Stats 170: Time Series
Homework 1

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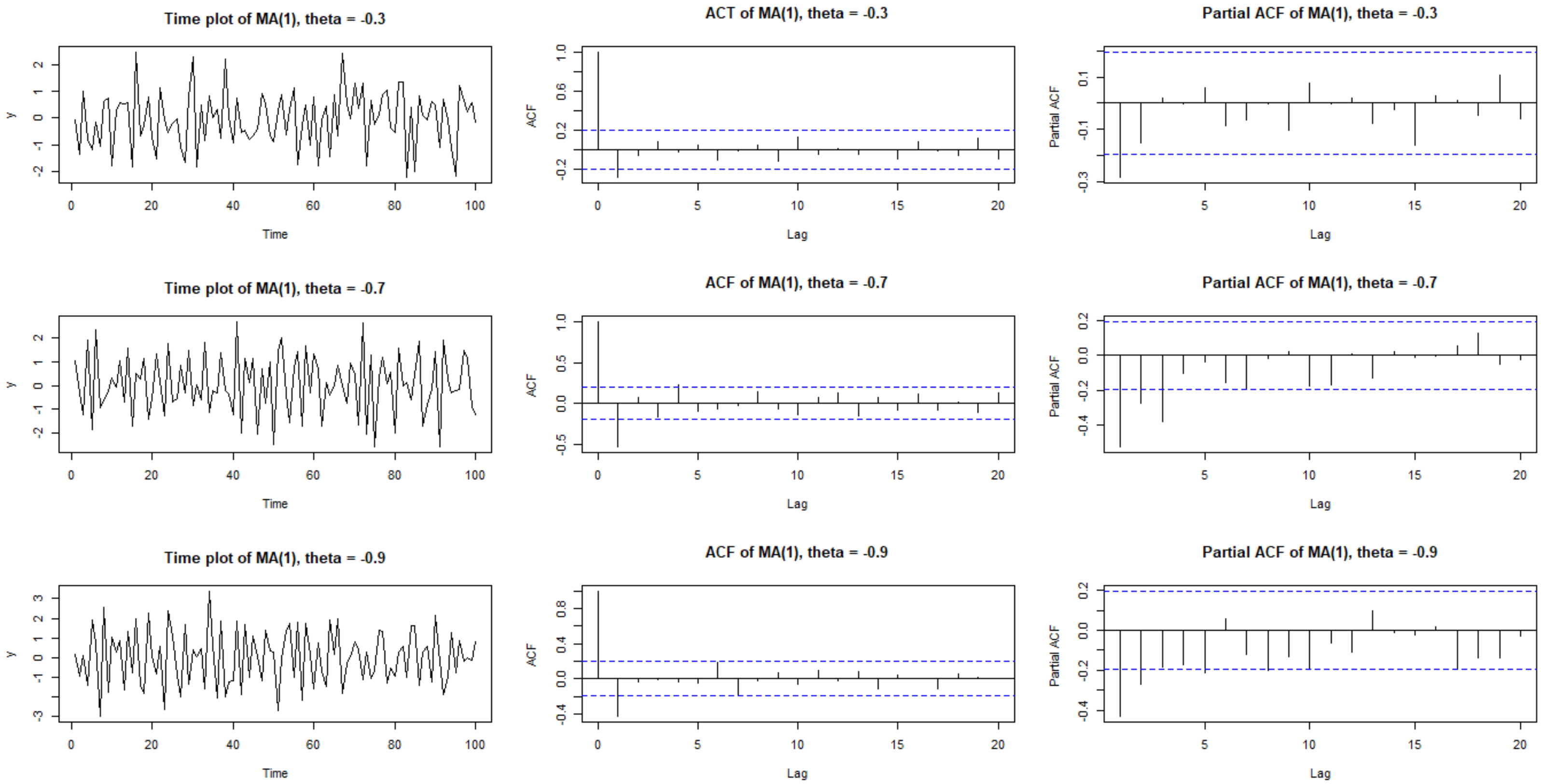
Question 1c



Question 1d

As the negative coefficient gets larger in absolute value, the time series plot appears to become more stationary with more constant variance over time. The ACF gradually decreases to 0, but at a faster rate with larger absolute value coefficients. The PACF has a sharper decline to 0 with larger absolute value coefficients.

Question 1e

All of the time series plots are relatively stationary and have no trend and similar seasonal/cyclical patterns. None of the lags in the ACF or in the PACF are significant besides due to random chance except for lags 1 and 2 in the ACF, and all of them tend to 0.

