



# Population Modeling

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# Consider

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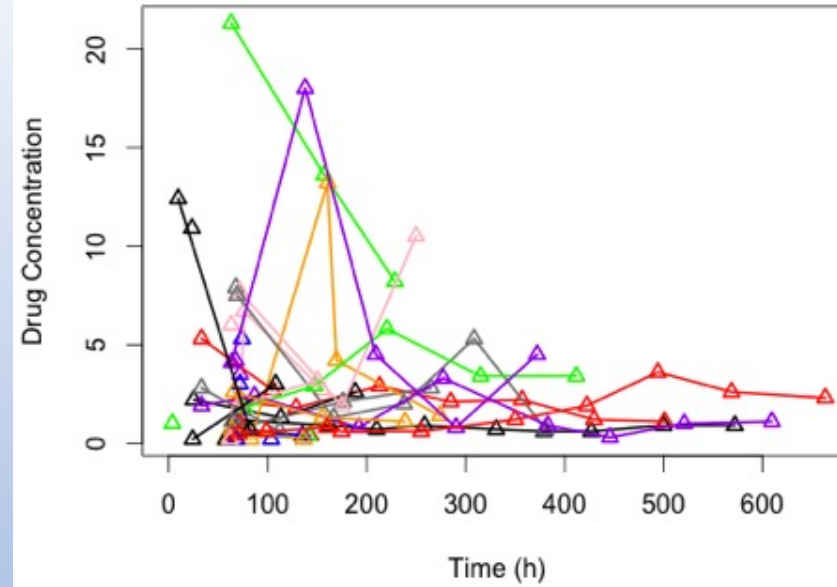
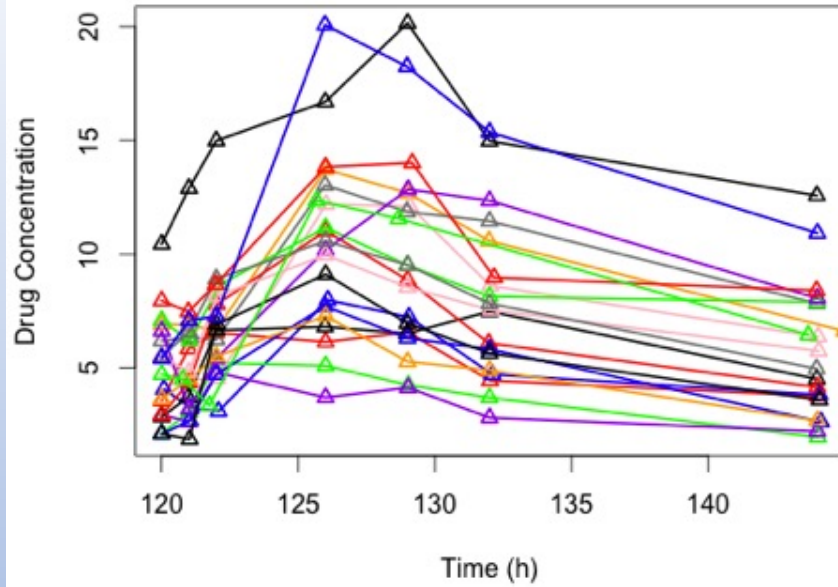
## Question 1

What is the initial drug dose most likely to achieve a safe and effective concentration in the maximum number of patients?

## Question 2

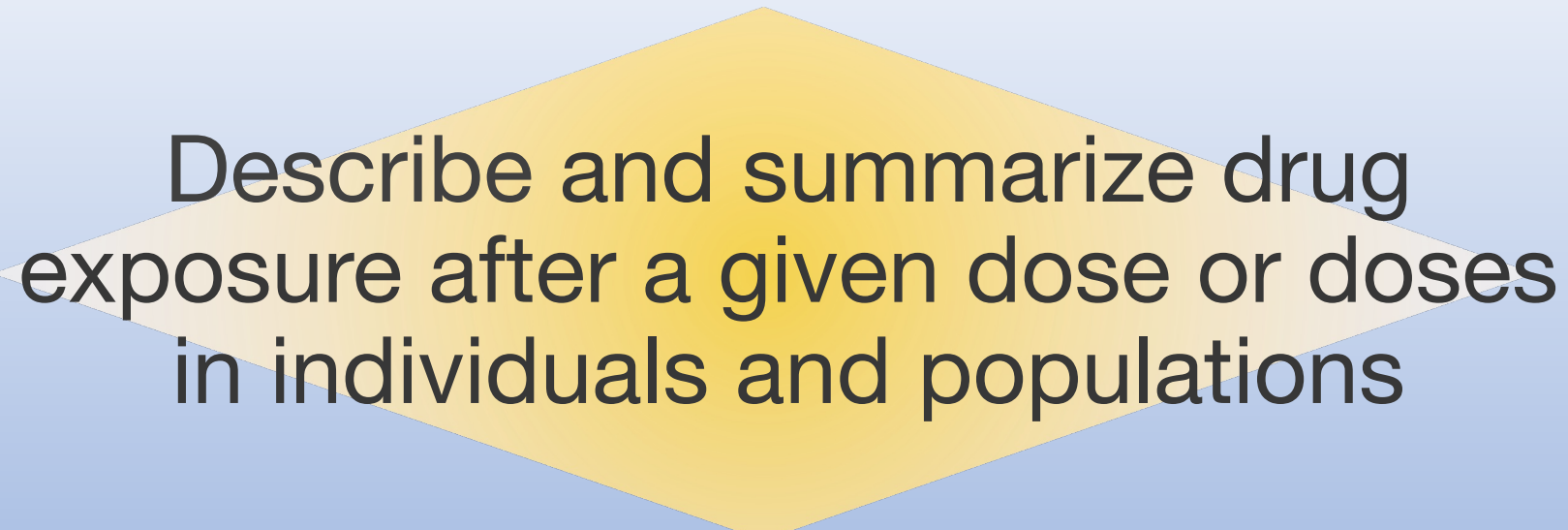
What is the drug dose most likely to achieve a safe and effective concentration in an individual patient?

