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Modeling Annular Substructures in the GW Lup Disk with MCMC

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1. INTRODUCTION

The DSHARP (*Disk Substructures at High Angular Resolution Project*) survey is an ALMA large survey designed to characterize substructures in the spatial distribution of protoplanetary disks. Their sample included 20 nearby protoplanetary disks, observed at very high resolution (35 mas or 5 au), in 240 GHz continuum emission (Andrews et al. 2018). I will take advantage of the public Data Release from the DSHARP survey for this MCMC term project (https://almascience.eso.org/almadata/lp/DSHARP/).

Today, we have observed many protoplanetary disk with "annular substructures". These substructures often manifest as bright rings accompanied with dark gaps. Additionally, numerical simulations predict that a forming planet will create similar substructures in the disk. This makes the detection of annular substructures particularly interesting with respect to planet formation theory; since these disk are likely hosting planets which are still forming. One of the interesting results from the DSHARP survey is that there is remarkable diversity in the morphology of observed disks. That is, annular substructures are common in all the disks but they vary drastically in radial location and size. Among the 20 disks in their sample, rings/gaps are found at almost all radii and have widths ranging from a few au to tens of au. Additionally, they find that spiral arms and azimuthal asymmetries exist although only in a minority of cases (Huang et al. 2018).

The physical parameters which describe the rings (width, amplitude, and radial location) can be helpful for further scientific analysis. For example, Dullemond et al. (2018) performed a Gaussian fitting of the rings associated with 5 disks from the DSHARP survey. They successfully recovered the width and amplitude of the rings, which ultimately revealed that the rings of these disks are consistent with dust trapping. This is just one example of how Gaussian fitting of disk rings can be a critical step for any analysis comparing protoplanetary disk observations to models.

For the purpose of this project, I use data from the DSHARP survey to model the annular substructures of a protoplanetary disk. I use MCMC to fit a model to the data and derive uncertainties in the model parameters.

2. METHODS

2.1. *GW Lup Protoplanetary Disk*

The target of my study is the GW Lup protoplanetary disk (Figure 1). GW Lup is a disk around a young T Tauri star in the Lup I region, located about 155 pc from Earth. The stellar properties for this system are listed in Table 1 of Andrews et al. (2018). I chose this disk in particular because it is a simple case of an extended disk with one gap, one outer ring, and no obvious azimuthal asymmetries. As provided from the Data Release, the deprojected azimuthally-averaged brightness temperature radial profile of GW Lup is shown in Figure 2. Huang et al. (2018) describes the procedure used to extract the radial profile from the continuum image. The disk is hottest in the inner region close to the host star, and the temperature decreases at larger radii. Also, there is a small "bump" located around 85 au which is associated with the bright outer ring.

2.2. *Gaussian Fitting*

Following the procedure in Appendix B of Dullemond et al. (2018), the bright rings in the brightness temperature profile are the shape of a Gaussian and can be modeled as such:

$$T_B^{gauss}(r) = A * exp(-\frac{(r-r_0)^2}{2\sigma^2}) \tag{1}$$

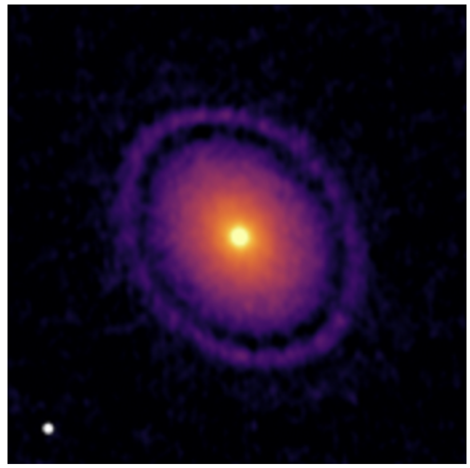

Figure 1. Continuum emission of the GW Lup disk. Data from Andrews et al. (2018).

