Lorraine Nicholson

Exoplanets Homework 3

1) Ternary diagram of K2-18b

a. Composition of K2-18b?

$\boxed{80\%\ H_2O,\ 10\%\ Fe,\ 10\%\ \text{Silicate}}$

b. at position b?

$\boxed{\sim 20\%\ H_2O,\ \sim 50\%\ Fe,\ \sim 30\%\ \text{Silicate}}$

c. $T_\star = 3450$ K
$M_\star = 0.495\ M_\odot$
$R_\star = 0.469\ R_\odot$
Host star parameters
$a = 0.159$ AU planets semi-major axis
$(a = 2.38 \times 10^{10}$ m$)$
$T_{eq} = ?$
$K = ?$, RV signal amplitude
$d = ?$, Transit depth

Start with equillibrium temperature,

$$T_{eq} = T_\star \sqrt{\frac{R_\star}{2a}} (1 - A_B)^{1/4}$$

Let's assume a reasonable albedo, $A_B = 0.3$

$$T_{eq} = (3450) \sqrt{\frac{0.469\ R_\odot}{2(2.38 \times 10^{10})}} (1 - 0.3)^{1/4}$$

$\Rightarrow \boxed{T_{eq} = 261.3\ K}$

$\longrightarrow$

Now RV signal amplitude,

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin(i)}{(M_* + M_p)^{2/3}} \cdot \frac{1}{\sqrt{1-e^2}}$$

Let's assume $M_p \ll M_*$ and $e = 0$ (circular orbit),

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin(i)}{(M_*)^{2/3}}$$

from NASA exoplanet archive,

$M_p = 7.96 \pm 1.91\ M_\oplus = 4.754 \times 10^{25}$ kg

$i = 89.5785$

$M_p \sin(i) \sim 7.96 \sin(89.5)$

I can determine period from semi-major axis,

$$\text{years} \rightarrow P^2 \propto a^3 \leftarrow \text{AU}$$

$$P = \sqrt{a^3}$$

$$P = \sqrt{(0.159\ \text{AU})^3}$$

$$P = 0.063\ \text{yr} = 1.98 \times 10^6\ \text{s}$$

Plug numbers into RV equation,

$$K = \left(\frac{2\pi G}{1.98 \times 10^6}\right)^{1/3} \frac{8.92\ M_\oplus \sin(89.5)}{(9.33 \times 10^{29})^{2/3}}$$

$$\Rightarrow \boxed{K \sim 3.324\ \text{m/s}}$$

Finally, the transit depth

$$d_{Transit} \sim \left(\frac{R_P}{R_\star}\right)^2$$

$R_P = 2.37\ R_\oplus$
$R_\star = 0.469\ R_\odot$

$$d_{Transit} \sim \left(\frac{2.37\ R_\oplus}{0.469\ R_\odot}\right)^2$$

$$\Rightarrow \boxed{d_{Transit} \sim 0.00216}$$

d. $K \sim 1\ m/s$, best RV instruments

Given the larger uncertainty on the mass currently, I think more radial velocity surveys to better constrain the mass would be helpful. However, even with a better constrained mass and radius, it will still be hard to determine the composition of the planet due to degeneracies.

e. $K = ?$

$e = 0.1$, $e = 0.3$

Again, the RV equation is

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{m_p \sin(i)}{(M_*)^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

$$\boxed{\begin{array}{l} \text{for } e = 0.1, \ K = 3.503 \text{ m/s} \\ \text{for } e = 0.3, \ K = 3.972 \text{ m/s} \end{array}}$$

This should be measurable with current RV precision of ~ 1 m/s.

Exoplanets: Homework 3

Lorraine Nicholson

Problem 2)

In [191]:
```
import pandas as pd
import matplotlib.pyplot as plt
```

In [192]:
```
# Read in the table from the website
url_to_webpage = 'https://lweb.cfa.harvard.edu/~lzeng/tables/mrtable
dt = pd.read_csv(url_to_webpage, engine='python', delimiter='\t', sk
dt
```

Out[192]:

