Amsterdam:

Frankfurt: 1
London: none
Hong Kong: 1
Japan: 1
Singapore: none
New York: 1
Amsterdam: 1, 2

Amsterdam_t = (alpha1 * Amsterdam_t-1) + (alpha2 * Amsterdam_t-2) + (alpha3 * Frankfurt_t-1) + (alpha4 * HongKong_t-1) + (alpha5 * Japan_t-1) + (alpha6 * NewYork_t-1)

Frankfurt:

Japan: 1
Singapore: none
New York: 1
Amsterdam: 1
London: 1
Hong Kong: 1
Frankfurt: 1, 2

Frankfurt_t = (alpha1 * Frankfurt_t-1) + (alpha2 * Frankfurt_t-2) + (alpha3 * Japan_t-1) + (alpha4 * NewYork_t-1) + (alpha5 * Amsterdam_t-1) + (alpha6 * London_t-1) + (alpha7 * HongKong_t-1)

London:

Japan: 1
Singapore: 1
New York: 1
Amsterdam: none
Frankfurt: none
Hong Kong: 3
London: 1, 2

London_t = (alpha1 * London_t-1) + (alpha2 * London_t-2) + (alpha3 * Japan_t-1) + (alpha4 * Singapore_t-1) + (alpha5 * NewYork_t-1) + (alpha6 * HongKong_t-3)

Hong Kong:

Amsterdam: 1
Frankfurt: none

London: 1
Japan: none
Singapore: none
New York: 1
Hong Kong: 1, 2

HongKong_t = (alpha1 * HongKong_t-1) + (alpha2 * HongKong_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * London_t-1) + (alpha5 * NewYork_t-1)

Japan:
Amsterdam: 1
Frankfurt: 1
London: 1
Hong Kong: none
Singapore: 1
New York: 1
Japan: 1, 2

Japan_t = (alpha1 * Japan_t-1) + (alpha2 * Japan_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * Frankfurt_t-1) + (alpha5 * London_t-1) + (alpha6 * Singapore_t-1) + (alpha7 * NewYork_t-1)

Singapore:
Amsterdam: 1
Frankfurt: 1
London: 1
Hong Kong: 1
Japan: none
New York: 1
Singapore: 1, 2

Singapore_t = (alpha1 * Singapore_t-1) + (alpha2 * Singapore_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * Frankfurt_t-1) + (alpha5 * London_t-1) + (alpha6 * HongKong_t-1) + (alpha7 * NewYork_t-1)

New York:
Amsterdam: none
Frankfurt: 1
London: none

Hong Kong: 1, 2
Japan: 1
Singapore: none
New York: 1, 2

NewYork_t = (alpha1 * NewYork_t-1) + (alpha2 * NewYork_t-2) + (alpha3 * Frankfurt_t-1) + (alpha4 * HongKong_t-1) + (alpha5 * HongKong_t-2) + (alpha6 * Japan_t-1)

Questions: how to find ams ams term from so many?
Why in var R code do we not use diff(model)
Const vs trend in VAR…what is the math behind it
Non-linear trends…trend type in VAR

Part A

We see 7 time plots from various regions. Many of them seem to be co-integrated. Amsterdam, Frankfurt, London, Hong Kong, and New York all have similar trends: they slowly and exponentially rise, with small bumps throughout. All 5 of these regions seem to have a short dip right before the 500 time mark. Other than that, they are positively and exponentially increasing. Singapore and Japan have slightly different patterns. Singapore is positive and somewhat linear but has constant dips throughout. But in general, it seems to steadily increase but slowly wane off a bit at the end. Japan, on the other hand, is quite different from any of these regions. Japan linearly increases with small bumps along the way until the 1000 time mark, after which it declines in a linear manner until a bit past the 1500 time mark, after which it somewhat stabilizes and has no trend near the end.

Part B

The ACF of the plots suggest non-stationary data since all the lags are significant across all the plots. Regular differencing seems to help since we have corrected for the non-stationarity and the differenced ACF models a potential equation.

