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Exoplanets: Homework 2

1) DHARMA TOU spectrograph
$\sigma = 0.7\ m/s = K$ (RV amplitude)
0.5 m telescope

a. $t = 3\ yr$, $e = 1$, $P_{max} = 1.5\ yr$
$= 4.73 \times 10^7 s$
$M_{min} = ?$
i. F-type star
ii. K-type star

find semi-major axis,
$P^2 \propto a^3$
$a \propto \sqrt[3]{P^2}$

In general we can determine the minimum mass of a planet with the RV eqn,

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{m_p \sin(i)}{(M_\star + M_p)^{2/3}} \cdot \frac{1}{\sqrt{1-e^2}}$$

Let's assume $M_p \ll M_\star$ and $e = 0$

$$\Rightarrow K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin(i)}{(M_\star)^{2/3}}$$

$$M_p \sin(i) = K (M_\star)^{2/3} \left(\frac{P}{2\pi G}\right)^{1/3}$$

$\longrightarrow$

i. for an F-type star,
$M_\star \sim 1.4\, M_\odot$ (wikipedia)
$= 2.784 \times 10^{30}$ kg

$$M_p \sin(i) = (0.7)(2.784\times10^{30})^{2/3}\left(\frac{4.73\times10^{7}}{2\pi G}\right)^{1/3}$$

$$\boxed{M_p \sin(i) = 6.70\times10^{25}\text{ kg (F-type star)}}$$

ii. for a K-type star,
$M_\star \sim 0.6\, M_\odot$ (wikipedia)
$= 1.193 \times 10^{30}$ kg

$$M_p \sin(i) = (0.7)(1.193\times10^{30})^{2/3}\left(\frac{4.73\times10^{7}}{2\pi G}\right)^{1/3}$$

$$\boxed{M_p \sin(i) = 3.81\times10^{25}\text{ kg (K-type star)}}$$

b. $T_{eq} = ?$

recall from the last homework,

$$T_{eq} = \left(\frac{(1-A_B)\,L_\star}{16\pi\,\sigma_{SB}\,a^2}\right)^{1/4}$$

semi-major axis

I'll assume a reasonable albedo of $A_B = 0.3$

→

estimate semi-major axis from orbital period,

$$P^2 \propto a^3 \Rightarrow a \propto \sqrt[3]{P^2}$$

$$a = \sqrt[3]{(1.5)^2} = 1.31\ AU = 1.96 \times 10^{13}\ cm$$

i. for an F-type star,

$L_\star \sim 5.13\ L_\odot$ (wikipedia)

$= 1.97 \times 10^{34}\ erg \cdot s^{-1}$

$$T_{eq} = \left(\frac{(1-0.3)(1.97 \times 10^{34})}{16\pi (5.67 \times 10^{-5})(1.96 \times 10^{13})^2} \right)^{1/4}$$

$$\Rightarrow \boxed{T_{eq} = 335\ K} \quad \text{(F-type)}$$

ii. for a K-type star,

$L_\star \sim 0.1\ L_\odot$ (wikipedia)

$= 3.846 \times 10^{32}\ erg \cdot s^{-1}$

$$T_{eq} = \left(\frac{(1-0.3)(3.846 \times 10^{32})}{16\pi (5.67 \times 10^{-5})(1.96 \times 10^{13})^2} \right)^{1/4}$$

$$\Rightarrow \boxed{T_{eq} = 125\ K} \quad \text{(K-type)}$$

c. $\sigma_K = \sigma_{RV} \cdot \left(\frac{2}{N_{RV}}\right)^{1/2}$

$N_{RV} = ?$; $\sigma_{RV} = 0.7\, m/s$

$\sigma_K = 10\%$

Survey duration = 9 hr per night

$t_{obs} \sim 1$ hr

$N_{RV} = ?$

70% clear nights

i. number of observations to determine mass measurement w/ 10% precision?

$\sigma_{RV} = K = 0.7\, m/s$, TOU precision

$\sigma_K = 0.10$, desired precision

$N_{RV} = ?$

$$\sigma_K = \sigma_{RV} \cdot \left(\frac{2}{N_{RV}}\right)^{1/2}$$

$$\left(\frac{\sigma_K}{\sigma_{RV}}\right)^2 = \frac{2}{N_{RV}} \Rightarrow N_{RV} = 2\left(\frac{\sigma_{RV}}{\sigma_K}\right)^2$$

$$N_{RV} = 2\left(\frac{0.7}{0.10}\right)^2$$

$$\Rightarrow \boxed{N_{RV} = 98 \text{ observations}}$$

ii. $t_{obs} \sim 1$ hr, time for each observation
$t_{night} \sim 9$ hr/night
$t_{TOT} = 3$ yrs $= 1095$ nights
70% clear nights,
$\Rightarrow t_{TOT} \approx 766$ nights

