Simple Harmonic Motion

We see different kinds of motion every day. The motion of the hands of a clock, motion of the wheels of a car, etc. Did you ever notice that these types of motion keep repeating themselves? Such motions are periodic in nature. One such type of periodic motion is simple harmonic motion (S.H.M.). But what is S.H.M.? Let's find out.

Periodic Motion and Oscillations

A motion that repeats itself in equal intervals of time is *periodic*. We need to know what periodic motion is to understand simple harmonic motion.

Periodic motion is the motion in which an object repeats its path in equal intervals of time. We see many examples of periodic motion in our day-to-day life. The motion of the hands of a clock is periodic motion. The rocking of a cradle, swinging on a swing, leaves of a tree moving to and fro due to wind breeze, these all are examples of periodic motion.

The particle performs the same set of movement again and again in a periodic motion. One such set of movement is called an Oscillation. A great example of oscillatory motion is Simple Harmonic Motion.

Browse more Topics under Oscillations

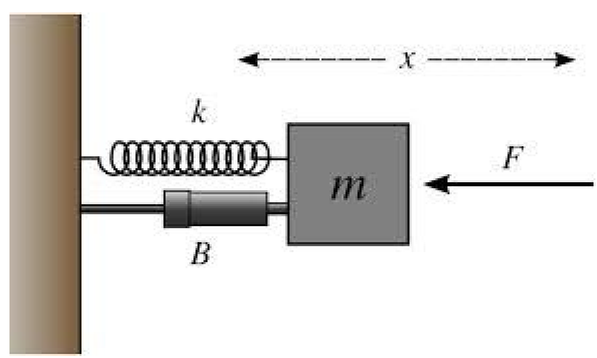
Let's learn about it below.

Simple Harmonic Motion (S.H.M.)

When an object moves to and fro along a line, the motion is called simple harmonic motion. Have you seen a pendulum? When we swing it, it moves to and fro along the same line. These are called oscillations. Oscillations of a pendulum are an example of simple harmonic motion.

Now, consider there is a spring that is fixed at one end. When there is no force applied to it, it is at its equilibrium position. Now,

- If we pull it outwards, there is a force exerted by the string that is directed towards the equilibrium position.
- If we push the spring inwards, there is a force exerted by the string towards the equilibrium position.



In each case, we can see that the force exerted by the spring is towards the equilibrium position. This force is called the restoring force. Let the force be F and the displacement of the string from the equilibrium position be x.

Therefore, the restoring force is given by, F= – kx (the negative sign indicates that the force is in opposite direction). Here, k is the constant called the force constant. Its unit is N/m in S.I. system and dynes/cm in C.G.S. system.

Learn how to calculate Energy in Simple Harmonic Motion here

Linear Simple Harmonic Motion

Linear simple harmonic motion is defined as the linear periodic motion of a body in which the restoring force is always directed towards the equilibrium position or mean position and its magnitude is directly proportional to the displacement from the equilibrium position. All simple harmonic motions are periodic in nature but all periodic motions are not simple harmonic motions.

Now, take the previous example of the string. Let its mass be m. The acceleration of the body is given by,

$$a = F/m = -kx/m = -\omega 2x$$

Here, $k/m = \omega 2$ (ω is the angular frequency of the body)

Learn the difference between Linear and Damped Simple Harmonic Motion here

Concepts of Simple Harmonic Motion (S.H.M)

- Amplitude: The maximum displacement of a particle from its equilibrium position or mean position is its amplitude. Its S.I. unit is the metre. The dimensions are $[L^1M^0 T^0]$. Its direction is always away from the mean or equilibrium position.
- Period: The time taken by a particle to complete one oscillation is its period. Therefore, period if S.H.M. is the least time after which the motion will repeat itself. Thus, the motion will repeat itself after nT. where n is an integer.
- Frequency: Frequency of S.H.M. is the number of oscillations that a particle performs per unit time. S.I. unit of frequency is hertz or r.p.s(rotations per second). Its dimensions are $[L^0M^0T^{-1}]$.
- Phase: Phase of S.H.M. is its state of oscillation. Magnitude and direction of displacement of particle represent the phase. The phase at the beginning of the motion is known as Epoch(α)

Learn how to find Velocity and Acceleration of Simple Harmonic Motion here

Note: The period of simple harmonic motion *does not* depend on amplitude or energy or the phase constant.

Difference between Periodic and Simple Harmonic Motion

Periodic Motion	Simple Harmonic Motion
In the periodic motion, the displacement of the object may or may not be in the direction of the restoring force.	In the simple harmonic motion, the displacement of the object is always in the opposite direction of the restoring force.
The periodic motion may or may not be oscillatory.	Simple harmonic motion is always oscillatory.
Examples are the motion of the hands of a clock, the motion of the wheels of a car, etc.	Examples are the motion of a pendulum, motion of a spring, etc.

Learn the difference between Periodic and Oscillatory Motion here.

Solved Questions for You

Q: Assertion(A): In simple harmonic motion, the motion is to and fro and periodic

Reason(R): Velocity of the particle $V = \omega\sqrt{A^2 - x^2}$ where x is displacement as measured from the extreme position

Chose the right answer:

a. Both a and B are true and R is the correct explanation of A.
b. Both A and B are true and R is not the correct explanation of A.
c. A is true and R is false.
d. A is false and R is true.

