$$A(x) = \sum_{i=0}^{n-1} a_i x^i, B(x) = \sum_{i=0}^{m-1} b_i x^i$$

$$C(x) = A(x) \cdot B(x) = \sum_{i=0}^{n+m-1} c_i x^i$$

$$\star \quad k = 2^{\lceil \log_2(n+m) \rceil}$$

$$\text{Matrix} \quad \text{Transform}$$

$$(x_0.x_1,..,x_{k-1}), \quad \{(a_0, a_1, ..., a_{n-1})\} \quad \xrightarrow{\underline{A(x)}} \quad (x_0, y_0), (x_1, y_1), ..., (x_{k-1}, y_{k-1})$$

$$(x_0.x_1,..,x_{k-1}), \quad \{(b_0, b_1, ..., b_{m-1})\} \quad \xrightarrow{\underline{B(x)}} \quad (x_0, z_0), (x_1, z_1), ..., (x_{k-1}, z_{k-1})$$

$$\mathbf{X_k} = \begin{pmatrix} 1 & 1 & ... & 1 \\ x_0^1 & x_1^1 & ... & x_{k-1}^1 \\ x_0^2 & x_1^2 & ... & x_{k-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_0^{k-1} & x_1^{k-1} & ... & x_{k-1}^{k-1} \end{pmatrix} \quad \vec{\mathrm{a}} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} \quad \vec{\mathrm{b}} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{m-1} \end{pmatrix}$$

$$\mathrm{a}^{\mathrm{T}} \mathbf{X_n} = \mathrm{y}^{\mathrm{T}}, \mathrm{b}^{\mathrm{T}} \mathbf{X_m} = \mathrm{z}^{\mathrm{T}}$$

$$\text{Matrix} \quad \text{Inverse} \quad \text{Transform}$$

$$(x_0, y_0 \cdot z_0), (x_1, y_1 \cdot z_1), ..., (x_{k-1}, y_{k-1} \cdot z_{k-1}) \to C(x)\{(c_0, c_1, ..., c_{n+m-1})\}$$

$$\mathrm{t} = \mathrm{hadamard}(\mathrm{y}, \mathrm{z}), \quad \mathrm{c}^{\mathrm{T}} \mathbf{X_k} = \mathrm{t}^{\mathrm{T}}, \to \mathrm{c}^{\mathrm{T}} = \mathbf{X_k^{-1}} \mathrm{t}^{\mathrm{T}}$$

$$\text{Euler} \quad \text{formula}$$
$$\mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$$
$$\mathrm{e}^{2\pi i} = 1, \forall\theta, \mathrm{e}^{i(\theta+2\pi)} = \mathrm{e}^{i\theta}$$
$$\omega_n^k = \mathrm{e}^{\frac{2\pi i}{n}k}, \quad (\omega_n^k)^j = \mathrm{e}^{\frac{2\pi i}{n}kj} = \cos(\frac{2\pi}{n}kj) + i\sin(\frac{2\pi}{n}kj)$$
$$(1).\omega_n^{k+\frac{n}{2}} = \mathrm{e}^{\frac{2\pi i}{n}(k+\frac{n}{2})} = -\mathrm{e}^{\frac{2\pi i}{n}k} = -\omega_n^k$$
$$(2).\omega_{2n}^{2k} = \omega_n^k$$
$$(3).\omega_n^{-k} = 1 * \mathrm{e}^{\frac{2\pi i}{n}(-k)} = \mathrm{e}^{\frac{2\pi i}{n}(n-k)} = \omega_n^{n-k}$$