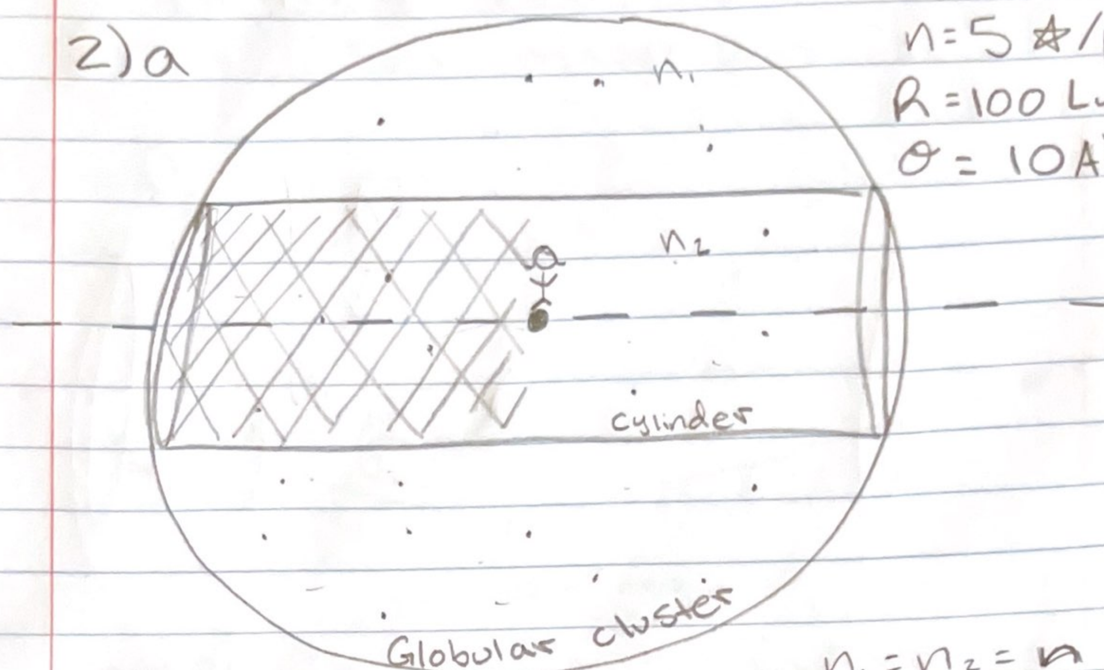
$t_{TOT} \cdot t_{night} = (766 \text{ nights})(9 \text{ hr/night})$
$= 6894$ hours to observe

$$\frac{6894 \text{ hours}}{1 \text{ hr/observation}} \sim 6894 \text{ observations}$$

$$\frac{6894 \text{ observations}}{N_{RV,10\%}} = \frac{6894}{98}$$

$\Rightarrow \boxed{\sim 70 \text{ planets}}$ (Number of planets you can measure the mass of with 10% presicion for survey duration)

2) a



$n = 5$ ★$/pc^3$
$R = 100$ Ly
$\theta = 10$ AU

$n_1 = n_2 = n$
(uniform number density)

number of stars in the globular cluster,

$V = \frac{4}{3}\pi R^3 \Rightarrow N = n \cdot \frac{4}{3}\pi R^3$

$N = (5)\frac{4}{3}\pi(100\,Ly)^3 = (5)\frac{4}{3}\pi(30.66)^3$

$\Rightarrow N_{cloud} = 6.04 \times 10^5$ stars

number of stars in the cylinder,

$V = \pi\theta^2 R \Rightarrow N = n \cdot \pi\theta^2 R$

$N = (5$ ★$/pc^3) \cdot \pi(10\,AU)^2(100\,Ly)$

$N = (5$ ★$/pc^3) \cdot \pi(4.85\times10^{-5}\,pc)^2(30.66\,pc)$

$\Rightarrow N_{cylinder} = 1.13\times10^{-6}$ stars

Probability,

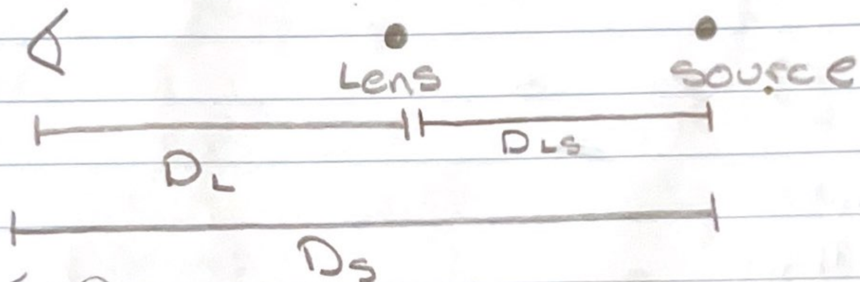
$P = \frac{N_{cylinder}}{N_{cloud}} \Rightarrow \boxed{P = 1.9\times10^{-12}}$

b. $\theta_E = ?$, size of Einstein ring

$d = 100$ Ly
$d_f = 10$ Ly , distance to forground star (Solar type)

