KOMYA, KITU
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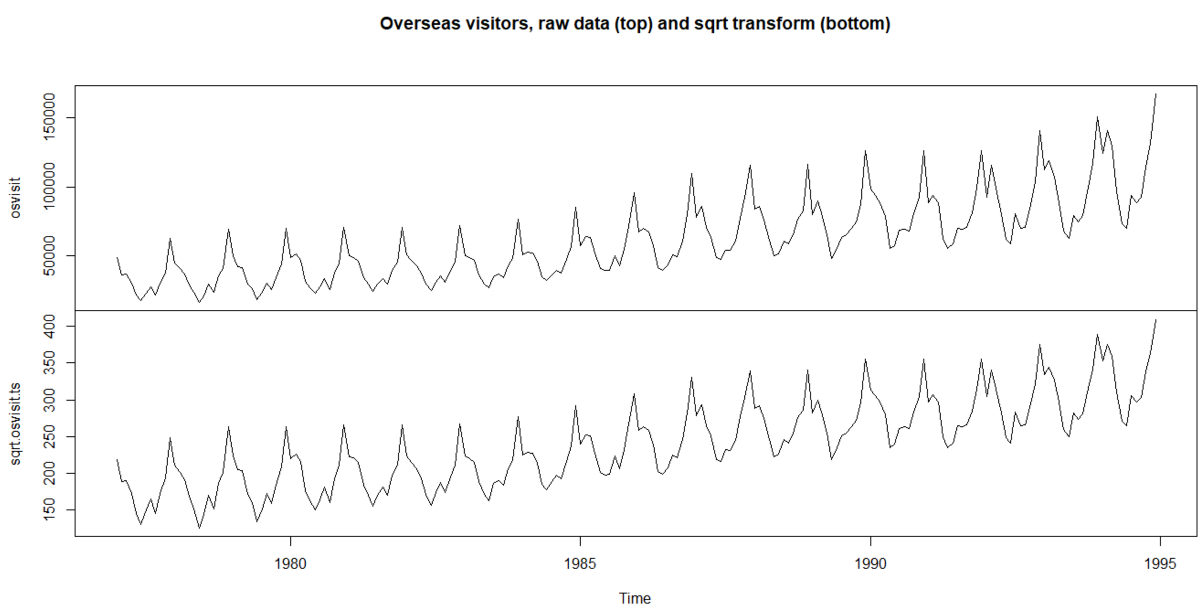
Due: 10/30/18
Homework 3

