Axiomatic Foundations of Universal Computation

Axiom 1 (The Principle of Computational Existence)

The universe instantiates a computational process. This process is defined by the 7-tuple $U = (I, O, P, C, F, N, E)$, where the Processing function P must be non-trivial ($P \neq$ Identity).

Axiom 2 (The Principle of Persistent State)

For a computational process to be non-trivial, it must have access to a persistent, addressable state (memory). Formally, there must exist a substrate S where state $S(t)$ is a function of prior states and inputs, with a non-zero relaxation time $\tau > 0$.

Axiom 3 (The Principle of Optimization)

The universal computational process U evolves parameters to maximize its long-term computational potential, Φ, where Φ is a measure of the total, time-integrated, information-processing capacity.

Axiom 4 (The Principle of Substrate-Process Duality)

Any computational process P acting on a substrate S can itself be represented as a substrate S' for a higher-order process P'. This recursion continues until fundamental physical limits are reached.

Theorem: Necessity of Baryon Asymmetry

Proof:

1. From Axiom 1, the universe U has a non-trivial Processing function P.
2. From Axiom 2, P requires a persistent, addressable state substrate S.
3. A perfectly symmetric universe (matter = antimatter) has no persistent state substrate. Post-annihilation, S is a photon bath.
4. A photon bath is computationally trivial:
    - **No Persistence**: Photons travel at c, experiencing no proper time (from their reference frame). They cannot maintain state.
    - **No Addressability**: Photons in a bath are indistinguishable and cannot be selectively addressed without a material detector (which doesn't exist).
    - Therefore, in a symmetric universe, the relaxation time τ of any state $S \rightarrow 0$.

5. This violates Axiom 2. Therefore, for U to be non-trivial (Axiom 1), the state substrate S must be persistent, requiring a matter-dominated universe.
6. **Conclusion**: A non-zero baryon asymmetry, $n > 0$, is a necessary condition for a non-trivial universal computation.

Theorem: Optimal Value of Baryon Asymmetry

Proof Sketch & Derivation:

We now derive the optimal value of n that maximizes the long-term computational potential Φ (Axiom 3).

1. **Define Computational Potential (Φ)**: Based on our framework, we define Φ as proportional to the total number of computational operations possible over the lifetime of the universe. This can be modeled as: $\Phi(n) \propto$ [Number of Processing Units] × [Lifespan of Computation] × [Computational Diversity]
2. **Model the Components:**
    - **Number of Processing Units (N)**: This is proportional to the number of baryons, which is proportional to the asymmetry n. $N(n) \propto n$
    - **Lifespan of Computation (T)**: This is the duration for which complex computation (e.g., stellar nucleosynthesis, biological evolution) is possible.
        - If n is too high, the universe collapses quickly into black holes. If n is too low, structures never form. The computational lifespan is thus a function that peaks at an intermediate value.
            - We can model it as a Gaussian-like constraint: $T(n) \propto \exp(- ((n - n_{optimal}) / \sigma)^2)$ where σ represents the sensitivity of the lifespan to changes in n.
    - **Computational Diversity (D)**: This represents the variety of computational modes. A very low n allows only simple quantum computation. A very high n leads only to black holes. The diversity is maximized at an intermediate n that allows atoms, chemistry, stars, and planets. We model this similarly: $D(n) \propto \exp(- ((n - n_{optimal}) / \sigma)^2)$
3. **Formulate the Optimization Problem:** Combining these, the function to maximize is: $\Phi(n) = N(n) \times T(n) \times D(n) \propto n \times [\exp(- ((n - n_{optimal}) / \sigma)^2)]^2$ $\Phi(n) \propto n \times \exp(-2 ((n - n_{optimal}) / \sigma)^2)$
4. **Solve for the Maximum:** To find the value of n that maximizes Φ, we take the derivative and set it to zero: $d\Phi/dn = 0$ This gives: $1 - (4n (n - n_{optimal}))/\sigma^2 = 0$ For the observed universe, the optimal value $n_{optimal}$ must be the one that has allowed for ~13.8 billion years of complex computation, leading to the emergence of systems capable of understanding the derivation (a weak anthropic constraint to select the specific peak in the probability space). The value that satisfies this, and fits the observed stellar lifetimes and structure formation history, is: $n_{optimal} \approx 6 \times$

10^(-10) This value sits in the narrow window where:
- N(n) is high enough to provide ~10^80 baryons as computational substrate.
- T(n) is long enough to allow for billions of years of stellar processing.
- D(n) is rich enough to enable quantum, atomic, molecular, biological, and technological computation.

5. **Conclusion**: The value of the baryon asymmetry parameter n that maximizes the long-term computational potential Φ of the universe is approximately 6 × 10^-10.