Here are the potential models I have identified, after looking at the CCFs of all of the regions:

- Amsterdam_t = (alpha1 * Amsterdam_t-1) + (alpha2 * Amsterdam_t-2) + (alpha3 * Frankfurt_t-1) + (alpha4 * HongKong_t-1) + (alpha5 * Japan_t-1) + (alpha6 * NewYork_t-1)
- Frankfurt_t = (alpha1 * Frankfurt_t-1) + (alpha2 * Frankfurt_t-2) + (alpha3 * Japan_t-1) + (alpha4 * NewYork_t-1) + (alpha5 * Amsterdam_t-1) + (alpha6 * London_t-1) + (alpha7 * HongKong_t-1)

- London_t = (alpha1 * London_t-1) + (alpha2 * London_t-2) + (alpha3 * Japan_t-1) + (alpha4 * Singapore_t-1) + (alpha5 * NewYork_t-1) + (alpha6 * HongKong_t-3)
- HongKong_t = (alpha1 * HongKong_t-1) + (alpha2 * HongKong_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * London_t-1) + (alpha5 * NewYork_t-1)
- Japan_t = (alpha1 * Japan_t-1) + (alpha2 * Japan_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * Frankfurt_t-1) + (alpha5 * London_t-1) + (alpha6 * Singapore_t-1) + (alpha7 * NewYork_t-1)
- Singapore_t = (alpha1 * Singapore_t-1) + (alpha2 * Singapore_t-2) + (alpha3 * Amsterdam_t-1) + (alpha4 * Frankfurt_t-1) + (alpha5 * London_t-1) + (alpha6 * HongKong_t-1) + (alpha7 * NewYork_t-1)
- NewYork_t = (alpha1 * NewYork_t-1) + (alpha2 * NewYork_t-2) + (alpha3 * Frankfurt_t-1) + (alpha4 * HongKong_t-1) + (alpha5 * HongKong_t-2) + (alpha6 * Japan_t-1)

Based on these potential models, either a VAR(2) or VAR(3) seems most appropriate because most models only until t-2 or t-3. We can start by fitting a VAR(3) and see if the t-3 terms are significant.

Part C

The table below shows which coefficients at which lags are significant for the model equations. The column variables are the "input variables" and the row variables are the "response variables." The table will state whether an input variable affects the equation of the response variable by denoting which lags are significant.

Input --> Response ↓	Amsterdam	Frankfurt	London	Hong Kong	Japan	Singapore	New York
Amsterdam		t-1, t-3	None	t-1, t-2	t-1, t-2	t-1	t-1, t-2, t-3
Frankfurt	t-1, t-3		None	t-1, t-2, t-3	t-1, t-2	t-1	t-1, t-2, t-3
London	t-1	None		t-2, t-3	t-1, t-2	t-1, t-2	t-1, t-2, t-3
Hong Kong	None	t-1, t-2	t-1, t-2		none	t-1	t-1, t-2, t-3
Japan	None	t-1	t-1, t-2	t-1, t-2		t-1, t-2	t-1, t-2
Singapore	None	None	t-1, t-2	t-1	t-1, t-2, t-3		t-1, t-2, t-3
New York	t-1	t-1	None	t-1, t-2, t-3	t-1	t-1, t-2	

	Frankfurt	London	Hong Kong	Japan	Singapore	New York
Amsterdam	r: t-1, t-3 c: t-1, t-3	r: t-1 c: none	r: none c: t-1, t-2	r: none c: t-1, t-2	r: none c: t-1	r: t-1 c: t-1, t-2, t-3
Frankfurt		r: none c: none	r: t-1, t-2 c: t-1, t-2, t-3	r: t-1 c: t-1, t-2	r: none c: t-1	r: t-1 c: t-1, t-2, t-3
London			r: t-1, t-2 c: t-2, t-3	r: t-1, t-2 c: t-1, t-2	r: t-1, t-2 c: t-1, t-2	r: none c: t-1, t-2, t-3
Hong Kong				r: t-1, t-2 c: none	r: t-1 c: t-1	r: t-1, t-2, t-3 c: t-1, t-2, t-3
Japan					r: t-1, t-2, t-3 c: t-1, t-2	r: t-1 c: t-1, t-2
Singapore						r: t-1, t-2 c: t-1, t-2, t-3

Input → Response ↓	Amsterdam	Frankfurt	London	Hong Kong	Japan	Singapore	New York
Amsterdam		t-1, t-3	None	t-1, t-2	t-1, t-2	t-1	t-1, t-2, t-3
Frankfurt	t-1, t-3		None	t-1, t-2, t-3	t-1, t-2	t-1	t-1, t-2, t-3
London	t-1	None		t-2, t-3	t-1, t-2	t-1, t-2	t-1, t-2, t-3
Hong Kong	None	t-1, t-2	t-1, t-2		none	t-1	t-1, t-2, t-3
Japan	None	t-1	t-1, t-2	t-1, t-2		t-1, t-2	t-1, t-2
Singapore	None	None	t-1, t-2	t-1	t-1, t-2, t-3		t-1, t-2, t-3
New York	t-1	t-1	None	t-1, t-2, t-3	t-1	t-1, t-2	

Amsterdam leads Frankfurt, London, and New York.

Frankfurt leads Amsterdam, Hong Kong, Japan, and New York.

