AMATH 515 Homework 0

Due: Friday, January 9th at Midnight.

The goal of this homework is to make sure you are comfortable with all prerequisites for this class, to set-up Python and Jupyter Notebook, and to try submitting your work to Gradescope Autograder. The theoretical portion of the homework will be graded based on completeness, and is intended as a primer on calculus and linear algebra.

1. Theory

(1) Submit your write-up to Gradescope. LaTeX is preferred. Look for the assignment "Homework 0 – theory".

(2) **Calculus primer**. For a function $f : \mathbb{R}^n \to \mathbb{R}$, we define the *gradient* to be the vector of partial derivatives:

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

and the *Hessian* to be the matrix of second partial derivatives:

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & & \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n \partial x_n} \end{bmatrix}$$

Compute the gradients and hessians of the following functions, with $x \in \mathbb{R}^4$ in all three examples.

(a) $f(x) = \sin(x_1 + x_2 + x_3 + x_4)$

(b) $f(x) = \|x\|^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

(c) $f(x) = \ln(x_1 x_2 x_3 x_4)$.

(3) **Linear algebra primer.**

(a) What are the eigenvalues of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \pi & 2 & 0 & 0 \\ 64 & -15 & 3 & 0 \\ 321 & 0 & 0 & 5 \end{bmatrix}$$

(b) Write down bases for the range and nullspace of the following matrix, written as the outer product of two vectors:

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(c) Let A be a 10×5 matrix, and b a vector in $\mathbb{R}^{10}$. The notation A^T denotes the *transpose* of A, where the columns of A are rows of A^T.

- What is the size of $A^T A$? What is the size of $A^T b$?
- How many solutions might there be to the system $Ax = b$?
- How many solutions might there be to the system $A^T Ax = A^T b$?
- Suppose the columns of A are linearly independent. How many solutions might there be to the system $Ax = b$? To the system $A^T Ax = A^T b$?

2. Practice

(1) Install Anaconda3 distribution. Instruction:
https://www.anaconda.com/products/individual

- If you've never used Python before – here is an excellent Python introduction: https://www.learnpython.org
- If you have experience with scientific computing in MATLAB, but you've never tried Python, here is a useful migration guide:
https://www.enthought.com/white-paper-matlab-to-python-a-migration-guide/

(2) Download "Homework0.ipynb" from Canvas, open it as a Jupyter Notebook, and complete all the tasks there.

- If you've never used Jupyter Notebooks then take a look at this tutorial: https://www.dataquest.io/blog/jupyter-notebook-tutorial/

(3) Submit your Jupyter Notebook to Gradescope. Look for the assignment "Homework 0 – code". There is no limit for the number of attempts for the coding part this time.