Solution: c) A is true and R is false. $V = \omega\sqrt{A^2 - x^2}$ is measured from the mean position. SHM involves to and fro periodic motion.

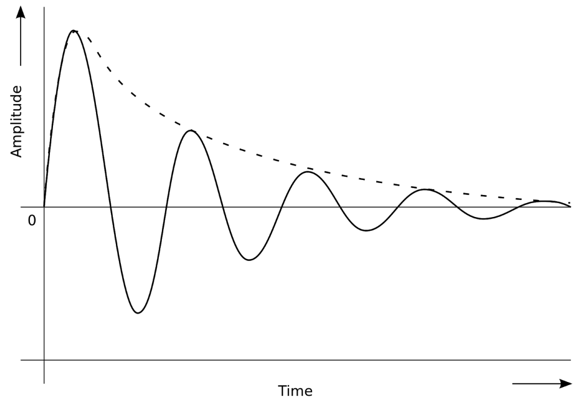
Damped Simple Harmonic Motion

We know that when we swing a pendulum, it will eventually come to rest due to air pressure and friction at the support. This motion is damped simple harmonic motion. Let’s understand what it is and how it is different from linear simple harmonic motion.

Damped Simple Harmonic Motion

When the motion of an oscillator reduces due to an external force, the oscillator and its motion are damped. These periodic motions of gradually decreasing amplitude are damped simple harmonic motion. An example of a damped simple harmonic motion is a simple pendulum.

In the damped simple harmonic motion, the energy of the oscillator dissipates continuously. But for a small damping, the oscillations remain approximately periodic. The forces which dissipate the energy are generally frictional forces.



Browse more Topics Under Oscillations

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion
- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Expression of damped simple harmonic motion

Let's take an example to understand what a damped simple harmonic motion is. Consider a block of mass *m* connected to an elastic string of spring constant k. In an ideal situation, if we push the block down a little and then release it, its angular frequency of oscillation is $\omega = \sqrt{k/m}$.

However, in practice, an external force (air in this case) will exert a damping force on the motion of the block and the mechanical energy of the block-string system will decrease. This energy that is lost will appear as the heat of the surrounding medium.

The damping force depends on the nature of the surrounding medium. When we immerse the block in a liquid, the magnitude of damping will be much greater and the dissipation energy is much faster. Thus, the damping force is proportional to the velocity of the bob and acts opposite to the direction of the velocity. If the damping force is F_d, we have,

$$F_d = -b\upsilon \qquad \text{(I)}$$

where the constant b depends on the properties of the medium(viscosity, for example) and size and shape of the block. Let's say O is the equilibrium position where the block settles after releasing it. Now, if we pull down or push the block a little, the restoring force on the block due to spring is $F_s = -kx$, where x is the displacement of the mass from its equilibrium position. Therefore, the total force acting on the mass at any time t is, F = -kx -bυ.

Now, if a(t) is the acceleration of mass *m* at time t, then by Newton's Law of Motion along the direction of motion, we have

$$ma(t) = -kx(t) - b\upsilon(t) \qquad \text{(II)}$$

Here, we are not considering vector notation because we are only considering the one-dimensional motion. Therefore, using first and second derivatives of s(t), v(t) and a(t), we have,

$$m(d^2x/dt^2) + b(dx/dt) + kx = 0 \qquad \text{(III)}$$

This equation describes the motion of the block under the influence of a damping force which is proportional to velocity. Therefore, this is the expression of damped simple harmonic motion. The solution of this expression is of the form

$$x(t) = Ae^{-bt/2m} \cos(\omega' t + ø) \qquad \text{(IV)}$$

where A is the amplitude and ω′ is the angular frequency of damped simple harmonic motion given by,

$$\omega' = \sqrt{(k/m - b^2/4m^2)} \qquad \text{(V)}$$

The function x(t) is not strictly periodic because of the factor $e^{-bt/2m}$ which decreases continuously with time. However, if the decrease is small in one-time period T, the motion is then approximately periodic. In a damped oscillator, the amplitude is not constant but depends on time. But for small damping, we may use the same expression but take amplitude as $Ae^{-bt/2m}$

$$\therefore E(t) = 1/2\ kAe^{-bt/2m} \qquad \text{(VI)}$$

This expression shows that the damping decreases exponentially with time. For a small damping, the dimensionless ratio $(b/\sqrt{km})$ is much less than 1. Obviously, if we put b = 0, all equations of damped simple harmonic motion will turn into the corresponding equations of undamped motion.

Solved Examples For You:

Q: When we immerse an oscillating block of mass in a liquid, the magnitude of damping will

a) decrease b) increasec) remain the same d) none of the above

Solution: b) The magnitude of damping will increase when we immerse the block in a liquid and its dissipation energy as well. Damping is proportional to the velocity of the block.

Forced Simple Harmonic Motion

When we displace a pendulum from its equilibrium position, it oscillates to and fro about its mean position. Eventually, its motion dies out due to the opposing forces in the medium. But can we force the pendulum to oscillate continuously? Yes. This type of motion is known as forced simple harmonic motion. Let’s find out what forced simple harmonic motion is.

Definition of Forced Simple Harmonic Motion

When we displace a system, say a simple pendulum, from its equilibrium position and then release it, it oscillates with a natural frequency ω and these oscillations are free oscillations. But all free oscillations eventually die out due to the everpresent damping forces in the surrounding.

