PRIEBEH FUNKCIE

PR $\ln\left(\frac{1}{1-x^2}\right)$

1) $D(f) = \{\forall x \in R;\ 1-x^2 \neq 0$

$\frac{1}{1-x^2} > 0 \iff 1-x^2 > 0$, $1 > x^2 \} = (-1, 1)$

2) PAR. NEPÁR.;

$f(-x) = \ln\left(\frac{1}{1-(-x)^2}\right) = \ln\left(\frac{1}{1-x^2}\right) = f(x)$ - PÁRNA

3) SPOJITOSŤ; ABS

$\lim_{x \to -1^+} \ln\left(\frac{1}{1-x^2}\right) = \infty$, $\frac{1}{1-x^2} \to \infty$, $1-x^2 \to 0^+$

$\lim_{x \to 1^-} \ln\left(\frac{1}{1-x^2}\right) = \infty$, $\frac{1}{1-x^2} \to \infty$, $1-x^2 \to 0^+$

ABS: $x = -1$ $x = 1$

4) ASS

$k_1 = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \nexists$ LEBO $D(f) \Rightarrow \nexists$ ASS

5) MONOTÓN.

$f(g(h))' = f'(g(h)) \cdot g'(h) \cdot h'$

$f'(x) = \left(\ln\left(\frac{1}{1-x^2}\right)\right)' = \frac{1}{\frac{1}{1-x^2} \cdot \underbrace{\ln e}_{=1}} \cdot (-1)(1-x^2)^{-2} \cdot (-2x) =$

$$= \cancel{(1-x^2)} \cdot \frac{\cancel{-}1}{(1-x^2)^{\cancel{2}}} \cdot (\cancel{-}2x) = \frac{2x}{1-x^2}$$

SB : $f'(x) = 0 \iff \frac{2x}{1-x^2} = 0 \iff 2x = 0 \iff x = 0$

	$(-1,0)$	$(0,1)$
$2x$	$-$	$+$
$1-x^2$	$+$	$+$
	$-$	$+$
	↘	↗

6) EXTRÉM : $f(x)$ JE SPOJ V $x = \boxed{0}$ ZĽAVA KLESÁ SPRAVA RASTIE => JE V TOMTO BODE LOK MIN.

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$f''(x) = \left(\frac{2x}{1-x^2}\right)' = \frac{2\cdot(1-x^2) - 2x\cdot(-2x)}{(1-x^2)^2} = \frac{2-2x^2+4x^2}{(1-x^2)^2}$$

$$= \frac{2+2x^2}{(1-x^2)^2}$$

$$f''(0) = \frac{2+2\cdot 0^2}{(1-0^2)^2} = \frac{2}{1} = 2 > 0 \Rightarrow \text{LOK. MIN.}$$

$$f(0) = \ln\left(\frac{1}{1-0^2}\right) = \ln 1 = 0$$

MIN $[0,0]$

7) KONV. KONK.

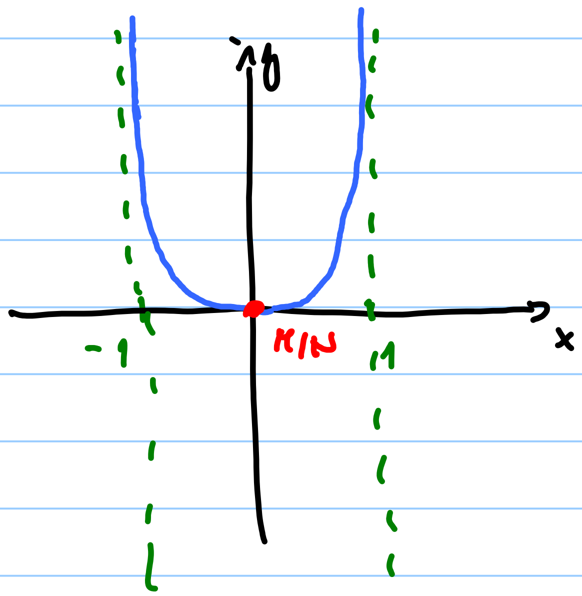
KAND. NA IB $f'' = 0 \iff \frac{2+2x^2}{(1-x^2)^2} = 0 \iff 2+2x^2 = 0$

NIKDY

$\Rightarrow \nexists$ IB

	$(-1, 1)$
$2+2x^2$	$+$
$(1-x^2)^2$	$+$
	$+$ $\cup$

$\ln\left(\frac{1}{1-x^2}\right)$



PR: $y = \sqrt{1-x^3}$

1) $D(f)$: $\left\{ \forall x \in \mathbb{R};\ \begin{matrix} 1-x^3 \geq 0 \\ x^3 \leq 1 \end{matrix} \right\} = (-\infty, 1\rangle$

2) PAR. ANI ANI LEBO $D(f)$ NIE JE SYMETRICKÝ

3) SPOJ ABS

$$\lim_{x \to 1^-} \sqrt{1-x^3} = \sqrt{1-(1^-)^3} = 0$$

ASS

$$k_2 = \lim_{x \to -\infty} \frac{\sqrt{1-x^3}}{x} = \lim_{x \to -\infty} \sqrt{\frac{1-x^3}{x^2}} =$$

$$= \lim_{x \to -\infty} \sqrt{\underbrace{\frac{1}{x^2}}_{\searrow} - \underbrace{x}_{\to -\infty}} = \sqrt{\infty} = \infty \Rightarrow \nexists \text{ ASS}$$

MONOT:

$$f' = \frac{1}{2}\left(1-x^3\right)^{-\frac{1}{2}} \cdot \left(-3x^2\right) = \frac{-3x^2}{2\sqrt{1-x^3}}$$

SB: $f' = 0 \iff \dfrac{-3x^2}{2\sqrt{1-x^3}} = 0 \iff -3x^2 = 0 \iff x = 0$

	$(-\infty, 0)$	$(0, 1\rangle$
$-3x^2$	$-$	$-$
$2\sqrt{1-x^3}$	$+$	$+$
	$\ominus$	$\ominus$
	$\searrow$	$\searrow$

6) EXTRÉMY

$$f'' = \frac{-6x \cdot 2\sqrt{1-x^3} - \left(-3x^2\right) \cdot \cancel{2} \cdot \cancel{\tfrac{1}{2}}\left(1-x^3\right)^{-\frac{1}{2}} \cdot \left(-3x^2\right)}{\left(2\sqrt{1-x^3}\right)^2} =$$

$$= \frac{\dfrac{-12x\left(1-x^3\right) - 9x^4}{\sqrt{1-x^3}}}{4\left(1-x^3\right)} = \frac{-3x\left(4-4x^3+3x^3\right)}{4\left(1-x^3\right)\sqrt{1-x^3}} = \frac{-3x\left(4-x^3\right)}{4\left(\sqrt{1-x^3}\right)^3}$$

$$f''(0) = \frac{-3 \cdot 0\left(4-0^3\right)}{4\left(\sqrt{1-0^3}\right)^3} = 0 \Rightarrow \text{KANDIDÁT NA IB}$$

$f'' = 0 \iff -3x\left(4-x^3\right) = 0 \iff -3x = 0$ ✓

$\left(4-x^3\right) = 0 \iff x = \sqrt[3]{4} \notin D(f)$

KONV. KONK

	$(-\infty, 0)$	$(0, 1)$
$-3x$	+	−
$4-x^3$	+	+
$4\left(\sqrt{1-x^3}\right)^3$	+	+
	+	−
	∪	∩