# PK Data



# PK Modeling Objective

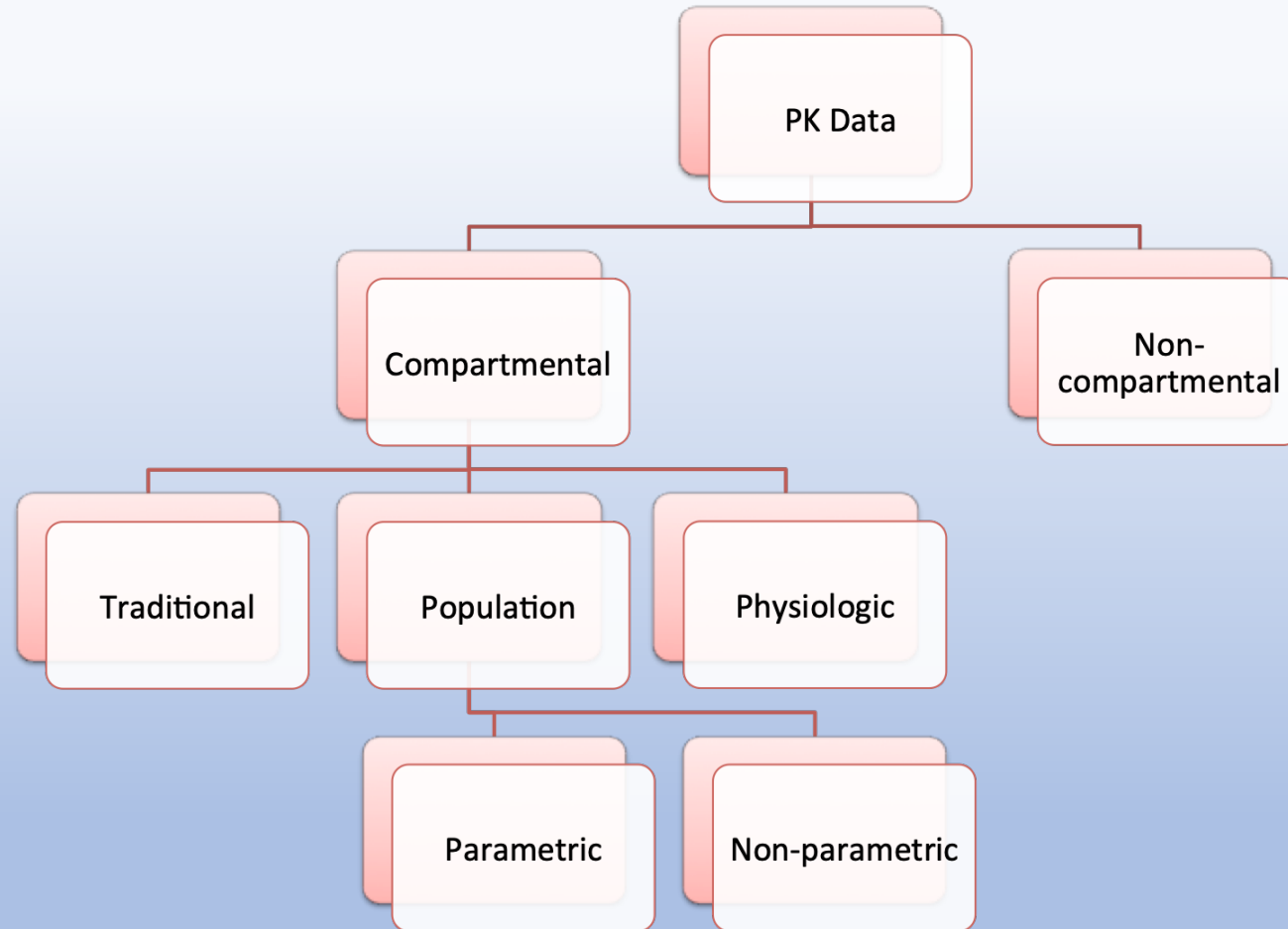
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Describe and summarize drug exposure after a given dose or doses in individuals and populations

# Modeling Approaches

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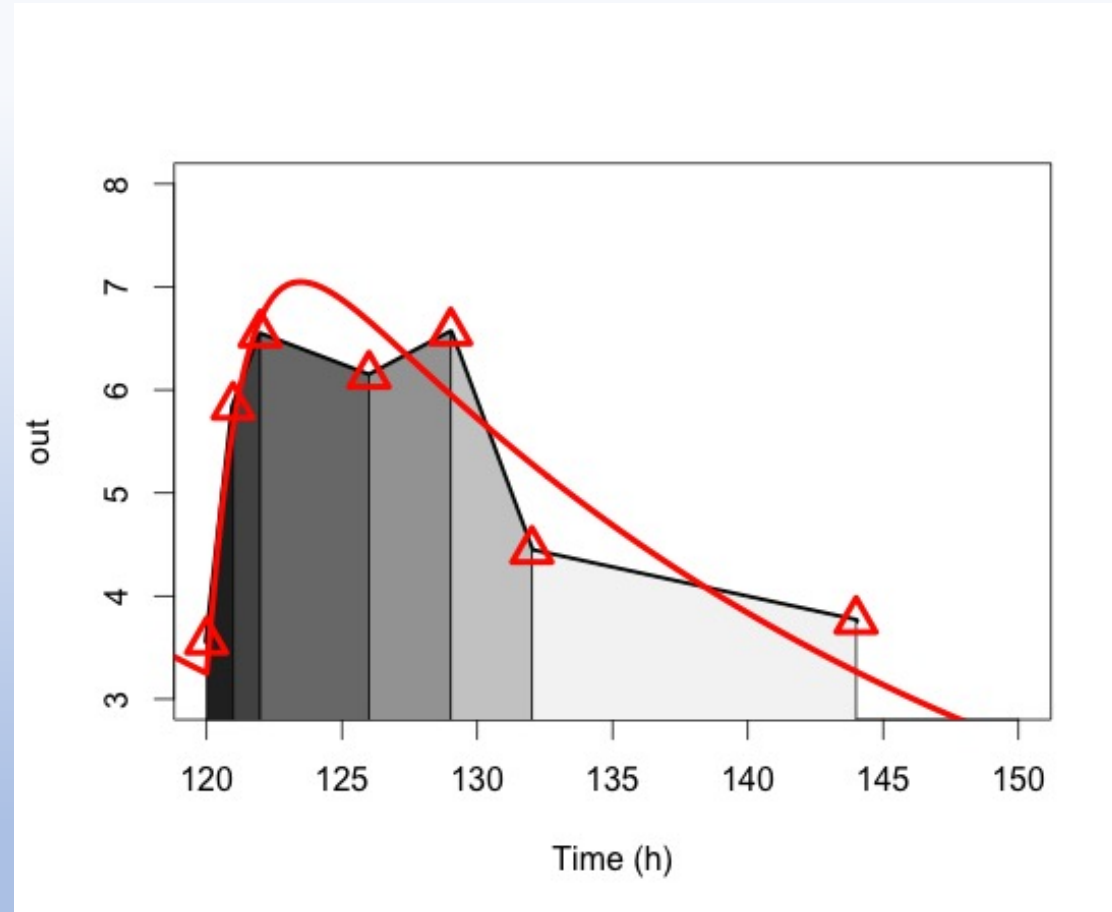