However, an external agency can maintain these oscillations. These oscillations are known as forced or driven oscillations. The motion that the system performs under this external agency is known as

Forced Simple Harmonic Motion. The external force is itself periodic with a frequency ωd which is known as the drive frequency.

A very important point to note is that the system oscillates with the driven frequency and not its natural frequency in Forced Simple Harmonic Motion. If it oscillates with its natural frequency, the motion will die out. A good example of forced oscillations is when a child uses his feet to move the swing or when someone else pushes the swing to maintain the oscillations.

Browse more Topics under Oscillations

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion
- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Expression of Forced Simple Harmonic Motion

Consider an external force F(t) of amplitude F_0 that varies periodically with time. This force is applied to a damped oscillator. Therefore, we can represent it as,

$$F(t) = F_0 \cos \omega_d t \qquad \text{(I)}$$

Thus, at this time, the forces acting on the oscillator are its restoring force, the external force and a time-dependent driving force. Therefore,

$$ma(t) = -kx(t) - b\upsilon(t) + F_0 \cos \omega_d t \qquad \text{(II)}$$

We know that acceleration = d^2x/dt^2. Substituting this value of acceleration in equation II , we get,

$$m(d^2x/dt^2) + b(dx/dt) + kx = F_0 \cos \omega_d t \qquad \text{(III)}$$

Equation III is the equation of an oscillator of mass m on which a periodic force of frequency ω_d is applied. Obviously, the oscillator first oscillates with its natural frequency. When we apply the external periodic force, the oscillations with natural frequency die out and the body then oscillates with the driven frequency. Therefore, its displacement after the natural oscillations die out is given by:

$$x(t) = A\cos(\omega_d + \o) \qquad \text{(IV)}$$

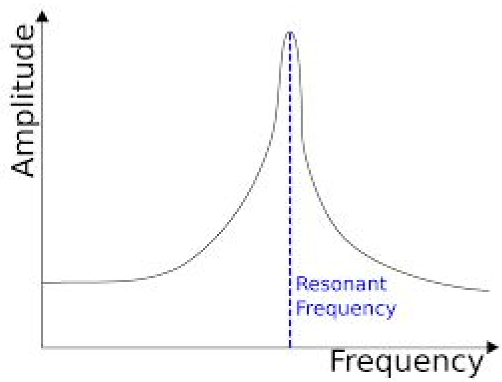
where t is the time from the moment we apply external periodic force.

Resonance

The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is known as resonance. To understand the phenomenon of resonance, let us consider two pendulums of nearly equal (but not equal) lengths (therefore, different amplitudes) suspended from the same rigid support.

When we swing the first pendulum which is greater in length, it oscillates with its natural frequency. The energy of this pendulum transfers through the rigid support to the second pendulum which is slightly smaller in length. Therefore, the second pendulum starts oscillating with its natural frequency first.

At one point, the frequency with the second pendulum vibrates becomes nearly equal to the first one. Therefore, the second pendulum now starts with the frequency of the first one, which is the driven frequency. When this happens, the amplitude of the oscillations is maximum. Thus, resonance takes place.



Here's a Solved Question for You:

Q: Resonance takes place only when the natural frequency of the oscillator is _________ the driven frequency of the external periodic force.

a) Less than b) More than c) Equal to d) None of the above

Solution: c) When the natural frequency is equal to the driven frequency, the amplitude of the oscillations is the maximum. Therefore, the phenomenon takes place.

Force Law for Simple Harmonic Motion

Do you ever wonder why, when we stretch an elastic band and then release it, it comes back to its original state? It is because of a force that forces it to return to its original state. But what is this force? Let's learn about this force and derive the force law for simple harmonic motion.

Periodic Motion

We already know what periodic motion is. A motion which repeats itself in equal intervals of time is periodic motion. For example, the motion of the hands of a clock, the motion of the wheels of a car and the motion of a merry-go-round. All these motions are repetitive in nature. They repeat themselves in a fixed amount of time.

Browse more Topics under Oscillations

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion

- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Oscillatory Motion

An oscillatory motion is a periodic motion in which an object moves to and fro about its equilibrium position. The object performs the same set of movements again and again after a fixed time. One such set of movements is an Oscillation. The motion of a simple pendulum, the motion of leaves vibrating in a breeze and the motion of a cradle are all examples of oscillatory motion.

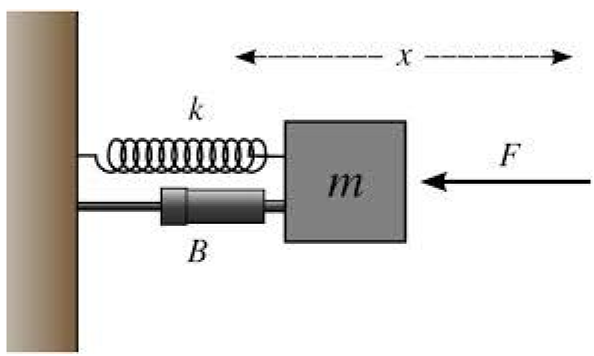
Simple Harmonic Motion (S.H.M.)

Simple harmonic motion is the simplest example of oscillatory motion. When an object moves to and fro along a straight line, it performs the simple harmonic motion. All examples of oscillatory motion are the examples of simple harmonic motion.