Question 1A

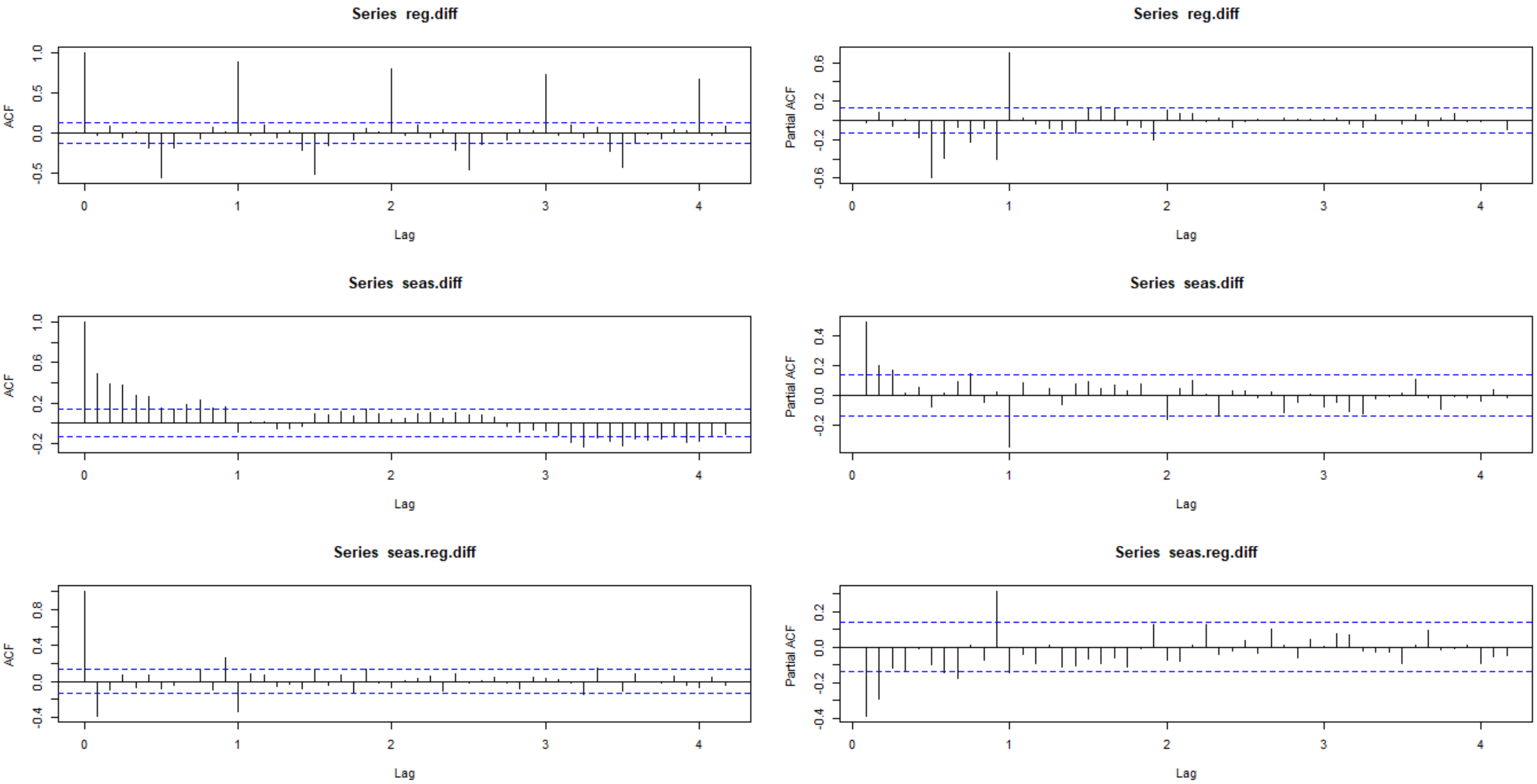
The square root function stabilizes the variance of the data points. y_t^* now has nearly equal variances without destroying the differences in the means between the data points. The time plot below visualizes the stable variance after transformation.



Question 1B

The ACF shows that the model is not stationary because all of the autocorrelations are significant and show a cyclical pattern. Further transformations are needed. We should start by differencing.

