Tutorial 3

Monday, April 1, 2019 5:25 PM

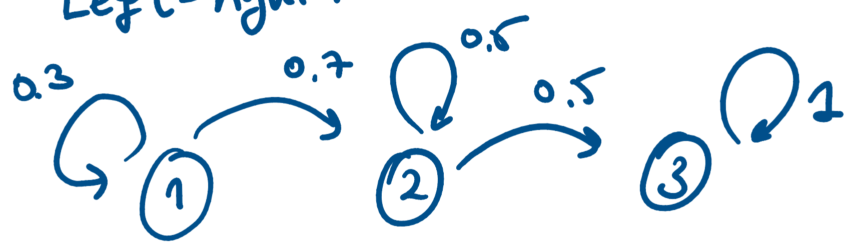
5.1

$$q = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad A = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{pmatrix}$$

$\underline{z} = (1,2,4,4,1)$

5.1.a. Left-right or ergodic?

Left-right. Because the chain cannot go back to 'states that are to the left'



5.1.b. Use forward algorithm to compute $P(\underline{z})$.

$j=1$

$$\alpha_{j,1}^{temp} = q_j b_j(1) = \begin{cases} 1 & j=1 \\ 0 & j=2 \\ 0 & j=3 \end{cases} \quad \text{because of } q_j.$$

$c_1 = 1 \qquad \hat{\alpha}_{j,1} = \alpha_{j,1}^{temp}.$

$$\alpha_{j,2}^{temp} = b_j(2)\,(1 \cdot a_{ij}\ \hat{\alpha}_{j,1}) = \begin{cases} 0 & j=1 \\ 0.5 \cdot 0.7 = 0.35 & j=2 \\ 0.1 \cdot 0 = 0 & j=3 \end{cases}$$

$j=2.$

$c_2 = 0.35$

$\hat{\alpha}_{2,2} = 1$, rest set to 0.

$$\alpha_{j,3}^{temp} = b_j(4)\left(1 \cdot a_{2j}\right) = \begin{cases} 0 & j=1 \\ 0.1 \cdot 0.5 = 0.05 & j=2 \\ 0.6 \cdot 0.5 = 0.3 & j=3 \end{cases}$$

(1 is $\hat{\alpha}_{2,2}$)

$j=3$

$c_3 = 0.35$

$\hat{\alpha}_{1,3} = 0$

$\hat{\alpha}_{2,3} = 5/35$

$\hat{\alpha}_{3,3} = 30/35$

$$\alpha_{j,4}^{temp} = b_j(4)\left(\frac{5}{35} a_{2j} + \frac{30}{35} a_{3j}\right) = \begin{cases} 0 & j=1 \\ (0.1)\left(\frac{5}{35} \cdot 0.5 + 0\right) = \frac{0.25}{35} & j=2 \\ (0.6)\left(\frac{5}{35} \cdot 0.5 + \frac{30}{35} \cdot 1\right) = \frac{19.5}{35} & j=3. \end{cases}$$

$j=4$

$c_4 = \frac{19.75}{35}$

$\hat{\alpha}_{1,4} = 0$, $\hat{\alpha}_{2,4} = \frac{0.25}{19.75}$, $\hat{\alpha}_{3,4} = \frac{19.5}{19.75}$

