FFT

$$A(x) = \sum_{i=0}^{n-1} a_i x^i, B(x) = \sum_{i=0}^{m-1} b_i x^i$$

$$C(x) = A(x) \cdot B(x) = \sum_{i=0}^{n+m-1} c_i x^i$$

$$\star \quad k = 2^{\lceil \log_2(n+m) \rceil}$$

Matrix Transform

$$(x_0.x_1, .., x_{k-1}), \quad \{(a_0, a_1, ..., a_{n-1})\} \quad \xrightarrow{A(x)} \quad (x_0, y_0), (x_1, y_1), ..., (x_{k-1}, y_{k-1})$$

$$(x_0.x_1, .., x_{k-1}), \quad \{(b_0, b_1, ..., b_{m-1})\} \quad \xrightarrow{B(x)} \quad (x_0, z_0), (x_1, z_1), ..., (x_{k-1}, z_{k-1})$$

$$\mathbf{X_k} = \begin{pmatrix} 1 & 1 & ... & 1 \\ x_0^1 & x_1^1 & ... & x_{k-1}^1 \\ x_0^2 & x_1^2 & ... & x_{k-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_0^{k-1} & x_1^{k-1} & ... & x_{k-1}^{k-1} \end{pmatrix} \quad \vec{\mathrm{a}} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} \quad \vec{\mathrm{b}} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{m-1} \end{pmatrix}$$

$$\mathrm{a}^{\mathrm{T}} \mathbf{X_n} = \mathrm{y}^{\mathrm{T}}, \mathrm{b}^{\mathrm{T}} \mathbf{X_m} = \mathrm{z}^{\mathrm{T}}$$

Matrix Inverse Transform

$$(x_0, y_0 \cdot z_0), (x_1, y_1 \cdot z_1), ..., (x_{k-1}, y_{k-1} \cdot z_{k-1}) \rightarrow C(x)\{(c_0, c_1, ..., c_{n+m-1})\}$$

$$\mathrm{t} = \mathrm{hadamard}(\mathrm{y}, \mathrm{z}), \quad \mathrm{c}^{\mathrm{T}} \mathbf{X_k} = \mathrm{t}^{\mathrm{T}}, \rightarrow \mathrm{c}^{\mathrm{T}} = \mathbf{X_k^{-1}} \mathrm{t}^{\mathrm{T}}$$

$$\text{Euler} \quad \text{formula}$$
$$\mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$$
$$\mathrm{e}^{2\pi i} = 1, \forall\theta, \mathrm{e}^{i(\theta+2\pi)} = \mathrm{e}^{i\theta}$$
$$\omega_n^k = \mathrm{e}^{\frac{2\pi i}{n}k}, \quad (\omega_n^k)^j = \mathrm{e}^{\frac{2\pi i}{n}kj} = \cos(\frac{2\pi}{n}kj) + i\sin(\frac{2\pi}{n}kj)$$
$$(1).\omega_n^{k+\frac{n}{2}} = \mathrm{e}^{\frac{2\pi i}{n}(k+\frac{n}{2})} = -\mathrm{e}^{\frac{2\pi i}{n}k} = -\omega_n^k$$
$$(2).\omega_{2n}^{2k} = \omega_n^k$$
$$(3).\omega_n^{-k} = 1 \cdot \mathrm{e}^{\frac{2\pi i}{n}(-k)} = \mathrm{e}^{\frac{2\pi i}{n}(n-k)} = \omega_n^{n-k}$$

$$\text{FFT}$$
$$\text{DFT}: \quad X_k = \sum_{j=0}^{n-1} x_j \mathrm{e}^{\frac{2\pi i}{n}kj}$$
$$\text{IDFT}: \quad x_j = \frac{1}{n}\sum_{k=0}^{n-1} X_k \mathrm{e}^{-\frac{2\pi i}{n}jk}$$
$$\omega = \mathrm{e}^{\frac{2\pi i}{N}}, N = n+m$$
$$c_j = f(j), C_k = C(\omega_k)$$
$$C_k = C(\omega_k) = \sum_{j=0}^{N-1} c_j(\omega^k)^j = \sum_{j=0}^{N-1} c_j \mathrm{e}^{\frac{2\pi i}{N}kj}$$
$$c_j = \frac{1}{N}\sum_{j=0}^{N-1} C_k \mathrm{e}^{-\frac{2\pi i}{N}jk}$$

$$\text{Div} \quad \text{and} \quad \text{conquer}$$
$$H(x) = \sum_{i=0}^{n-1} a_i x^i = \sum_{i=0}^{\frac{n}{2}-1} a_{2i}x^{2i} + x\left(\sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}x^{2i}\right)$$
$$H(x) = H_1(x^2) + xH_2(x^2)$$
$$A(\omega_n^k) = A_1(\omega_n^{2k}) + xA_2(\omega_n^{2k}) = A_1(\omega_{\frac{n}{2}}^k) + \omega_n^k A_2(\omega_{\frac{n}{2}}^k)$$
$$A(\omega_n^{k+\frac{n}{2}}) = A(-\omega_n^k) = A_1(w_{\frac{n}{2}}^k) - \omega_n^k A_2(\omega_{\frac{n}{2}}^k)$$

Strassen Matrix Multiply

$$A_{2^k \cdot 2^k} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_{(2^{k-1} \cdot 2^{k-1}) \cdot 4} \quad B_{2^k \cdot 2^k} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$S_1 = B_{12} - B_{22} \quad S_2 = A_{11} + A_{12}$$

$$S_3 = A_{21} + A_{22} \quad S_4 = B_{21} - B_{11}$$

$$S_5 = A_{11} + A_{22} \quad S_6 = B_{11} + B_{22}$$

$$S_7 = A_{12} - A_{22} \quad S_8 = B_{21} + B_{22}$$

$$S_9 = A_{11} - A_{21} \quad S_{10} = B_{11} + B_{12}$$

$$P_1 = A_{11} \cdot S_1 \quad P_2 = S_2 \cdot B_{22}$$

$$P_3 = S_3 \cdot B_{11} \quad P_4 = A_{22} \cdot S_4$$

$$P_5 = S_5 \cdot S_6 \quad P_6 = S_7 \cdot S_8 \quad P_7 = S_9 \cdot S_{10}$$

$$C_{11} = P_5 + P_4 - P_2 + P_6$$

$$C_{12} = P_1 + P_2$$

$$C_{21} = P_3 + P_4$$

$$C_{22} = P_5 + P_1 - P_3 - P_7$$