Elementary Function

$$(a+b)^n=\sum_{k=0}^{n}\boldsymbol{C_n^k}a^{n-k}b^k$$

$$a^n-b^n=(a-b)\cdot\sum_{k=0}^{n-1}a^{n-1-k}b^k$$

$$\arcsin x+\arccos x=\arctan x+\mathrm{arccot}x=\frac{\pi}{2}$$

$$f(x)=\int_a^x f'(t)\mathrm{d}t+f(a)$$

$$K=\frac{|y''x'-y'x''|}{(x'^2+y'^2)^{\frac{3}{2}}},\quad \rho=\frac{1}{K}$$

$$\int_a^b f(x)\mathrm{d}x=\lim_{\lambda\to 0}\sum_{i=1}^{n}f(\xi_i)\Delta x_i$$

$$\oint_\alpha^\beta f(x(t),y(t))\mathrm{d}s=\int_a^b f(x,y(x))\sqrt{x'^2+y'^2}\mathrm{d}x=\int_\alpha^\beta f(r,\theta)\sqrt{r^2+r'^2}\mathrm{d}\theta\quad t\in\alpha\to\beta\Rightarrow x\in a\to b$$

$$r=r(\theta),\theta=\alpha\to\beta,S=\frac{1}{2}\int_\alpha^\beta r^2(\theta)\mathrm{d}\theta$$

$$S_{side}=\int_a^b 2\pi y(x)\sqrt{x'^2(x)+y'^2(x)}\mathrm{d}x=\int_\alpha^\beta 2\pi r(\theta)\sin\theta\sqrt{r^2(\theta)+r'^2(\theta)}\mathrm{d}\theta$$

$$[\overrightarrow{abc}]=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}$$

$$\cos\theta=\frac{\vec{a}}{|\vec{a}\cdot\vec{b}|}$$

$$\boldsymbol{Line-Area\quad Integral}$$

$$\frac{\partial f}{\partial l}|(x_0,y_0,z_0)=f_x\cos\alpha+f_y\cos\beta+f_z\cos\gamma$$

$$\exists\Omega=8\Omega_0,f(x,y,z)=f(y,z,x)=f(z,x,y)$$

$$\iiint_{\Omega}f(x,y,z)\mathrm{d}v=24\iiint_{\Omega_0}f(z)\mathrm{d}z=24\iint_{D_{xy}}\mathrm{d}\sigma\int f(z)\mathrm{d}z$$

$$\mathbf{grad}f(x_0,y_0)=f_x(x_0,y_0)\vec{\boldsymbol{i}}+f_y(x_0,y_0)\vec{\boldsymbol{j}}$$

$$S=\iint_{D_{xy}}\mathrm{d}S=\iint_{D_{xy}}\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}\mathrm{d}\sigma$$

$$\iint_{\Sigma}f\mathrm{d}S=\iint_{\Sigma_1}f\mathrm{d}S+\iint_{\Sigma_2}f\mathrm{d}S$$

$$\iint_{-\Sigma}P\mathrm{d}y\mathrm{d}z+Q\mathrm{d}z\mathrm{d}x+R\mathrm{d}x\mathrm{d}y=-\iint_{\Sigma}P\mathrm{d}y\mathrm{d}z+Q\mathrm{d}z\mathrm{d}x+R\mathrm{d}x\mathrm{d}y$$

$$\oint_{\Sigma}P\mathrm{d}y\mathrm{d}z+Q\mathrm{d}z\mathrm{d}x+R\mathrm{d}x\mathrm{d}y=\iiint_{\Omega}(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z})\mathrm{d}v$$

$$\mathrm{div}\vec{\boldsymbol{A}}=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$$

$$\oint_{\Gamma}P\mathrm{d}x+Q\mathrm{d}y+R\mathrm{d}z=\iint_{\Sigma}(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z})\mathrm{d}y\mathrm{d}z+(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x})\mathrm{d}z\mathrm{d}x+(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y})\mathrm{d}x\mathrm{d}y$$

$$\mathbf{rot}\vec{\boldsymbol{A}}=(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z})\vec{\boldsymbol{i}}+(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x})\vec{\boldsymbol{j}}+(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y})\vec{\boldsymbol{k}}$$

$$\boldsymbol{Infinite\quad Series}$$

$$\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}|=\rho\quad,R=\frac{1}{\rho}$$

$$S(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l})$$

$$a_0=\frac{1}{l}\int_{-l}^{l}f(x)\mathrm{d}x,\quad a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}\mathrm{d}x,\quad b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}\mathrm{d}x$$

$$S(x)=\begin{cases} f(x),x\in consecutive\\ \frac{f(x-0)+f(x+0)}{2},x\in discontinuity\\ \frac{f(l-0)+f(l+0)}{2},x=\{-l,l\}\end{cases}$$

Probability Theory

$$A-B=A\bar{B}=A\cap\bar{B}$$

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}},x\in(-\infty,+\infty)$$

$$\Phi(x)=\int_{-\infty}^{x}f(t)\mathrm{d}t,X\sim N(0,1)$$

$$\Phi(a)+\Phi(-a)=1$$

$$F(x)=\Phi(\frac{x-\mu}{\sigma})$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)},f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}$$

$$(X,Y)\sim U(D),f(x,y)=\begin{cases}\frac{1}{S_D},(x,y)\in D\\ 0,others\end{cases}$$

$$f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\mathrm{e}^{-\frac{1}{2(1-\rho^2)}[(\frac{x-\mu_1}{\sigma_1})^2-\frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+(\frac{x-\mu_2}{\sigma_2})^2]}$$

