$$\lim_{n\to\infty}\sum_{k=1}^{n} k(1-p)^{k-1}p = 3$$

$$p\sum_{k=1}^{n} kq^{k-1} = pf(q) = (1-q)f(q) = g(q)$$

$$f(q) = \frac{g(q)}{1-q} \quad \xrightarrow{integral} \quad \int f(q)\mathrm{d}q = \int \frac{g(q)}{1-q}\mathrm{d}q$$

$$\sum_{k=1}^{n} q^k = \frac{q}{1-q} = \int \frac{g(q)}{1-q}\mathrm{d}q \quad \xrightarrow{derivation} \quad \frac{(1-q)+q}{(1-q)^2} = \frac{g(q)}{1-q}$$

$$p = \frac{1}{3}$$

$$\lim_{n\to\infty}\sum_{k=2}^{n} k[(1-p)^{k-1}p + p^{k-1}(1-p)] = 3$$

$$G(p) = p\sum_{k=2}^{n} k(1-p)^{k-1} + (1-p)\sum_{k=2}^{n} kp^{k-1}$$

$$F(x) = (1-x)\sum_{k=2}^{n} kx^{k-1}$$

$$\frac{F(x)}{1-x} = \sum_{k=2}^{n} kx^{k-1} \quad \xrightarrow{integral}$$

$$\int \frac{F(x)}{1-x}\mathrm{d}x = \frac{x^2}{1-x} \quad \xrightarrow{derivation} \quad \frac{F(x)}{1-x} = \frac{2x(1-x)+x^2}{(1-x)^2}$$

$$F(x) = \frac{(2-x)x}{1-x}, \quad G(p) = F(1-p) + F(p) = \frac{(1-p)(1+p)}{p} + \frac{(2-p)p}{1-p} = 3$$

$$p = \frac{1}{2}$$

$$f(x,y)=\begin{cases} \frac{1}{2}\mathrm{e}^{-y}, y>|x|, x\in(-\infty,+\infty) \\ 0, others \end{cases}$$

$$f_X(x)=\int_{|x|}^{+\infty}\frac{1}{2}\mathrm{e}^{-y}\mathrm{d}y=-\frac{1}{2}\mathrm{e}^{-y}|_{|x|}^{+\infty}=\frac{1}{2}\mathrm{e}^{-|x|}, x\in(-\infty,+\infty)$$

$$f_Y(y)=\begin{cases} \int_{-y}^{+y}\frac{1}{2}\mathrm{e}^{-y}\mathrm{d}x=\frac{1}{2}\mathrm{e}^{-y}\cdot 2y=y\mathrm{e}^{-y}, y\in(0,+\infty) \\ 0, y\in(-\infty,0) \end{cases}$$

$$obviously \quad f(x,y)\neq f_X(x)\cdot f_Y(y)$$

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{\frac{1}{2}\mathrm{e}^{-y}}{\frac{1}{2}\mathrm{e}^{-|x|}}=\mathrm{e}^{-y+|x|}, y\in(|x|,+\infty)$$

$$f(x,y)=\begin{cases} \frac{1}{x}\mathrm{e}^{-2x}, y\in(0,2x), x\in(0,+\infty) \\ 0, others \end{cases}$$

$$\begin{cases} U=2X \\ V=X+Y \end{cases}\rightarrow\begin{cases} X=\frac{1}{2}U+0\cdot V \\ Y=-\frac{1}{2}U+V \end{cases}$$

$$J=\begin{vmatrix} \frac{\partial X}{\partial U} & \frac{\partial X}{\partial V} \\ \frac{\partial Y}{\partial U} & \frac{\partial Y}{\partial V} \end{vmatrix}=\begin{vmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{vmatrix}=\frac{1}{2}$$

