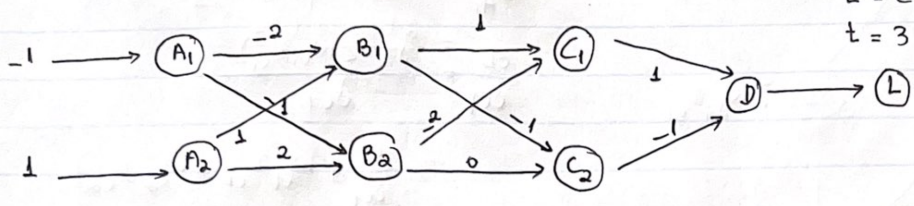
Q3 neural networks:

1). Calculate the value of L. Use the weights and the input values to calculate the value of L using forward propagation

$L = (D-t)^2$

$t = 3$



2). Compute the derivitives. Use backward propagation to find the derivatives of the following variables with respect to each input and weight (A1, A2, ~~A3~~, B1, B2, C1, C2, D). Use the chain rule to find the derivatives of each variable with respect to the inputs and weights.

Answer 1 :

$B_1 = w_1x_1 + w_2x_2$ (assuming there are no bias)

$= (-1)(-2) + (1)(1)$

$\Rightarrow B_1 = 2+1 = 3$

$B_2 = w_1x_1 + w_2x_2$

$= (1)(-1) + (2)(1)$

$\Rightarrow B_2 = 1$

$\Rightarrow C_1 = w_1B_1 + w_2B_2$

$= (1)(3) + (-2)(1)$

$\Rightarrow C_1 = 3-2 = 1$

$\Rightarrow C_2 = w_1B_1 + w_2B_2$

$= (-1)(3) + (0)(1)$

$\Rightarrow C_2 = -3$

$\Rightarrow D = w_1C_1 + w_2C_2$

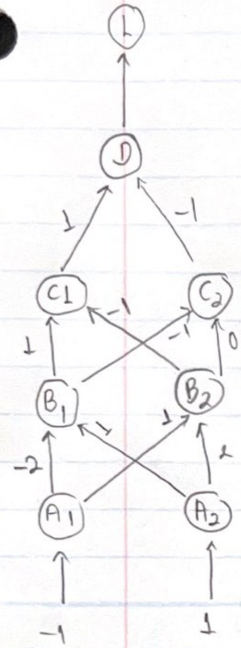
$= (1)(1) + (-1)(-3)$

$\Rightarrow D = (1) + 3 = 4$

$\Rightarrow L = (D-t)^2$

$= (4-3)$

$\boxed{L = 1}$



$$\frac{\delta L}{\delta D} = \frac{\delta}{\delta D}(D-3)^2 = (2)(D-3)^1(D-3)' = 2(D-3) = 2D-6 \quad \{ D=4$$

prob 1

$$\Rightarrow \frac{\delta L}{\delta D} = (2)(4)-6 = \boxed{2}$$

$$\frac{\delta L}{\delta C_1} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_1} \quad \left\{ \begin{array}{l} \frac{\delta L}{\delta D} = 2 \\ \frac{\delta D}{\delta C_1} = \frac{\delta}{\delta C_1}(W^T x_{C_i}) = \frac{\delta}{\delta C_1}(C_1 W_{C1} + C_2 W_{C2}) \\ \Rightarrow \frac{\delta D}{\delta C_1} = W_{C1} = 1 \quad (a) \end{array} \right.$$

$$\frac{\delta L}{\delta C_1} = (2)(1)$$

$$\Rightarrow \frac{\delta L}{\delta C_1} = \boxed{2}$$

$$\frac{\delta L}{\delta C_2} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_2} \quad \left\{ \begin{array}{l} \frac{\delta L}{\delta D} = 2 \\ \frac{\delta D}{\delta C_2} = \frac{\delta}{\delta C_2}(W^T x_{C_i}) = \frac{\delta}{\delta C_2}(W_{C1} C_1 + W_{C2} C_2) \\ \Rightarrow \frac{\delta D}{\delta C_2} = -1 \quad (b) \end{array} \right.$$

$$= (2)(-1)$$

$$\Rightarrow \frac{\delta L}{\delta C_2} = \boxed{-2}$$

$$\frac{\delta L}{\delta B_1} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_1}\cdot\frac{\delta C_1}{\delta B_1} \quad \left\{ \begin{array}{l} \frac{\delta L}{\delta D} = 2 \; ; \; \frac{\delta D}{\delta C_1} = 1 \\ \frac{\delta C_1}{\delta B_1} = \frac{\delta}{\delta B_1}(B_1 W_1 + B_2 W_2) \quad \{ W_1 = 1, \; W_2 = -1 \\ = \frac{\delta}{\delta B_1}((B_1)(1) + (-1)(B_2)) \\ \frac{\delta C_1}{\delta B_1} = 1 \quad (c) \end{array} \right.$$

$$= (2)(1)(1)$$

$$\Rightarrow \frac{\delta L}{\delta B_1} = \boxed{2}$$

$$\frac{\delta L}{\delta B_2} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_2}\cdot\frac{\delta C_2}{\delta B_2} \quad \left\{ \begin{array}{l} \frac{\delta C_2}{\delta B_2} = \frac{\delta}{\delta B_2}(B_1 W_1 + B_2 W_2) \quad \{ W_1 = -1, \; W_2 = 0 \\ = \frac{\delta}{\delta B_2}(-B_1) = 0 \quad (d) \end{array} \right.$$

$$\Rightarrow \frac{\delta L}{\delta B_2} = (2)(-1)(0) = \boxed{0}$$

$$\frac{\delta L}{\delta A_1} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_1}\cdot\frac{\delta C_1}{\delta B_1}\cdot\frac{\delta B_1}{\delta A_1} \quad \left\{ \begin{array}{l} \frac{\delta B_1}{\delta A_1} = \frac{\delta}{\delta A_1}(W_1 A_1 + W_2 A_2) \\ \text{since } W_1 = -2, \; W_2 = 1 \\ \Rightarrow \frac{\delta}{\delta A_1}(-2A_1 + A_2) = -2 \end{array} \right.$$

$$= (2)(1)(1)(-2)$$

$$\Rightarrow \frac{\delta L}{\delta A_1} = \boxed{-4}$$

$$\frac{\delta L}{\delta A_2} = \frac{\delta L}{\delta D}\cdot\frac{\delta D}{\delta C_2}\cdot\frac{\delta C_2}{\delta B_2}\cdot\frac{\delta B_2}{\delta A_2} \quad \left\{ \begin{array}{l} \frac{\delta B_2}{\delta A_2} = \frac{\delta}{\delta A_2}(W_1 A_1 + W_2 A_2) \\ \text{since } W_1 = 1 \; ; \; W_2 = 2 \\ \Rightarrow \frac{\delta}{\delta A_2}(A_1 + 2A_2) = 2 \end{array} \right.$$

$$= (2)(-1)(0)(2)$$

$$\Rightarrow \frac{\delta L}{\delta A_2} = \boxed{0}$$