		100%fe	95%fe	90%fe	85%fe	80%fe	75%fe	70%fe	65%fe	60%fe	...	65%h2
	Mearth	Rearth	Rearth	Rearth	Rearth	Rearth	Rearth	Rearth	Rearth	Rearth	...	Rearth
0	0.06699	0.3651	0.3735	0.3804	0.3872	0.3938	0.4003	0.4065	0.4126	0.4183	...	0.599
1	0.07179	0.3733	0.3819	0.3890	0.3959	0.4027	0.4093	0.4156	0.4218	0.4276	...	0.612
2	0.07695	0.3818	0.3905	0.3978	0.4049	0.4118	0.4186	0.4250	0.4312	0.4372	...	0.625
3	0.08247	0.3903	0.3992	0.4066	0.4139	0.4211	0.4281	0.4346	0.4410	0.4472	...	0.638
4	0.08839	0.3989	0.4080	0.4156	0.4232	0.4305	0.4376	0.4444	0.4509	0.4573	...	0.651
...	...	...	...	...	...	...	...	...	...	...	...	
85	24.25000	1.7280	1.7900	1.8370	1.8780	1.9180	1.9550	1.9910	2.0260	2.0600	...	2.961
86	25.99000	1.7510	1.8140	1.8620	1.9040	1.9440	1.9820	2.0190	2.0550	2.0890	...	3.005
87	27.86000	1.7730	1.8380	1.8870	1.9300	1.9710	2.0100	2.0470	2.0840	2.1190	...	3.050
88	29.86000	1.7950	1.8620	1.9120	1.9560	1.9980	2.0370	2.0760	2.1130	2.1490	...	3.094
89	32.00000	1.8180	1.8860	1.9370	1.9820	2.0250	2.0650	2.1040	2.1420	2.1780	...	3.137

90 rows × 44 columns

In [193]:
```
dt.keys()
```

Out[193]:
```
MultiIndex([(        '        ',        'Mearth '),
            (        '100%fe',          'Rearth'),
```

In [193]:
```
dt.keys()
```

Out[193]:
```
MultiIndex([(       '        ',        'Mearth '),
            (        '100%fe',         'Rearth'),
            (        '95%fe ',         'Rearth'),
            (        '90%fe ',         'Rearth'),
            (        '85%fe ',         'Rearth'),
            (        '80%fe ',         'Rearth'),
            (        '75%fe ',         'Rearth'),
            (        '70%fe ',         'Rearth'),
            (        '65%fe ',         'Rearth'),
            (        '60%fe ',         'Rearth'),
            (        '55%fe ',         'Rearth'),
            (        '50%fe ',         'Rearth'),
            (        '45%fe ',         'Rearth'),
            (        '40%fe ',         'Rearth'),
            (        '35%fe ',         'Rearth'),
            (        '30%fe ',         'Rearth'),
            (        '25%fe ',         'Rearth'),
            (        '20%fe ',         'Rearth'),
            (        '15%fe ',         'Rearth'),
```

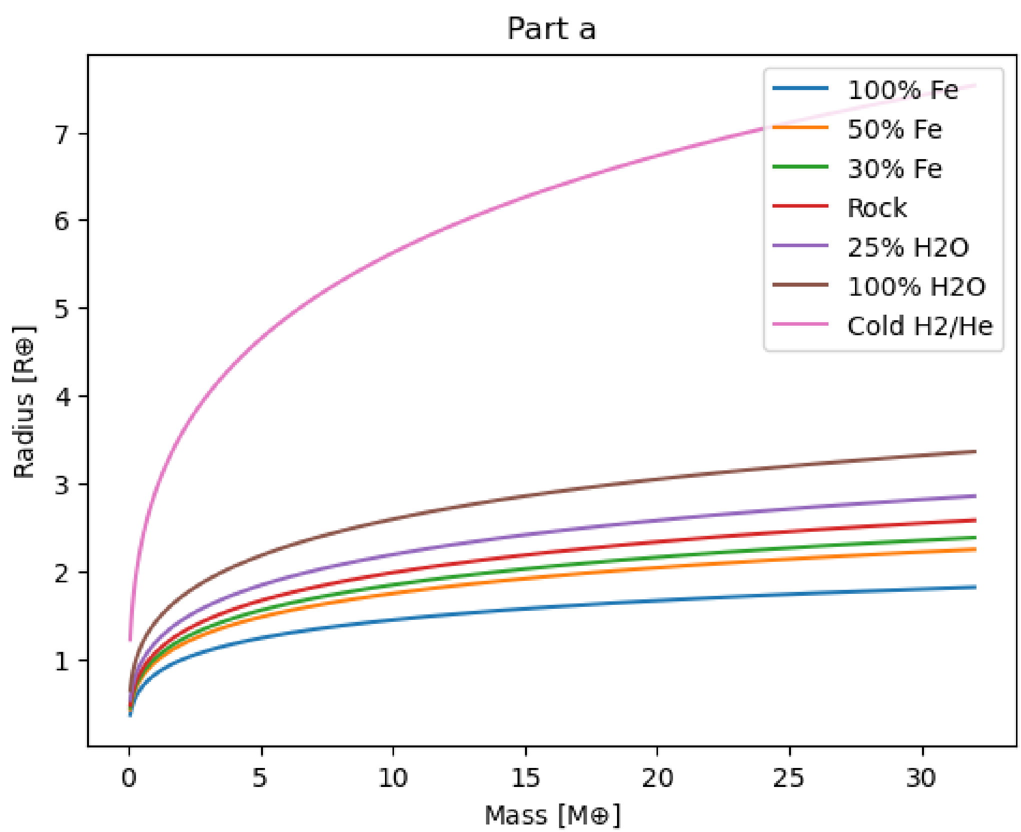
a. Using the model tracks you downloaded, generate a diagram showing the mass-radius curves for planets with the following