Question 1f

In the ACF model, the spikes cut-off to 0, whereas in the PACF, the spikes decay towards 0.

Question 1g

Besides lag 0, lag 1 is also statistically significant across all ACF plots. The autocorrelation coefficients of each, from theta = -0.3, -0,7, -0.9 respectively are: -0.2946, -0.5045, -0.4194.

Question 1h

Besides lag 0, lag 1 is also statistically significant across all ACF plots. The autocorrelation coefficients of each, from theta = -0.3, -0,7, -0.9 respectively are: -0.2946, -0.5045, -0.4194.

The theoretical values, respectively, at lag 1 are: -0.2752, -0.4698, -0.4972.

Question 2

$Y_t = e_t + 0.5e_{t-1}$

Because for MA(1), rho = beta1/(1 +beta1$^{2)}$ = 0.4 → 0.4 + beta1^2 = beta1 →

-0.4beta1^2 + beta1 – 0.4 = 0 → quadratic equation solves that beta1 = 0.5 or beta1 = 2

But since theoretically beta1 must be < 1, beta1 = 0.5

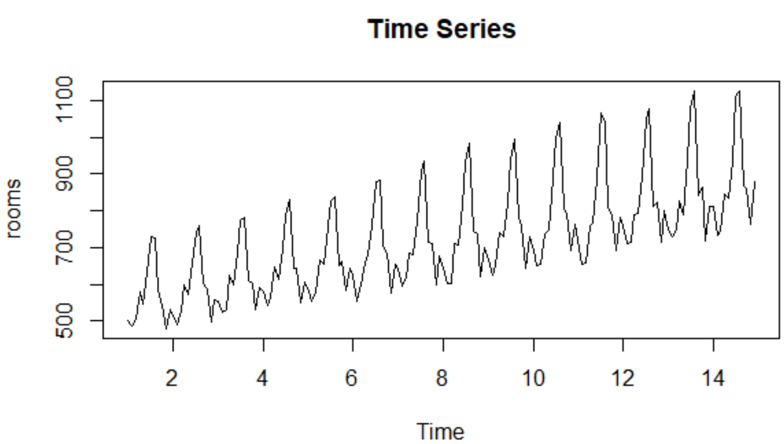
Question 3

Expectation: $E[5 + w_t - 0.5w_{t-1} + 0.25w_{t-2}] = E[5] + E[w_t] - E[0.5w_{t-1}] + E[0.25w_{t-2}] = 5$ since w is white noise and expectation of that will be 0.

Variance: $Var[5 + w_t - 0.5w_{t-1} + 0.25w_{t-2}] = Var[5] + Var[w_t] - Var[0.5w_{t-1}] + Var[0.25w_{t-2}] = 0 + \sigma^2 - 0.25\sigma^2 + 0.0625\sigma^2 = 0.8125\sigma^2$

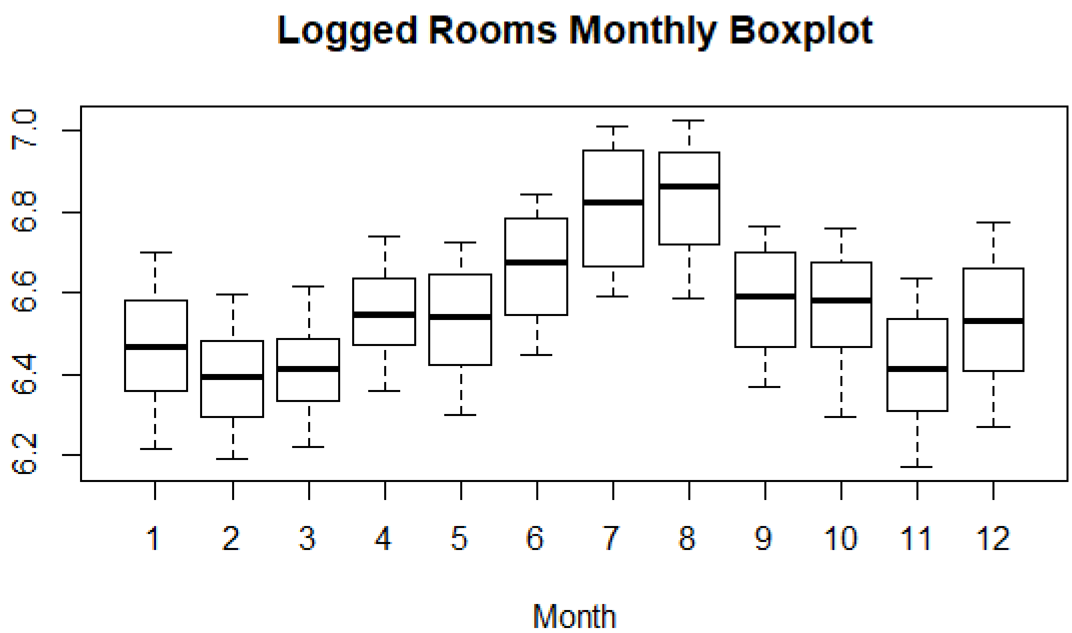
Thus, since expectation and variance are both constant over time and does not depend on lag ks, this is first order stationary.