$$\text{FFT}$$
$$\text{DFT}: \quad X_k = \sum_{j=0}^{n-1} x_j \mathrm{e}^{\frac{2\pi i}{n}kj}$$
$$\text{IDFT}: \quad x_j = \frac{1}{n}\sum_{k=0}^{n-1} X_k \mathrm{e}^{-\frac{2\pi i}{n}jk}$$
$$\omega = \mathrm{e}^{\frac{2\pi i}{N}}, N = n+m$$
$$c_j = f(j), C_k = C(\omega_k)$$
$$C_k = C(\omega_k) = \sum_{j=0}^{N-1} c_j(\omega^k)^j = \sum_{j=0}^{N-1} c_j \mathrm{e}^{\frac{2\pi i}{N}kj}$$
$$c_j = \frac{1}{N}\sum_{j=0}^{N-1} C_k \mathrm{e}^{-\frac{2\pi i}{N}jk}$$

$$\text{Div} \quad \text{and} \quad \text{conquer}$$
$$H(x) = \sum_{i=0}^{n-1} a_i x^i = \sum_{i=0}^{\frac{n}{2}-1} a_{2i}x^{2i} + x\left(\sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}x^{2i}\right)$$
$$H(x) = H_1(x^2) + xH_2(x^2)$$
$$A(\omega_n^k) = A_1(\omega_n^{2k}) + xA_2(\omega_n^{2k}) = A_1(\omega_{\frac{n}{2}}^k) + \omega_n^k A_2(\omega_{\frac{n}{2}}^k)$$
$$A(\omega_n^{k+\frac{n}{2}}) = A(-\omega_n^k) = A_1(w_{\frac{n}{2}}^k) - \omega_n^k A_2(\omega_{\frac{n}{2}}^k)$$

Strassen Matrix Multiply

$$A_{2^k \cdot 2^k} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_{(2^{k-1} \cdot 2^{k-1})*4} \quad B_{2^k \cdot 2^k} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$S_1 = B_{12} - B_{22} \quad S_2 = A_{11} + A_{12}$$

$$S_3 = A_{21} + A_{22} \quad S_4 = B_{21} - B_{11}$$

$$S_5 = A_{11} + A_{22} \quad S_6 = B_{11} + B_{22}$$

$$S_7 = A_{12} - A_{22} \quad S_8 = B_{21} + B_{22}$$

$$S_9 = A_{11} - A_{21} \quad S_{10} = B_{11} + B_{12}$$

$$P_1 = A_{11} \cdot S_1 \quad P_2 = S_2 \cdot B_{22}$$

$$P_3 = S_3 \cdot B_{11} \quad P_4 = A_{22} \cdot S_4$$

$$P_5 = S_5 \cdot S_6 \quad P_6 = S_7 \cdot S_8 \quad P_7 = S_9 \cdot S_{10}$$

$$C_{11} = P_5 + P_4 - P_2 + P_6$$

$$C_{12} = P_1 + P_2$$

$$C_{21} = P_3 + P_4$$

$$C_{22} = P_5 + P_1 - P_3 - P_7$$

$$f(x,y)=\begin{cases}\frac{1}{2}\mathrm{e}^{-y}, y>|x|, x\in(-\infty,+\infty)\\ 0, others\end{cases}$$

$$f_X(x)=\int_{|x|}^{+\infty}\frac{1}{2}\mathrm{e}^{-y}\mathrm{d}y=-\frac{1}{2}\mathrm{e}^{-y}|_{|x|}^{+\infty}=\frac{1}{2}\mathrm{e}^{-|x|}, x\in(-\infty,+\infty)$$

$$f_Y(y)=\begin{cases}\int_{-y}^{+y}\frac{1}{2}\mathrm{e}^{-y}\mathrm{d}x=\frac{1}{2}\mathrm{e}^{-y}\cdot 2y=y\mathrm{e}^{-y}, y\in(0,+\infty)\\ 0, y\in(-\infty,0)\end{cases}$$

$$obviously \quad f(x,y)\neq f_X(x)\cdot f_Y(y)$$

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{\frac{1}{2}\mathrm{e}^{-y}}{\frac{1}{2}\mathrm{e}^{-|x|}}=\mathrm{e}^{-y+|x|}, y\in(|x|,+\infty)$$