a. Using the model tracks you downloaded, generate a diagram showing the mass-radius curves for planets with the following compositions: 100% Fe, 50% Fe, 30%Fe, Rock, 25%H2O, 100%H2O, and cold H2/He.

In [194]:

```
plt.plot(dt['       '][['Mearth '], dt['100%fe']['Rearth'], label='10(
plt.plot(dt['       ']['Mearth '], dt['50%fe ']['Rearth'], label='50%
plt.plot(dt['       ']['Mearth '], dt['30%fe ']['Rearth'], label='30%
plt.plot(dt['       ']['Mearth '], dt['rocky ']['Rearth'], label='Ro(
plt.plot(dt['       ']['Mearth '], dt['25%h2o']['Rearth'], label='25%
plt.plot(dt['       ']['Mearth '], dt['100%h2o']['Rearth '], label='
plt.plot(dt['       ']['Mearth '], dt['cold_h2/he']['Rearth    '], la

plt.xlabel('Mass [M$\oplus$]')
plt.ylabel('Radius [R$\oplus$]')
plt.legend()
plt.title('Part a');
```



b. Why aren't these curves simply lines of constant density?

The more massive the planet, the higher gravity will be, so material should condense. So, given two planets of the same composition but with different masses, the more massive planet should always be denser.

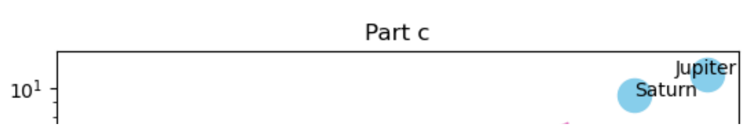
c. Add the Solar System planets and your favorite solar system moon to your diagram. How well do the Zeng et al. models describe the

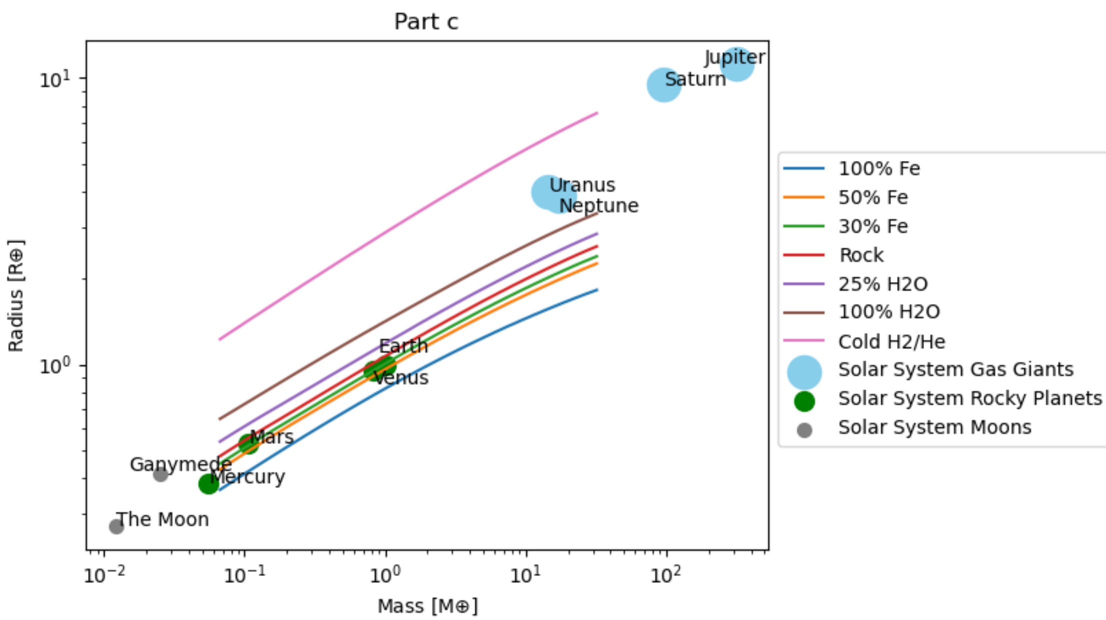
c. Add the Solar System planets and your favorite solar system moon to your diagram. How well do the Zeng et al. models describe the actual compositions of these planets? (i.e., are the planets closest to the model tracks that best match their compositions?)