When we swing a simple pendulum, it moves away from its mean equilibrium position. After it reaches its extreme position, where it has maximum displacement, it stops, that is, its velocity becomes zero. It returns to its equilibrium position due to a force which acts in towards the equilibrium position.

Now, it passes through its mean position but doesn't stop. It moves towards its other extreme position. After which, it again returns to its equilibrium position. One such complete motion is known as an Oscillation. The motion of a simple pendulum is a great example of simple harmonic motion.

Force Law For Simple Harmonic Motion



Let's derive the force law for simple harmonic motion with an example. A spring-block system is the simplest example of simple harmonic motion. Consider a block of mass m attached to a spring, which in turn is fixed to a rigid wall. The block rests on a frictionless surface. When we do not pull the spring, that is, no force is applied on it, it is in its equilibrium position. In this state, the net force acting on it is zero. Now, let's perform two actions and observe what happens.

- When we pull the block outwards, there is a force acting on the block that tries to pull it inwards, that is, towards its equilibrium position.
- When we push the block inwards, there is a force acting on the block tries to push it outwards, that is, towards its equilibrium position.

In both the cases, we can see that there is a force acting on the block that tries to return the block to its equilibrium position. This force is the restoring force and the force law for simple harmonic motion is based on this force. Let's learn how to calculate this law.

Derivation of Force Law for Simple Harmonic Motion

Let the restoring force be F and the displacement of the block from its equilibrium position be x. Therefore, from the cases we observed, we can say that the restoring force is directly proportional to the displacement from the mean position.

$$\therefore F = -kx \qquad \text{(I)}$$

where k is a constant known as the force constant.Its unit is N/m in S.I. system and dynes/cm in C.G.S. system. The negative sign indicates that the restoring force and the displacement are always in opposite direction. The equation I is the simplest form of force law for simple harmonic motion. It proves the basic rule of simple harmonic motion, that is, force and displacement should be in opposite direction.

Further, we know that F = ma. Therefore, a = F/m. Substituting this value in Equation I, we get,

$$a = -kx/m = -\omega 2x \quad (\text{where } k/m = \omega 2) \qquad \text{(II)}$$

Hence, Equations I and II are the forms of force law of simple harmonic motion. Note that the restoring force is always towards the mean position and in the opposite direction of that of displacement.

Solved Examples For You

Q1: A hydrogen atom has mass 1.68×10^{-27} kg. When attached to a certain massive molecule it oscillates with a frequency 10^{14} Hz and with an amplitude 10^{-9} cm. Find the force acting on the hydrogen atom.

A) 2.21×10^{-9} N B) 3.31×10^{-9} N C) 4.42×10^{-9} N D) 6.63×10^{-9} N

Solution: We know that F = kx. Let's first find the value of k. We know that $\omega^2 = (k/m)$; where the letters have their usual meaning. Also, we know that $\omega = 2\pi\nu$, therefore $\omega^2 = (2\pi\nu)^2 = (k/m)$.

Therefore, $k = 4\pi^2\nu^2 m$. Hence, $F = k\,x = 4\pi^2\nu^2 m\,x$

Therefore, $F = 6.63 \times 10^{-9}$ N

Velocity and Acceleration in Simple Harmonic Motion

A motion is said to be accelerated when its velocity keeps changing. But in simple harmonic motion, the particle performs the same motion again and again over a period of time. Do you think it is accelerated? Let's find out and learn how to calculate the acceleration and velocity of SHM.

Acceleration in SHM

We know what acceleration is. It is velocity per unit time. We can calculate the acceleration of a particle performing S.H.M. Lets learn how. The differential equation of linear S.H.M. is $d^2x/dt^2 + (k/m)x = 0$ where d^2x/dt^2 is the acceleration of the particle, x is the displacement of the particle, m is the mass of the particle and k is the force constant. We know that $k/m = \omega^2$ where ω is the angular frequency.

Therefore, $d^2x/dt^2 + \omega^2 x = 0$

Hence, acceleration of S.H.M. = $d^2x/dt^2 = -\omega^2 x$

(I)

The negative sign indicated that acceleration and displacement are in opposite direction of each other. Equation I is the expression of acceleration of S.H.M. Practically, the motion of a particle performing S.H.M. is accelerated because its velocity keeps changing either by a constant number or varied number.

Take a simple pendulum for example. When we swing a pendulum, it moves to and fro about its mean position. But after some time, it eventually stops and returns to its mean position. This type of simple harmonic motion in which velocity or amplitude keeps changing is damped simple harmonic motion.

Browse more Topics under Oscillations

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion

- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Learn more about Damped Simple Harmonic Motion in detail here.

Velocity in SHM

Velocity is distance per unit time. We can obtain the expression for velocity using the expression for acceleration.Let's see how.

Acceleration $d^2x/dt^2 = dv/dt = dv/dx \times dx/dt$. But dx/dt = velocity 'v'

Therefore, acceleration = $v(dv/dx)$

(II)

When we substitute equation II in equation I, we get, $v(dv/dx) = -\omega^2 x$.

$\therefore\ vdv = -\omega^2\, xdx$

After integrating both sides, we get,

$\int vdv = \int -\omega^2\, xdx = -\omega^2 \int xdx$

Hence, $v^2/2 = -\omega^2 x^2/2 + C$ where C is the constant of integration.

Now, to find the vaue of C, lets consider boundary value condition.