# Non-Compartmental

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- Based on the shape of the time concentration curve
- Driven by estimation of  $AUC_{0-\infty}$  and  $AUMC_{0-\infty}$
- Typically used in bioequivalence, dose proportionality, and drug interaction studies (e.g. drug-food, drug-drug)

# NCA AUC Estimation



# Challenges for NCA

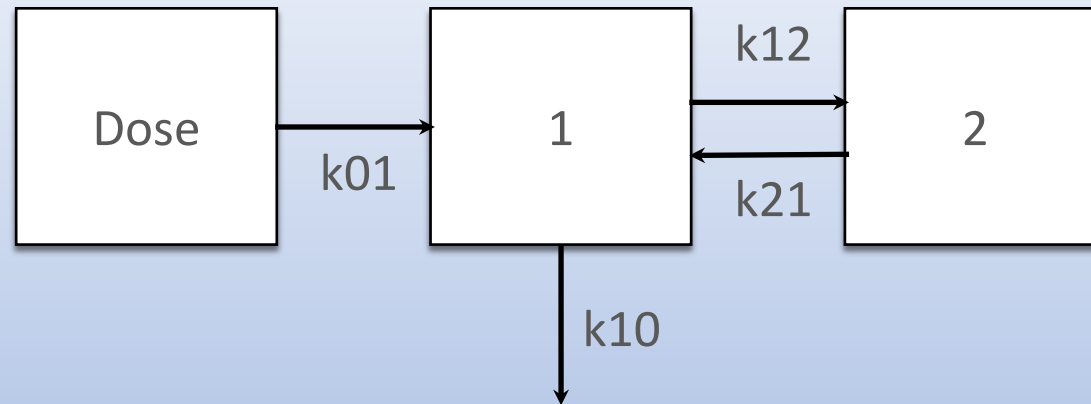
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- Analysis of sparse or unbalanced data
- Complex dosage regimens
- Non-linear PK
- Simulation of exposure from different regimens than those studied
- Elimination of drug other than from sampling pool

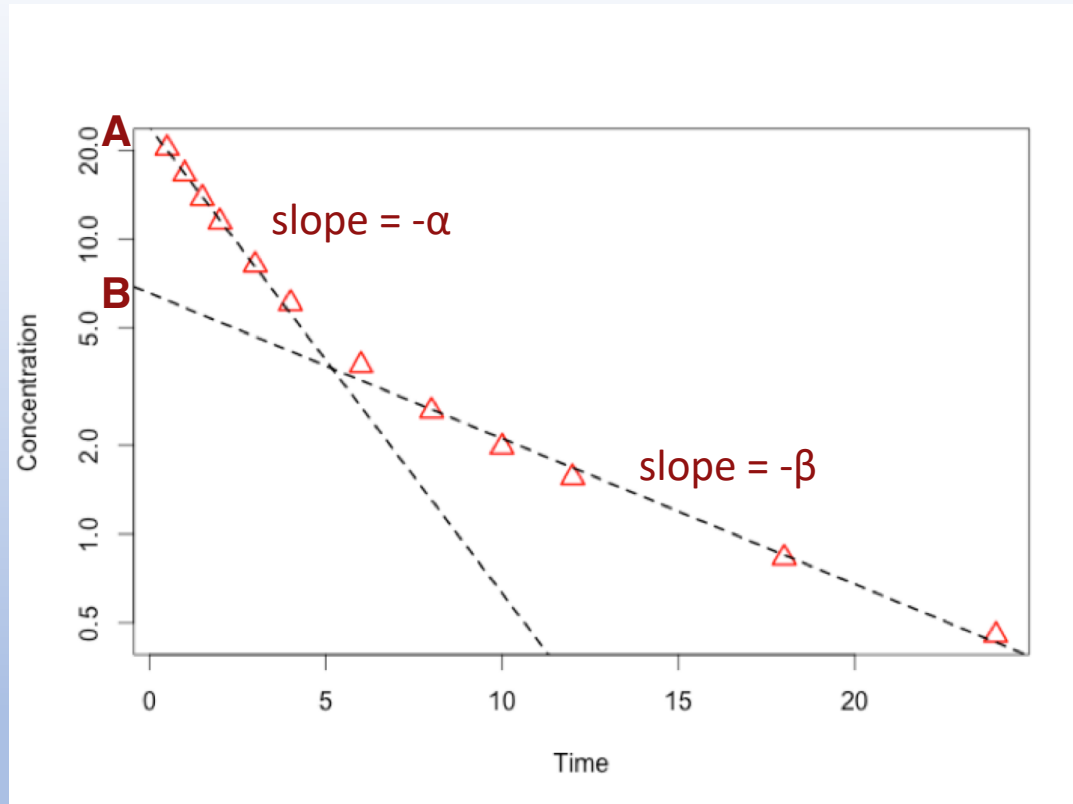


# Compartmental Models

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# Traditional Compartmental



$$C_t = A * e^{-\alpha t} + B * e^{-\beta t}$$

# Challenges

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- Only uses information from one dosing interval
- Can be biased by sparse or unbalanced data
- Assumes parameters remain constant
- Neglects errors in observations (measurement or timing)
- Need at least one drug level per parameter in the model (eg. peak and trough to estimate volume and clearance)
- Does not distinguish sources of variability (e.g. interpatient from inpatient )

# Population Modeling

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aka “Pharmacometrics”

# Why Pop Model?

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- **To understand and describe...**
  - the time course of drug concentrations in the body
  - relationships between drug concentration and effects, both desired and undesired
  - effects of covariates, e.g. renal function, on these relationships
  - sources of PK variability in the population

# Why Pop Model?

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- **To simulate new scenarios which is useful for...**
  - hypothesis generation, study design, dose finding
  - extrapolation to dosing in novel populations
- **To optimize and personalize therapy for individual patients**





# The Best Population Model

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- The correct structural PK/PD model, e.g. one-, two-compartment, inclusion of relevant covariates,...
- The collection of each subject's exactly known parameter values for that model, e.g. absorption, volume, clearance



# Approximating the best

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- $\text{observed} = f(\text{pred}, \text{error})$ 
  - For example,  $\text{obs} = \text{pred} + \text{error}$

# Approximating the Best

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$$y_{ij} = f(x_j, \beta_j) + \varepsilon_{ij}$$