In [265]:
```
#Plotting the mass-radius curves
plt.plot(dt['       '][‘Mearth '], dt['100%fe']['Rearth'], label='10(
plt.plot(dt['       ']['Mearth '], dt['50%fe ']['Rearth'], label='50%
plt.plot(dt['       ']['Mearth '], dt['30%fe ']['Rearth'], label='30%
plt.plot(dt['       ']['Mearth '], dt['rocky ']['Rearth'], label='Ro(
plt.plot(dt['       ']['Mearth '], dt['25%h2o']['Rearth'], label='25%
plt.plot(dt['       ']['Mearth '], dt['100%h2o']['Rearth '], label='
plt.plot(dt['       ']['Mearth '], dt['cold_h2/he']['Rearth    '], la

#Plotting the Solar System planets
plt.scatter(317.83*1, 11.209*1, s=300, color='skyblue', label='Solar
plt.scatter(1,1, s=100, color='green', label='Solar System Rocky Plan
plt.scatter(0.0553*1, 0.383*1, s=100, color='green')
plt.scatter(0.815*1, 0.949*1, s=100, color='green')
plt.scatter(0.107*1, 0.532*1, s=100, color='green')
plt.scatter(95.16*1, 9.449*1, s=300, color='skyblue')
plt.scatter(14.54*1, 4.007*1, s=300, color='skyblue')
plt.scatter(17.15*1, 3.883*1, s=300, color='skyblue')
plt.scatter(0.0123*1, 0.2725*1, s=50, color='grey', label='Solar Syst
plt.scatter(0.025*1, 0.413*1,s=50,color='grey')
#Plotting the names of the planets on the figure
plt.text(1-0.1,1+0.1,s='Earth')
plt.text(0.0553,0.383,s='Mercury')
plt.text(0.815,0.949-0.1, s='Venus')
plt.text(0.107,0.532,s='Mars')
plt.text(317.83-130.0,11.209,s='Jupiter')
plt.text(95.16,9.449,s='Saturn')
plt.text(14.54,4.007,s='Uranus')
plt.text(17.15,3.883-0.5,s='Neptune')
plt.text(0.0123,0.2725,s='The Moon')
plt.text(0.025-0.01,0.413+0.01,s='Ganymede')


plt.xlabel('Mass [M$\oplus$]')
plt.ylabel('Radius [R$\oplus$]')
plt.legend(bbox_to_anchor=(1.0, 0.8))
plt.xscale('log')
plt.yscale('log')
plt.title('Part c');
```





The models do a good job of describing the actual composition of the terrestrial planets in the Solar System. All the terrestrial planets are close to, but not directly on, the "Rock" curve which makes sense. Also, the gas giant planets don't fall along these curves because their composition are quite different. I can imagine if I extended the pink "Cold H2/He" curve that it would fall on Jupiter and Saturn.

d. Consulting the Confirmed Planets table on the NASA Exoplanet Archive (https://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=PS (https://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=PS)), make a second version of your planet composition plot showing all of the exoplanets with both mass and radius estimates. Include the mass and radius errors reported in the table as error bars and make some cut (your choice!) on quality (ie – error bars have to be better than 50% or bulk density has to be better than 30% after propagating mass and radius error bars together).