When a particle performing SHM is at the extreme position, displacement of the particle is maximum and velocity is zero. (a is the amplitude of SHM)

Therefore, At $x = \pm a$, $v = 0$

And $0 = -\omega^2 a^2/2 + C$

Hence, $C = \omega^2 a^2/2$

Let's substitute this value of C in equation $v^2/2 = -\omega^2 x^2/2 + C$

$\therefore v^2/2 = -\omega^2 x^2/2 + \omega^2 a^2/2$

$\therefore v^2 = \omega^2 (a^2 - x^2)$

Taking square root on both sides, we get,

$v = \pm \omega \sqrt{(a^2 - x^2)}$

(III)

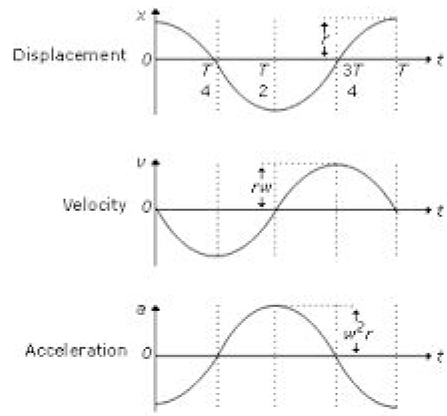
Equation III is the expression of the velocity of S.H.M. The double sign indicates that when a particle passes through a given point in the positive direction of *x*, v is positive, and when it passes through the same point in opposite direction of *x*, v is negative.

Learn the difference between Periodic and Oscillatory Motion.

Maximum and Minimum velocity

We know the velocity of a particle performing S.H.M. is given by, $v = \pm \omega \sqrt{a^2 - x^2}$. At mean position, $x = 0$. Therefore, $v = \pm \omega \sqrt{a^2 - 0^2} = \pm \omega \sqrt{a^2} = \pm a\omega$. Therefore, at mean position, velocity of the particle performing S.H.M. is maximum which is $V_{max} = \pm a\omega$. At extreme position, $x = \pm a$

Therefore, $v = \pm \omega \sqrt{a^2 - a^2} = \omega \times 0 = 0$. Therefore, at extreme position, velocity of the particle performing S.H.M. is minimum which is $V_{min} = 0$



Solved Examples For You

Q: What is the value of acceleration at the mean position?

Solution: At mean position, $x = 0$

∴ acceleration = $-\omega 2\ x = -\omega 2 \times 0 = 0$. Therefore, the value of acceleration at the mean position is minimum and it is zero.

Some Systems Executing Simple Harmonic Motion

We have already studied Simple Harmonic Motion (S.H.M.). But do systems execute purely simple harmonic motion in practice? No, but under certain conditions, they execute simple harmonic motion approximately. For example, a simple pendulum. Let's learn about some of these systems and their harmonic motion.

Systems executing S.H.M.

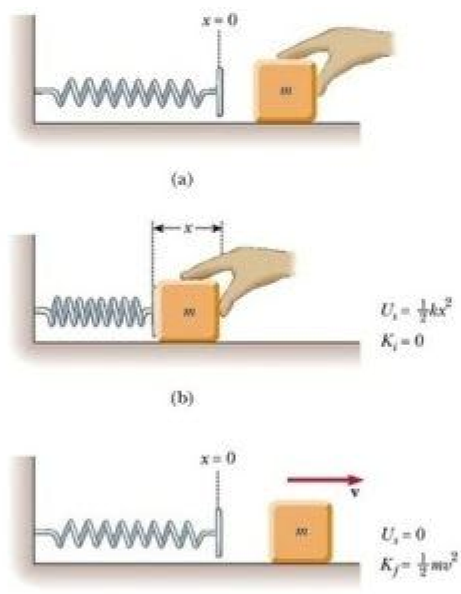
We are going to study about two systems below that perform Simple Harmonic Motion. These systems include- a spring-block system and a simple pendulum. Let's understand one by one how these systems perform simple harmonic motion and under what conditions.

Browse more Topics under Oscillations

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion

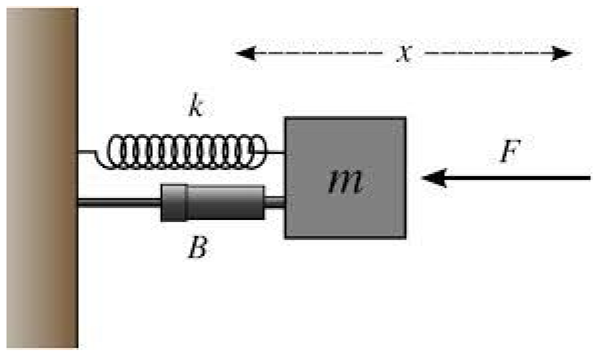
- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Oscillations due to a Spring



The small oscillations of a block fixed to a spring, which in turn is fixed to a wall are the simplest example of simple harmonic motion. Let the mass of the block be 'm'. The block is placed on a frictionless horizontal surface. When we pull the block and then release it, it performs to and fro motion about its mean position. Let x=0 be the position of the block when it is at its equilibrium position. Now,

- If we pull the block outwards, there is a force exerted by the string that is directed towards the equilibrium position.
- If we push the block inwards, there is a force exerted by the string towards the equilibrium position.



In each case, we can see that the force exerted by the spring is towards the equilibrium position. This force is called the restoring force. Let the restoring force be F and the displacement of the string from the equilibrium position be x.