Question 4a



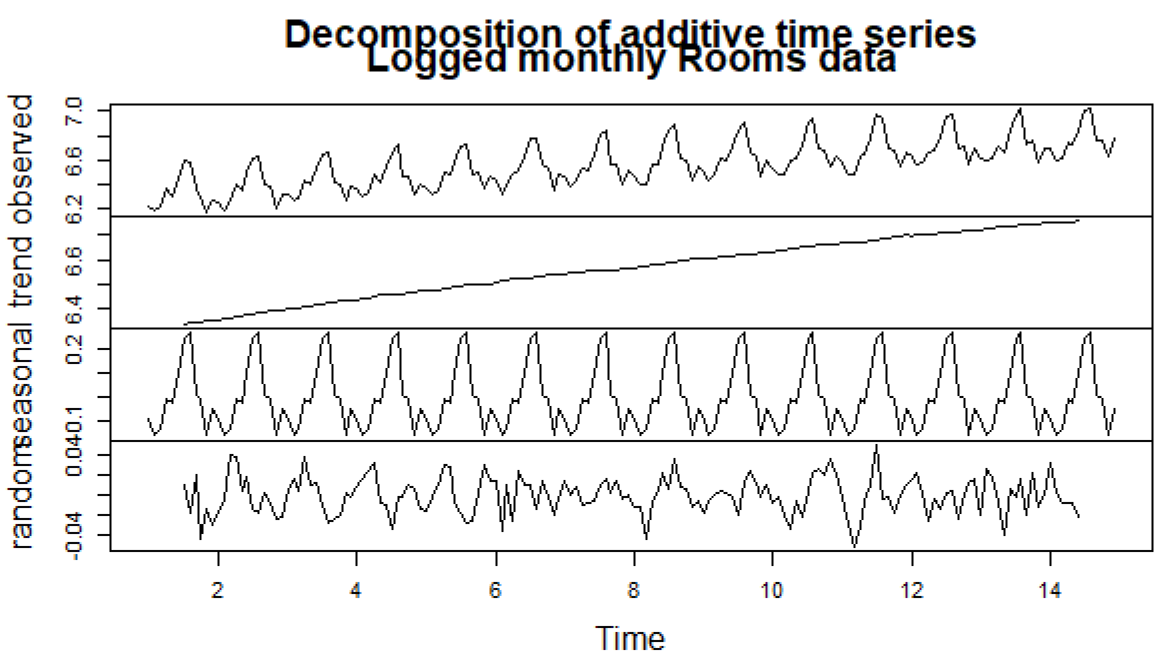
The trend is positive and linear, and there is a seasonal component. The amplitude slightly increases over time as well.