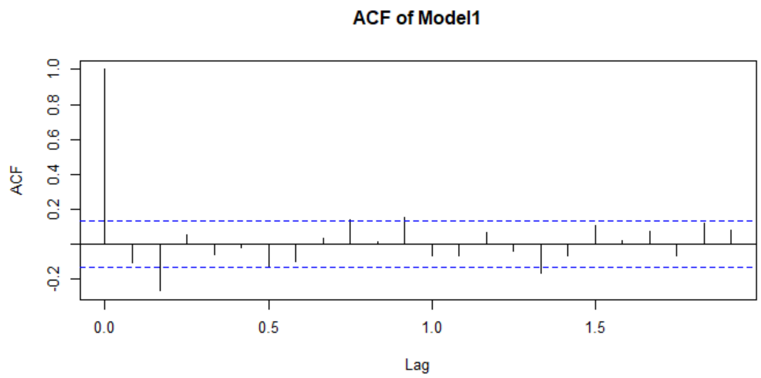
Question 1C

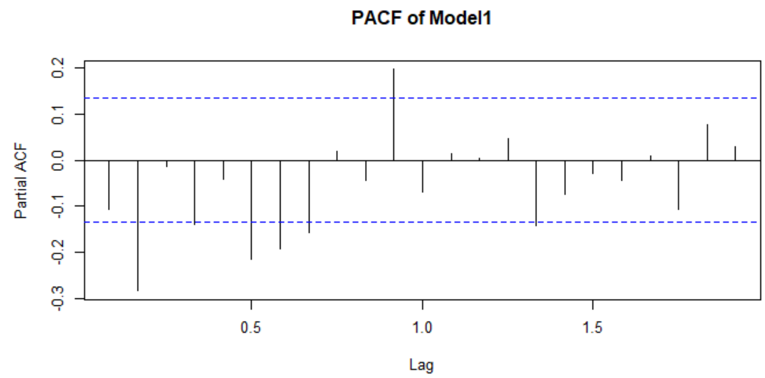


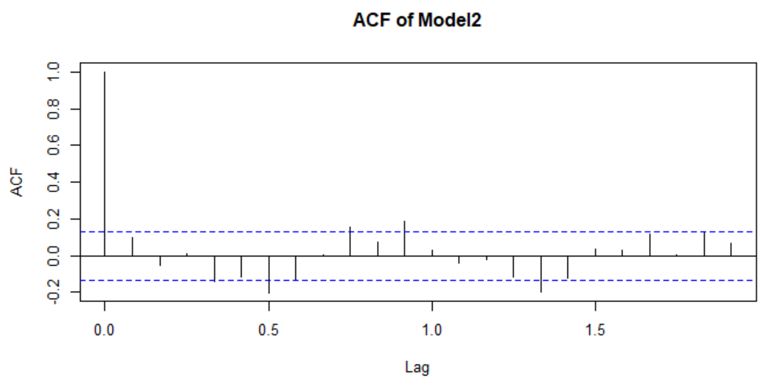
To make the data stationary, we will choose seasonal differencing of the regular difference (the last row). y_t^{**} is chosen because there are not many significant autocorrelations in the earlier lags and is less auto-correlated in the earlier lags than the other options in the ACF. Interpreting the PACF is irrelevant when looking at stationarity of models.

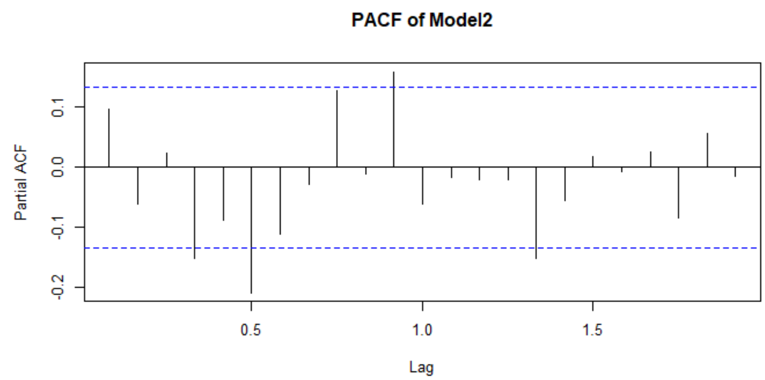
Question 1D

The first model, SARIMA(1, 1, 0)(1, 1, 0)$_{12}$ is more suitable because the ACF and PACF are less auto-correlated. Both, however, are not white noise residuals because there is a cyclical pattern in the ACF, meaning that the models are not a good fit since they do not take into account the seasonality.