Therefore, we can deduce from this system that the magnitude of restoring force exerted by a system is directly proportional to the displacement of the system from its equilibrium position.The restoring force always acts in the opposite direction of that of displacement. Therefore,

$$F = -kx$$

Here, k is the constant which is called the force constant. In this system, it is called the spring system. The value of k depends on the stiffness of the spring. A stiff spring will have larger k and a soft spring will have small k. You can see that this equation is the same as the Force law of Simple Harmonic Motion. Therefore, a spring system executes simple harmonic motion. From equation I, we have, $\omega = \sqrt{k/m}$

$$\therefore \text{ The time period (T) of the oscillator} = 2\pi\sqrt{m/k}$$

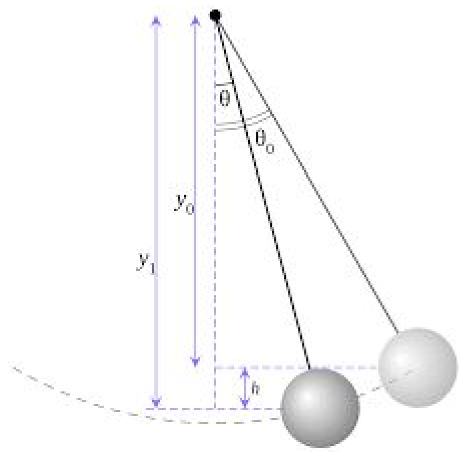
The Simple Pendulum

A practical simple pendulum is a small heavy sphere(bob) suspended by a light and inextensible string from a rigid support. The length of the simple pendulum is the distance between the point of suspension and centre of gravity of heavy sphere.

The Simple Harmonic Motion Pendulum

The motion of Simple Pendulum as Simple Harmonic Motion

When we pull a simple pendulum from its equilibrium position and then release it, it swings in a vertical plane under the influence of gravity. It begins to oscillate about its mean position. Therefore, the motion is periodic and oscillatory. Now, if we displace the pendulum by a very small angle Θ, then it performs the simple harmonic motion.



Consider a simple pendulum having mass 'm', length L and displaced by a small angle Θ with the vertical. Thus, it oscillates about its mean position. In the displaced position, two forces are acting on the bob,

- Gravitational force, which is the weight of the bob – 'mg' acting in the downward direction.
- Tension T' in the string.

Now, weight mg is resolved in two components,

- Radial component $mg\cos\Theta$ along the string.
- Tangential component $mg\sin\Theta$ perpendicular to the string.

Radial component $mg\cos\Theta$ is balanced by Tension T' in the string and the Tangential component $mg\sin\Theta$ is the restoring force acting on mass m towards the equilibrium position. Therefore,

$$\text{Restoring force, } F = -mg\sin\Theta$$

The negative sign indicates that F and Θ are in opposite directions. Note that the restoring force is proportional to $\sin\Theta$ instead of Θ. Therefore, it is not yet a simple harmonic motion. However, if the angle is very small, we can assume that $\sin\Theta$ is nearly equal to Θ in radian. Thus, the displacement $x = L\Theta$ and for a small angle, it is nearly a straight line.

$$\therefore \Theta = x/L$$

Hence, assuming $\sin\Theta = \Theta$, $F = -mg\Theta = -mgx/L$. As m,g, and L are constant, $F \propto -x$. For a small displacement, the restoring force is directly proportional to the displacement and its direction is opposite to that of displacement. Therefore, Simple pendulum performs linear S.H.M.

As we know, acceleration= force/mass = -mgx/L/m

∴ acceleration= -gx/L

∴ acceleration per unti displacement will be,

acceleration/x = -g/L

Also, acceleration = ω2x

∴ ω = √acceleration per unit displacement

Considering magnitude, acceleration/x = g/L = ω2

Period of a Simple Pendulum

Now, the period of a simple pendulum is, T = 2π/ω = 2π/√acceleration per unit displacement

$$\therefore\ T = 2\pi/\sqrt{g/L} = 2\pi\sqrt{L/g}$$

The above equation is the equation of period of a simple pendulum

The frequency of Simple Pendulum

Now, frequency of a simple pendulum = $f = 1/T = 1/2\pi\sqrt{g/L}$. This equation is the equation of frequency of a simple pendulum.

Solved Question for You on Simple Pendulum Equations

Q: When does a simple pendulum perform Linear Simple Harmonic Motion?

Solution: In an ideal situation, a simple pendulum will execute periodic oscillation with constant amplitude. But in practice, the amplitude of oscillation will gradually decrease until the body comes to rest. The restoring force is proportional to $\sin\Theta$ and not Θ(Θ is the angular displacement).

Therefore, this motion is not S.H.M. But for Θ as large as $10°$,$\sin\Theta$ can be approximated by Θ to an error in fourth decimal point. For Θ less than $6°$, $\sin\Theta \approx \Theta$. Thus, for small displacement, the restoring force is directly proportional to the displacement and the pendulum performs linear S.H.M.

Energy in Simple Harmonic Motion

Each and every object possesses energy, either while moving or at rest. In the simple harmonic motion, the object moves to and fro along the same path. Do you think an object possesses energy while travelling the same path again and again? Yes, it is energy in simple harmonic motion. Let's learn how to calculate this energy and understand its properties.