Question 4c



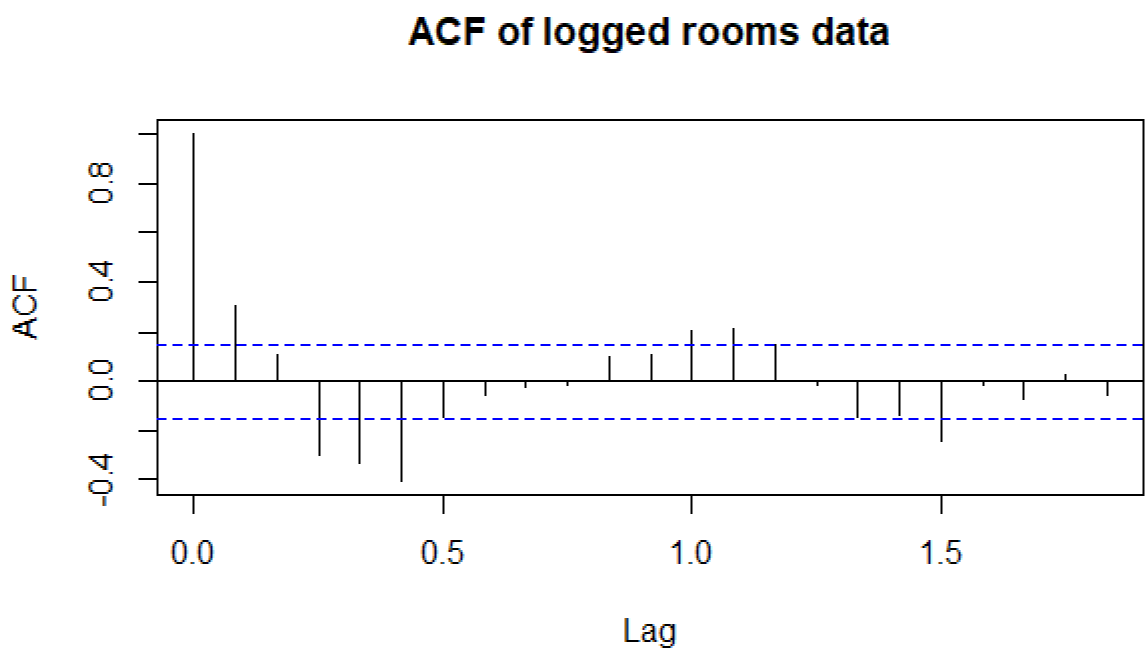
There is a clear seasonality across the months. There is a peak in the summer months from June to August, and then a smaller peak in the winter, in December and January.

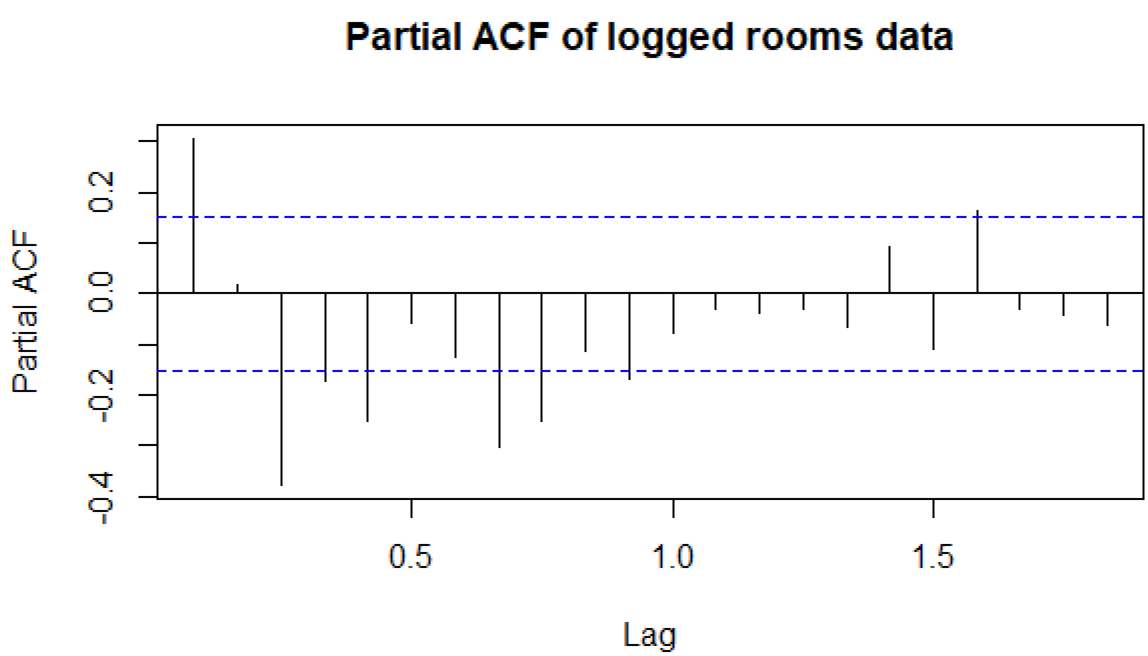
Question 4d



The trend is positive and linear, as I predicted. There is also a seasonal component, as predicted. The random noise seems to be white noise. The observed is the seasonal + trend patterns.