$$f(x,y)=\begin{cases}\frac{1}{x}\mathrm{e}^{-2x}, y\in(0,2x), x\in(0,+\infty)\\ 0, others\end{cases}$$

$$\begin{cases}U=2X\\ V=X+Y\end{cases}\rightarrow\begin{cases}X=\frac{1}{2}U+0\cdot V\\ Y=-\frac{1}{2}U+V\end{cases}$$

$$J=\begin{vmatrix}\frac{\partial X}{\partial U} & \frac{\partial X}{\partial V}\\ \frac{\partial Y}{\partial U} & \frac{\partial Y}{\partial V}\end{vmatrix}=\begin{vmatrix}\frac{1}{2} & 0\\ -\frac{1}{2} & 1\end{vmatrix}=\frac{1}{2}$$

$$\frac{u}{2}\in(0,+\infty), -\frac{u}{2}+v\in(0,u)$$

$$g(u,v)=f(\frac{u}{2},-\frac{u}{2}+v)|J|=\frac{2}{u}\mathrm{e}^{-u}\cdot\frac{1}{2}=\frac{\mathrm{e}^{-u}}{u}, u\in(0,+\infty), v\in(\frac{u}{2},\frac{3u}{2})$$

$$g(u,v)=0, others$$

$$Y\sim U(0,1)$$

$$F_Y(y)=P(Y<=y)=P(g(X)<=y)=F_X(g^{-1}(y))$$

$$F_Y(y)=F_X(F^{-1}(y))=y, y\in(0,1)$$

$$\Pr[X = k] = \frac{2^{k-1} + 2}{3^k}$$

$$\begin{aligned}
\mathrm{Ex}[X] &= \lim_{n\to\infty}\sum_{k=2}^{n}\Pr[X=k]\cdot k = \sum_{k=2}^{\infty}\left[\frac{1}{2}\left(\frac{2}{3}\right)^k k + 2\left(\frac{1}{3}\right)^k k\right] \\
&= \frac{1}{2}\left(\frac{\frac{2}{3}}{\left(1-\frac{2}{3}\right)^2} - \frac{2}{3}\right) + 2\left(\frac{\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2} - \frac{1}{3}\right) = \frac{7}{2} = 3.5
\end{aligned}$$

$$\phi_X(t) = \mathrm{E}(\mathrm{e}^{itx})$$

$$f(x) = \frac{1}{b-a}, x \in (a,b) \quad \phi_x(t) = \int_a^b \mathrm{e}^{itx}\frac{1}{b-a}\mathrm{d}x = \frac{-i}{(b-a)t}\mathrm{e}^{itx}|_a^b = \frac{-i(\mathrm{e}^{itb} - \mathrm{e}^{ita})}{(b-a)t}$$

$$t \sim N(0,1) \quad \phi(t) = \mathrm{e}^{-\frac{t^2}{2}}$$

$$Y = aX + b$$

$$\phi_Y(t) = \phi_{(aX+b)}(t) = \mathrm{E}(\mathrm{e}^{it(ax+b)}) = \mathrm{e}^{itb}\phi_X(at)$$

$$x^2 + y^2 = 1, \quad x + \frac{1}{y+2} \in \quad ?$$

$$= \cos\theta + \frac{1}{\sin\theta + 2}, \quad \theta \in [0, 2\pi]$$

$$f(\theta) = F(x(\theta), y(\theta))$$

$$f'(\theta) = -\sin\theta - \frac{\cos\theta}{(\sin\theta+2)^2} = -\cos\theta\left(\tan\theta + \frac{1}{(\sin\theta+2)^2}\right)$$

$$\frac{xy + 2x + 1}{y+2}$$

$$2x + 2y(x)y'(x) = 0, y'(x) = -\frac{x}{y(x)}, y(x) \neq 0$$

$$F(x) = x + \frac{1}{y(x)+2}, F'(x) = 1 - \frac{y'(x)}{(y(x)+2)^2}$$

$$F'(x) = 1 + \frac{x}{y(x)(y(x)+2)^2}$$