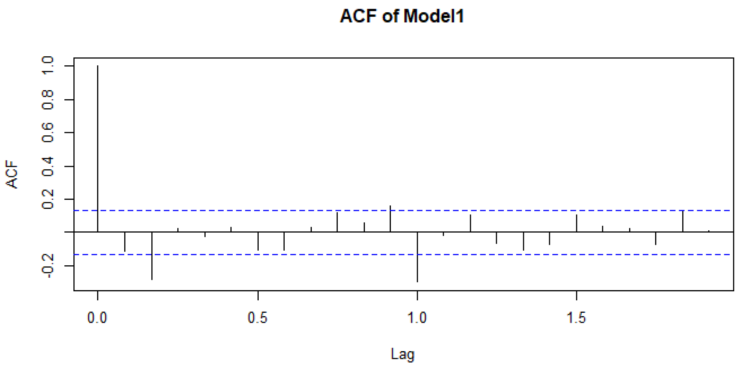


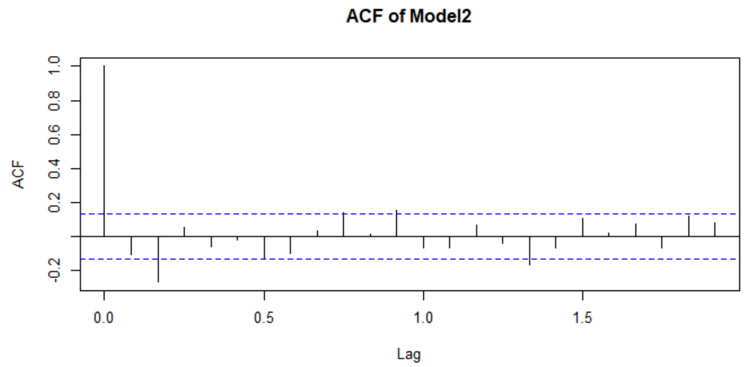


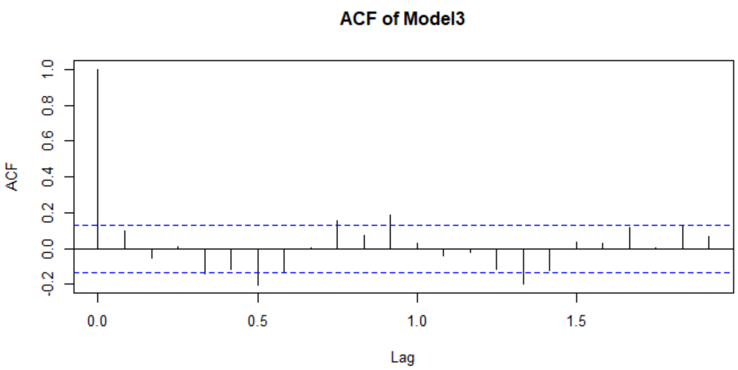


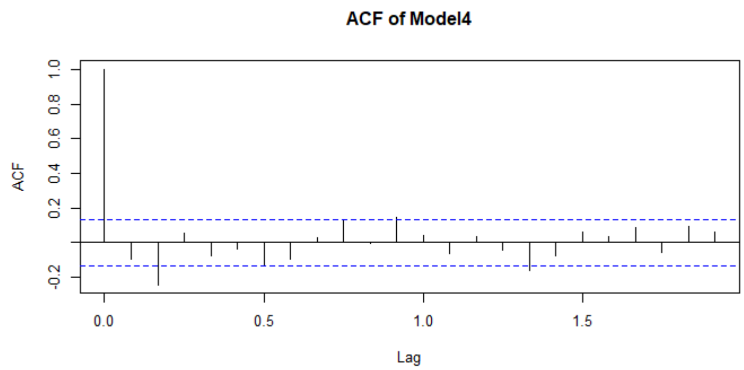


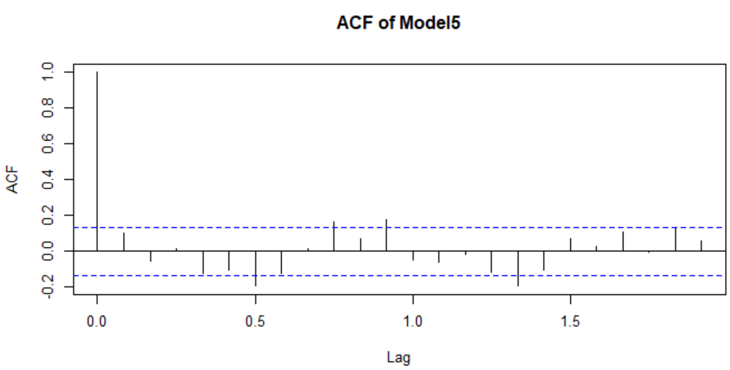
Question 1E

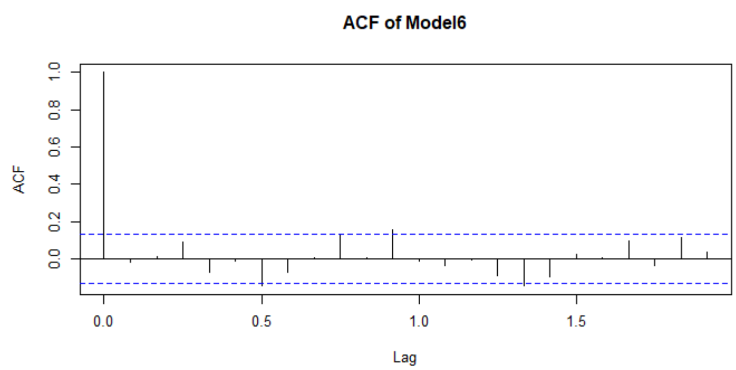

ACF of Model1
ACF
-0.2 0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag

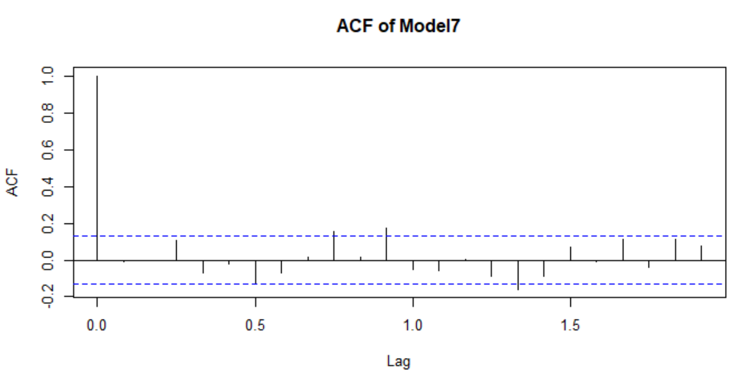

ACF of Model2
ACF
-0.2 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag

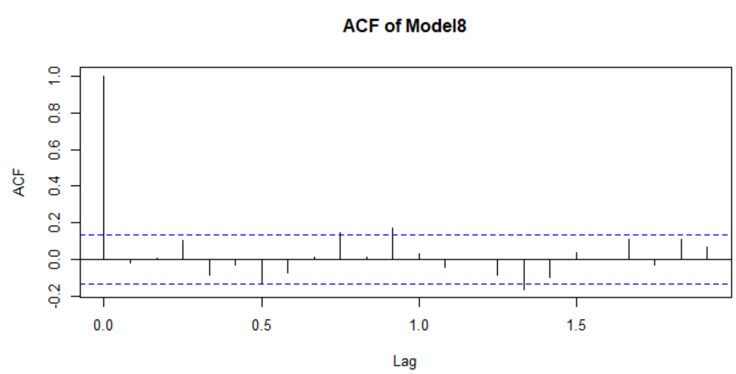

ACF of Model3
ACF
-0.2 0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag


ACF of Model4
ACF
-0.2 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag


ACF of Model5
ACF
-0.2 0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag


ACF of Model6
ACF
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag


ACF of Model7
ACF
-0.2 0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag


ACF of Model8
ACF
-0.2 0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.5 1.0 1.5
Lag

Selecting models based on ACFs that look like "white noise"
Models that can be taken into consideration that have residuals closer to white noise include models 2, 4, 6, 7, and 8. However, none of these models resemble white noise that well. To prove that, using the Ljung-Box test we see that none of the models have p-values greater than 0.05 except for Model 6 (0.07 p-value) which proves that Model 6's residuals are white noise, but the other models indicated above have p-values closest to 0.05.

Significant coefficients determined by whether 0 exists in its 95% confidence interval, constructed using the estimate and standard error (if 0 does not exist, then it is significant)

Model 2: two/two significant coefficients (AR1, SAR1)
Model 4: two/two significant coefficients (AR1, SMA1)
Model 6: three/four significant coefficients (AR1, MA1, SMA1)
Model 7: three/three significant coefficients (AR1, MA1, SAR1)
Model 8: three/three significant coefficients (AR1, MA1, SMA1)

I will still include all 5 models because all of the coefficients are significant except for Model 6, which we can still include because Model 6's residuals resemble white noise using the Ljung-Box, so we can still include it. Besides, ¾ significant coefficients is still good.