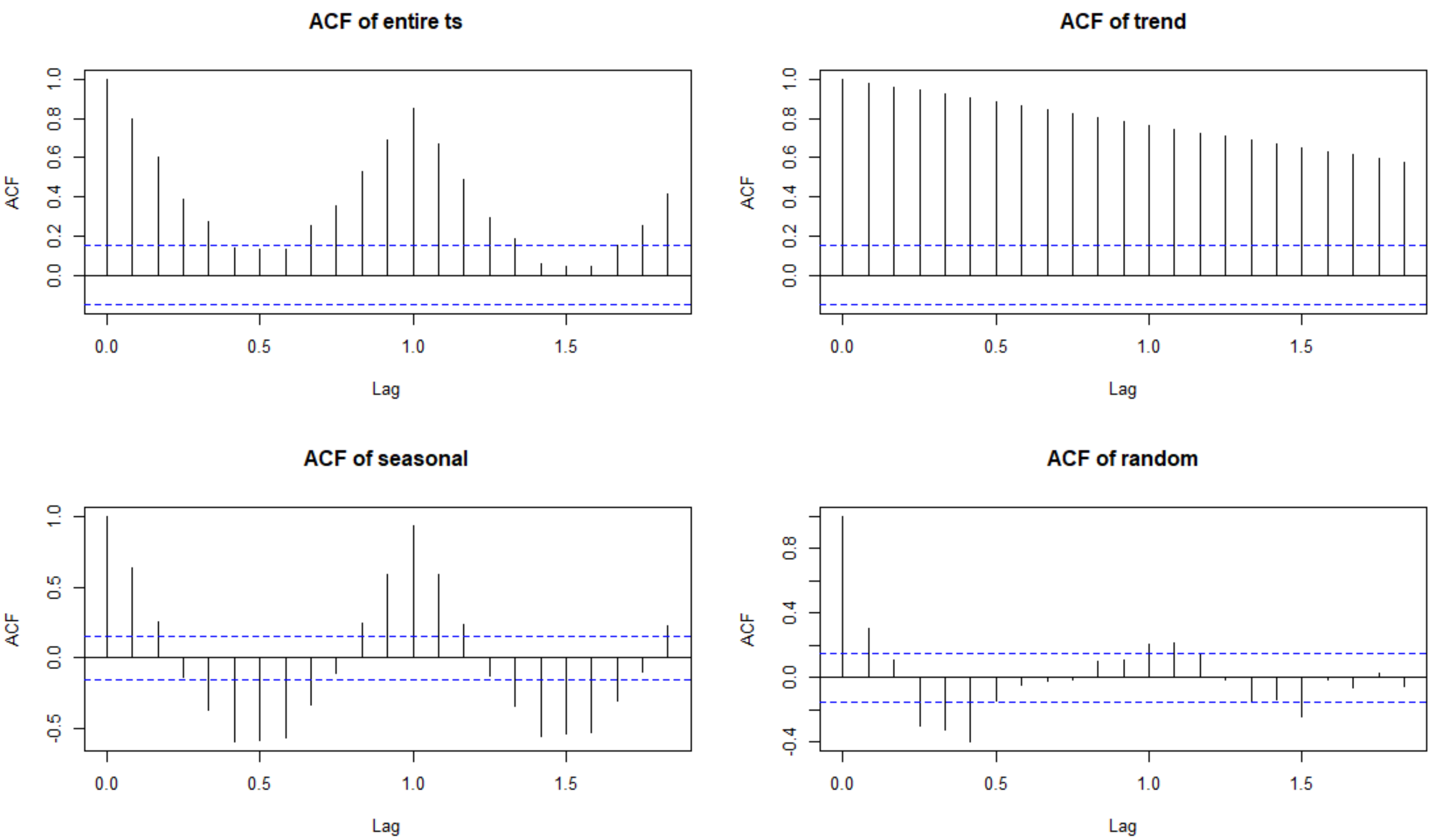
Question 4f



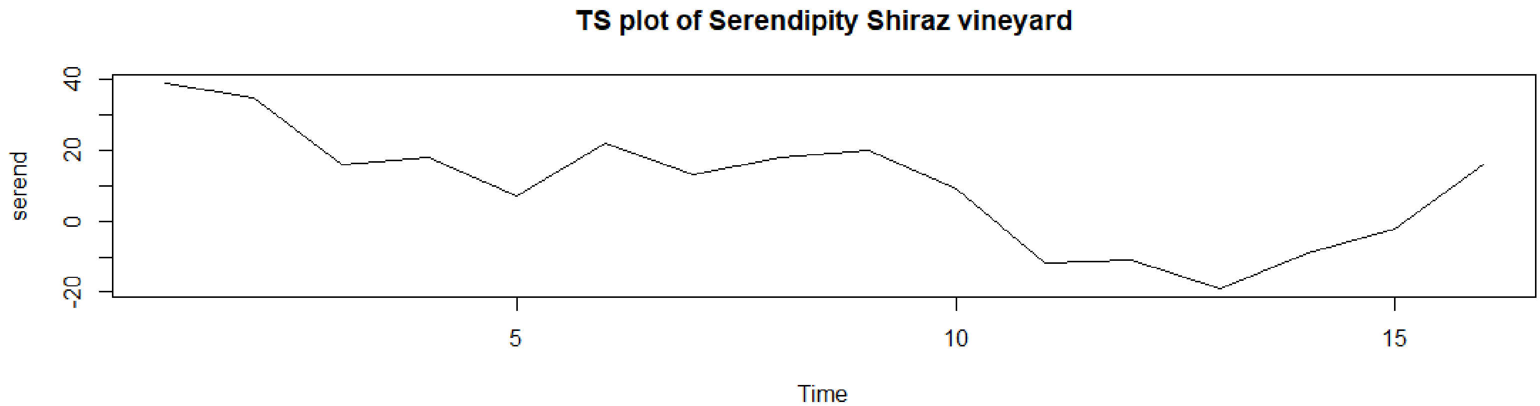


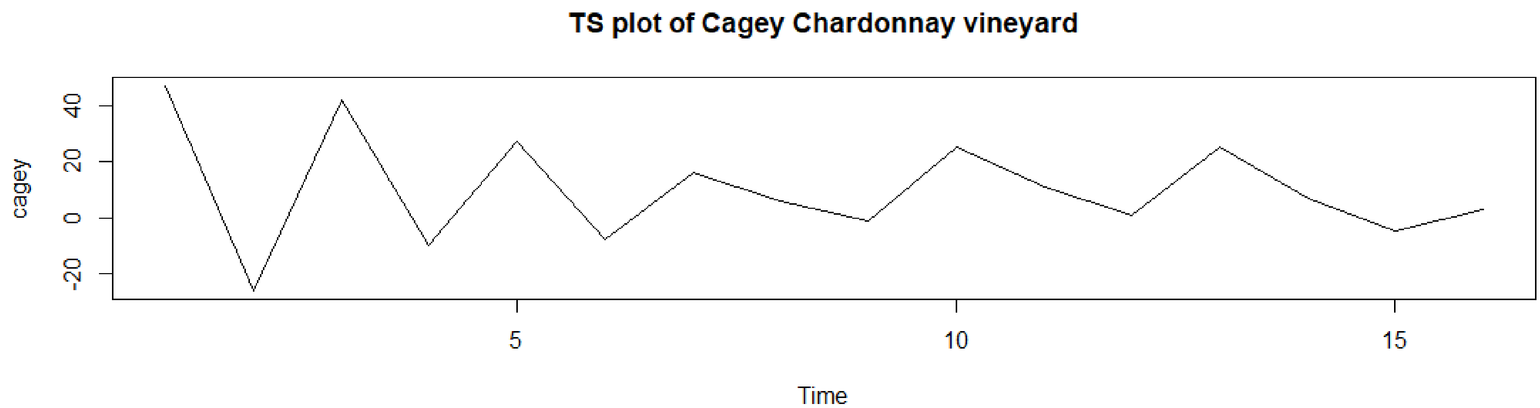
There seems to be a cyclical pattern in the ACF. It seems like an MA(1) model since in the ACF, the spikes cutoff to 0 and in the Partial ACF, the spikes decay toward 0. The significant lags in the ACF are at lags 0, 1, 3, 4, 5, 12, 13, and 18. Their autocorrelation values, respectively, are: 1, 0.3047, -0.3034, -0.3285, -0.4028, 0.2065, 0.2151, -0.2433.

