(X)| \geq \epsilon\} \leq \frac{D(X)}{\epsilon^2}$$

$$\lim_{n\to\infty} P\{|\frac{X_n}{n} - p| < \epsilon\} = 1$$

$$\lim_{n\to\infty} P\{|\frac{1}{n}\sum_{k=1}^{n} X_k - \mu| < \epsilon\} = 1$$

$$\lim_{n\to\infty} P\{|\frac{1}{n}\sum_{k=1}^{n} X_k - \frac{1}{n}\sum_{k=1}^{n} E(X_k)| < \epsilon\} = 1$$

$$\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2), \lim_{n\to\infty} P\{\frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma} \leq x\} = F_n(x) = \Phi(x)$$

$$X \sim N(np, np(1-p)), \lim_{n\to\infty} P\{\frac{X_n - np}{\sqrt{np(1-p)}} \leq x\} = \Phi(x)$$

**Mathematical Statistics**

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}), \frac{\bar{X}-u}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}[\sum_{i=1}^{n} X_i^2 - n\bar{X}^2]$$

$$E(S^2) = \sigma^2, D(S^2) = \frac{2\sigma^4}{n-1}$$

$$\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2 \sim \chi^2(n), E(\chi^2(n)) = n, D(\chi^2(n)) = 2n$$

$$T \sim t(n), T = \frac{X}{\sqrt{Y/n}} \frac{\sim}{\sim} \frac{N(0,1)}{\sqrt{\chi^2(n)/n}}$$

$$F \sim F(n1,n2), F = \frac{X/n1}{Y/n2} \frac{\sim}{\sim} \frac{\chi^2(n1)/n1}{\chi^2(n2)/n2}$$

$$(X,Y) \sim N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2, 0)$$

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2}/(n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2}/(n_2-1)} \sim F(n_1-1, n_2-1)$$

$$\frac{\frac{\sum_{i=1}^{n_1}(X_i-\mu_1)^2}{\sigma_1^2}/n_1}{\frac{\sum_{i=1}^{n_2}(X_i-\mu_2)^2}{\sigma_2^2}/n2} \sim F(n1,n2)$$

$$\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n2}}} \sim N(0,1)$$

$$\exists \sigma_1 = \sigma_2, \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{S_\omega\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

$$S_\omega = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$$