

## Structure of the Output Returned by `run_gibbs_cpp()`

The C++ Gibbs sampler returns a list containing three main objects:

- `table_of`: a list of length  $S$  (number of saved iterations). Each element is an integer vector of length  $n$ , where

$$\text{table\_of}[[s]][i] = \text{table assigned to customer } i$$

at saved iteration  $s$ .

- `dish_of`: a list of length  $S$ . Each `dish_of`[[ $s$ ]] is itself a list of length  $d$  (number of views). For each view  $v$ , `dish_of`[[ $s$ ]][[ $v$ ]] is an integer vector of length  $T_s$ , where  $T_s$  is the number of tables at iteration  $s$ . The entry

$$\text{dish\_of}[[s]][[v]][t] = \text{dish assigned to table } t \text{ in view } v.$$

- `loglik`: a numeric vector of length  $S$  containing the log-likelihood at each saved iteration.

This output structure mirrors the hierarchical nature of the model: customers → tables → dishes (one per view). Since the number of tables changes over iterations, lists must be used instead of fixed-size arrays.

## Example: Interpreting the Structure with a Small Illustration

Assume the following setting:

$$n = 3, \quad d = 2, \quad S = 2.$$

Suppose the saved states returned by the sampler are:

```
res$table_of =
[[1]]  [1, 1, 2]
[[2]]  [2, 2, 1]

res$dish_of =
[[1]]
  [[1]]  [10, 20]
  [[2]]  [11, 21]

[[2]]
  [[1]]  [30, 40, 50]
  [[2]]  [31, 41, 51]
```

This corresponds to the following interpretation:

### **State 1 ( $s = 1$ ).**

- Customers:

$$1 \rightarrow t_1, \quad 2 \rightarrow t_1, \quad 3 \rightarrow t_2.$$

- Number of tables:  $T_1 = 2$ .

- Dishes:

View 1: (10, 20)

View 2: (11, 21)

meaning table 1 uses dishes (10, 11), while table 2 uses (20, 21).

### **State 2 ( $s = 2$ ).**

- Customers:

$$1 \rightarrow t_2, \quad 2 \rightarrow t_2, \quad 3 \rightarrow t_1.$$

- Number of tables:  $T_2 = 3$ .

- Dishes:

View 1: (30, 40, 50)

View 2: (31, 41, 51)

meaning each table has its own dish in both views.

## **Why This Structure is Appropriate**

- The number of tables  $T_s$  varies across iterations; a list-of-lists allows dynamic sizes.
- Accessing the dish used by customer  $i$  at iteration  $s$  only requires:

$$k_{svi} = \text{dish\_of}[[s]][[v]][\text{table\_of}[[s]][i]].$$

- The structure is compact, flexible, and captures the full restaurant–table–dish hierarchy.