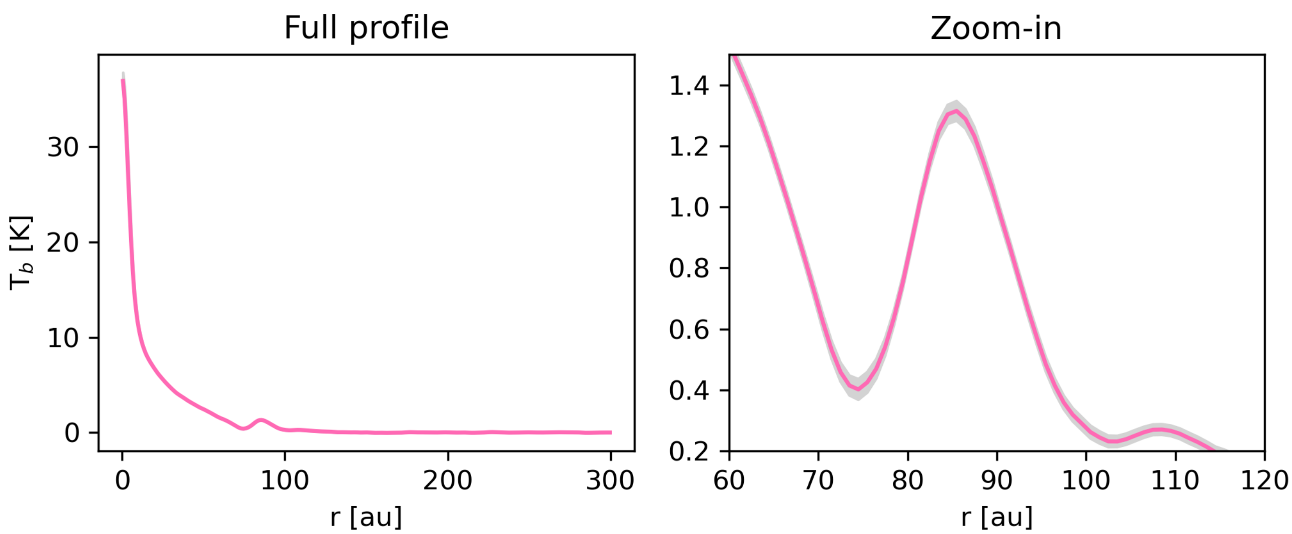

Figure 2. Visualizing the Data. The deprojected azimuthally-averaged brightness temperature radial profile of GW Lup (data from Andrews et al. (2018)). The grey shaded region represents the 1σ uncertainty of the data. Left panel: Full profile of the disk. Right panel: Zoom-in of the profile centered around the radial location of the ring.

where, A describes the amplitude of the ring, r_0 describes the radial location of the center of the ring, and σ is the standard deviation or width of the ring. Resulting in 3 model parameters (A, r_0, and σ) to fit and derive uncertainties for using MCMC.

In the case of a disk with multiple rings, you can just add another Gaussian to the model:

$$T_b^{gauss}(r) = A_1 * exp(-\frac{(r - r_{0,1})^2}{2\sigma_1^2}) + A_2 * exp(-\frac{(r - r_{0,2})^2}{2\sigma_2^2}) + ... \tag{2}$$

where the first term describes the first ring, the second term describes the next ring, and so on. So, the number of model parameters scales with 3N, where N is the number of rings in the disk. For the purposes of this project, GW Lup only has one ring so I will use Equation 1.

To come up with a good initial guess for the model parameters, I did a Maximum Likelihood Estimation. This method numerically optimizes the likelihood function for a given model. The likelihood function is:

$$-\frac{1}{2}\sum_n [\frac{(y_n - model)^2}{s_n^2} + ln(2\pi s_n^2)] \tag{3}$$

Disk (1)	Ring (2)	A [K] (3)	r_0 [au] (4)	σ [au] (5)	Beam FWHM [mas] (6)	σ_b [au] (7)	w_d [au] (8)	A_{dec} [K] (9)
GW Lup	1	$1.281^{+0.014}_{-0.014}$	$85.874^{+0.071}_{-0.069}$	$6.511^{+0.103}_{-0.095}$	49	3.225	5.656	1.475

Table 1. (1) Name of the source. (2) Number of rings in the disk. (3-5) Resulting amplitude, radial location, and width of the ring from Gaussian fitting. Most likely model parameters and their 1σ uncertainties are reported. (6) Effective FWHM beam size reported by Dullemond et al. (2018). (7) Gaussian beam width. (8) Deconvolved width of the dust ring (Equation 4). (9) Deconvolved amplitude of the dust ring (Equation 5).

where, $s_n^2 = \sigma_n^2 + f^2(model)^2$ and "model" is Equation 1. The estimated model parameters from this method are: A = 1.282, r_0 = 85.870, and σ = 6.496. This result will serve as the "initial guess" input for the forthcoming MCMC.

Next, to further set-up the MCMC architecture, I defined some uniform priors for each parameter. Specifically the values must only exist in these regions: $0.0 < A < 10.0$, $50.0 < r_0 < 150.0$, and $1.0 < \sigma < 30.0$. With the Python package *emcee* (Foreman-Mackey et al. 2013) I ran an MCMC composed of 100 walkers and 10000 steps. Based on the integrated auto-correlation time for each parameter (< 54 steps for each parameter), I apply a generous burn-in of 150 steps, for a total chain of 9850 steps to use for my statistics. The resulting model parameters with uncertainty are reported in Table 1.

