$$E(\Delta x, \Delta y) = \sum w(x,y) \left[I(x,y) - I(x+\Delta x, y+\Delta y) \right]$$

gaussian → denoise

$f(x) = f(a) \quad f'(a)(x-a)$

$f(x+\Delta x) = f(x) + f'(x)\,\Delta x$

$$E(\Delta x, \Delta y) \approx \sum w(x,y) \left[I(x,y) - I(x,y) - I_x(x,y)\Delta x - I_y(x,y)\Delta y \right]^2$$

$$\Rightarrow \sum w(x,y) \left(I_x^2 \Delta x^2 + 2 I_x I_y \Delta x \Delta y + I_y^2 \Delta y^2 \right)$$

$$= [\Delta x \ \ \Delta y] \, M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$M = \sum w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$ → square matrix → $U D U^T$

orthogonal $U^{-1} = U^T$

$$U \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} U^T$$

$$E(\Delta x, \Delta y) = \underbrace{[\Delta x \ \ \Delta y] \, U \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix}}_{A^T} \underbrace{\begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_1} \end{bmatrix} U^T \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}}_{A}$$

stretch

$= A^T A = \|A\|$

the eigenvalue correlation with E

$\underset{2\times 2}{v_1^T} \, \underset{2\times 1}{\Delta x} \rightarrow$ scalar a

在 $\vec{v_1}$ 的 a unit

U : rotation to different basis

eigenvalue λ_1 λ_2 $\to \Sigma$

λ_1 λ_2	$\uparrow$	corner
one of λ	$\uparrow$	edge
λ_1 λ_2	$\downarrow$	flat

compute eigenvalue too expensive

$$r = \det M - K(\operatorname{trace} M)^2$$

$\swarrow$

$[0.04, 0.06]$

$$= \lambda_1\lambda_2 - K(\lambda_1 + \lambda_2)^2$$

pf $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = U \Sigma U^T$

$\det(M) = \det(\Sigma) = \lambda_1 \lambda_2 = \underline{AD - BC}$

$\det(M - \lambda I) = \lambda^2 - (A+D)\lambda + AD - BC$

$\lambda_1 + \lambda_2 = A + D$

$\operatorname{trace}(M) = \lambda_1 + \lambda_2 = \underline{A+D}$

don't need to do decomposition

R large	λ_1 λ_2 large	corner
$R < 0$	$\lambda_1 \gg \lambda_2$ or $\lambda_1 \ll \lambda_2$	edge
$\lvert R \rvert$ small	λ_1 λ_2 small	flat

Harris Corner Detection

1. Calculate the gradient I_x I_y
2. apply Gaussian filter to denoise
3. Get M
4. $R = Det(M) - k \cdot Trace(M)^2$ $k = [0.04, 0.06]$

checking the threshold