$$\begin{cases} 0 & j=1 \\ 0 & j=2 \end{cases}$$

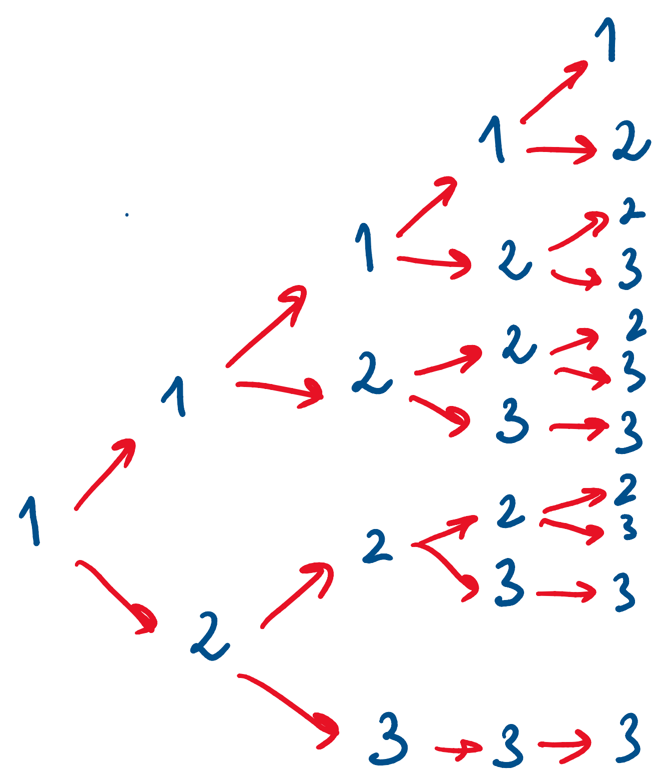
temp $\left(0.25 \quad 19.5\right)$

$$\alpha_{j,5}^{temp} = b_j(1)\left(\frac{0.25}{19,75}\, a_{2j} + \frac{19.5}{19,75}\, a_{3j}\right) = \begin{cases} 0 & j=2 \\ (0.1)\left(\frac{0.25}{19.75}\, 0.5 + \frac{19.5}{19.75}\cdot 1\right) = \boxed{\frac{1.9625}{19.75}} \end{cases}$$

$$\boxed{c_1 c_2 c_3 c_4 c_5 = 0.00686875}$$

$$\underbrace{P(z_1|\emptyset)}_{c_1}\ \underbrace{P(z_2|z_1)}_{c_2}\ \underbrace{P(z_3|z_2,z_1)}_{c_3}\ \underbrace{P(z_4|z_3,z_2,z_1)}_{c_4}\ \underbrace{P(z_5|z_4,z_3,z_2,z_1)}_{c_5}$$

S.l.c.



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S.l.d.

$x_{1,1}=1$, $x_{2,1}=0$, $x_{3,1}=0$ $\quad (\alpha_{1,1}\, b_1(2)) = 0 \quad j=1 \quad \{ \ 0 \ (i=1)$

$j = 1$

$x_{1,1} = 1, \quad x_{2,1} = 0, \quad x_{3,1} = 0$

$$x_{j,2} = \max_i x_{i,1} a_{ij} b_j(x_2) = \begin{cases} 1 a_{11} b_1(2) = 0 & j=1 \\ 1 a_{12} b_2(2) = 1 \cdot 0{,}7 \cdot 0{,}5 = 0{,}35 & j=2 \\ 1 \cdot a_{13}\, b_3(2) = 1 \cdot 0 & j=3 \end{cases} \qquad \max_i x_{i,1} a_{ij} = \begin{cases} 1 \cdot 1 = 1 \ (i=1 \\ 1 \cdot 0 = 0 \ (i=1 \end{cases}$$

$$x_{j,3} = b_j(4) \max_i x_{i,2} a_{ij} = \begin{cases} 0 & j=1 \\ 0.1 \ (0.35)\ 0.5 = 0.0175 & j=2 \\ \text{(max pick 0.35, not zero) (max pick 0.5, not 0)} & \\ 0.6\,(0.35)\,0.5 = 0.105 & j=3 \end{cases} \qquad \max_i x_{i,2} a_{ij} = \begin{cases} x_{2,2} a \\ x_{2,2} a \\ x_{2,2} a \end{cases}$$

$$x_{j,4} = b_j(4) \max_i x_{i,3} a_{ij} = \begin{cases} 0 & j=1 \\ (0.1)(0.0175)(0.5) \quad (0.105) \cdot 0 & j=2 \\ 0.6\,(0.105)\,(1) & j=3 \end{cases} \qquad \max_i x_{i,3} a_{ij} = \begin{cases} 0 \\ 0.008 \\ 0.105 \end{cases}$$

$$x_{j,5} = b_j(1) \max_i x_{i,4} a_{ij} = \begin{cases} (1) \cdot 0 \ \text{(nothing but 1 can go to 1)} & j=1 \\ 0 & j=2 \\ \boxed{(0.1)(0.6)(0.105)(1)} & j=3 \end{cases} \qquad \max_i x_{i,4} a_{ij} = \begin{cases} 0 \\ 0.00 \\ 0.0 \end{cases}$$

$j=2 \rightarrow \tau_{2,2} = 1$ 1

$j=3$.