Eqn for the Einstein ring,

$$\theta_E = \left(\frac{4GM_\odot}{c^2} \frac{D_{LS}}{D_L D_S} \right)^{1/2}$$



given $\begin{cases} D_L = 10 \text{ Ly} = 9.46 \times 10^{18} \text{ cm} \\ D_S = 100 \text{ Ly} = 9.46 \times 10^{19} \text{ cm} \end{cases}$

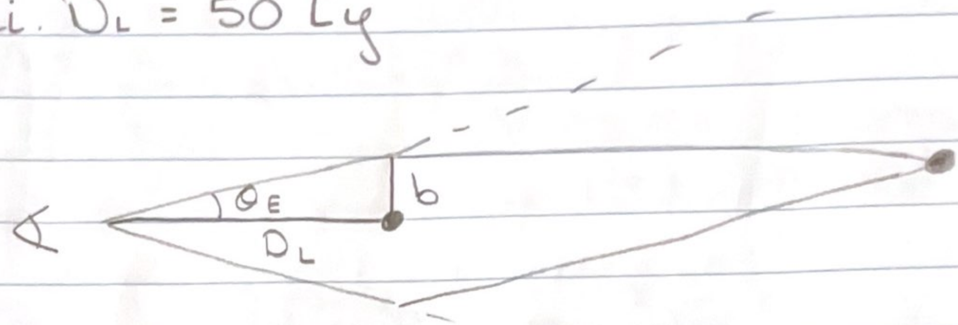
$D_{LS} = D_S - D_L = 90 \text{ Ly} = 8.52 \times 10^{19} \text{ cm}$

$$\Rightarrow \theta_E = \left(\frac{4(6.67\times10^{-8})(2\times10^{33})}{(3\times10^{10})^2} \frac{8.52\times10^{19}}{(9.46\times10^{18})(9.46\times10^{19})} \right)^{1/2}$$

$$\Rightarrow \boxed{\theta_E = 2.38 \times 10^{-7}}$$

C. $a_p = ?$, semi-major axis

i. $D_L = 10$ Ly
ii. $D_L = 50$ Ly



semi-major axis

$$\tan(\theta_E) = \frac{b}{D_L} \Rightarrow b = D_L \tan(\theta_E)$$

for $D_L = 10$ Ly, $\theta_E = 2.38 \times 10^{-7}$ (part b):

$$b = (10)\tan(2.38 \times 10^{-7})$$

$$\Rightarrow \boxed{b = 2.38 \times 10^{-6} \text{ Ly}} \sim .15 \text{ AU}$$

for $D_L = 50$ Ly:

$$D_S = 100 \text{ Ly} \Rightarrow D_{LS} = 50 \text{ Ly}$$

$$\theta_E = \left(\frac{4(6.67 \times 10^{-8})(2 \times 10^{33})}{(3 \times 10^{10})^2} \cdot \frac{4.73 \times 10^{19}}{4.73 \times 10^{19}(9.46 \times 10^{19})} \right)^{1/2}$$

$$\Rightarrow \theta_E = 7.92 \times 10^{-8}$$

$$b = (50)\tan(7.92 \times 10^{-8})$$

$$\Rightarrow \boxed{b = 3.96 \times 10^{-6} \text{ Ly}} \sim .25 \text{ AU}$$

It would be hard to observe planets outside the snow line in this case.

3) a. $t_{transit} = ?$

(calculations in Jupyter notebook)

use vis viva eqn to calculate velocity,

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

distance from star

semi-major axis

$$\Rightarrow v = 4.80 \times 10^6 \text{ cm/s}$$

and the length of transit is,

$$l = 2R_\odot$$

$$v = \frac{\text{distance}}{\text{time}} = \frac{l}{t}$$

$$\Rightarrow t = \frac{l}{v}$$

$$\Rightarrow t = \frac{2R_\odot}{v}$$

$$\boxed{t = 2.9 \times 10^4 \text{ s}}$$

b. $\boxed{t = 2.6 \times 10^4 \text{ s}}$

c. from the textbook,

$$T_{TOT} = \frac{P}{\pi} \sin^{-1}\left[\frac{R_\star}{a} \frac{\sqrt{(1+k)^2 - b^2}}{\sin(i)}\right]$$

$$k = \frac{R_p}{R_\star}$$

Impact parameter,

$$b = \frac{a \cos i}{R_\star}\left(\frac{1-e^2}{1+e\sin\omega}\right)$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\boxed{T_{TOT} = 39432 \text{ s}}$$

d. consider apoapse instead of periapse

$$\boxed{t = 52713 \text{ s}}$$