In [133]:
```
#Read table that I downloaded from NASA Exoplanet Archive
archive = pd.read_csv('PS_2023.11.08_14.40.44.csv',delimiter=',', sk
```

In [134]:
```
archive
```

Out[134]:

In [134]:
```
archive
```

Out[134]:

	pl_rade	pl_radeerr1	pl_radeerr2	pl_radelim	pl_masse	pl_masseerr1	pl_masseerr2	pl_ma
0	18.647	NaN	NaN	0	2543.00000	1271.00000	-636.00000	
1	16.141	0.336	-0.336	0	4417.83700	349.61300	-349.61300	
2	1.920	0.080	-0.080	0	8.08000	0.31000	-0.31000	
3	1.910	0.080	-0.080	0	8.08000	0.31000	-0.31000	
4	2.080	0.160	-0.170	0	7.81000	0.58000	-0.53000	
...	...	...	...	...	...	...	...	
2463	15.390	0.291	-0.291	0	225.34147	10.80622	-10.80622	
2464	18.495	0.673	-0.673	0	2224.81000	1271.32000	-953.49000	
2465	16.800	2.200	-2.200	0	6356.00000	NaN	NaN	
2466	2.060	0.030	-0.030	0	4.52000	0.81000	-0.81000	
2467	2.042	0.050	-0.050	0	4.82000	0.84000	-0.86000	

2468 rows × 8 columns

In [157]:
```
#Make a cut for error bars that are better than 50%
archive_cut = archive[archive['pl_radeerr1'] < archive['pl_rade']*0.!
archive_cut = archive_cut[archive_cut['pl_masseerr1'] < archive_cut[

archive_cut
```

Out[157]:

	pl_rade	pl_radeerr1	pl_radeerr2	pl_radelim	pl_masse	pl_masseerr1	pl_masseerr2	pl_ma
1	16.141	0.336	-0.336	0	4417.83700	349.61300	-349.61300	
2	1.920	0.080	-0.080	0	8.08000	0.31000	-0.31000	
3	1.910	0.080	-0.080	0	8.08000	0.31000	-0.31000	
4	2.080	0.160	-0.170	0	7.81000	0.58000	-0.53000	
5	1.897	0.044	-0.046	0	7.74000	0.37000	-0.30000	
...	...	...	...	...	...	...	...	
2460	12.890	1.345	-1.345	0	365.50450	25.42640	-25.42640	
2461	12.431	0.560	-0.560	0	336.56800	8.89900	-8.89900	
2463	15.390	0.291	-0.291	0	225.34147	10.80622	-10.80622	
2466	2.060	0.030	-0.030	0	4.52000	0.81000	-0.81000	
2467	2.042	0.050	-0.050	0	4.82000	0.84000	-0.86000	

2024 rows × 8 columns

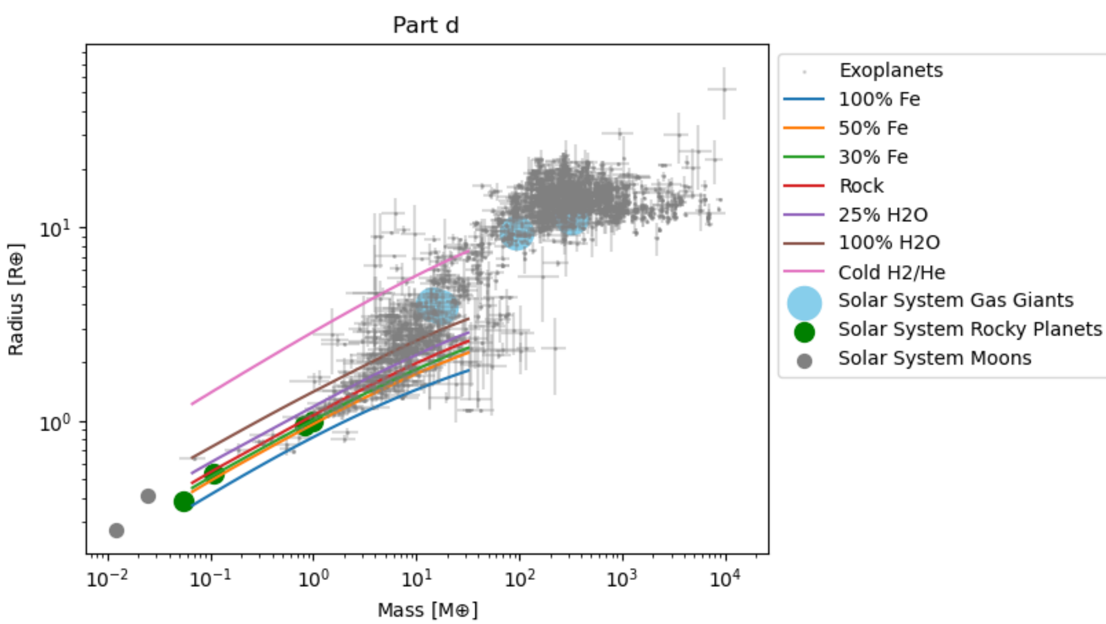
In [263]:
```
#Plotting all the exoplanets with mass and radius measurements
plt.scatter(archive_cut['pl_masse'], archive_cut['pl_rade'], s=1, co
plt.errorbar(archive_cut['pl_masse'], archive_cut['pl_rade'], xerr=a
```