Notation of the models we will choose

Model 2: SARIMA(1, 1, 0)(1, 1, 0)$_{12}$
Model 4: SARIMA(1, 1, 0)(0, 1, 1)$_{12}$
Model 6: SARIMA(1, 1, 1)(1, 1, 1)$_{12}$
Model 7: SARIMA(1, 1, 1)(1, 1, 0)$_{12}$
Model 8: SARIMA(1, 1, 1)(0, 1, 1)$_{12}$

Question 1F

Model	**AIC**	**Estimated Sigma^2**	**RMSE**
Model 2: SARIMA(1, 1, 0)(1, 1, 0)$_{12}$	1450.531	71.61326	19316.73
Model 4: SARIMA(1, 1, 0)(0, 1, 1)$_{12}$	1440.316	67.53381	16131.14
Model 6: SARIMA(1, 1, 1)(1, 1, 1)$_{12}$	1415.504	**58.3783**	**11296.16**
Model 7: SARIMA(1, 1, 1)(1, 1, 0)$_{12}$	1422.092	61.41928	17310.10
Model 8: SARIMA(1, 1, 1)(0, 1, 1)$_{12}$	**1415.157**	58.98177	12050.42

Question 1G

We see that an ARMA(1, 1) for y_t^{**} is appropriate for the lower lags because both the ACF and PACF have significant spikes at lag 1 and then cut off or die down quickly after that. We also see that an ARMA(1, 1) for y_t^{**} for the seasonal lags is also appropriate because both the ACF and PACF only have a significant spike at the seasonal lag 12. This is illustrated in Model 6: SARIMA$(1, 1, 1)(1, 1, 1)_{12}$

The equation for Model 6

$(1 - \alpha_{12}B^{12})(1 - \alpha_1 B)(1 - B^{12})(1 - B)y_t^* = (1 + \beta_{12}B^{12})(1 + \beta_1 B)w_t$

$(1 - \alpha_1 B - \alpha_{12}B^{12} + \alpha_{12}\,\alpha_1 B^{13})y_t^{**} = (1 + \beta_1 B + \beta_{12}B^{12} + \beta_1\beta_{12}B^{13})w_t$

$y_t^{**} = \alpha_1\, y_{t-1}^{**} + \alpha_{12}\, y_{t-12}^{**} + \alpha_{12}\alpha_1 y_{t-13}^{**} + w_t + \beta_1 w_{t-1} + \beta_{12}w_{t-12} + \beta_{12}\beta_1 w_{t-13}$

Best model using RMSE (lowest score)

Model 6: SARIMA$(1, 1, 1)(1, 1, 1)_{12}$

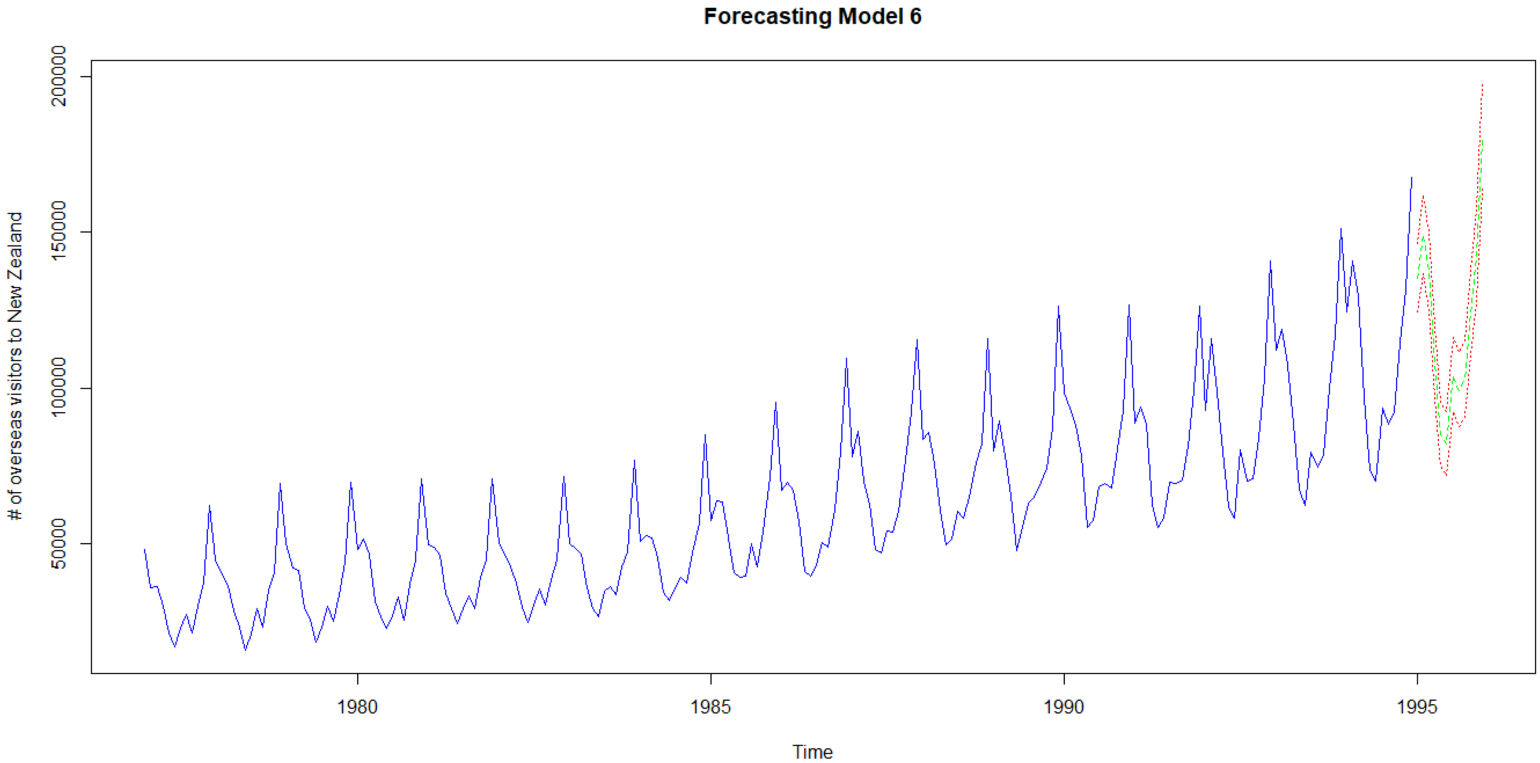
Best model using AIC (lowest score)

