7.7 HMM with GMMs

$$\gamma_{im,t} = P[S_t=i \cap U_t=m \mid x, \lambda] = \gamma_{i,t} \cdot \frac{w_{im}\, g(x_t, \mu_{im}, C_{im})}{\sum_{k=1}^{M} w_{ik}\, g(x_t, \mu_{ik}, C_{ik})}$$

$$\gamma_{i,t} = P(S_t=i \mid x, \lambda)$$

$$P(S_t=i, U_t=m \mid x, \lambda) = P(S_t=i \mid U_t=m, x, \lambda)\, P(U_t=m \mid x, \lambda)$$

$$= P(U_t=m \mid S_t=i, x, \lambda)\, \underbrace{P(S_t=i \mid x, \lambda)}$$

$$f(x \mid u_t=m, S_t=i, \lambda) = g(x, \mu_{im}, C_{im})$$

$$P(U_t=m \mid S_t=i, x)\, f(x \mid S_t=i) = f(x \mid U_t=m, S_t=i)\, P(U_t=m \mid S_t=i)$$

$$P(U_t=m \mid S_t=i, x) = \frac{g(x_t, \mu_{im}, C_{im})\, w_{im}}{\sum_{k=1}^{M} w_{ik}\, g(x_t, \mu_{ik}, C_{ik})}$$

$$\Rightarrow \gamma_{im,t} = \gamma_{i,t} \cdot$$