2.3. *Deconvolving the Model Parameters*

This section describes how to get physically meaningful values from the Gaussian fitting result. In reality, the results of the Gaussian fitting procedure is a convolution of the thermal emission of a dust ring and the ALMA beam. So, I must deconvolve the result to obtain the width, w_d, of the underlying ring. To do so, I assume both a Gaussian ring and a Gaussian beam and then apply the "rule of convolution" for two Gaussian's such that,

$$w_d = \sqrt{\sigma^2 - \sigma_b^2} \tag{4}$$

where σ_b is the beam width in au. I can calculate the beam width based on the reported FWHM of the beam through $\sigma_b = d_{pc}\frac{b_{fwhm,as}}{2.355}$, where d_{pc} is the distance to the disk in parsec. Additionally, the deconvolved ring will have a slightly higher amplitude, A_{dec},

$$A_{dec} = \frac{\sigma}{w_d}A \tag{5}$$

where σ and A are results from my Gaussian fitting procedure and w_d is the result from Equation 4. The resulting deconvolved values as well as beam parameters are reported in Table 1.

3. RESULTS

The most likely model parameters and their 1σ uncertainties are given in Table 1. The resulting Gaussian fit for the ring in GW Lup is shown in Figure 3. The model is a good fit to the data. Additionally, the corner plot illustrating the MCMC covariance is shown in Figure 4. Noticeably, the model parameters do converge. Figure 5 is another helpful diagnostic plot which shows how the walkers are moving at each step in the chain.

4. CONCLUSION

The annular substructures of protoplanetary disks, namely bright rings, can be modeled very well with a Gaussian fit. MCMC is a powerful technique which I employed to derived uncertainty in the model parameters. As a result of this study, I found that the disk GW Lup has one bright ring with A ≈ 1.281 K, r_0 ≈ 85.874 au, and σ ≈ 6.511 au. Similar analyses may be helpful in understanding the underlying physics of protoplanetary disks in general.

Software: emcee (Foreman-Mackey et al. 2013),

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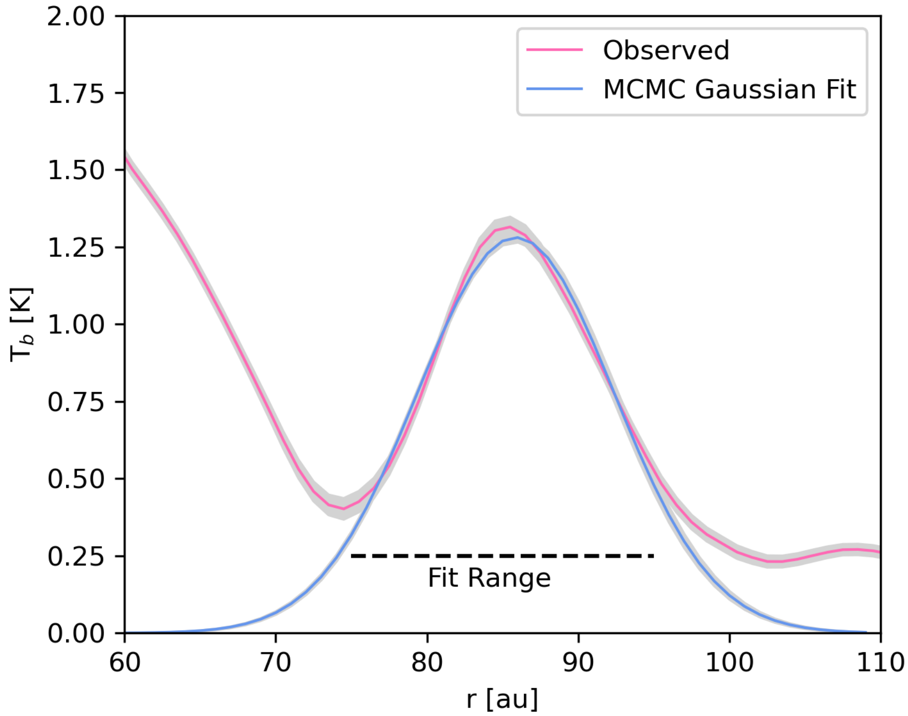

Figure 3. MCMC Gaussian Fit. The resulting Gaussian fit from the MCMC method (blue) and the data (pink). The grey shaded region represents the 1σ uncertainty on the model and data respectively. The dashed "fit range" bar shows the radial range for which the Gaussian curve was fit to the data. The most likely parameters and their 1σ uncertainties are reported in Table 1.

Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306, doi: 10.1086/670067

Huang, J., Andrews, S. M., Dullemond, C. P., et al. 2018, ApJL, 869, L42, doi: 10.3847/2041-8213/aaf740

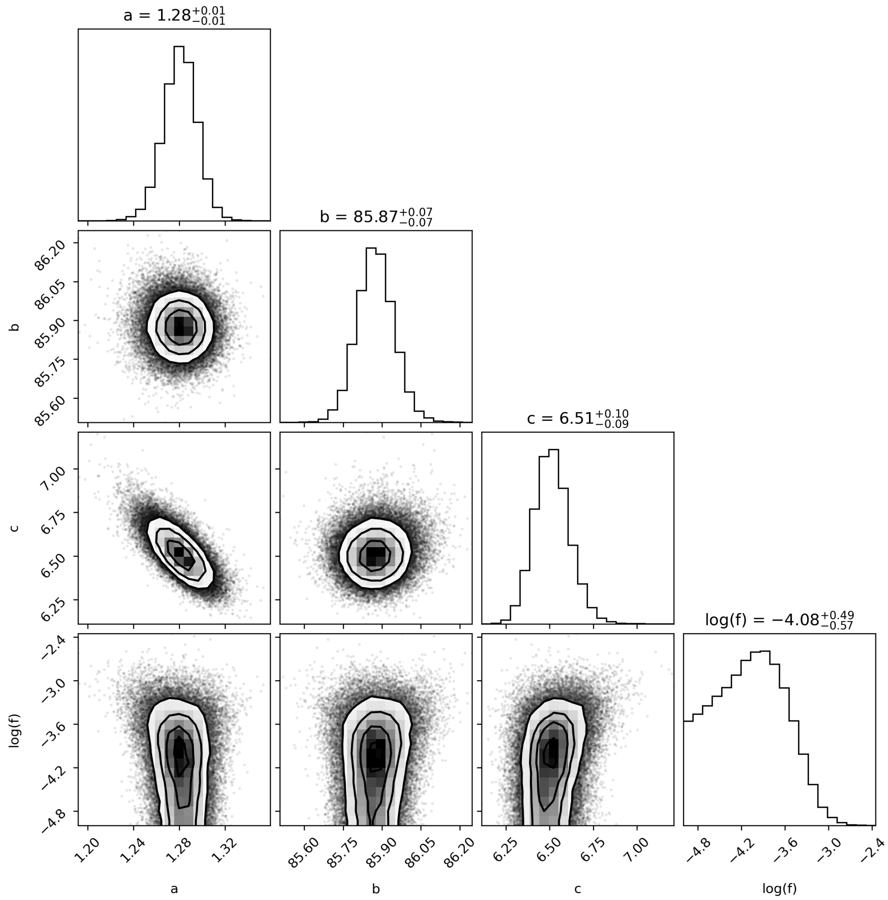

Figure 4. Corner Plot. Corner plot for the MCMC Gaussian fit, helpful for visualizing the covariance of model parameters.

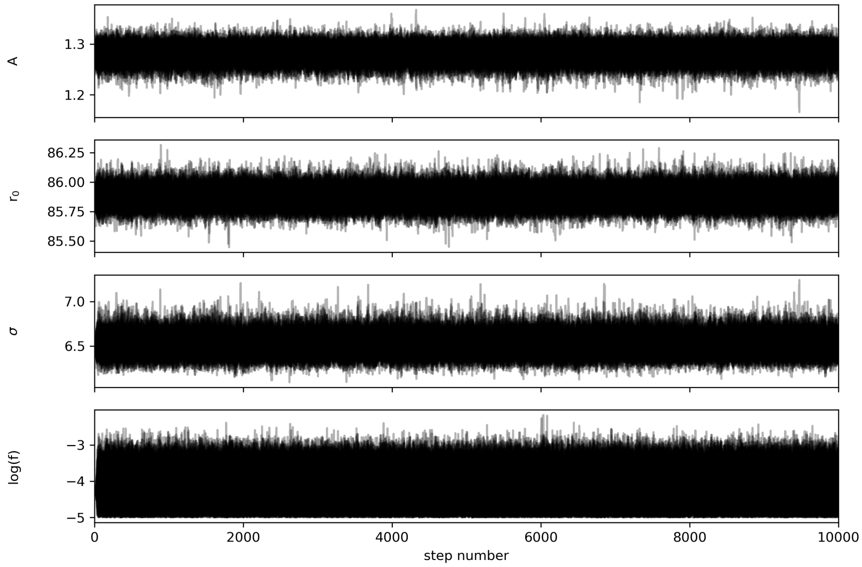

Figure 5. Walkers. Walker predictions for each parameter at each step in the MCMC. The parameters converge to a median value very early (< 100 steps) in the chain.