**Parametric**

$$\beta_j = h(\theta, \eta)$$

$$h = \theta + \eta, \theta^* e^\eta, \text{ or } \theta^*(1 + \eta)$$

$$\eta \sim N(0, \omega)$$

**Non-Parametric**

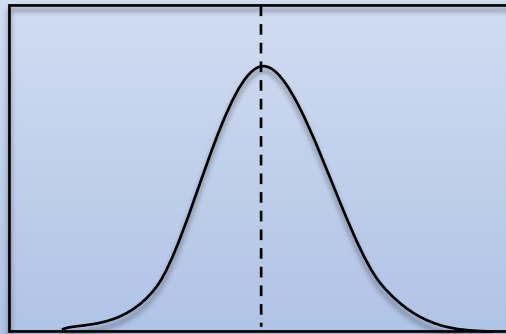
$$FML(\beta) = p_1 \delta_1(\beta_1) + \dots + p_K \delta_K(\beta_K), K \leq N$$

$$\varepsilon \sim N(0, \sigma)$$

# Parametric

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- Familiar, easy to summarize
  - e.g. Clearance =  $0.7 \pm 0.3$  L/min





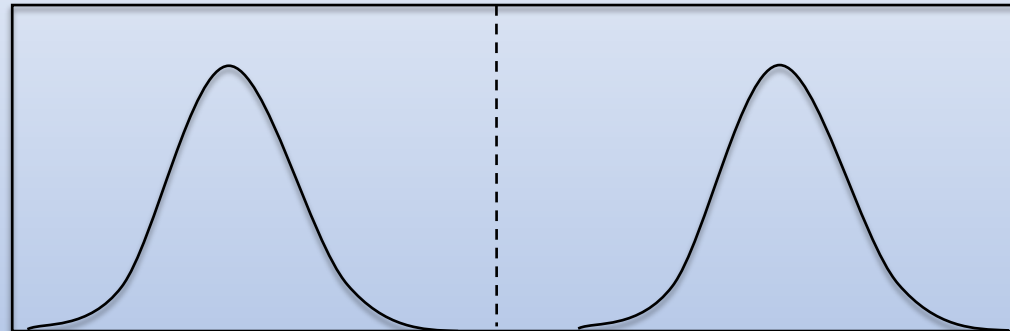
Video courtesy of Marc Lavielle, Ph.D.

Institut national de recherche en informatique et en automatique (INRIA), Paris, France,  
available at <https://team.inria.fr/popix/files/2011/11/PopulationApproach.swf>

# Non-Parametric

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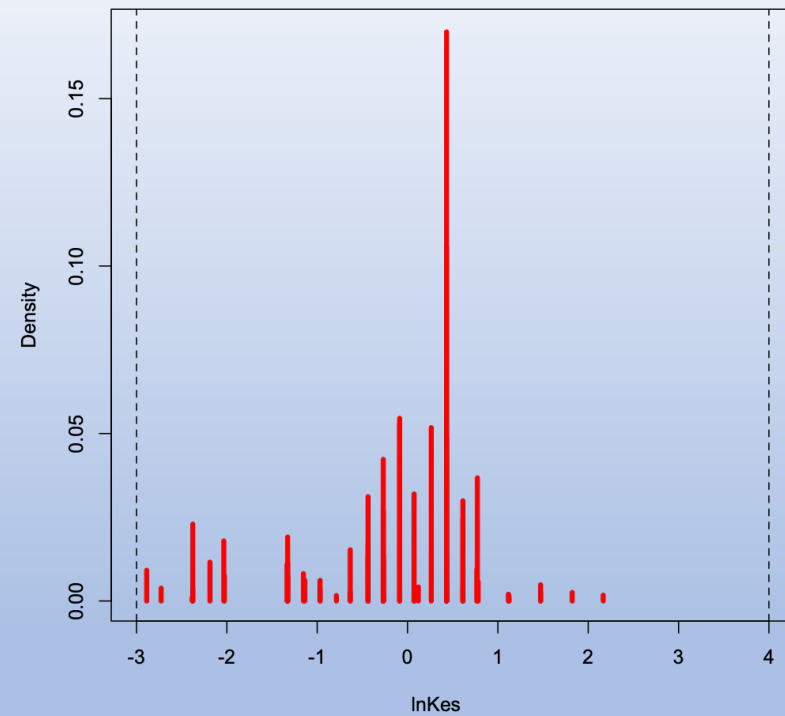
But what if the population distribution of clearance is this?



# Non-Parametric

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Or this?



# Non-Parametric

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- We don't need to look at the infinity of all continuous distributions.
- The most likely distribution, given a set of data, can be found in a discrete collection of points, up to one per subject.
- Each (support) point is a vector of estimates for each parameter value, and of the probability of those values.

# Non-Parametric

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- The shape of this distribution is determined only by the data itself, not by an equation.
- This forms a natural basis for optimal control of dosage regimen with estimates of precision.



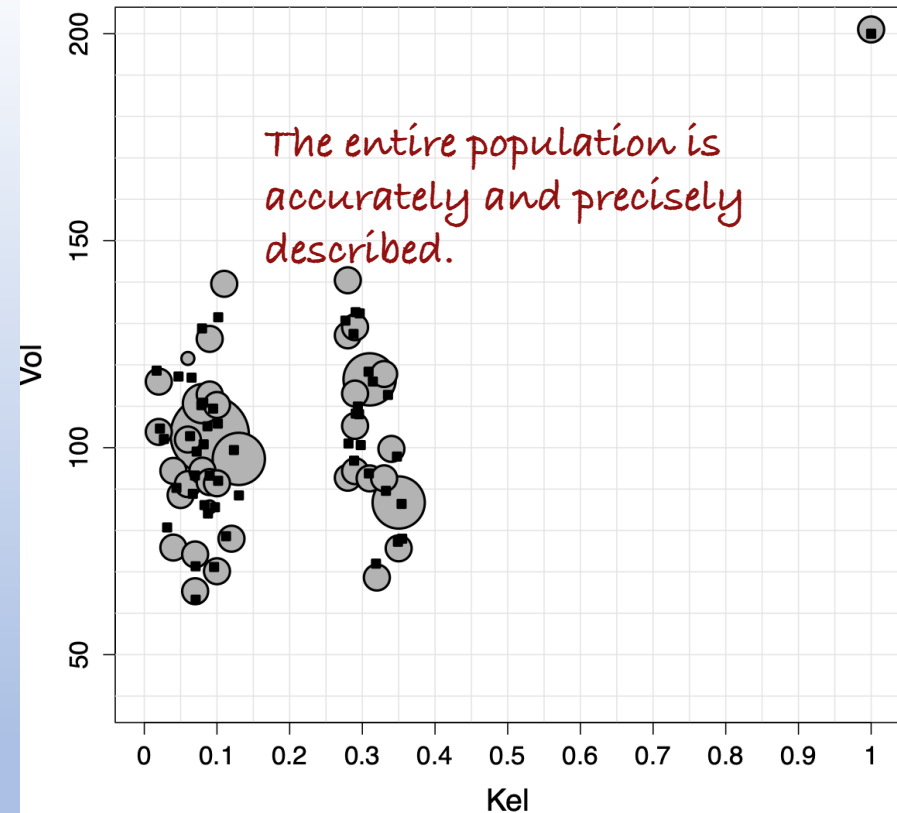
# Non-Parametric

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- Nonparametric – unfamiliar, harder to conceptualize
- Makes no assumptions about underlying parameter distributions
- Assigns a probability to each parameter value in the population based on the frequency of occurrence
- Can detect unexpectedly different subpopulations