Energy in Simple Harmonic Motion

The total energy that a particle possesses while performing simple harmonic motion is energy in simple harmonic motion. Take a pendulum for example. When it is at its mean position, it is at rest. When it moves towards its extreme position, it is in motion and as soon as it reaches its extreme position, it comes to rest again. Therefore, in order to calculate the energy in simple harmonic motion, we need to calculate the kinetic and potential energy that the particle possesses.

Browse more Topics under Oscillations

- Simple Harmonic Motion

- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion
- Energy in Simple Harmonic Motion
- Periodic and Oscillatory Motion

Kinetic Energy (K.E.) in S.H.M

Kinetic energy is the energy possessed by an object when it is in motion. Let's learn how to calculate the kinetic energy of an object. Consider a particle with mass *m* performing simple harmonic motion along a path AB. Let O be its mean position. Therefore, OA = OB = a.

The instantaneous velocity of the particle performing S.H.M. at a distance x from the mean position is given by

$v = \pm\omega \sqrt{a^2 - x^2}$

$\therefore\ v^2 = \omega^2 (a^2 - x^2)$

∴ Kinetic energy= $1/2\ mv^2 = 1/2\ m\ \omega^2\ (a^2 - x^2)$

As, $k/m = \omega^2$

∴ $k = m\ \omega^2$

Kinetic energy= $1/2\ k\ (a^2 - x^2)$. The equations Ia and Ib can both be used for calculating the kinetic energy of the particle.

Learn how to calculate Velocity and Acceleration in Simple Harmonic Motion.

Potential Energy(P.E.) of Particle Performing S.H.M.

Potential energy is the energy possessed by the particle when it is at rest. Let's learn how to calculate the potential energy of a particle performing S.H.M. Consider a particle of mass *m* performing simple harmonic motion at a distance x from its mean position. You know the restoring force acting on the particle is F= -kx where k is the force constant.

Now, the particle is given further infinitesimal displacement dx against the restoring force F. Let the work done to displace the particle be dw. Therefore, The work done dw during the displacement is

$dw = – fdx = – (- kx)dx = kxdx$

Therefore, the total work done to displace the particle now from 0 to x is

$\int dw= \int kxdx = k \int x\ dx$

Hence Total work done = $1/2\ K\ x^2 = 1/2\ m\ \omega^2 x^2$

The total work done here is stored in the form of potential energy.

Therefore Potential energy = $1/2\ kx^2 = 1/2\ m\ \omega^2 x^2$

Equations IIa and IIb are equations of potential energy of the particle. Thus, potential energy is directly proportional to the square of the displacement, that is P.E. $\alpha\ x^2$.

Learn the Difference between Periodic and Oscillatory Motion.

Total Energy in Simple Harmonic Motion (T.E.)

The total energy in simple harmonic motion is the sum of its potential energy and kinetic energy.

Thus, T.E. = K.E. + P.E. = $1/2\ k\ (a^2 - x^2) + 1/2\ K\ x^2 = 1/2\ k\ a^2$

Hence, T.E.= E = $1/2\ m\ \omega^2 a^2$

Equation III is the equation of total energy in a simple harmonic motion of a particle performing the simple harmonic motion. As ω^2, a^2 are constants, the total energy in the simple harmonic motion of a particle performing simple harmonic motion remains constant. Therefore, it is independent of displacement x.

As $\omega = 2\pi f$, $E = 1/2\ m\ (2\pi f)^2 a^2$

$\therefore E = 2m\pi^2 f^2 a^2$

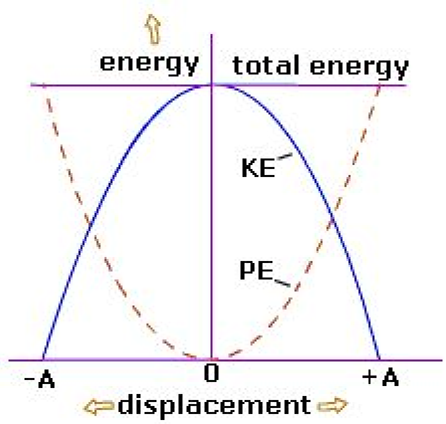
As 2 and π^2 constants, we have T.E. ~ m, T.E. ~ f^2, and T.E. ~ a^2

Thus, the total energy in the simple harmonic motion of a particle is:

- Directly proportional to its mass

- Directly proportional to the square of the frequency of oscillations and
- Directly proportional to the square of the amplitude of oscillation.

The law of conservation of energy states that energy can neither be created nor destroyed. Therefore, the total energy in simple harmonic motion will always be constant. However, kinetic energy and potential energy are interchangeable. Given below is the graph of kinetic and potential energy vs instantaneous displacement.



In the graph, we can see that,

- At the mean position, the total energy in simple harmonic motion is purely kinetic and at the extreme position, the total energy in simple harmonic motion is purely potential energy.
- At other positions, kinetic and potential energies are interconvertible and their sum is equal to $1/2\ k\ a^2$.
- The nature of the graph is parabolic.

Learn how Damped Simple Harmonic Motion is different than Linear Simple Harmonic Motion.