Model 8: SARIMA$(1, 1, 1)(0, 1, 1)_{12}$

Best model using estimated sigma^2 (lowest score)

Model 6: SARIMA$(1, 1, 1)(1, 1, 1)_{12}$

Question 1H



We have forecasted Model 6 on the original time series.

Forecasting Model Equation

Using equation of Model 6

$$(1-\alpha_{12}B^{12})(1-\alpha_1 B)(1-B^{12})(1-B)y_t^* = (1+\beta_{12}B^{12})(1+\beta 1B)w_t$$

Using coefficients from R

$$(1-0.2078273B^{12})(1-0.2840407B)(1-B^{12})(1-B)y_t^* = (1+(-0.6175477)B^{12})(1+(-0.8337891)B)w_t$$

Multiplying polynomial out

$$(0.0590314\,B^{26} - 0.266859\,B^{25} + 0.207827\,B^{24} - 0.343072\,B^{14} + 1.5509\,B^{13} - 1.20783\,B^{12} + 0.284041\,B^{2} - 1.28404\,B + 1)\,y_t^* = (1-0.833789\,B - 0.617548\,B^{12} - 0.514905\,B^{13})\,w_t$$

Simplifying polynomial

$$0.0590314\,y_{t-26}^* - 0.266859\,y_{t-25}^* + 0.207827\,y_{t-24}^* - 0.343072\,y_{t-14}^* + 1.5509\,y_{t-13}^* - 1.20783\,y_{t-12}^* + 0.284041\,y_{t-2}^* - 1.28404\,y_{t-1}^* + y_t^* = w_t - 0.833789\,w_{t-1} - 0.617548\,w_{t-12} - 0.514905 w_{t-13}$$

Final Model

$$\hat{y}_t^* = 1.28404\, y_{t-1}^* - 0.284041\, y_{t-2}^* + 1.20783\, y_{t-12}^* - 1.5509\, y_{t-13}^* + 0.343072\, y_{t-14}^* - 0.207827\, y_{t-24}^* + 0.266859\, y_{t-25}^* - 0.0590314\, y_{t-26}^* + w_t - 0.833789\, w_{t-1} - 0.617548\, w_{t-12} - 0.514905\, w_{t-13}$$

Square the value of $\hat{y}_t^*$ to get $\hat{y}_t$, since we had a square root transformation in the beginning.