Large Number Law Central Limit Theorem

$$P\{|X-E(X)|\geq\epsilon\}\leq\frac{D(X)}{\epsilon^2}$$

$$\lim_{n\to\infty}P\{|\frac{X_n}{n}-p|<\epsilon\}=1$$

$$\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{k=1}^{n}x_k-\mu|<\epsilon\}=1$$

$$\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{k=1}^{n}X_k-\frac{1}{n}\sum_{k=1}^{n}E(X_k)|<\epsilon\}=1$$

$$\sum_{i=1}^{n}X_i\sim N(n\mu,n\sigma^2),\lim_{n\to\infty}P\{\frac{\sum_{i=1}^{n}X_i-n\mu}{\sqrt{n}\sigma}\leq x\}=F_n(x)=\Phi(x)$$

$$X\sim N(np,np(1-p)),\lim_{n\to\infty}P\{\frac{X_n-np}{\sqrt{np(1-p)}}\leq x\}=\Phi(x)$$

Mathematical Statistics

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}), \frac{\bar{X}-u}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}[\sum_{i=1}^{n} X_i^2 - n\bar{X}^2]$$

$$E(S^2) = \sigma^2, D(S^2) = \frac{2\sigma^4}{n-1}$$

$$\frac{\sum_{i=1}^{n}(X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\chi^2 \sim \chi^2(n), E(\chi^2(n)) = n, D(\chi^2(n)) = 2n$$

$$T \sim t(n), T = \frac{X}{\sqrt{Y/n}}\frac{\sim}{\sim}\frac{N(0,1)}{\sqrt{\chi^2(n)/n}}$$

$$F \sim F(n1, n2), F = \frac{X/n1}{Y/n2}\frac{\sim}{\sim}\frac{\chi^2(n1)/n1}{\chi^2(n2)/n2}$$

$$(X, Y) \sim N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2, 0)$$

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2}/(n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2}/(n_2-1)} \sim F(n_1-1, n_2-1)$$

$$\frac{\frac{\sum_{i=1}^{n_1}(X_i-\mu_1)^2}{\sigma_1^2}/n_1}{\frac{\sum_{i=1}^{n_2}(X_i-\mu_2)^2}{\sigma_2^2}/n2} \sim F(n1, n2)$$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n2}}} \sim N(0,1)$$

$$\exists\sigma_1 = \sigma_2, \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{S_\omega\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

$$S_\omega = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

Constant Series

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n} i)^2 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=0}^{n} a_i \cdot \sum_{j=0}^{n} b_j = \sum_{i=0}^{n}\sum_{j=0}^{n} (a_i \cdot b_j)$$

$$\sum_{n=s}^{t} \ln f(n) = \ln \prod_{n=s}^{t} f(n)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln 2$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n}}{2n-1} = -\frac{\pi}{4}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i} = \sum_{i=0}^{\infty} \frac{1}{(2i+1)(2i+2)} = \sum_{i=1}^{\infty} \frac{1}{2^i i} = \sum_{n=1}^{\infty} (\frac{1}{3^i} + \frac{1}{4^i})\frac{1}{i} = \ln 2$$

$$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$$

$$\sum_{k=1}^{\infty} \frac{1}{k^6} = \frac{\pi^6}{945}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sum_{i=1}^{k} i} = 2$$

Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \ln(1+x)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x)$$

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan x$$

$$\sum_{n=0}^{\infty} (n+1) x^n = \frac{1}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) x^n = \frac{2}{(1-x)^3}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n} = -\ln(1-x^2)$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} = \frac{1}{2}\ln(\frac{1+x}{1-x})$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sin x$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos x$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \mathrm{e}^x$$

Transcendental Equation

$$\sum_{i=0}^{n} a_i x^i = 0 \Longrightarrow \prod_{i=0}^{n} (x - x_i) = 0$$

$$\prod_{i=0}^{n} x_i = (-1)^n \frac{a_0}{a_n}, \quad \sum_{i=0}^{n} \frac{\prod_{i=0}^{n} x_i}{x_i} = (-1)^{n-1} \frac{a_1}{a_n}$$

$$\sum_{i=0}^{n} \frac{1}{x_i} = -\frac{a_1}{a_0}$$

eg. $\tan x = x \Longrightarrow \sin x = \cos x \cdot x$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + ... = x \cdot (1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + ...)$$

$$\frac{1}{3}x^3 - \frac{1}{30}x^5 + ... = 0 \quad \xrightarrow{x \neq 0} \quad \frac{1}{3}x^2 - \frac{1}{30}x^4 + ... = 0$$

$$\aleph x^2 = t, \quad \sum_{i=0}^{n} \frac{1}{t_i} = \frac{1}{10} \quad \xrightarrow{t = x^2} \quad \lim_{n \to \infty} \sum_{i=0}^{n} \frac{1}{x_i^2} = \frac{1}{10}$$