$$\frac{u}{2}\in(0,+\infty), -\frac{u}{2}+v\in(0,u)$$

$$g(u,v)=f(\frac{u}{2},-\frac{u}{2}+v)|J|=\frac{2}{u}\mathrm{e}^{-u}\cdot\frac{1}{2}=\frac{\mathrm{e}^{-u}}{u}, u\in(0,+\infty), v\in(\frac{u}{2},\frac{3u}{2})$$

$$g(u,v)=0, others$$

$$Y\sim U(0,1)$$

$$F_Y(y)=P(Y<=y)=P(g(X)<=y)=F_X(g^{-1}(y))$$

$$F_Y(y)=F_X(F^{-1}(y))=y, y\in(0,1)$$

$$\Pr[X=k]=\frac{2^{k-1}+2}{3^k}$$

$$\begin{aligned}\mathrm{Ex}[X]=\lim_{n\to\infty}\sum_{k=2}^{n}\Pr[X=k]\cdot k&=\sum_{k=2}^{\infty}\left[\frac{1}{2}\left(\frac{2}{3}\right)^k k+2\left(\frac{1}{3}\right)^k k\right]\\&=\frac{1}{2}\left(\frac{\frac{2}{3}}{\left(1-\frac{2}{3}\right)^2}-\frac{2}{3}\right)+2\left(\frac{\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}-\frac{1}{3}\right)=\frac{7}{2}=3.5\end{aligned}$$

$$\begin{aligned}x^2+y^2=1,\quad x+\frac{1}{y+2}&\in\quad ?\\=\cos\theta+\frac{1}{\sin\theta+2},&\quad\theta\in[0,2\pi]\\f(\theta)&=F(x(\theta),y(\theta))\\f'(\theta)=-\sin\theta-\frac{\cos\theta}{(\sin\theta+2)^2}&=-\cos\theta\left(\tan\theta+\frac{1}{(\sin\theta+2)^2}\right)\\&\frac{xy+2x+1}{y+2}\\2x+2y(x)y'(x)=0,y'(x)&=-\frac{x}{y(x)},y(x)\neq 0\\F(x)=x+\frac{1}{y(x)+2},F'(x)&=1-\frac{y'(x)}{(y(x)+2)^2}\\F'(x)&=1+\frac{x}{y(x)(y(x)+2)^2}\end{aligned}$$

!!!Error Expand!!!

$$X_1, X_2, Y > 0$$

$$F_{X_1}(x) = 1 - \mathrm{e}^{-\lambda_1 x}, F_{X_2}(x) = 1 - \mathrm{e}^{-\lambda_2 x}, F_Y(y) = 1 - \mathrm{e}^{-\mu y}$$

$$f_{X_1}(x) = \lambda_1 \mathrm{e}^{-\lambda_1 x}, f_{X_2}(x) = \lambda_2 \mathrm{e}^{-\lambda_2 x}, f_Y(y) = \mu \mathrm{e}^{-\mu y}$$

$$F_X(x) = P\{X \leq x\} = P\{X \leq x, Y \leq X_1\} + P\{X \leq x, Y > X_1\}$$

$$= P\{X \leq x | Y \leq X_1\} P\{Y \leq X_1\} + P\{X \leq x | Y > X_1\} P\{Y > X_1\}$$

$$= P\{X_1 \leq x\} P\{Y \leq X_1\} + P\{X_1 + X_2 \leq x\} P\{Y > X_1\}$$

$$= F_{X_1}(x) P\{Y - X_1 \leq 0\} + P\{X_1 + X_2 \leq x\} P\{Y - X_1 > 0\}$$

$$Z = Y - X_1$$

$$Y = Z + X_1, \quad y \in (z, z + x), x \in (0, +\infty)$$

$$F_Z(z) = P\{Z \leq z\} = P\{Y - X_1 \leq z\} = \iint_{Y - X_1 \leq z} f(x_1, y) \mathrm{d}x_1 \mathrm{d}y$$