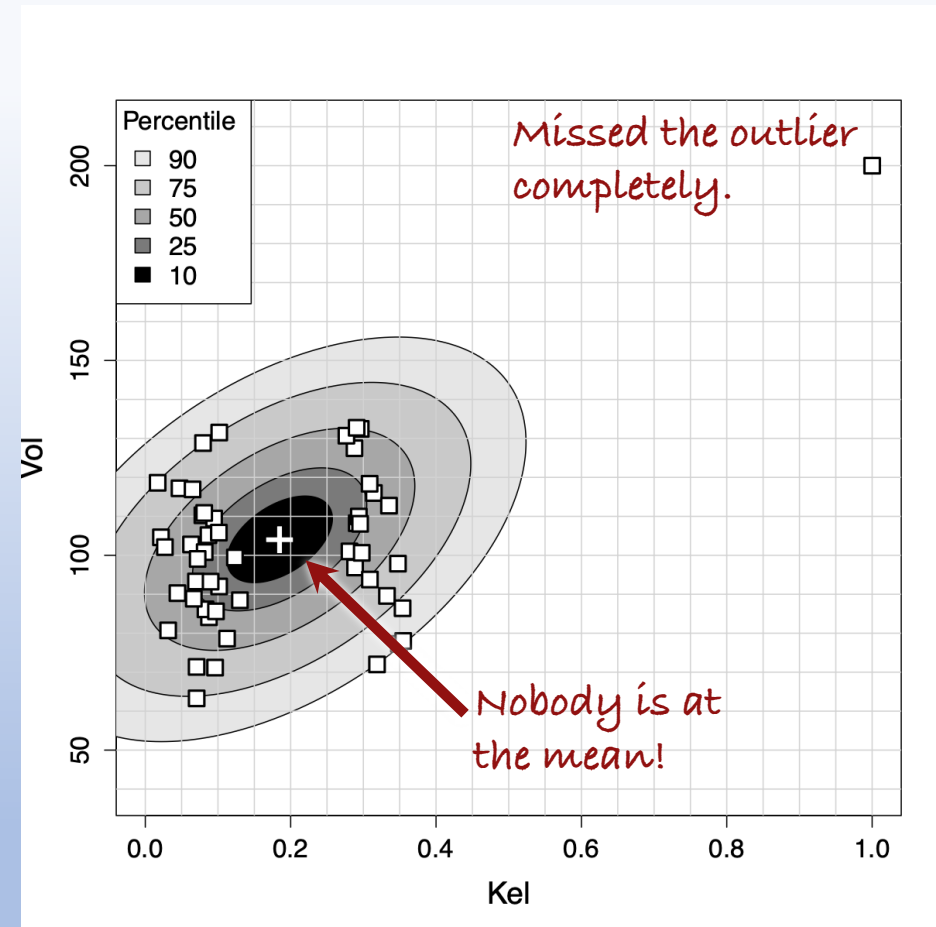
# Non-Normal Populations

- Simulated population (■)
- Non-parametric estimation of population values (●)
  - Size proportional to probability

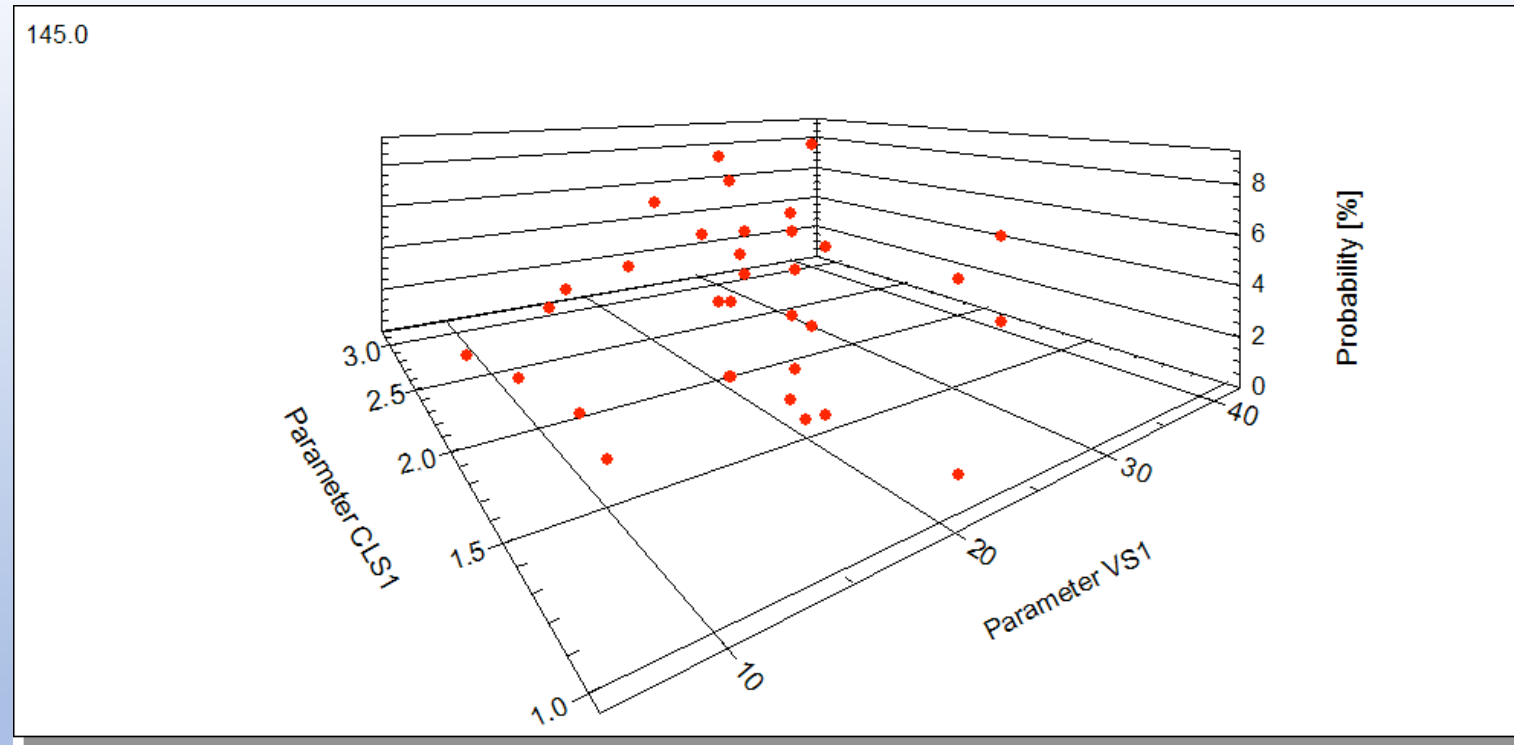


# Non-Normal Populations

- Simulated population (□)
- Mean (+) and percentile distributions of parametric population parameter estimates



