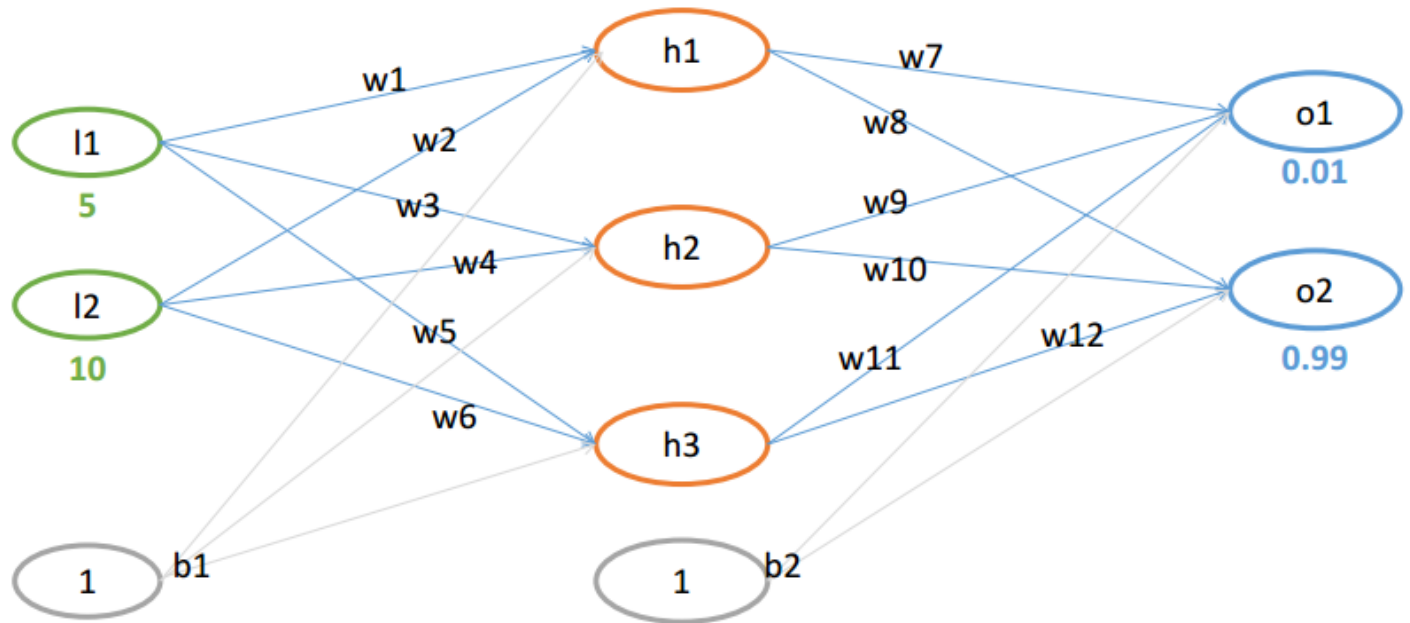


神经网络 backpropagation



$w = (0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65)$

$b = (0.35, 0.65)$

FP 过程

隐藏层的输入与输出公式：

$$net_{h1} = w_1 * l_1 + w_2 * l_2 + b_1 * 1$$

$$out_{h1} = \frac{1}{1 + e^{-net_{h1}}}$$

计算可得：

$$net_{h1} = 2.35$$

$$out_{h1} = 0.912934$$

$$out_{h2} = 0.979164$$

$$out_{h3} = 0.995275$$

输出层：

$$net_{o1} = w_7 * out_{h1} + w_9 * out_{h2} + w_{11} * out_{h3}$$

$$out_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

计算可得：

$$out_{o1} = 0.891090$$

$$out_{o2} = 0.904330$$

输出层误差：

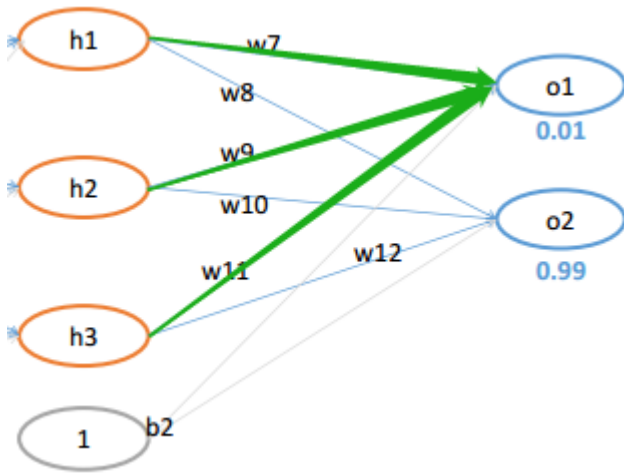
$$E_{total} = E_{o1} + E_{o2}$$

计算可得：

$$E_{total} = 0.391829$$

BP 过程

1. 对 o1 做 bp 运算



$$E_{o1} = \frac{1}{2} * (target_{o1} - out_{o1})^2$$

$$\partial E_{total} / \partial w_7 = \partial E_{total} / \partial out_{o1} * \partial out_{o1} / \partial net_{o1} * \partial net_{o1} / \partial w_7$$

$$\partial E_{total} / \partial out_{o1} = 2 * \frac{1}{2} * (target_{o1} - out_{o1})^{2-1} - 1 + 0 = 0.88109$$

$$\partial out_{o1} / \partial net_{o1} = out_{o1} * (1 - out_{o1}) = 0.097049$$

$$\partial net_{o1} / \partial w_7 = 1 * out_{h1} = 0.912934$$

得到：

$$\partial E_{total} / \partial w_7 = 0.88109 * 0.097049 * 0.912934 = 0.078064$$

权重更新：

$$w_7^+ = w_7 + \nabla w_7 = w_7 - \eta * \partial E_{total} / \partial w_7$$

同理可得

$$w_8^+ = 0.453383$$

$$w_9^+ = 0.458137$$

$$w_{10}^+ = 0.553629$$

$$w_{11}^+ = 0.557448$$

$$w_{12}^+ = 0.653688$$

2. 对 w1 做 bp 运算

$$\begin{aligned} \partial E_{total} / \partial w_1 &= \partial E_{total} / \partial out_{h1} * \partial out_{h1} / \partial net_{h1} * \partial net_{h1} / \partial w_1 \\ &= (\partial E_{o1} / \partial out_{h1} + \partial E_{o2} / \partial out_{h1}) * \partial out_{h1} / \partial net_{h1} * \partial net_{h1} / \partial w_1 \end{aligned}$$

其中：

$$\partial E_{o1} / \partial out_{h1} = \partial E_{o1} / \partial out_{o1} * \partial out_{o1} / \partial net_{o1} * \partial net_{o1} / \partial out_{h1} = -(target_{o1} - out_{o1}) * out_{o1} * (1 - out_{o1}) * w_7$$

得到：

$$\partial E_{total} / \partial w_1 = 0.011204$$

$$w_1^+ = w_1 + \nabla w_1 = w_1 - \eta * \partial E_{total} / \partial w_1 = 0.094534$$

同理反推其他权重。