$$= \int_z^{z+x} \int_0^{+\infty} f(x_1, y) \mathrm{d}x_1 \mathrm{d}y = \int_z^{z+x} \int_0^{+\infty} f_{X_1}(x_1) f_Y(y) \mathrm{d}x_1 \mathrm{d}y$$

$$= \frac{\mu}{\lambda_1 + \mu} \mathrm{e}^{-\mu z}, \quad z \in (0, +\infty)$$

$$P\{Y - X_1 \leq 0\} = F_Z(0) = \frac{\mu}{\lambda_1 + \mu}, P\{Y - X_1 > 0\} = \frac{\lambda_1}{\lambda_1 + \mu}$$

$$W = X_1 + X_2$$

$$X_2 = W - X_1, \quad v \in (0, w - u), u \in (0, w)$$

$$F_W(w) = P\{W \leq w\} = P\{X_1 + X_2 \leq w\} = \int_0^w f_{X_1}(u) \left[\int_0^{w-u} f_{X_2}(v) \mathrm{d}v \right] \mathrm{d}u$$

$$= 1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} \mathrm{e}^{-\lambda_1 w} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \mathrm{e}^{-\lambda_2 w}, \quad w \in (0, +\infty)$$

$$F_X(x) = \left(1 - \mathrm{e}^{-\lambda_1 x}\right) \cdot \left(\frac{\mu}{\lambda_1 + \mu}\right) + \left(1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} \mathrm{e}^{-\lambda_1 x} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \mathrm{e}^{-\lambda_2 x}\right) \cdot \left(\frac{\lambda_1}{\lambda_1 + \mu}\right)$$

$$max(X_1, X_2) = \begin{cases} X, & X \geq Y \\ Y, & X < Y \end{cases} \longrightarrow$$

$$max(X_1, X_2) = \frac{X_1 + X_2 + |X_1 - X_2|}{2}$$

$$X_1 \sim (a, \sigma^2), \quad X_2 \sim (a, \sigma^2)$$

$$E(X_1 - X_2) = E(X_1 - X_2) = E(X_1) - E(X_2) = a - a = 0$$

$$D(X_1 - X_2) = D(X_1) + D(X_2) = 2\sigma^2$$

$$X_1 - X_2 \sim (0, 2\sigma^2) \longrightarrow$$

$$f_{X_1 - X_2}(x) = \frac{1}{\sqrt{2\pi}\sqrt{2}\sigma} \mathrm{e}^{-\frac{x^2}{2 \cdot 2\sigma^2}}, \quad x \in (-\infty, +\infty)$$

$$Z = X_1 - X_2$$

$$E\{|X_1 - X_2|\} = E\{X_1 - X_2, X_1 \geq X_2\} + E\{X_2 - X_1, X_1 < X_2\}$$

because of symmetric

$$E\{Z|z \geq 0\} + E\{-Z|z < 0\} = 2E\{Z|z \geq 0\}$$

$$= 2\int_0^{+\infty} \frac{1}{\sqrt{2\pi}\sqrt{2}\sigma} \mathrm{e}^{-\frac{z^2}{2 \cdot 2\sigma^2}} \mathrm{d}z$$

$$= \int_0^{+\infty} \frac{2\sigma}{\sqrt{\pi}} \mathrm{e}^{-\frac{z^2}{4\sigma^2}} \mathrm{d}\left(\frac{z^2}{4\sigma^2}\right) = \frac{2\sigma}{\sqrt{\pi}}$$

$$E\{max(X_1, X_2)\} = E\{\frac{X_1 + X_2 + |X_1 - X_2|}{2}\}$$

$$= \frac{1}{2} \cdot E\{X_1\} + \frac{1}{2} \cdot E\{X_2\} + \frac{1}{2}E\{|X_1 - X_2|\}$$

$$= \frac{1}{2}a + \frac{1}{2}a + \frac{1}{2}\frac{2\sigma}{\sqrt{\pi}} = a + \frac{\sigma}{\sqrt{\pi}}$$