# Non-Parametric Model



# Comparison

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	<b>Parametric</b>	<b>Non-parametric</b>
<b>Software</b>	NONMEM, Monolix, ADAPT, S-Adapt, ITS, Phoenix	Pmetrics
<b>Algorithms</b>	FOCE, SAEM, MLEM, QRPEM, ITS	NPAG
<b>Fixed effects</b>	Population “typical” PK parameter values (TV)	Process and observation noise
<b>Random effects</b>	Inter-individual variability (IIV) and residual (intra-individual) variability (RV)	Population PK parameter values and residual variability
<b>Assumptions</b>	Normally distributed IIV and RV	Normally distributed RV

# Software Tools

	<b>Pmetrics</b>	<b>NONMEM</b>	<b>ADAPT</b>	<b>Phoenix</b>	<b>Monolix</b>
<b>Mode</b>	NP, P	P, NP	P	P, NP	P
<b>Cost</b>	0	\$\$\$/\$ <sup>b</sup>	\$\$\$ <sup>a</sup>	\$\$\$/0 <sup>b</sup>	\$\$\$/0 <sup>b</sup>
<b>Simulate</b>	Y	Y	Y	Y	Y
<b>GUI</b>	+	+	+	+++	+++
<b>Platforms</b>	W, U, L	W	W	W	W, L
<b>All-in-one</b>	+++	+	+	+++	++
<b>Clinical</b>	+++ <sup>c</sup>	+	+	+	+

<sup>a</sup>Adapt is free, but it only uses the Intel Fortran compiler, which is \$\$\$

<sup>b</sup>Academic license available

<sup>c</sup>With BestDose

# Terminology

# Model

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- The collection of equations that relates the input to the output, AKA the “structural model”
- $C = \text{Dose}/V * e^{-k_e * t}$
- Also can refer to the probability distribution of parameter values, which is more properly termed the joint probability density



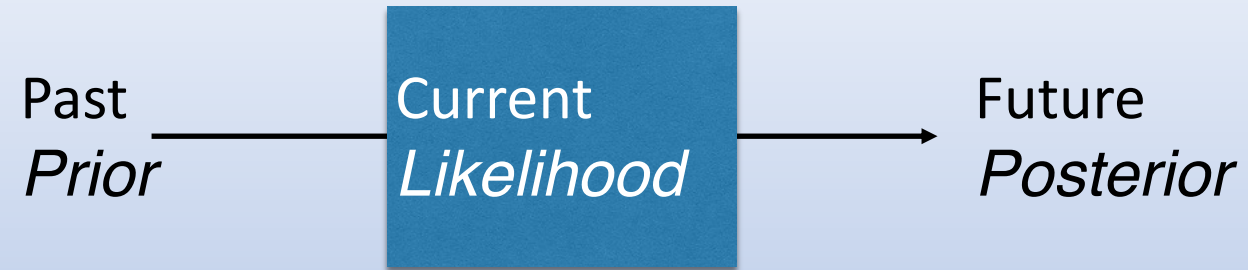
# Parameters

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- Variables in the structural model equations
- e.g.  $C = \text{Dose}/V * e^{-ke*t}$

# Bayes' Theorem

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# Bayesian Prior

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- The probability distribution of parameter values without consideration of current data
- AKA “the model”, “the population prior”

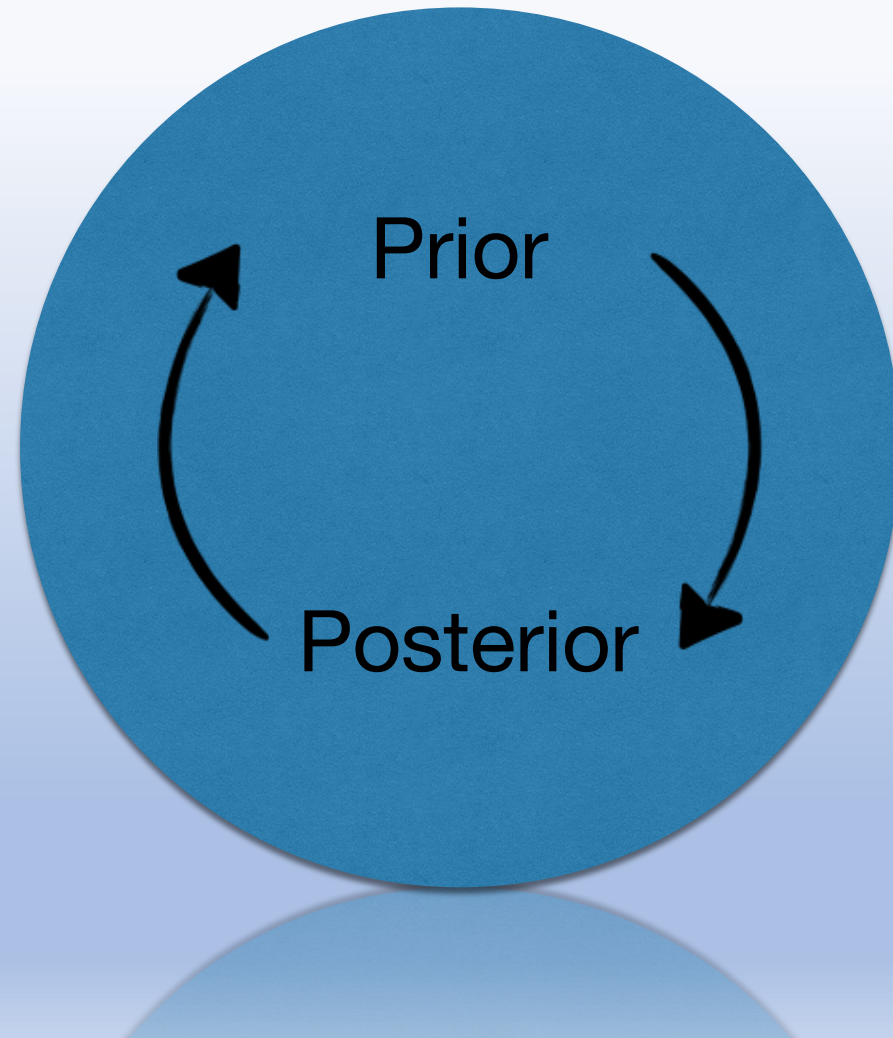
# Bayesian Posterior

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- The probability distribution of parameter values which has been updated based on new data
- AKA “individual distribution”