$12 = 0.35 \quad j=1 \quad \tau_{1,3} = 2$

$22 = 0.175 \quad j=2 \quad \tau_{2,3} = 2$

$23 = 0.175 \quad j=3 . \quad \tau_{3,3} = 2$ 2

$j = 1$

$75\ (i=2) \quad j = 2 \quad \tau_{2,4} = 2$

$(i=2) \quad j = 3 \quad \tau_{3,4} = 3$ 3

$j = 1$

$0.175 \quad j = 2 \quad \tau_{2,5} = 2$

$63 \quad j = 3 \quad \tau_{3,5} = 3$

$\boxed{(0.1)(0.6)(0.105)(1)}$ $j=3$. L 0.0

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1, 2, 3, 3, 3

5.5 $\lambda = (q, A, B)$ $q = \begin{pmatrix} 0.6 \\ 0.1 \\ 0.3 \end{pmatrix}$ $A = \begin{pmatrix} 0.9 & 0 & 0.1 \\ 0 & 1 & 0 \\ 0.2 & 0 & 0.8 \end{pmatrix}$

5.5.a. Infinite. No exit state.

5.5.b. $A^T = \begin{pmatrix} 0.9 & 0 & 0.2 \\ 0 & 1 & 0 \\ 0.1 & 0 & 0.8 \end{pmatrix}$

Find λ s.t. $\begin{vmatrix} 0.9-\lambda & 0 & 0.2 \\ 0 & 1-\lambda & 0 \\ 0.1 & 0 & 0.8-\lambda \end{vmatrix} = 0$. $(\lambda_1, \lambda_2, \lambda_3)$

Then, find v_1, v_2, v_3

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Then, find v_1, v_2, v_3

$$A^T v_i = \lambda_i v_i \quad i = 1,2,3.$$

$$(V_1\ V_2\ V_3) = \begin{pmatrix} 0.8944 & -0.7071 & 0 \\ 0 & 0 & 1 \\ 0.4472 & 0.7071 & 0 \end{pmatrix} \quad (\lambda_1, \lambda_2, \lambda_3) = (1, 0.7, 1).$$

5.5.c: stationary?

$$p^\infty = c_1 V_1 + c_2 V_3 \qquad c_1, c_2 \geq 0, \quad c_1 + c_2 = 1.$$

5.5.d. nope! more than one stationary distribution.

5.6 $a_{ij} = P(S_{t+1} = j \mid S_t = i)$

5.6.a

$$P(D_i = d) = P(S_{t+d} \neq i, S_{t+d-1} = i, \dots, S_t = i+1 \mid S_t = i, S_{t-1} \neq i) =$$

$$= P(S_{t+d} \neq i, S_{t+d-1} = i, \dots, S_t = i+1 \mid S_t = i)$$

$$\dots P(S_{t+d-1} = i \mid S_{t+d-2} = i) \dots P(S_t = i+1 \mid S_t = i)$$

$$= P(S_{t+d} \neq i \mid S_{t+d-1} = i) \underbrace{P(S_{t+d-1} = i \mid S_{t+d-2} = i) \ldots P(S_{t} = i+1 \mid S_t = i)}_{d-1 \text{ times}}$$

$$= (1-a_{ii})\, a_{ii}^{\,d-1}$$

5.6.b.

$$E[D_i] = \sum_{i=1}^{\infty} (d)(a_{ii})^{(d-1)} (1-a_{ii}) = (1-a_{ii}) \frac{d}{da_{ii}} \sum_{i=1}^{\infty} (a_{ii})^d = (1-a_{ii}) \frac{d}{da_{ii}} \left(\frac{a_{ii}}{1-a_{ii}} \right)$$

$$= (1-a_{ii}) \cdot \frac{(1-a_{ii}) + a_{ii}}{(1-a_{ii})^2} = \frac{1}{1-a_{ii}}.$$

6.7

(1) silence 5 visits on average. 1

(2) 3 phoneme states each 10 visits average. 2,3,4

(3) silence 5 visits on average 5.

a_{11} is s.t. $E[D_1] = 5 \Rightarrow \frac{1}{1-a_{11}} = 5 \rightarrow a_{11} = 0.8.$

$a_{22} = a_{33} = a_{44} \Rightarrow E[D_i] = 10 \quad i = 2, 3,$

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$$
\begin{pmatrix}
0.8 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0.9 & 0.1 & 0 & 0 & 0 \\
0 & 0 & 0.9 & 0.1 & 0 & 0 \\
0 & 0 & 0 & 0.9 & 0.1 & 0 \\
0 & 0 & 0 & 0 & 0.8 & 0.2
\end{pmatrix}
$$

exit

$a_{22} = a_{33} = a_{44} \Rightarrow$

$a_{ii} = 0.9. \quad i = 2, 3, 4.$