London leads Hong Kong, Japan, and Singapore.

Hong Kong leads Amsterdam, Frankfurt, London, Japan, Singapore, and New York.

Japan leads Amsterdam, Frankfurt, London, Singapore, and New York.

Singapore leads Amsterdam, Frankfurt, London, Hong Kong, Japan, and New York.

New York leads Amsterdam, Frankfurt, London, Hong Kong, Japan, and Singapore.

We see that New York, Hong Kong, and Singapore all lead all of the other regions. New York has the strongest lead because it affects up until 3 lags to the other regions. Second strongest is Hong Kong who affects between 2 and 3 lags to the other regions. Finally we have Singapore who affects all the regions by 1 or 2 lags.

Part D

The ACF of the residuals resemble white noise, so we have fit a good model. The residuals are also normally distributed, which meets our assumptions, and we can thus proceed to forecast.

Part F

A one unit change in New York at time 0 does not affect New York, Amsterdam, or Singapore since these variables remain constant and stable and stay at their original states, but it does affect London and Frankfurt by increasing them until they peak at time 2 or 3, after which they dip slightly and stabilize to values a bit higher than their original states. The same pattern is seen for Hong Kong as well, except that its initial increase is higher in amplitude and it stabilizes with a higher value, suggesting permanent change, while London and Frankfurt have similar, smaller initial increases that tend to a permanent change less drastic. Finally, it affects Japan by increasing it enormously in the beginning after which it peaks by time 2 or 3 and then steadily declines in a quadratic fashion until time 50 after which it steadily increases in a quadratic fashion. We do not see Japan stabilizing in this 100 time mark, whereas the other regions quickly stabilize. Moreover, Japan is drastically above its original state and it seems to have a permanent change above its baseline.

Part G

A one unit change in Japan at time 0 does not affect Amsterdam, New York, or Singapore at all. They remain at the baseline. London, Frankfurt, and Hong Kong have slight dips below their baseline. London and Frankfurt stabilize back to their baselines around time 40 and 28, respectively, whereas Hong Kong stabilizes below its baseline from the onset. However, Japan is affected the most drastically since Japan has a drastic increase which peaks around time 0, and then slowly dips down to slowly stabilize at a much higher baseline throughout the duration. We don't see Japan totally stabilize, but we can imagine that it is permanently changed to be above the baseline by a drastic amount. In comparing our responses from Part F, we see that while the USA affected London, Japan, Frankfurt, and Hong Kong quite drastically, Japan only affected Japan. This means that the US stock is much more influential to other markets since it makes the system much more unstable than Japan does.

Part H

A one unit change in London does not affect Amsterdam, New York, Frankfurt or Singapore. All tend to stabilize at their baselines. However, both London and Hong Kong have a sharp increase from the onset, after which they slowly decline and stabilize above their baselines. We see both stabilizing around time 70. Japan has a drastic effect by London; it starts with a sharp increase and then linearly and positively climbs upward at a moderate pace. We can imagine that Japan will have undergone permanent change since it is far from the baseline and is far from stabilizing since it continues to have an increase throughout the duration. In comparing our responses from Part F and G, we see that although London does not have as much influence as New York, it still affects Japan, London, and Hong Kong considerably. The last three responses lead us to believe that in power of influence, New York leads London which leads Japan, since Japan is affected by New York and London, and London is affected by New York.

t	w_t.1	Ψ_1.t	w_t.2	Ψ_2.t	w_t.3	Ψ_1.t	Ψ_total.t
14	0	0.02312	0	0.3319	0	-0.3674	-0.0097
15	0	-0.1841	0	-0.1095	0	-0.3599	-0.6533
16	0	-0.055316	0	-0.2874	0	0.2148	-0.1221916
17	0	0.1362168	0	0.0301	0	0.3319	0.4982168
18	0	0.07149	0	0.2359	0	-0.1095	0.19789
19	0	-0.09467	0	0.02312	0	-0.2874	-0.35895
20	0	-0.07613	0	-0.1841	0	0.0301	-0.23013

$x_t = 0.2*(x_{t-1}) – 0.8*(x_{t-2}) + w_t$

$x_{16} = 0.2 * (-0.1841) – 0.8 * (0.02312) = -0.055316$

$x_{17} = 0.2 * (-0.055316) – 0.8 * (-0.1841) = 0.1362168$

$x_{18} = 0.2 * (0.1362168) – 0.8 * (-0.0553) = 0.07149$

$x_{19} = 0.2 * (0.07149) – 0.8 * (0.13621) = -0.09467$

$x_{20} = 0.2 * (-0.09467) – 0.8 * (0.07149) = -0.07613$