# Iterative

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# Convergence

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- In the eye of the beholder
- Iterate and search for new parameter value distributions
- Stop when likelihood changes less than a specified threshold

# AIC

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- Akaike Information Criterion
- $AIC = -2 \cdot \log \text{likelihood} + 2K$
- Penalizes for the number of parameters in the model
- Useful for comparing any two models, selecting the one with the lowest AIC

# BIC

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- Bayesian or Schwartz Information Criterion
- Similar to AIC, but greater penalty on parameters



# Covariates

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- Subject/patient specific factors which are linked to PK/PD behavior
- E.g. volume of distribution linked to body weight or clearance linked to genotype

# Covariates

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- Typically included in a model if they improve the AIC or some other objective function

# Assay Errors

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The myth of  
quantification limits...



# Assay Precision

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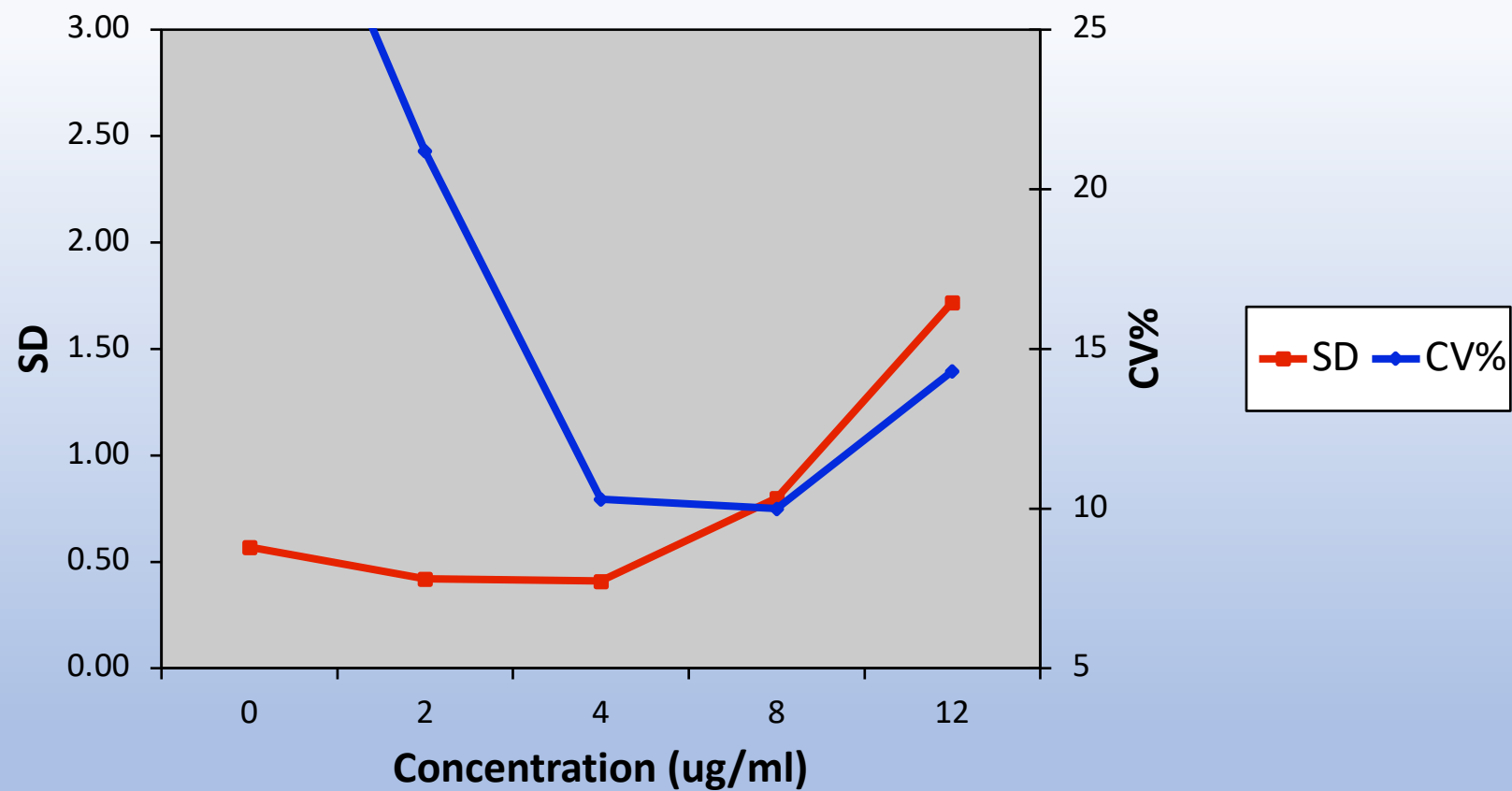
- Estimate the SD of every measured observation
- Fisher Information =  $1 / \text{Variance} = \text{Weight}$
- Variance =  $\text{SD}^2$
- Assay Error Polynomial (AEP):  $\text{SD} = C_0[\text{drug}]^0 + C_1[\text{drug}]^1 + C_2[\text{drug}]^2 + C_3[\text{drug}]^3$

# CV% vs Fisher

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- Assume CV% = 10 when concentrations  $\geq 10$ , and constant SD = 2 when concentrations  $< 10$ .
- If conc = 10, SD = 1, var = 1, weight = 1
- If conc = 20, SD = 2, var = 4, weight =  $\frac{1}{4}$
- If conc = 0.1, SD = 2, var = 4, weight =  $\frac{1}{4}$  but CV% = 2000%
- As concentration approaches zero, CV% approaches infinity