In [263]:
```
#Plotting all the exoplanets with mass and radius measurements
plt.scatter(archive_cut['pl_masse'], archive_cut['pl_rade'], s=1, co
plt.errorbar(archive_cut['pl_masse'], archive_cut['pl_rade'], xerr=a

#Plotting the mass-radius curves
plt.plot(dt['       ']['Mearth '], dt['100%fe']['Rearth'], label='10(
plt.plot(dt['       ']['Mearth '], dt['50%fe ']['Rearth'], label='50%
plt.plot(dt['       ']['Mearth '], dt['30%fe ']['Rearth'], label='30%
plt.plot(dt['       ']['Mearth '], dt['rocky ']['Rearth'], label='Ro(
plt.plot(dt['       ']['Mearth '], dt['25%h2o']['Rearth'], label='25%
plt.plot(dt['       ']['Mearth '], dt['100%h2o']['Rearth '], label='
plt.plot(dt['       ']['Mearth '], dt['cold_h2/he']['Rearth    '], la

#Plotting the Solar System planets
plt.scatter(317.83*1, 11.209*1, s=300, color='skyblue', label='Solar
plt.scatter(1,1, s=100, color='green', label='Solar System Rocky Pla
plt.scatter(0.0553*1, 0.383*1, s=100, color='green')
plt.scatter(0.815*1, 0.949*1, s=100, color='green')
plt.scatter(0.107*1, 0.532*1, s=100, color='green')
plt.scatter(95.16*1, 9.449*1, s=300, color='skyblue')
plt.scatter(14.54*1, 4.007*1, s=300, color='skyblue')
plt.scatter(17.15*1, 3.883*1, s=300, color='skyblue')
plt.scatter(0.0123*1, 0.2725*1, s=50, color='grey', label='Solar Sys
plt.scatter(0.025*1, 0.413*1,s=50,color='grey')


plt.xscale('log')
plt.yscale('log')
plt.legend(bbox_to_anchor=(1.0, 1.0))
plt.xlabel('Mass [M$\oplus$]')
plt.ylabel('Radius [R$\oplus$]')
plt.title('Part d');
```



e. Write a few sentences describing how the measured masses and radii of the exoplanets compare to the expectations from theoretical

e. Write a few sentences describing how the measured masses and radii of the exoplanets compare to the expectations from theoretical models. Which planetary compositions best describe planets of different masses? Are any planets in “forbidden” regions? Why might that be?

In the mass range between ~0.1-120.0 $M_{\oplus}$, the measured masses and radii follow the trends of the model curves very well. That is, there is a general upward (positive) trend. They fall along a range of composition curves ranging from 100% Fe to 100% H2O, and many of the exoplanets fall along the "Rock" curve aswell. It's hard to determine a distinct composition based on this figure though because there is a lot of degeneracy concerning what a planets composition could be given a mass and radius.

There are also a lot of exoplanets discovered which are more similar in mass and radius (M > 120 $M_{\oplus}$, R > 100 $RM_{\oplus}$) to the Solar Systems gas giants; I would need a model which extends to larger masses and radii for a comparison of these.

It seems to me that the area under the 100% Fe curve would be considered "forbidden" since I can't imagine a planet more dense than one that is 100% Fe. It's curious that there are some exoplanets in this region on my figure.

In [ ]:
```
1
```