CS6140 - Machine Learning (Spring 23)

Assignment 2

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Problem 1. (30 points)

Consider a logistic regression problem where $\mathcal{X} = \mathbb{R}^d$ and $\mathcal{Y} = \{0, 1\}$. Derive the weight update rule that maximizes the conditional likelihood assuming that a data set $\mathcal{D} = \{(\boldsymbol{x}*_i, y_*i)\}_{i=1}^n$ is given. Add explanation of each step.

Problem 2. (30 points)

The sigmoid function is given as: $\sigma(a) = \frac{1}{1+e^{-a}}$ Solve the following questions.

1. (5 points) Compute $\frac{\delta\sigma(a)}{\delta w}$ when $a = w^T x$, where $w, x \in R^m$
2. (10 points) For logistic regression with target variable $y \in \{0, 1\}$. Show the posterior of y with given x and w, i.e. $P(y|x, w)$:
3. (15 points) Show the loss function for logistic regression and explain how do we learn w.

# Problem 3 (40 points)

Access the [data set]. It contains 100 data points [50 for each class]. Implement Gaussian Discriminant Analysis algorithm to correctly classify each data point. Write code from scratch without using any library except common data manipulation libraries like numpy, pandas etc. Submit your code a python notebook (ipynb).

1. (5 points) Manipulate data as necessary & create a scatter plot using any plotting library [preferably 'matplotlib'].

2. (15 points) Use the following class definition as skeleton of your code.

```python
class GDA():
    def __init__(self):
        self.pi = None
        self.mu0 = None
        self.mu1 = None
        self.sigma = None

    def train(self, x, y):
        '''
        estimate GDA parameters
        '''

    def predict(self, x):
        '''
        returns predictions vector
        '''
        pass
```

3. (15 points) Write a function to draw the decision boundary and contours of each class along with the data points. The output would look like as following.
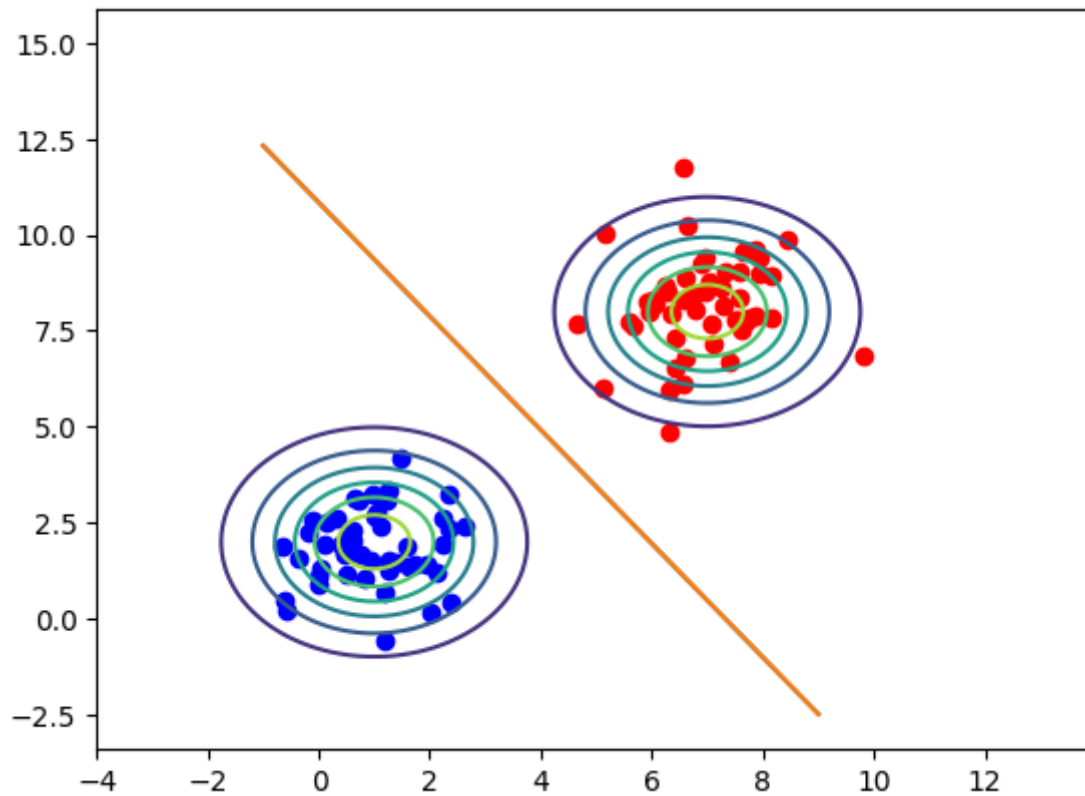
4. (5 points) Explain the difference between **Gaussian Discriminant Analysis** and **Logistic Regression** as a classifier ?