# Assay error

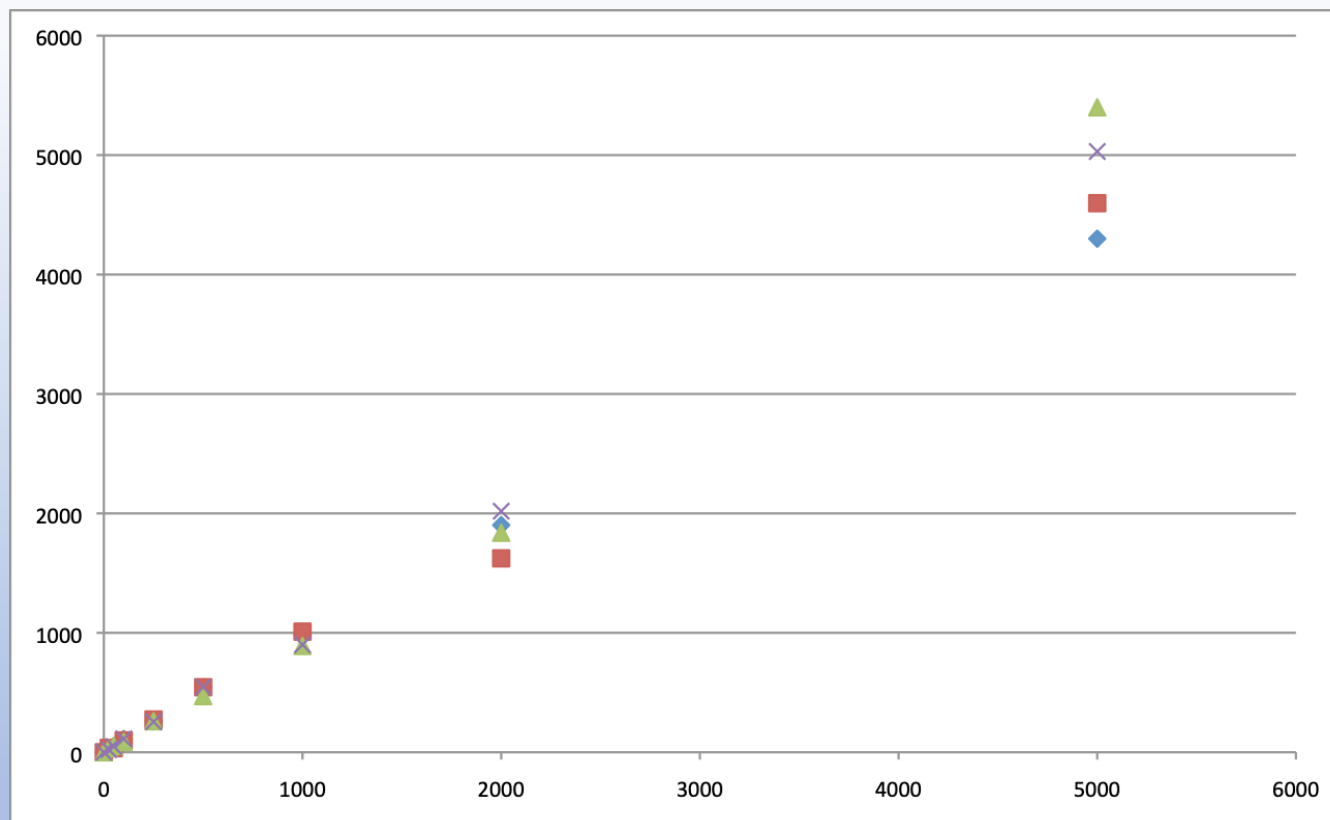


# CV% vs Fisher

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- No LOQ with Fisher
- Assay SD, variance, and weight are always finite.

# Fit assay error

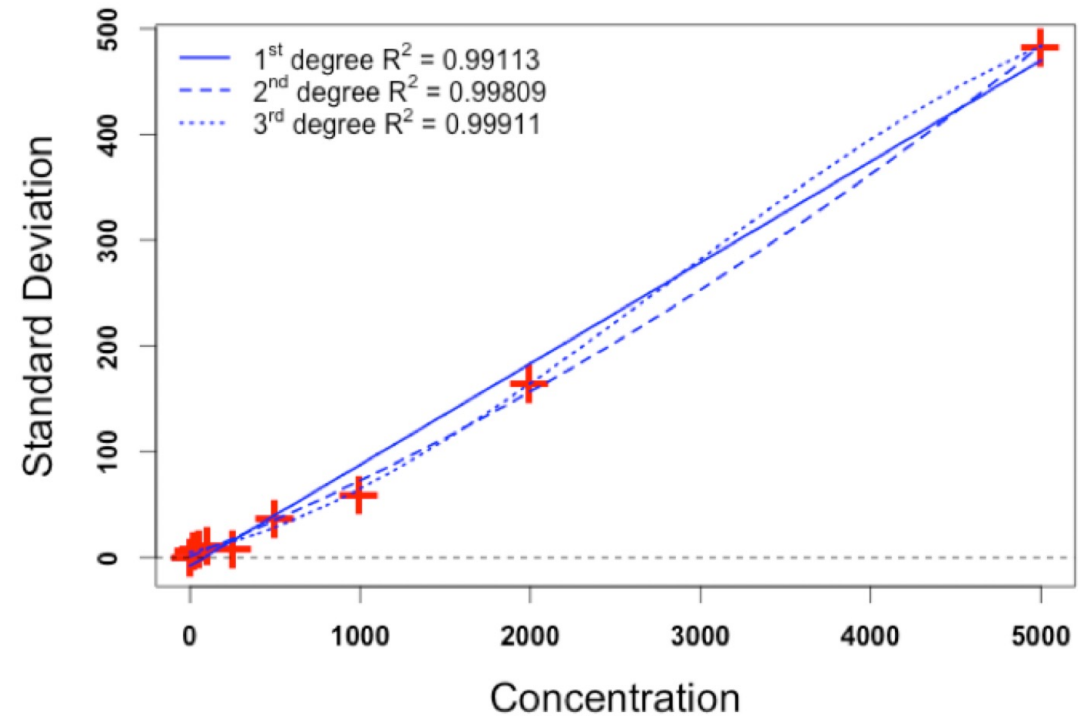


Conc	SD	CV%	Wt
0	0.5	$\infty$	4.000000
25	6.4	26%	0.024414
50	8.6	17%	0.013521
100	12	12%	0.006944
250	8.6	3%	0.013521
500	37.2	7%	0.000723
1000	60.1	6%	0.000277
2000	165.7	8%	0.000036
5000	483	10%	0.000004



# Fit assay error

	C0	C1	C2	C3
First	-7.9	0.1		
Second	1.0	6.5E-02	6.2E-06	
Third	4.8	3.3E-02	3.1E-05	-3.6E-09



# Additional error

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- $SD = C_0[\text{drug}]^0 + C_1[\text{drug}]^1 + C_2[\text{drug}]^2 + C_3[\text{drug}]^3$
- Use additive (lambda) or multiplicative (gamma) model for weight:
  - $\text{weight} = 1/(\lambda + SD)^2$
  - $\text{weight} = 1/(\gamma \times SD)^2$

# Modeling scheme

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