<u>Question 4g</u>



Question 5a

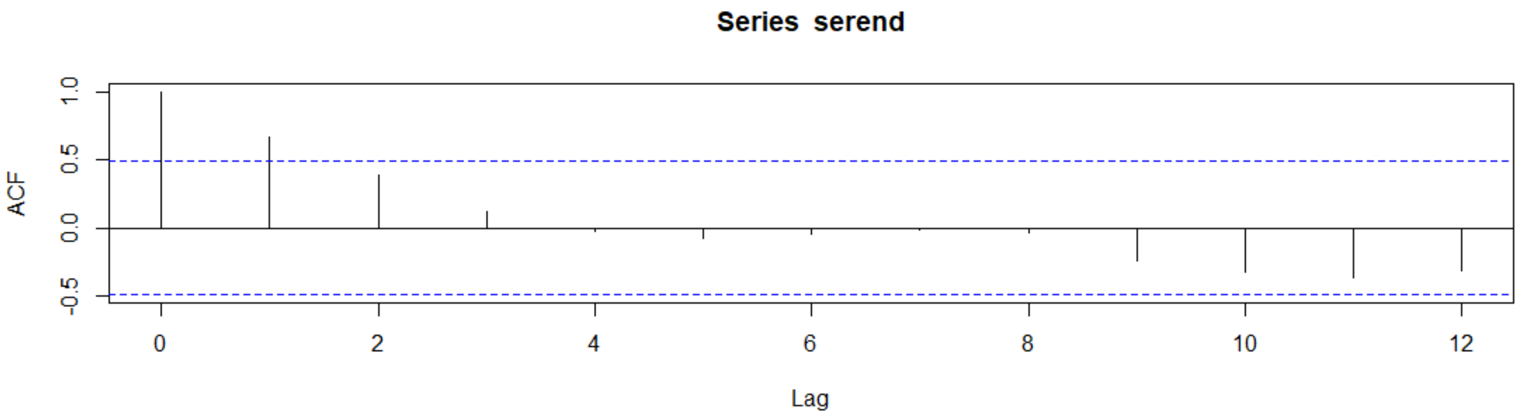


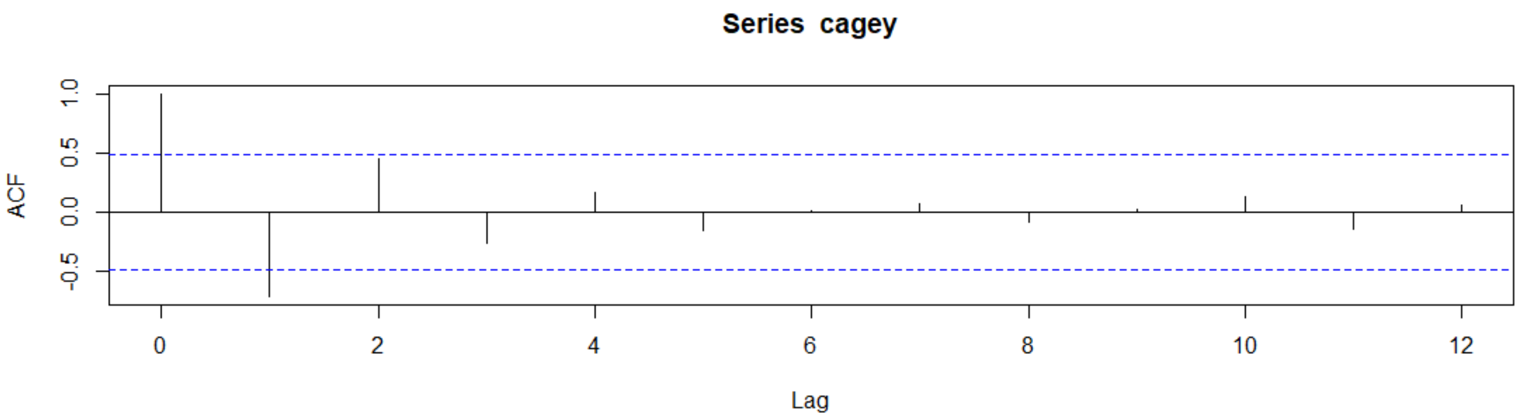


Serendipity Shiraz: seems to have a trend that is downward in the beginning and then upward at the end; unsure if quadratic. No apparent seasonal trend.

Cagey Chardonnay: seems to be stationary, with variance decreasing over time and with apparent seasonal trend.

Question 5b





The Serendipity ACF seems to have a downward trend in its ACF.
The Cagey ACF cut off to 0 over time.

Question 5c
I would trust Cagey, since the plot is somewhat stationary and has a decreasing variance over time. Serendipity seems to be quite inconsistent and thus unreliable.