Homework 2: Microlensing, Transits, RVs

1. Let's imagine that you're thinking about conducting a radial velocity survey of exoplanets with the DHARMA TOU spectrograph that UF owns (https://ui.adsabs.harvard.edu/abs/2016SPIE.9908E..6IG/abstract). TOU has a precision of 0.7 m/s and sits at a 0.5 meter telescope in Arizona and is currently being brought back into operation and roboticized.

(a) If you had 3 years to conduct a survey, what is the coldest minimum mass planet you could possibly detect around an F-type star? What about a K-type star? Assume a circular orbit and that you have to see the planet orbit its star twice.

P_max = 1.5 yr

M_min -> K (amplitude of signal) = precision (sigma_R) of Dharma

(b) What are these planets' equilibrium temperatures?

(c) One of the hardest parts of survey design is deciding how best to strategically allocate your resources – IE how many exposures are needed to detect a signal and with what precision. We have recently heard a thesis plan that examines this problem in detail! Let's do something a little simpler though.

The amount of information that a measurement contains about a model parameter (in our case, the semi-amplitude of the RV curve) is described by the Fisher information matrix. You can read more about it here (https://iopscience.iop.org/article/10.3847/1538-3881/aacea9) but let's just steal their derived equation under the assumption of uncorrelated noise and that measurements are uniformly sampled over the orbit (this is never true!)

$\sigma_K = \sigma_{RV} * (2 / N_{RV})^{1/2}$

Where σ_K is the error on the RV semi-amplitude (ie planet mass, assuming we know the stellar parameters well enough) and σ_{RV} is the precision of our spectrograph.

For your planets in part b, how many observations would you need to determine a mass measurement with a precision of 10%? If each observation is ~1 hour of telescope time for dim stars, and we average 9 hours of night time per night, how many of these planets can we measure the mass of with this instrument over the survey duration? Assume 70% of nights are clear per year. Remember, this is also the best case scenario and we would likely need more observations than this.

2. As mentioned in lecture, gravitational microlensing is considered a rare phenomenon, which is why we have to observe near the galactic center where star populations are denser and we increase out probability of alignment. Still, you have to monitor on the order of hundreds of thousands of stars in order to find a pair aligned closely enough for lensing to be detectable. However, that was assuming stellar densities in the Solar neighborhood looking toward the galactic center (the distances in our discussion was focused on the scale of the Milky Way). The story might be very different if we lived in the middle of a globular cluster. In these regions, the density of stars per pc^3 can be 100 times higher than the solar neighborhood!

(a) What are the odds, when you look up at the nightsky in the middle of a globular cluster, that one of the stars associated with your cluster is being strongly lensed by another star in the cluster? Make the simplifying assumption that the number density of stars is constant throughout the cluster at 5 stars pc^{-3} and that cluster has a radius of 100 lightyears. Assume that the cross-section for a strong lensing interaction is a circle with radius 10 AU. IE – calculate the area of the lensing circle and using the number density of stars in the cluster determine if that area is likely to overlap with another star if you project it outwards across the cluster.

(b) Imagine you are lucky enough to see a foreground star aligned so precisely with a background star that lensing is occurring. Let's figure out how big the Einstein ring associated with the lensed light would be. Remember from lecture, the equation for the Einstein ring:

$$\theta_E = \left(\frac{4GM}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}}} \right)^{1/2}$$

Use this formula to determine the Einstein ring radius, in arcseconds, of a star 100 lightyears away, being lensed by a foreground star 10 lightyears away. You may assume the foreground lens star is a Solar type star.

(c) To now detect a planet orbiting the foreground star using microlensing, it would have to be located right on top of where the image of the Einstein ring appears! Then the planet's presence would bend still more photons into your line of sight, causing a brightening on top of a brightening. What semimajor axis does this correspond to for a star 10 lightyears away? How about 50 lightyears away? Would astronomers in the middle of globular cluster be able to use microlensing to find planets beyond the snow line, like we do?

3. (Computation-based)

Let's go back to your orbital code from Homework 1. We're going to estimate the transit duration of your planet (with eccentricity determined by your birthday, and periapse pointed toward the viewer) in a couple of different ways. There's a lot of cool things we can do with transits and measuring eccentricity from timing in their orbits.

(a) First, let's use the velocity of the planet in its orbit. We can approximate the transit duration as the time it takes the planet to travel $2R_{star}$ at the time of transit– why is this? Use your planet velocity at $\theta = 0$ to approximate what the duration should be (I recommend using the vis viva equation).

(b) Second, we should determine where in the orbit the +/- r_y components of the planet's position are equal to R_{star}. Make your timestep in your code, Δt, small enough that you get at least 10 measurements of r_y that are less than or equal to R_{star} when the planet is near transit. You'll want to make sure r_x is positive when you do this– otherwise you'll mix up transit (when the planet is passing in front of the star) with eclipse (when the planet is passing behind the star)! When you get at least 10 measurements of the planet's orbit during transit, figure out the maximum and minimum time when $r_y < R_{star}$. Is this similar to your estimate from (a)?

(c) Third, use the equation for transit duration in the Seager textbook (Equation 14 in the "Tran- sits and Occultations" chapter) to calculate the duration exactly. How much faster is this, compared to the transit duration if the orbit were circular?

(d) In your code, Rotate the orientation of the planet by π radians, so that now apoapse is pointed toward the viewer. Use any of the three methods above to determine the transit duration. Imagine that you have the ability to measure the transit duration of an exoplanet to a precision of 10%. How eccentric would the planet need to be, in order for you to know for sure that the eccentricity is >0? (That is, how eccentric would the planet need to be in order for $T_{ecc} > 1.1T_{circ}$?). Why would it be much harder to infer $e > 0$ for a transiting planet if periapse is pointed toward the viewer? (Using transit durations to constrain eccentricity is a neat trick that was first proposed to the community in a 2012 paper by Bekki Dawson, now a professor at Penn State, and John Johnson, now a professor at Harvard).