Here's a Solved Question for You

Q: At the mean position, the total energy in simple harmonic motion is ________

a) purely kinetic b) purely potential c) zero d) None of the above

Answer: a) purely kinetic. At the mean position, the velocity of the particle in S.H.M. is maximum and displacement is minimum, that is, x=0. Therefore, P.E. $=1/2\ K\ x^2 = 0$ and K.E. $= 1/2\ k\ (a^2 - x^2) = 1/2$

$k\,(a^2 - o^2) = 1/2\ ka^2$. Thus, the total energy in simple harmonic motion is purely kinetic.

Periodic and Oscillatory Motion

We come across various kinds of motions in our daily life. You have already studied some of them like linear and projectile motion. However, these motions are non-repetitive. Here, we are going to learn about periodic motion and oscillatory motion. Let's find out what they are.

Periodic Motion

What is common in the motion of the hands of a clock, motion of the wheels of a car and motion of a planet around the sun? They all are repetitive in nature, that is, they repeat their motion after equal intervals of time. A motion which repeats itself in equal intervals of time is periodic.

A body starts from its equilibrium position(at rest) and completes a set of movements after which it will return to its equilibrium position. This set of movements repeats itself in equal intervals of time to perform the periodic motion.

Circular motion is an example of periodic motion. Very often the equilibrium position of the body is in the path itself. When the body is at this position, no external force is acting on it. Therefore, if it is left at rest, it remains at rest.

Browse more Topics under Oscillations

Period and Frequency of Periodic Motion

- Simple Harmonic Motion
- Damped Simple Harmonic Motion
- Forced Simple Harmonic Motion
- Force Law for Simple Harmonic Motion
- Velocity and Acceleration in Simple Harmonic Motion
- Some Systems executing Simple Harmonic Motion
- Energy in Simple Harmonic Motion

We know that motion which repeats itself after equal intervals of time is periodic motion. The time interval after which the motion repeats itself is called time period (T) of periodic motion. Its S.I.unit is second.

The reciprocal of T gives the number of repetitions per unit time. This quantity is the frequency of periodic motion. The symbol υ represents frequency. Therefore, the relation between υ and T is

$$\upsilon = 1/T$$

Thus, the unit of υ is s^{-1} or hertz(after the scientist Heinrich Rudolf Hertz). Its abbreviation is Hz. Thus, 1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1} The frequency of periodic motion may not be an integer. It can be a fraction.

Oscillatory Motion

Every body at rest is in its equilibrium position. At this position, no external force is acting on it. Therefore, the net force acting on the body is zero. Now, if this body is displaced a little from its equilibrium position, a force acts on the body which tries to bring back the body to its equilibrium position. This force is the restoring force and it gives rise to oscillations or vibrations.

For example, consider a ball that is placed in a bowl. It will be in its equilibrium position. If displaced a little from this position, it will perform oscillations in the bowl. Therefore, every oscillatory motion is periodic but all periodic motions are not oscillatory. Circular motion is a periodic motion but not oscillatory motion.

There is no significant difference between oscillations and vibrations. When the frequency is low, we call it oscillatory motion and when the frequency is high, we call it vibrations. Simple harmonic motion is the simplest form of oscillatory motion. This motion takes place when the restoring force acting on the body is directly proportional to its displacement from its equilibrium position.

In practice, Oscillatory motion eventually comes to rest due to damping or frictional forces. However, we can force them by means of some external forces. A number of oscillatory motions together form waves like electromagnetic waves.

Displacement in Oscillatory Motion

Displacement of a particle is a change in its position vector. In an oscillatory motion, it simply displacement means a change in any physical property with time.

Consider a block attaches to a spring, which in turn is fixed to a rigid wall. We measure the displacement of the block from its equilibrium position. In an oscillatory motion, we can represent the displacement by a mathematical function of time. One of the simplest periodic functions is given by,

$$f(t) = A\cos\omega t$$

If the argument, ωt, is increased by an integral multiple of 2π radians, the value of the function remains the same. Therefore, it is periodic in nature and its period T is given by,

$$T = 2\pi/\omega$$

Thus, the function f(t) is periodic with period T. $\therefore$ $f(t) = f(t + T)$. Now, if we consider a sine function, the result will be the same. Further, taking a linear combination of sine and cosine functions is also a periodic function with period T.

$$f(t) = A\sin\omega t + B\cos\omega t$$

Taking $A = D\cos\varnothing$ and $B = D\sin\varnothing$ equation V becomes, $f(t) = D\sin(\omega t + \varnothing)$. In this equation D and $\varnothing$ are constant and they are given by,

$$D = \sqrt{A^2 + B^2} \text{ and } \varnothing = \tan^{-2}(B/A)$$

Therefore, we can express any periodic function as a superposition of sine and cosine functions of different time periods with suitable coefficients. The period of the function is $2\pi/\omega$.

Difference Between Periodic and Oscillatory Motion

The *main* difference is that oscillatory motion is always periodic but a periodic motion may or may not be oscillatory motion. For example, the motion of a pendulum is both oscillatory motion and periodic motion but the motion of the wheels of a car is only periodic because the wheels rotate in a circular motion. Circular motion is only periodic motion and not oscillatory motion. The wheels do not move to and fro about a mean position.

Learn about Damped Simple Harmonic Motion here.

Here's a Solved Question for You

Q: In periodic motion, the displacement is

a) directly proportional to the restoring force b) inversely proportional to the restoring force

c) independent of restoring force d) none of them

Solution: a) directly proportional to the restoring force. The displacement of the body from its equilibrium is directly proportional to the restoring force. Therefore, higher the displacement, higher is the restoring force.